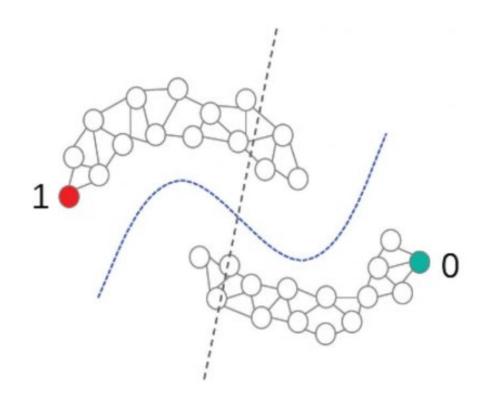
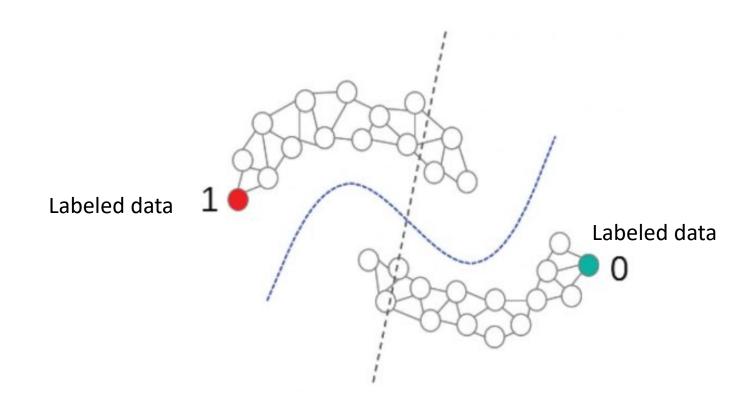
Open Problems in Deep Learning

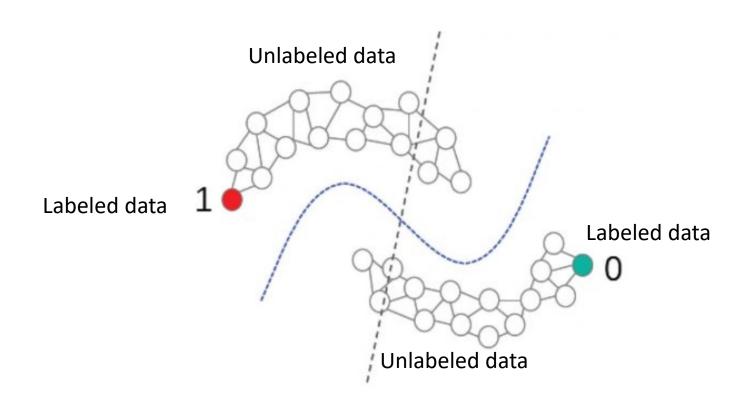
Neural Networks Design And Application

Open problems in deep learning

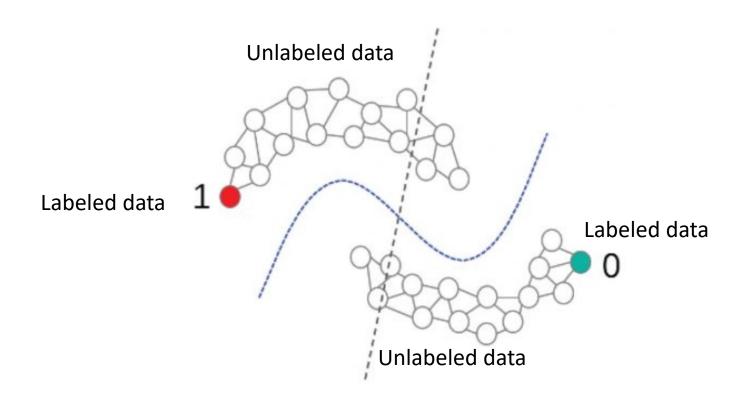
- Weak supervised learning
- Causal modeling
- Curriculum learning
- Over-parameterized modeling
- Self-supervised learning
- Meta-learning
- Federated learning
- •

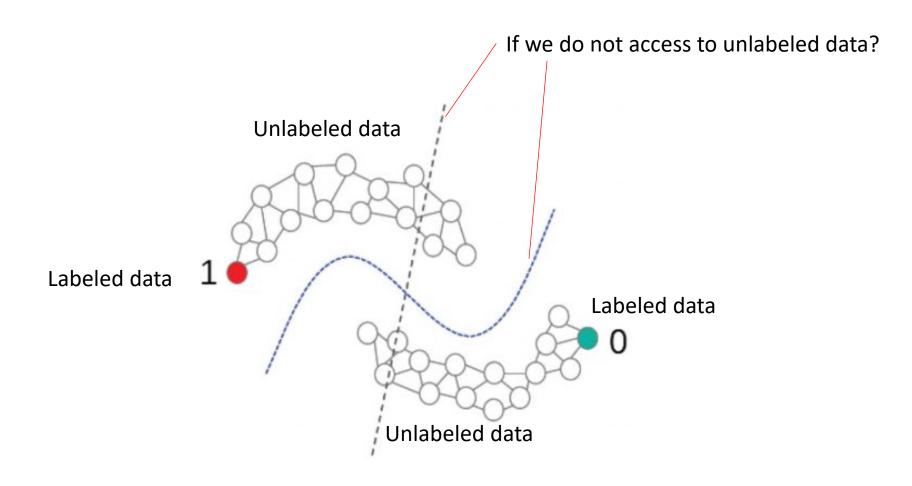


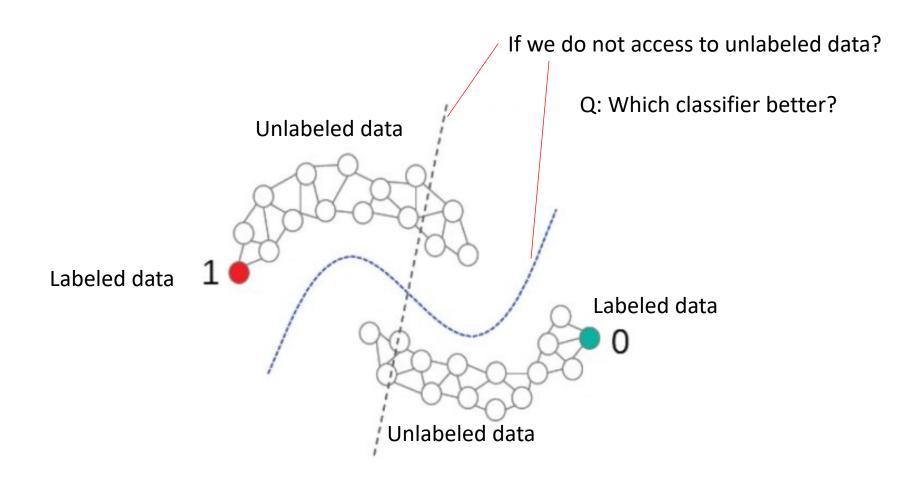


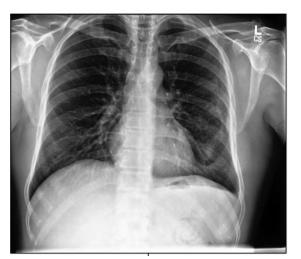


If we do not access to unlabeled data?











X-ray image analysis

Irvin, Jeremy, Pranav Rajpurkar, Michael Ko, Yifan Yu, Silviana Ciurea-Ilcus, Chris Chute, Henrik Marklund et al. "Chexpert: A large chest radiograph dataset with uncertainty labels and expert comparison." In *Proceedings of the AAAI Conference on Artificial Intelligence*, vol. 33, no. 01, pp. 590-597. 2019.

Some classification tasks are very difficult to acquire a lot of labeled data





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X-ray image analysis

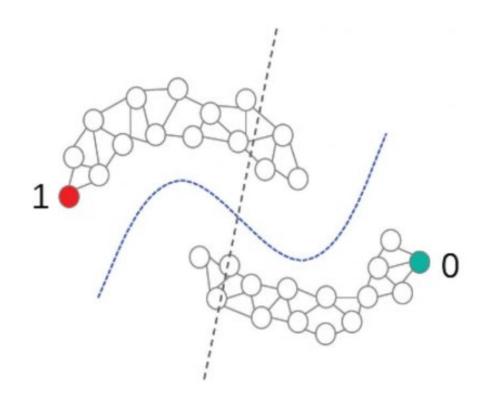
Identify: which x-ray image may imply life threatening diseases?

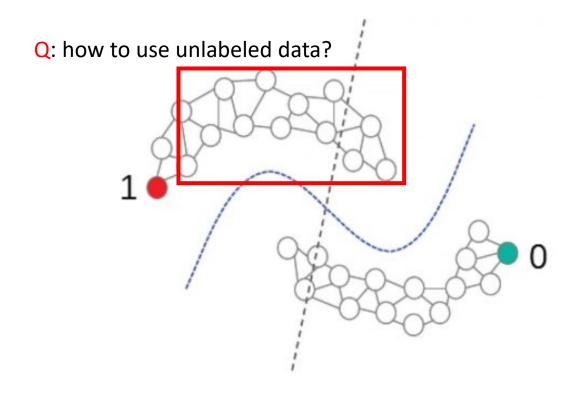
Irvin, Jeremy, Pranav Rajpurkar, Michael Ko, Yifan Yu, Silviana Ciurea-Ilcus, Chris Chute, Henrik Marklund et al. "Chexpert: A large chest radiograph dataset with uncertainty labels and expert comparison." In *Proceedings of the AAAI Conference on Artificial Intelligence*, vol. 33, no. 01, pp. 590-597. 2019.

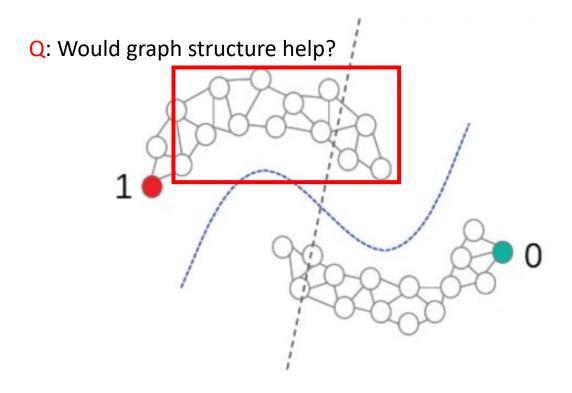


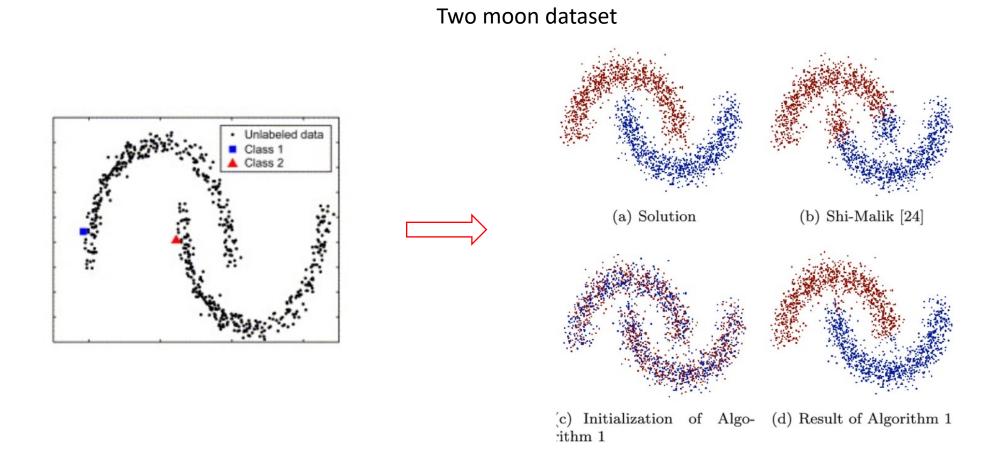


Leaderboard Will your model perform as well as radiologists in detecting different pathologies in chest X-rays?				
Rank	Date	Model	AUC	Num Rads Below Curve
1	Aug 31, 2020	DeepAUC-v1 <i>ensemble</i> https://arxiv.org/abs/201 2.03173	0.930	2.8
2	Sep 01, 2019	Hierarchical-Learning- V1 (ensemble) <i>Vingroup</i> <i>Big Data Institute</i> https://arxiv.org/abs/191 1.06475	0.930	2.6
3	Oct 15, 2019	Conditional-Training- LSR <i>ensemble</i>	0.929	2.6
4	Dec 04, 2019	Hierarchical-Learning- V4 (ensemble) <i>Vingroup</i>	0.929	2.6





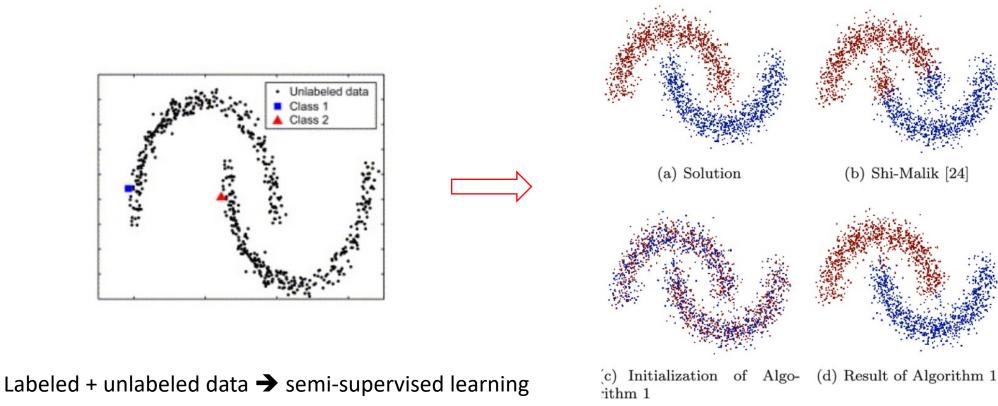




Multi-class Transductive Learning Based on & 1 Relaxations of Cheeger Cut and Mumford-Shah-Potts Model. May 2014

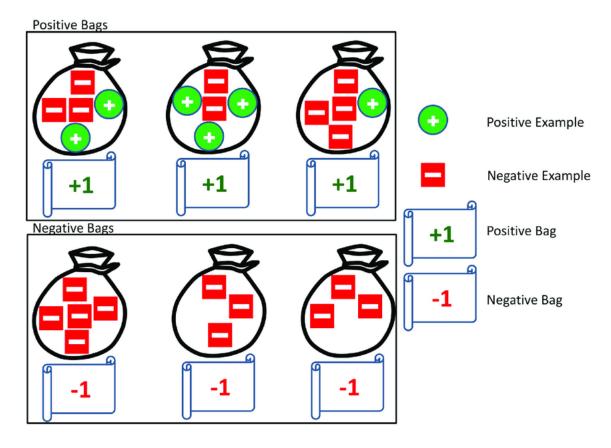
<u>Journal of Mathematical Imaging and Vision</u> 49(1). DOI: 10.1007/s10851-013-0452-5

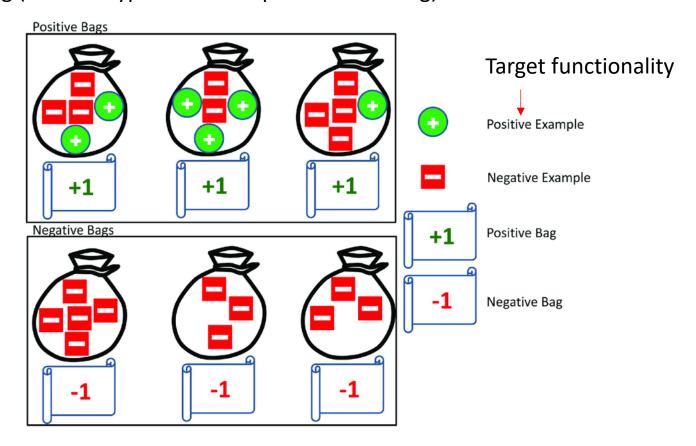
Two moon dataset

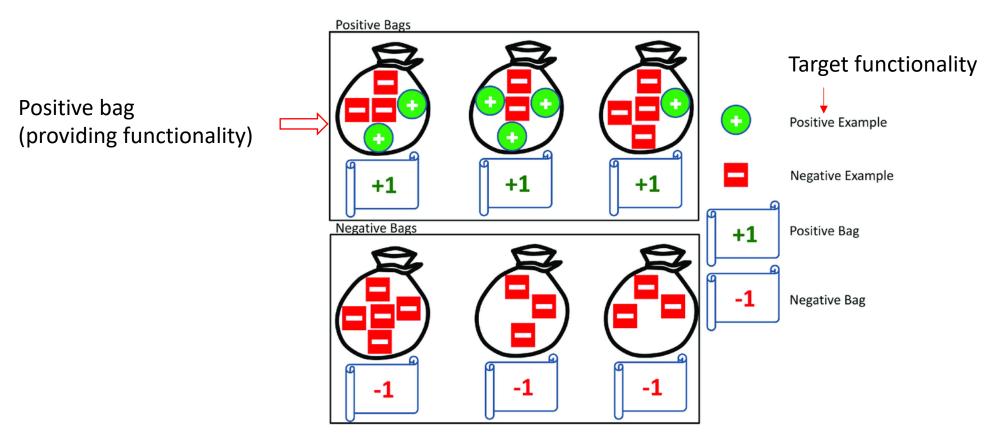


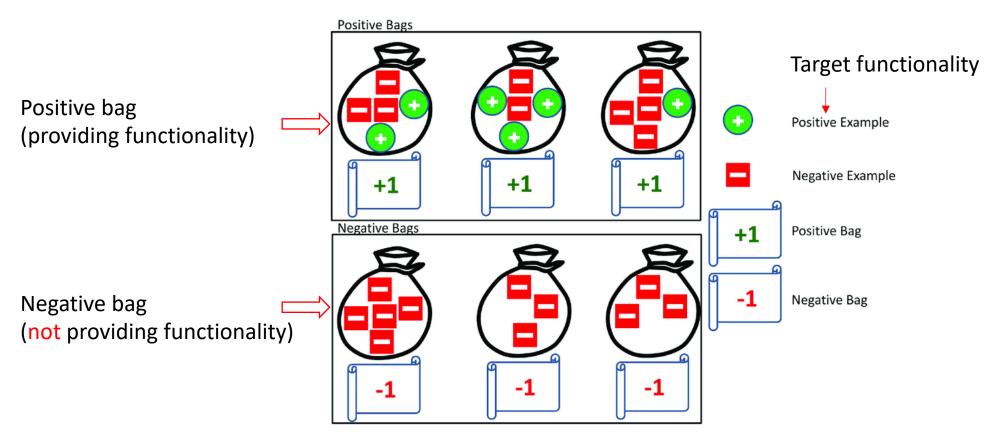
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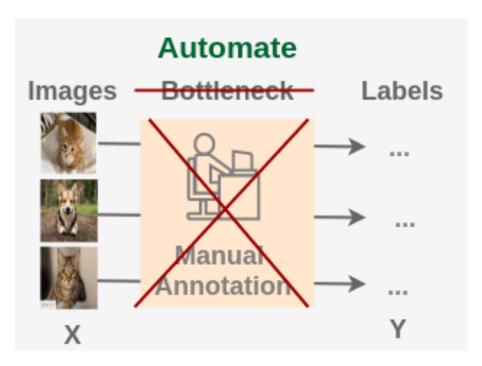
<u>Journal of Mathematical Imaging and Vision</u> 49(1). DOI: 10.1007/s10851-013-0452-5



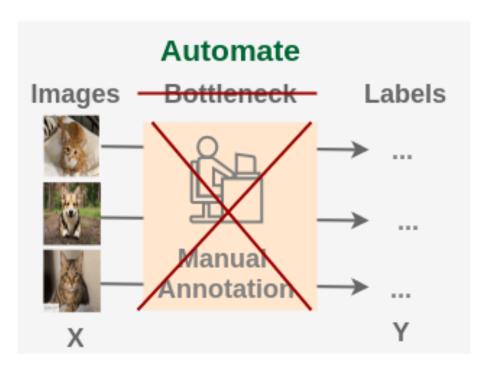








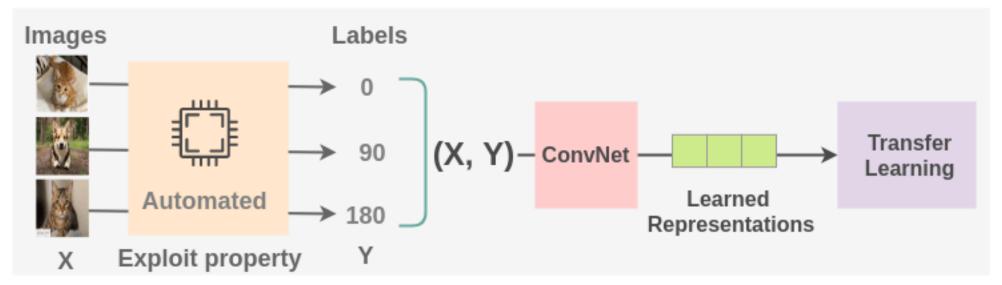
Without human effort for labeling

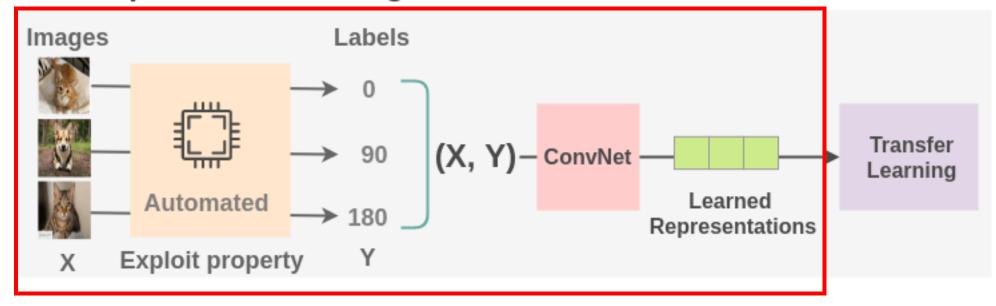


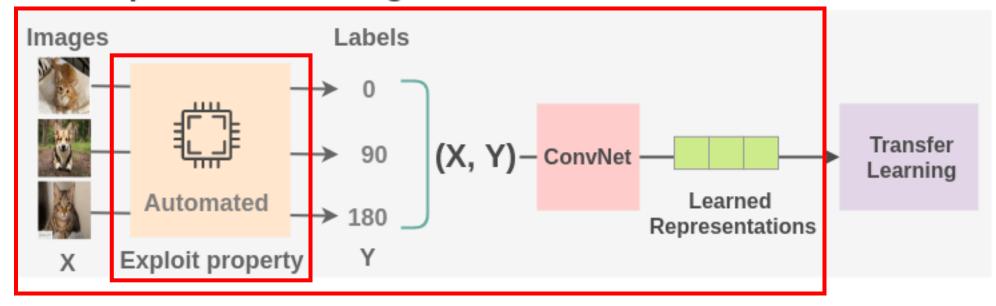
Without human effort for labeling

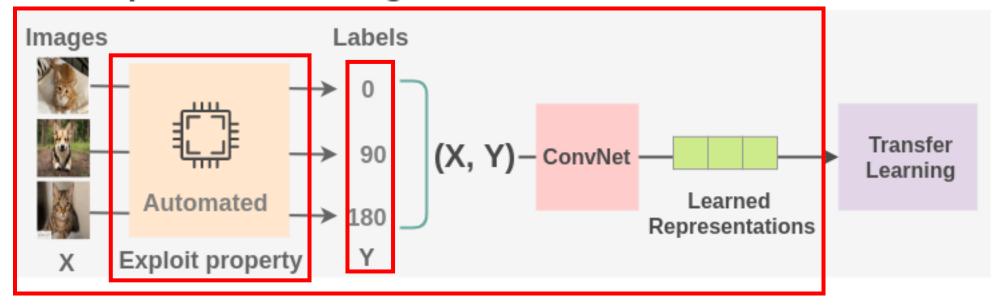
Q: can we automate labeling to get approximation to label information?



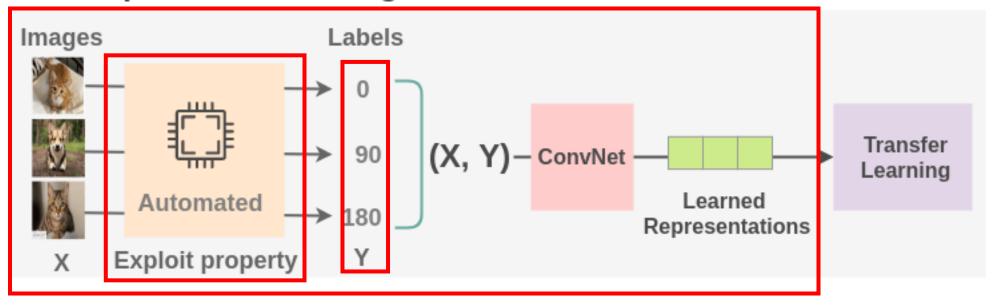








Self-Supervised Learning Workflow

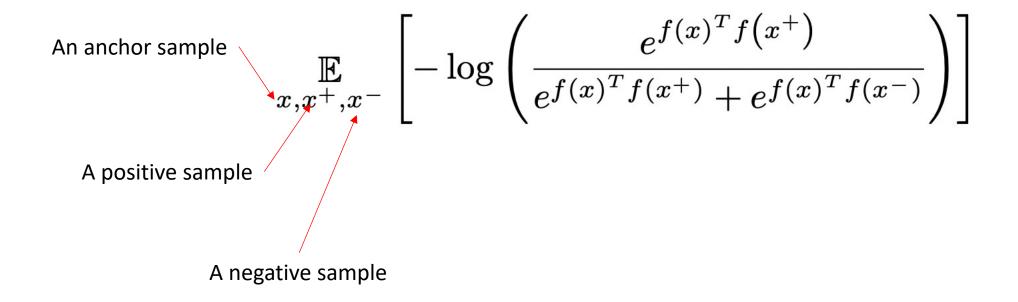


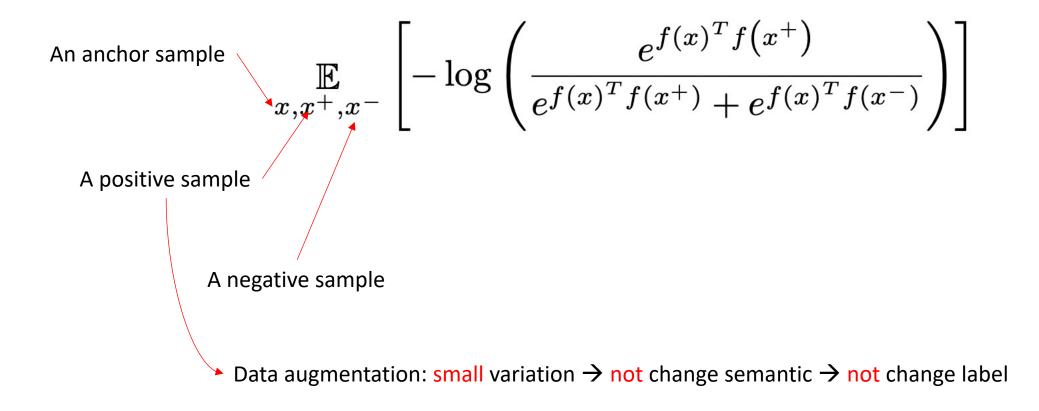
Hypothesis: if the approximated labels behave exactly the same with the underlying labels, then we recover a fully supervised learning

$$\mathbb{E}_{x,x^{+},x^{-}} \left[-\log \left(\frac{e^{f(x)^{T} f(x^{+})}}{e^{f(x)^{T} f(x^{+})} + e^{f(x)^{T} f(x^{-})}} \right) \right]$$

An anchor sample
$$\mathbb{E}_{x,x^+,x^-} \left[-\log \left(\frac{e^{f(x)^T f\left(x^+\right)}}{e^{f(x)^T f(x^+)} + e^{f(x)^T f(x^-)}} \right) \right]$$

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 A positive sample





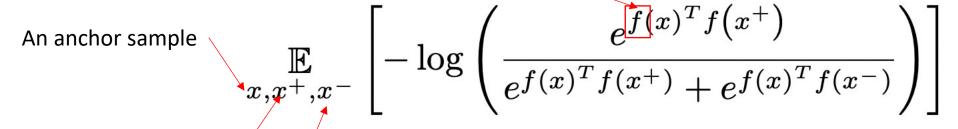
An anchor sample
$$\mathbb{E}_{x,x^+,x^-} \left[-\log \left(\frac{e^{f(x)^T f(x^+)}}{e^{f(x)^T f(x^+)} + e^{f(x)^T f(x^-)}} \right) \right]$$
A positive sample \rightarrow random sample data and directly treat them as negative samples

Data augmentation: small variation \rightarrow not change semantic \rightarrow not change label

An anchor sample $\mathbb{E}_{x,x^+,x^-} \left[-\log \left(\frac{e^{f(x)^T f(x^+)}}{e^{f(x)^T f(x^+)} + e^{f(x)^T f(x^-)}} \right) \right]$ A positive sample

A negative sample

A representation function What is it?



A positive sample

A negative sample

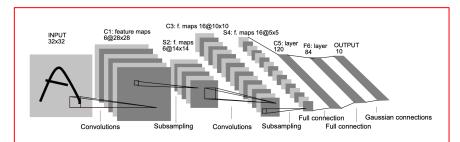
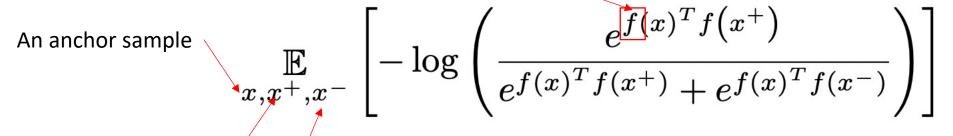


Fig. 1. Architecture of LeNet-5, a Convolutional Neural Network, here for digits recognition. Each plane is a feature map, i.e. a set of units whose weights are constrained to be identical.

A representation function What is it?



A positive sample

A negative sample

Each layer's feature map can be used as representation of an input data sample

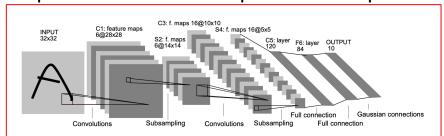


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A representation function What is it?

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A positive sample

A negative sample

Each layer's feature map can be used as representation of an input data sample

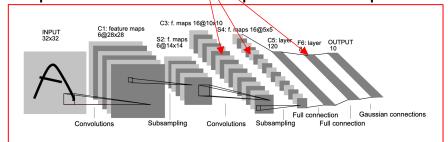
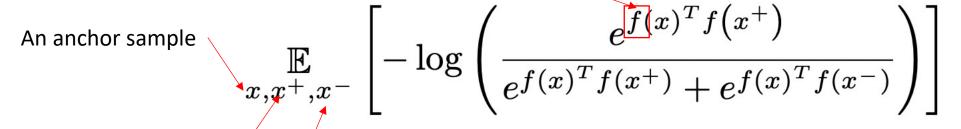


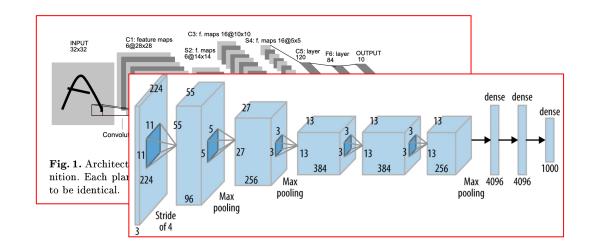
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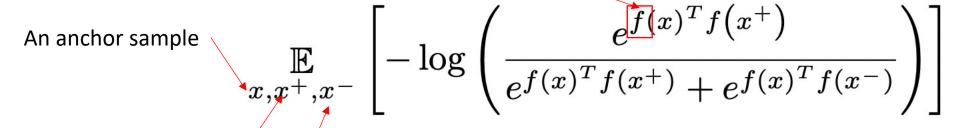


A positive sample

A negative sample

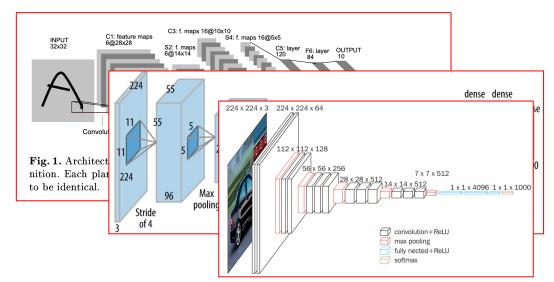


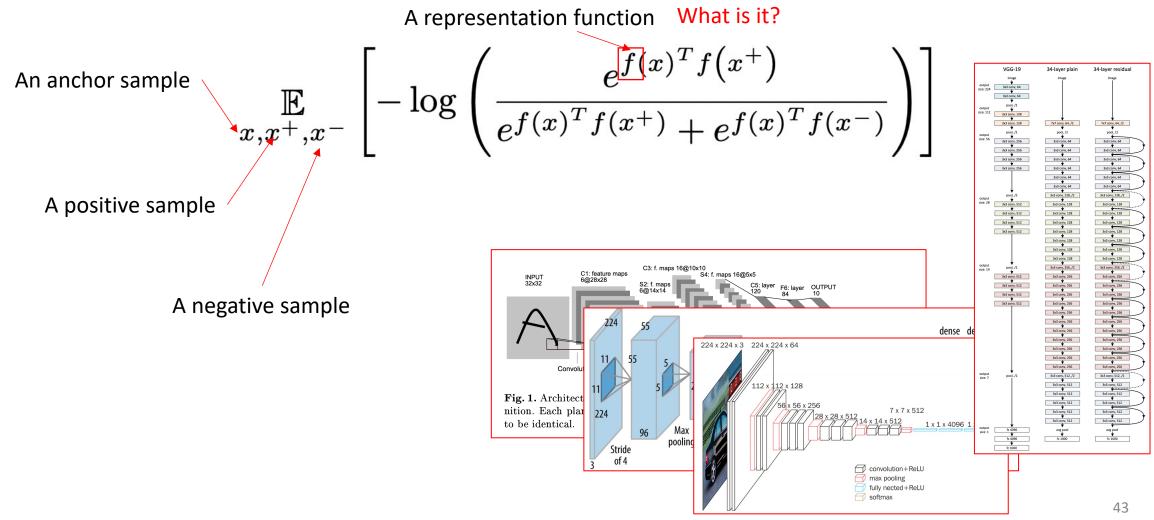
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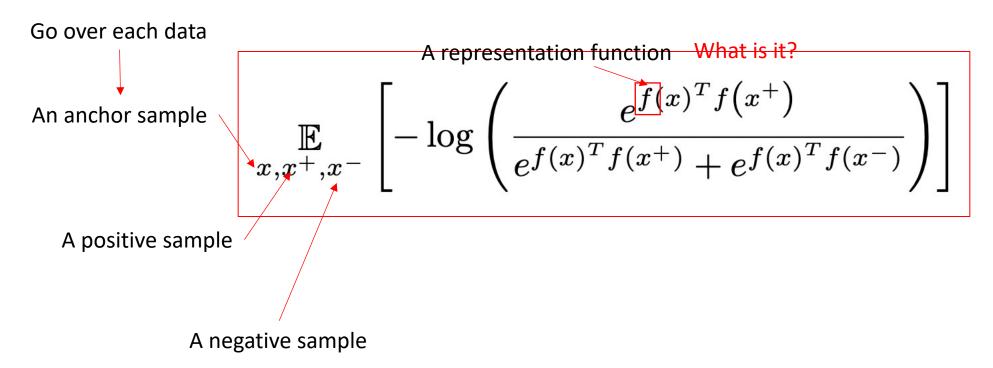


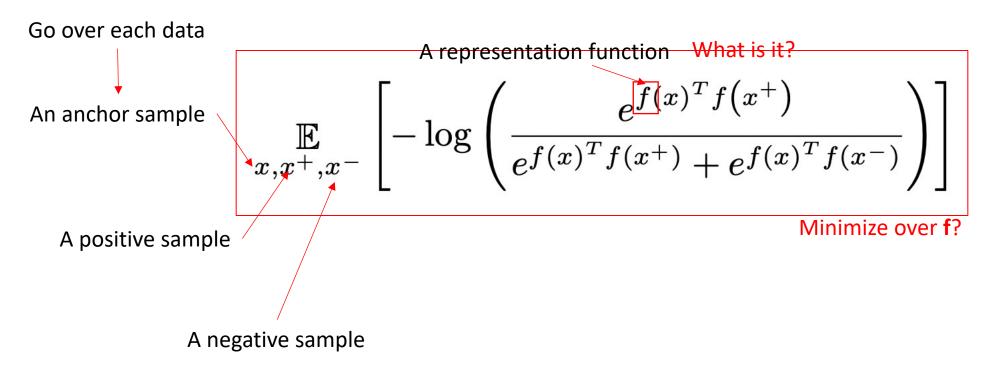
A positive sample

A negative sample





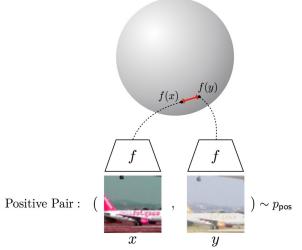




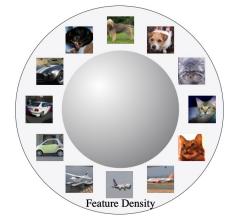


Uniformity: Preserve maximal information.

Figure 1: Illustration of alignment and uniformity of feature distributions on the output unit hypersphere. STL-10 (Coates et al., 2011) images are used for demonstration.

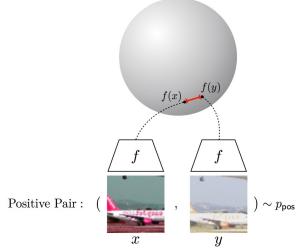


Data samples from different classes

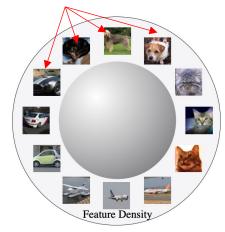


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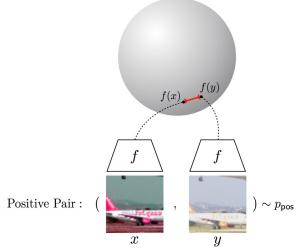


Data samples from different classes

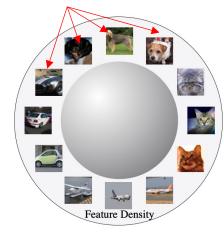


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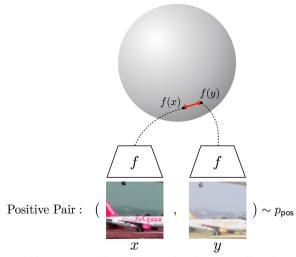
Data samples from different classes



Uniformity: Preserve maximal information.

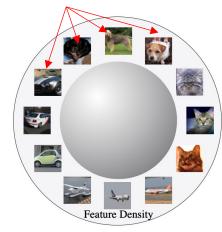
Figure 1: Illustration of alignment and uniformity of feature distributions on the output unit hypersphere. STL-10 (Coates et al., 2011) images are used for demonstration.

Data samples from the same class:



Alignment: Similar samples have similar features. (Figure inspired by Tian et al. (2019).)

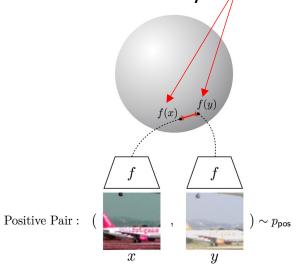
Data samples from different classes



Uniformity: Preserve maximal information.

Figure 1: Illustration of alignment and uniformity of feature distributions on the output unit hypersphere. STL-10 (Coates et al., 2011) images are used for demonstration.

Data samples from the same class: stay close to each other



Random init

Supervised learning

Contrastive learning (unsupervised, self-supervised)

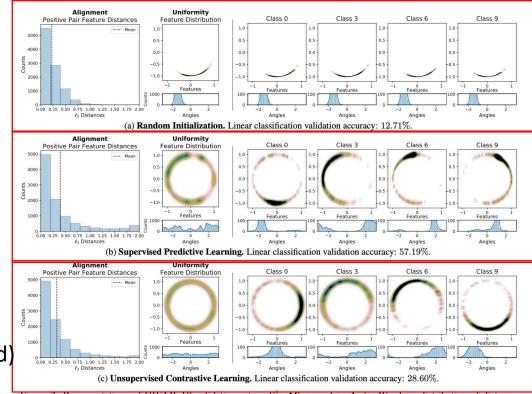


Figure 3: Representations of CIFAR-10 validation set on S^1 . Alignment analysis: We show distribution of distance between features of positive pairs (two random augmentations). Uniformity analysis: We plot feature distributions with Gaussian kernel density estimation (KDE) in \mathbb{R}^2 and von Mises-Fisher (vMF) KDE on angles (i.e., $\arctan 2(y,x)$ for each point $(x,y) \in S^1$). Four rightmost plots visualize feature distributions of selected specific classes. Representation from contrastive learning is both *aligned* (having low positive pair feature distances) and *uniform* (evenly distributed on S^1).

The gap between supervised learning and contrastive learning?

The gap between supervised learning and contrastive learning?

$$\mathbb{E}_{x,x^{+},x^{-}} \left[-\log \left(\frac{e^{f(x)^{T} f(x^{+})}}{e^{f(x)^{T} f(x^{+})} + e^{f(x)^{T} f(x^{-})}} \right) \right]$$

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Review: A negative sample \rightarrow random sample data and directly treat them as negative samples

The gap between supervised learning and contrastive learning?

$$\mathbb{E}_{x,x^+,x^-} \left[-\log \left(\frac{e^{f(x)^T f\left(x^+\right)}}{e^{f(x)^T f\left(x^+\right)} + e^{f(x)^T f\left(x^-\right)}} \right) \right]$$

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What if "negative" samples are actually from the same class?

The gap between supervised learning and contrastive learning?

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$$L_{un}(f) = \tau L_{un}^{=}(f) + (1 - \tau)L_{un}^{\neq}(f)$$
 (8)

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The gap between supervised learning and contrastive learning?

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The gap between supervised learning and contrastive learning?

The prob: we have true negative

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$$L_{un}(f) = \tau L_{un}^{=}(f) + (1 - \tau)L_{un}^{\neq}(f)$$
 (8)

What if "negative" samples are actually from the same class?

$$L_{un}^{=}(f) = \underset{\substack{c \sim \nu \\ x, x^+, x^- \sim \mathcal{D}_c^3}}{\mathbb{E}} \left[\ell(f(x)^T (f(x^+) - f(x^-))) \right]$$

$$\geq \underset{\substack{c \sim \nu, x \sim \mathcal{D}_c}}{\mathbb{E}} \left[\ell(f(x)^T (\mu_c - \mu_c)) \right] = 1$$

The gap between supervised learning and contrastive learning?

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$$L_{un}^{=}(f) = \mathbb{E}_{\substack{c \sim \nu \\ x, x^+, x^- \sim \mathcal{D}_c^3}} \left[\ell(f(x)^T (f(x^+) - f(x^-))) \right]$$

$$\geq \mathbb{E}_{c \sim \nu, x \sim \mathcal{D}_c} \left[\ell(f(x)^T (\mu_c - \mu_c)) \right] = 1$$

$$L_{un}^{\neq}(f) = \underset{\substack{c^+, c^- \sim \rho^2 \\ x, x^+ \sim \mathcal{D}_{c^+}^2 \\ x^- > \mathcal{D}}}{\mathbb{E}} \left[\ell(f(x)^T (f(x^+) - f(x^-))) | c^+ \neq c^- \right]$$

False negative

62

The gap between supervised learning and contrastive learning?

The prob: we have true negative

$$\mathbb{E}_{x,x^+,x^-} \left[-\log \left(\frac{e^{f(x)^T f\left(x^+\right)}}{e^{f(x)^T f\left(x^+\right)} + e^{f(x)^T f\left(x^-\right)}} \right) \right]$$

$$L_{un}(f) = \tau L_{un}^{=}(f) + (1 - \tau) L_{un}^{\neq}(f)$$
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What if "negative" samples are actually from the same class?

$$L_{un}^{=}(f) = \underset{\substack{c \sim \nu \\ x, x^{+}, x^{-} \sim \mathcal{D}_{c}^{3}}}{\mathbb{E}} \left[\ell(f(x)^{T}(f(x^{+}) - f(x^{-}))) \right]$$

$$\geq \underset{\substack{c \sim \nu, x \sim \mathcal{D}_{c}}}{\mathbb{E}} \left[\ell(f(x)^{T}(\mu_{c} - \mu_{c})) \right] = 1$$

$$L_{un}^{\neq}(f)$$

$$= \underset{\substack{c^{+}, c^{-} \sim \rho^{2} \\ x, x^{+} \sim \mathcal{D}_{c^{+}}^{2} \\ x^{-} \sim \mathcal{D}_{c^{-}}}}{\mathbb{E}} \left[\ell(f(x)^{T}(f(x^{+}) - f(x^{-}))) \right] c^{+} \neq c^{-}$$

False negative

63

The gap between supervised learning and contrastive learning?

The prob: we have true negative

$$\mathbb{E}_{x,x^{+},x^{-}} \left[-\log \left(\frac{e^{f(x)^{T} f(x^{+})}}{e^{f(x)^{T} f(x^{+})} + e^{f(x)^{T} f(x^{-})}} \right) \right]$$

$$L_{un}(f) = \tau L_{un}^{=}(f) + (1 - \tau)L_{un}^{\neq}(f)$$
 (8)

$$L_{un}^{=}(f) = \mathbb{E}_{\substack{c \sim \nu \\ x, x^{+}, x^{-} \sim \mathcal{D}_{c}^{3}}} \left[\ell(f(x)^{T}(f(x^{+}) - f(x^{-}))) \right]$$

$$\geq \mathbb{E}_{\substack{c \sim \nu, x \sim \mathcal{D}_{c}}} \left[\ell(f(x)^{T}(\mu_{c} - \mu_{c})) \right] = 1$$

$$L_{un}^{\neq}(f) = \underset{\substack{c^{+}, c^{-} \sim \rho^{2} \\ x, x^{+} \sim \mathcal{D}_{c^{+}}^{2}}}{\mathbb{E}} \left[\ell(f(x)^{T} (f(x^{+}) - f(x^{-}))) | c^{+} \neq c^{-} \right]$$

Theorem 4.5. With probability at least $1 - \delta$, $\forall f \in \mathcal{F}$

$$L_{sup}(\widehat{f}) \le L_{sup}^{\mu}(\widehat{f}) \le L_{un}^{\neq}(f) + \beta s(f) + \eta Gen_M$$

where
$$\beta = c' \frac{\tau}{1-\tau}$$
, $\eta = \frac{1}{1-\tau}$ and c' is a constant.

64

The gap between supervised learning and contrastive learning?

The prob: we have true negative

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If we have a large number of classes and we perform uniform random, sampling?

and Nikunj Saunshi. "A theoretical analysis of contrastive unsupervised representation learning." arXiv preprint arXiv:1902.09229 (2019).

Meta-learning

Multi-Task Learning

Solve multiple tasks $\mathcal{T}_1, \cdots, \mathcal{T}_T$ at once.

$$\min_{\theta} \sum_{i=1}^{T} \mathcal{L}_{i}(\theta, \mathcal{D}_{i})$$

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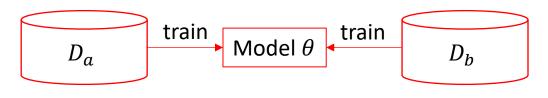


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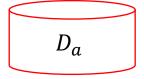
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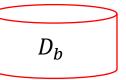


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Transfer Learning

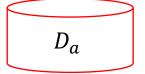
Solve target task \mathcal{T}_b after solving source task \mathcal{T}_a by transferring knowledge learned from \mathcal{T}_a

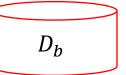




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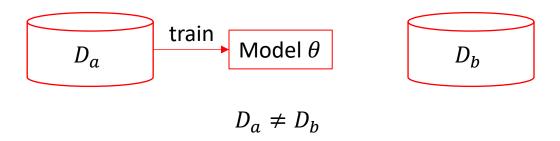




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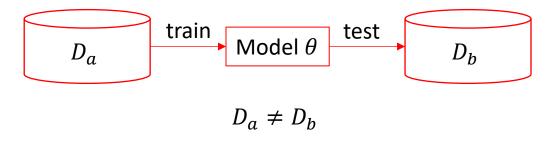
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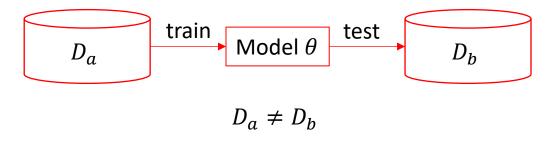
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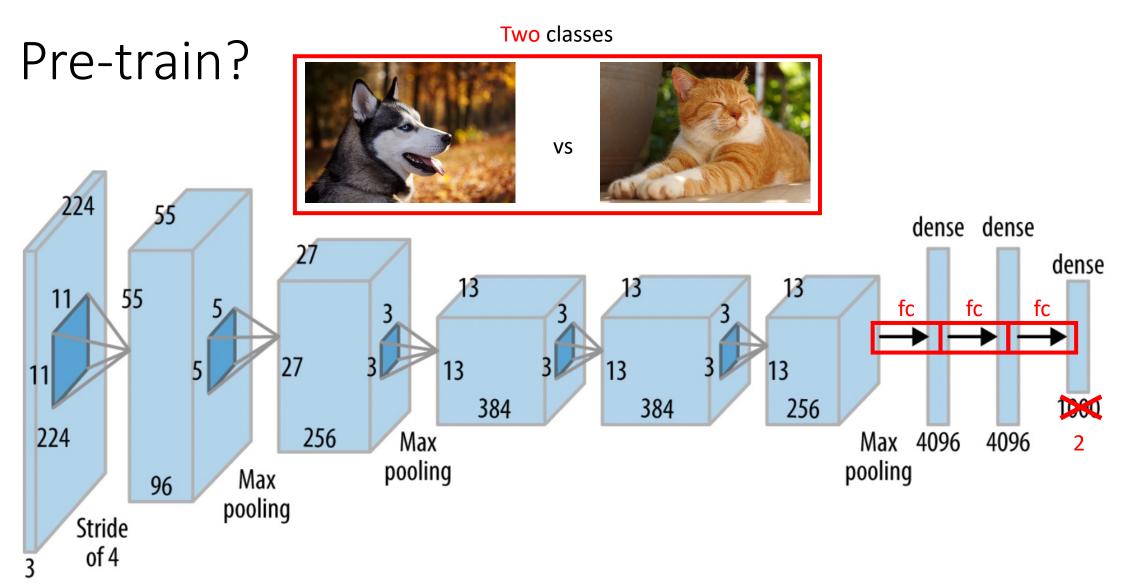


Q: anything we learned can be regarded as "transfer learning"?

Transfer Learning

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Suppose we have a learned model → weight parameters are determined and fixed

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Given data from $\mathcal{T}_1,...,\mathcal{T}_n$, quickly solve new task $\mathcal{T}_{\text{test}}$

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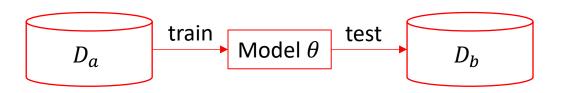
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 ${\mathscr T}_{ ext{test}}$ can be one of many target tasks

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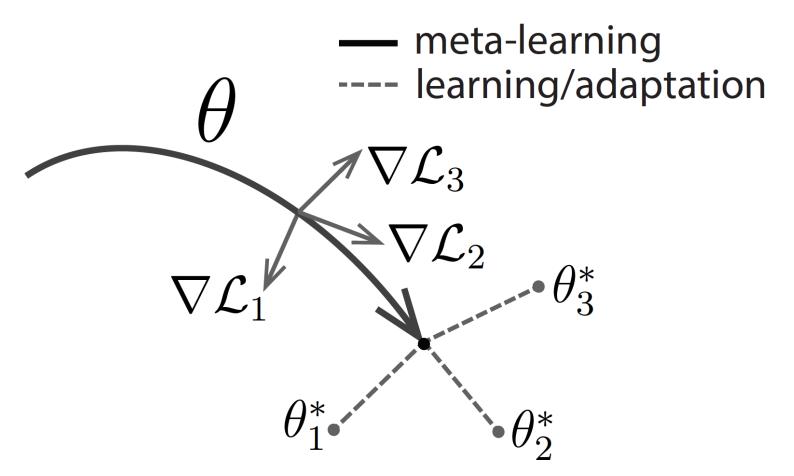
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Key assumption: Cannot access data \mathscr{D}_a during transfer.

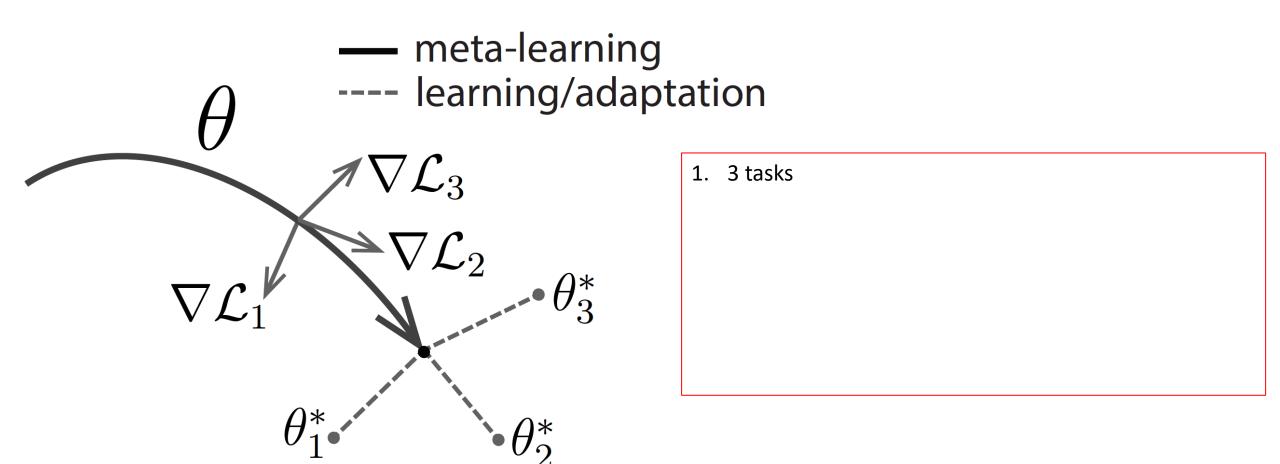
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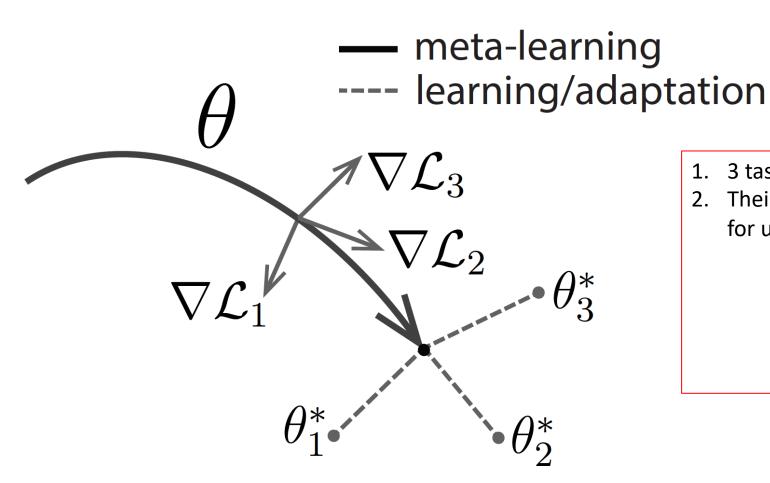
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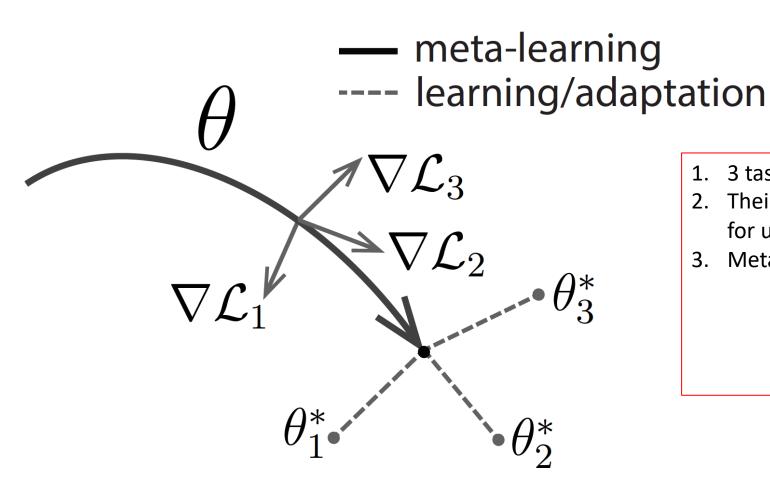
Finn, Chelsea, Pieter Abbeel, and Sergey Levine. "Model-agnostic meta-learning for fast adaptation of deep networks." In *International Conference on Machine Learning*, pp. 1126-1135. PMLR, 2017.

http://proceedings.mlr.press/v70/finn17a/finn17a.pdf

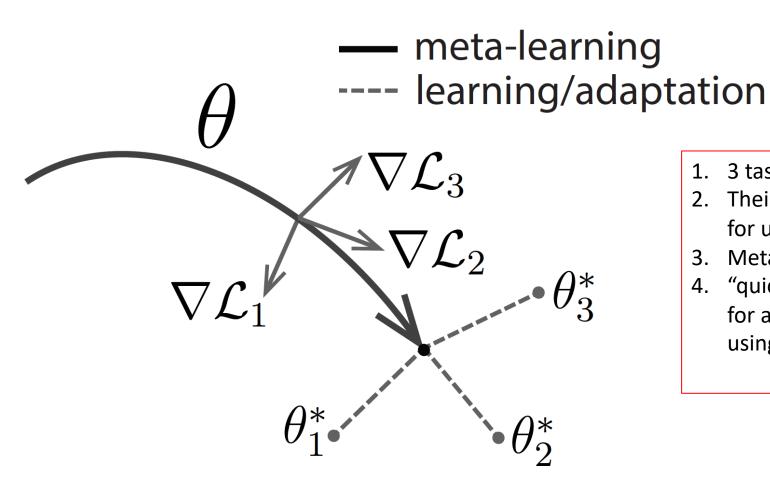




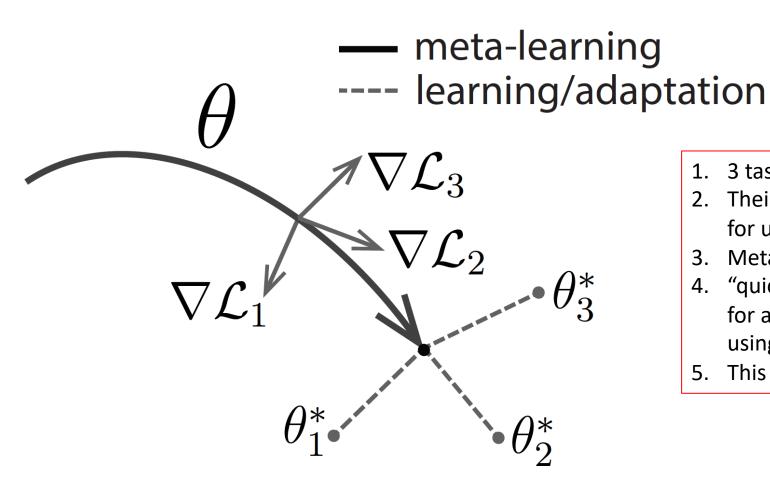
- 3 tasks
- Their datasets may have different directions/gradients for updating current model



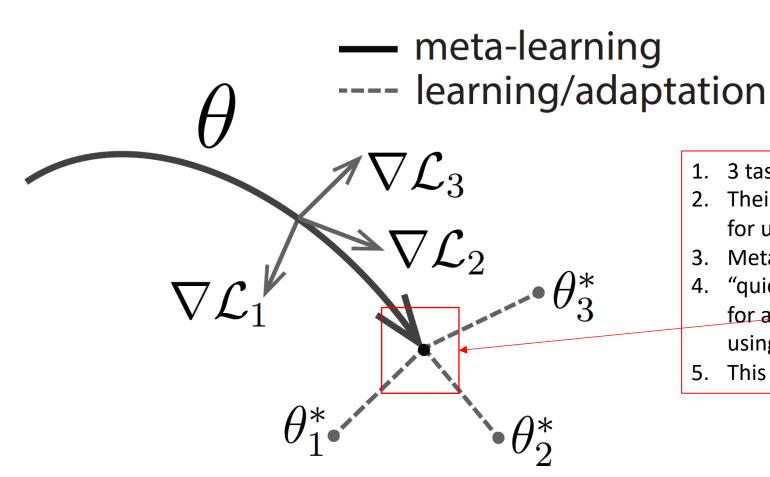
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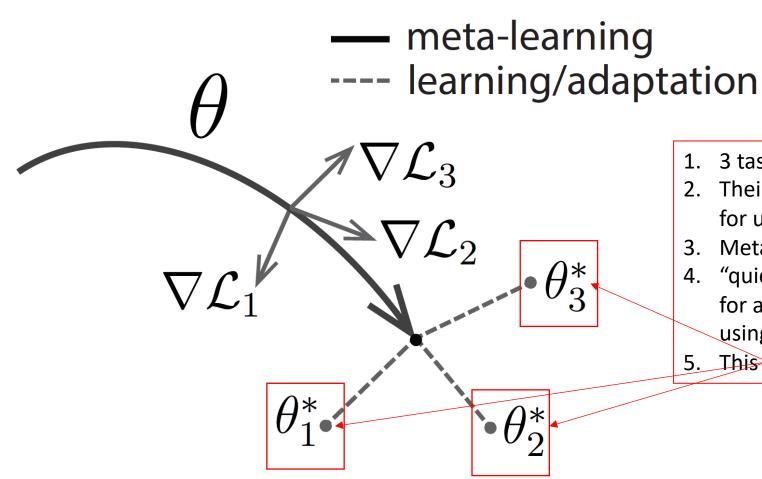
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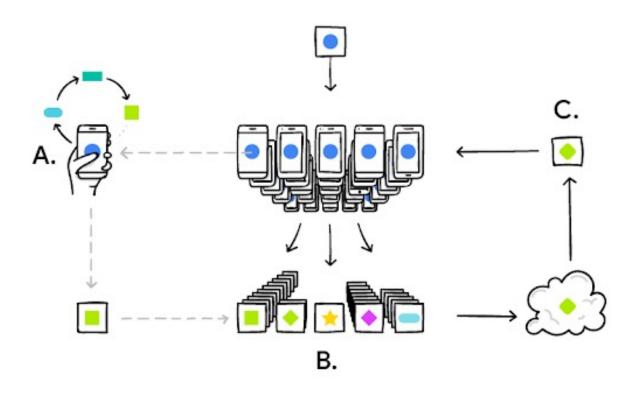
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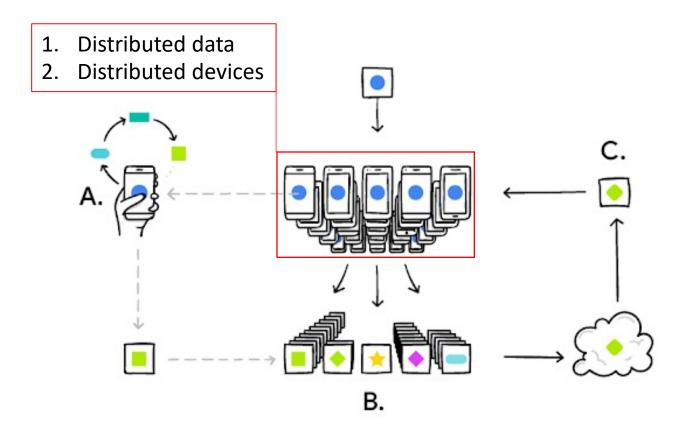


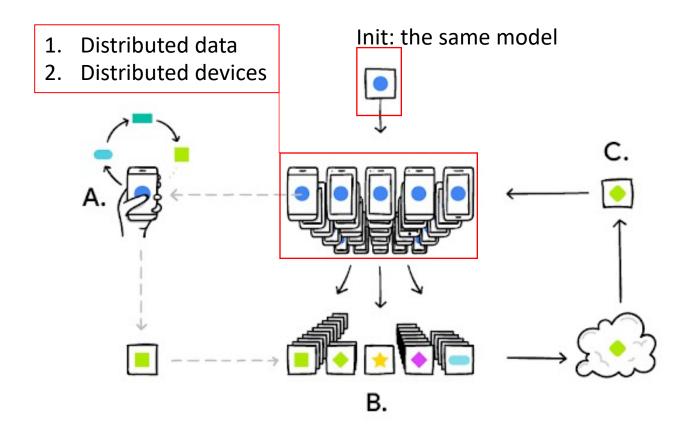
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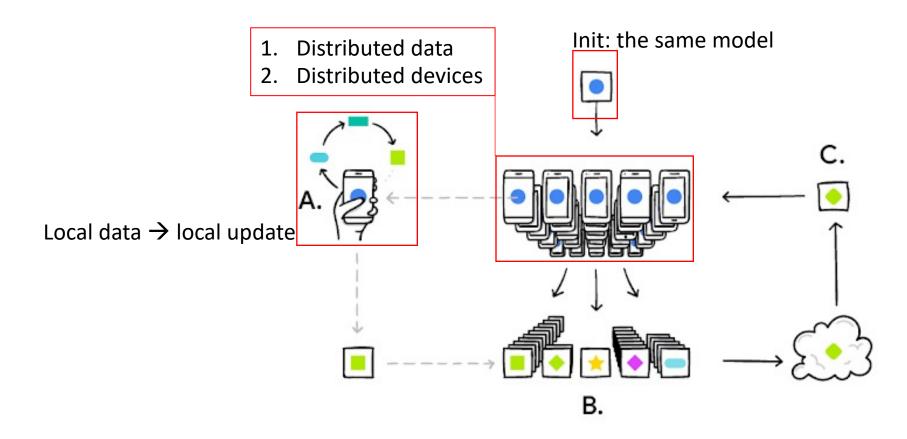


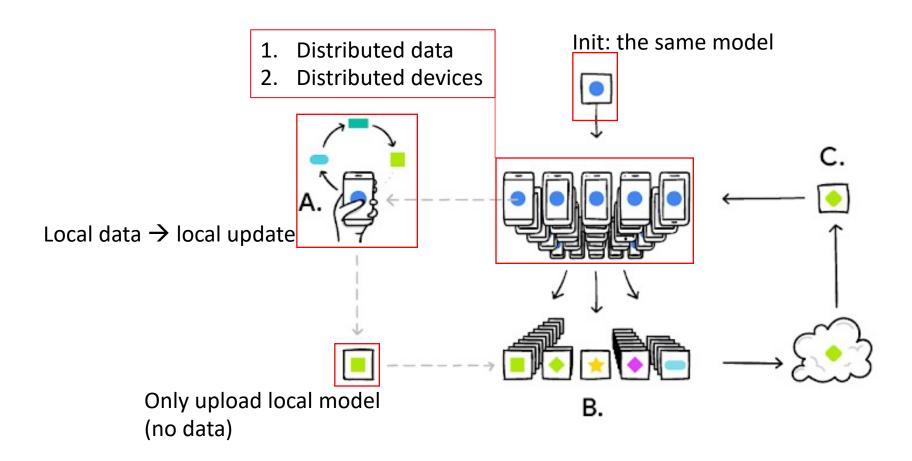
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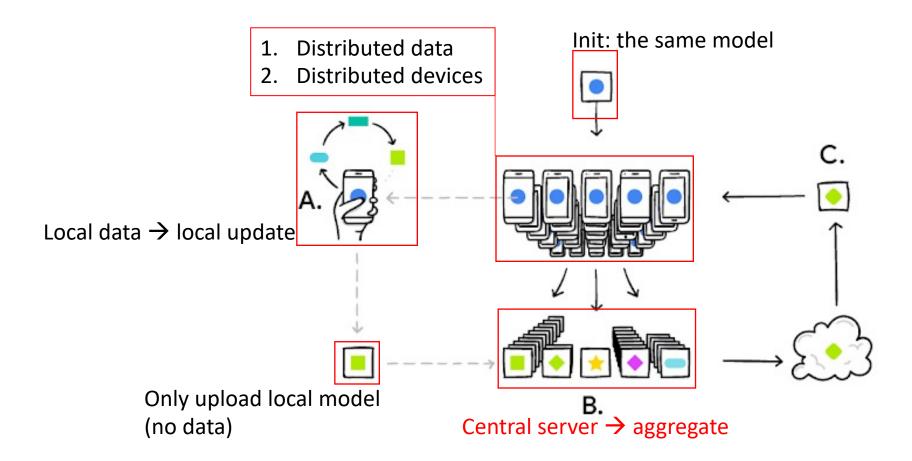


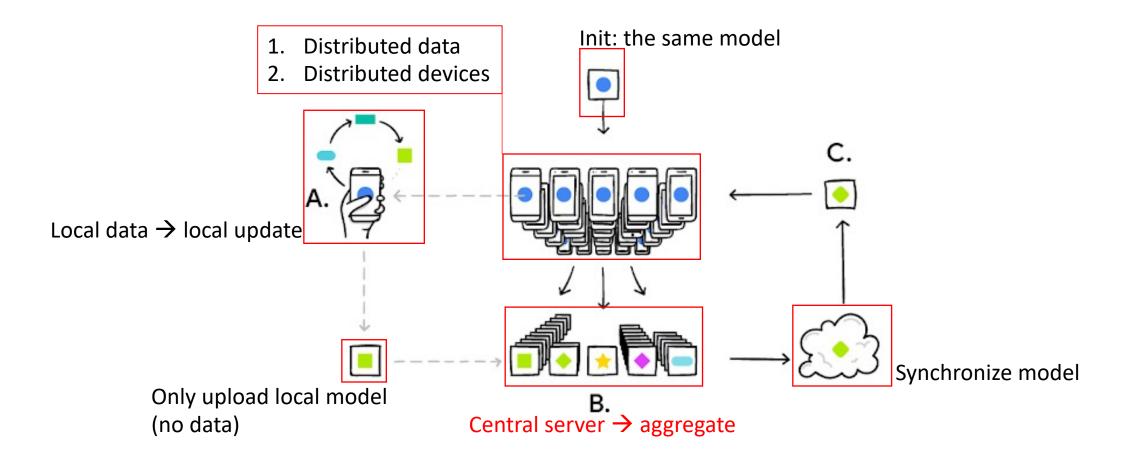


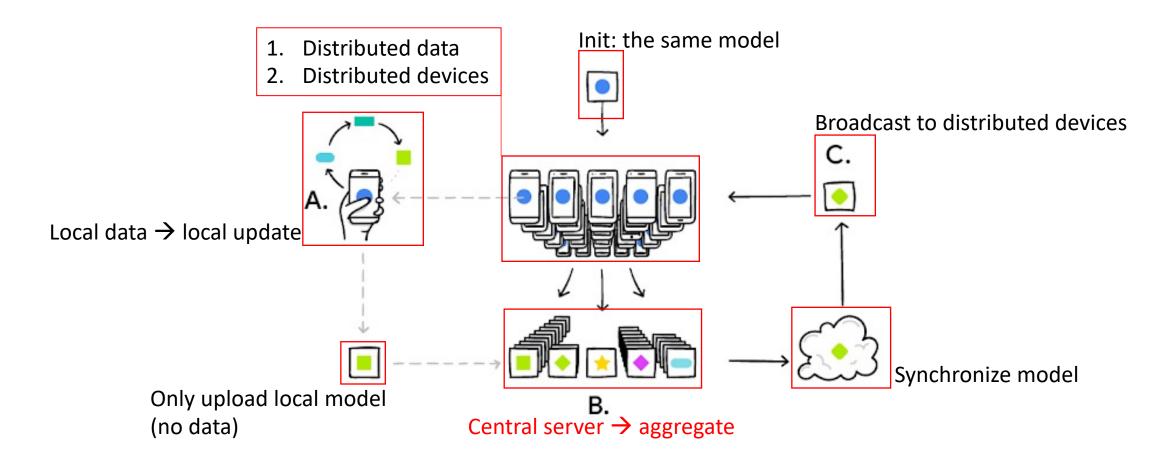












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Local model using local data for updating (a single step)

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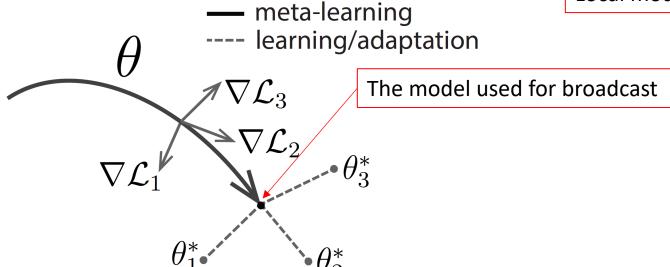
 $\theta^* \stackrel{\text{meta-learning}}{\sim} \theta^*$ $\nabla \mathcal{L}_1$ $\theta_1^* \qquad \theta_2^*$

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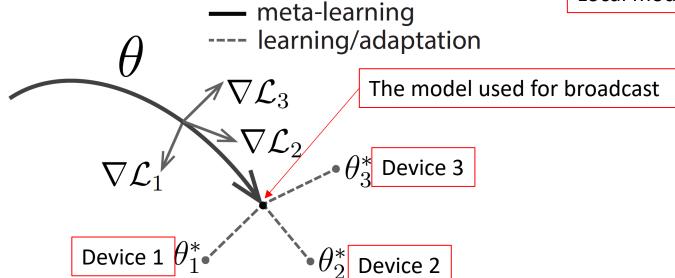
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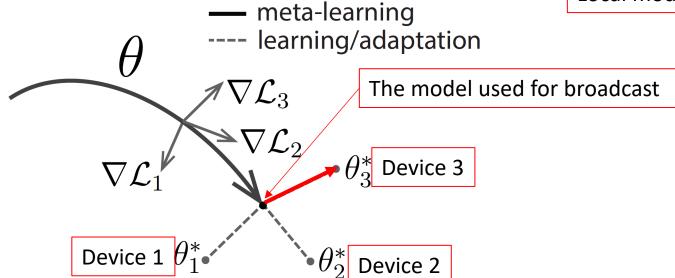
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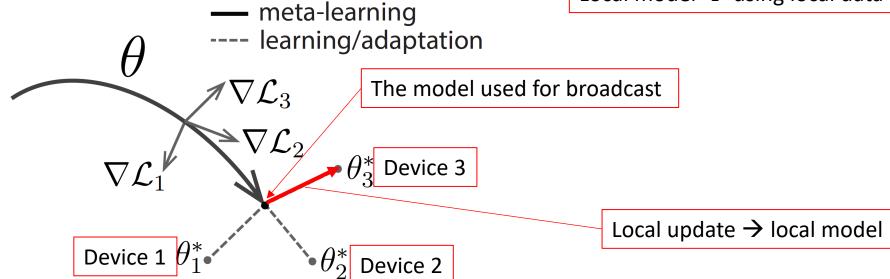
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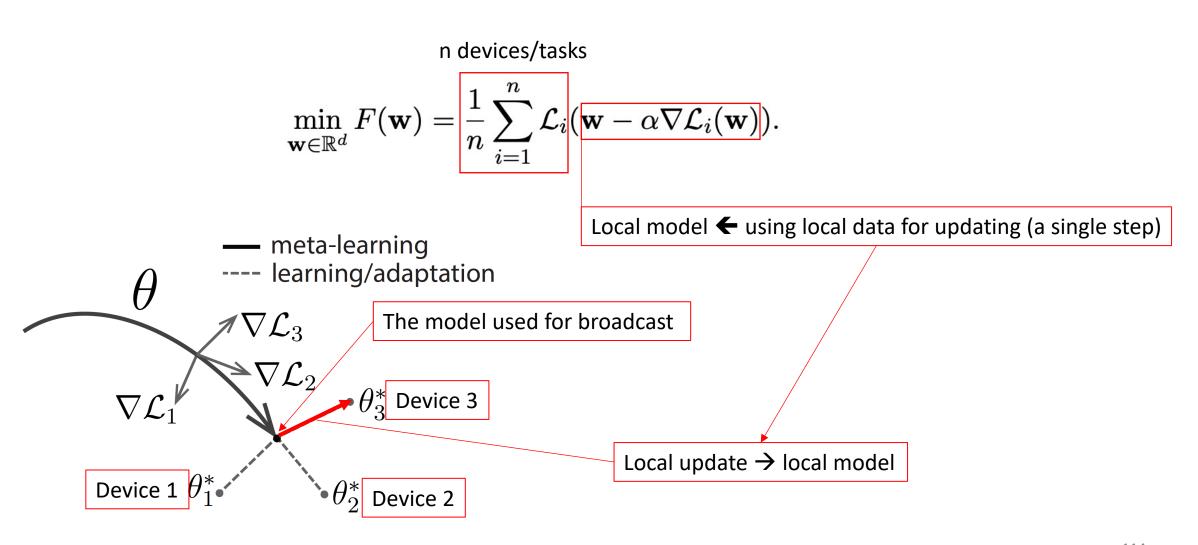


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Personalized Federated Learning

Algorithm	Client Sampling	Sample Complexity	Communication Complexity	Avg. $\#$ Data points (K) Per Iteration
Per-FedAvg (Fallah et al., 2020b)	X	$\mathcal{O}\left(n\epsilon^{-6} ight)$	$\mathcal{O}(\epsilon^{-3})$	$\mathcal{O}(\epsilon^{-2})$
Per-FedAvg (This work)	$\checkmark^{(3)}$	$\mathcal{O}(\epsilon^{-7})$	$\mathcal{O}(\epsilon^{-4})$	$\mathcal{O}(\epsilon^{-2})$
LocalMOML (This work)	X	$\mathcal{O}\left(n\epsilon^{-5} ight)$	$\mathcal{O}(\epsilon^{-3})$	$\mathcal{O}(1)$
LocalMOML (This work)	✓	$\mathcal{O}(\epsilon^{-6})$	$\mathcal{O}(\epsilon^{-4})$	$\mathcal{O}(1)$

Personalized Federated Learning

Algorithm	Client Sampling	Sample Complexity	Communication Complexity	Avg. $\#$ Data points (K) Per Iteration
Per-FedAvg (Fallah et al., 2020b) Per-FedAvg (This work)	X ✓(3)	$\mathcal{O}\left(n\epsilon^{-6} ight) \ \mathcal{O}(\epsilon^{-7})$	$egin{aligned} \mathcal{O}(\epsilon^{-3}) \ \mathcal{O}(\epsilon^{-4}) \end{aligned}$	$\mathcal{O}(\epsilon^{-2}) \ \mathcal{O}(\epsilon^{-2})$
LocalMOML (This work) LocalMOML (This work)	X ✓	$egin{array}{c} \mathcal{O}\left(n\epsilon^{-5} ight) \ \mathcal{O}\left(\epsilon^{-6} ight) \end{array}$	$egin{aligned} \mathcal{O}(\epsilon^{-3}) \ \mathcal{O}(\epsilon^{-4}) \end{aligned}$	$egin{aligned} \mathcal{O}(1) \ \mathcal{O}(1) \end{aligned}$

Number of data samples (on local devices)

Personalized Federated Learning

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Per-FedAvg (Fallah et al., 2020b) Per-FedAvg (This work)	X ✓(3)	$\mathcal{O}\left(n\epsilon^{-6} ight) \ \mathcal{O}(\epsilon^{-7})$	$egin{aligned} \mathcal{O}(\epsilon^{-3}) \ \mathcal{O}(\epsilon^{-4}) \end{aligned}$	$egin{aligned} \mathcal{O}(\epsilon^{-2}) \ \mathcal{O}(\epsilon^{-2}) \end{aligned}$
LocalMOML (This work) LocalMOML (This work)	X ✓	$egin{array}{c c} \mathcal{O}\left(n\epsilon^{-5} ight) \ \mathcal{O}\left(\epsilon^{-6} ight) \end{array}$	$egin{array}{c} \mathcal{O}(\epsilon^{-3}) \ \mathcal{O}(\epsilon^{-4}) \end{array}$	$egin{aligned} \mathcal{O}(1) \ \mathcal{O}(1) \end{aligned}$

Number of data samples (on local devices)

Number of broadcasts

- 1. drop the dependence on n
- 2. reduce order of \epsilon

Personalized Federated Learning

Algorithm	Client Sampling	Sample Complexity	Communication Complexity	Avg. $\#$ Data points (K) Per Iteration
Per-FedAvg (Fallah et al., 2020b) Per-FedAvg (This work)	X ✓(3)	$\mathcal{O}\left(n\epsilon^{-6} ight) \ \mathcal{O}(\epsilon^{-7})$	$\mathcal{O}(\epsilon^{-3}) \ \mathcal{O}(\epsilon^{-4})$	$egin{array}{c} \mathcal{O}(\epsilon^{-2}) \ \mathcal{O}(\epsilon^{-2}) \end{array}$
LocalMOML (This work) LocalMOML (This work)	×	$egin{array}{c} \mathcal{O}\left(n\epsilon^{-5} ight) \ \mathcal{O}\left(\epsilon^{-6} ight) \end{array}$	$egin{array}{c} \mathcal{O}(\epsilon^{-3}) \ \mathcal{O}(\epsilon^{-4}) \end{array}$	$egin{array}{c} \mathcal{O}(1) \ \mathcal{O}(1) \end{array}$

Number of data samples (on local devices)

Number of broadcasts