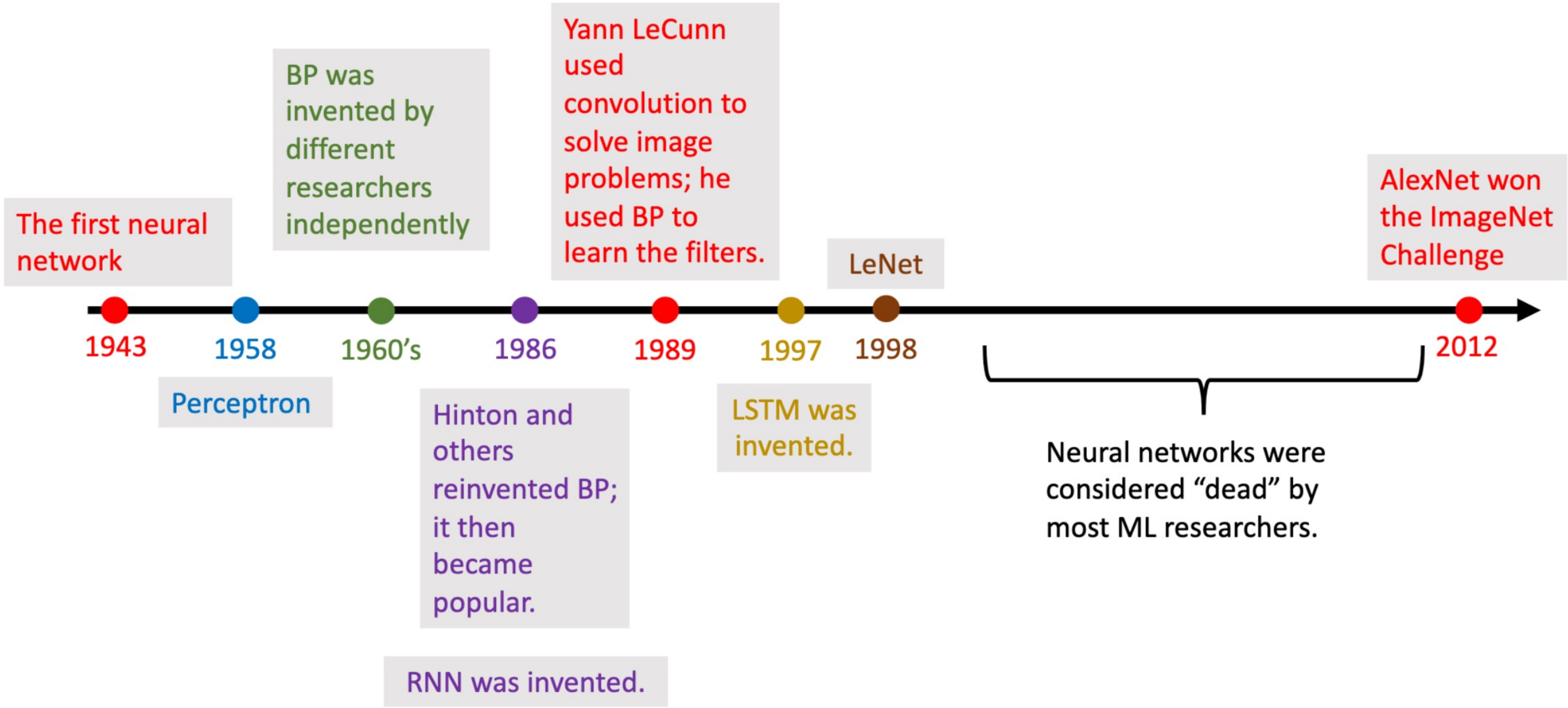
Convolutional Neural Networks

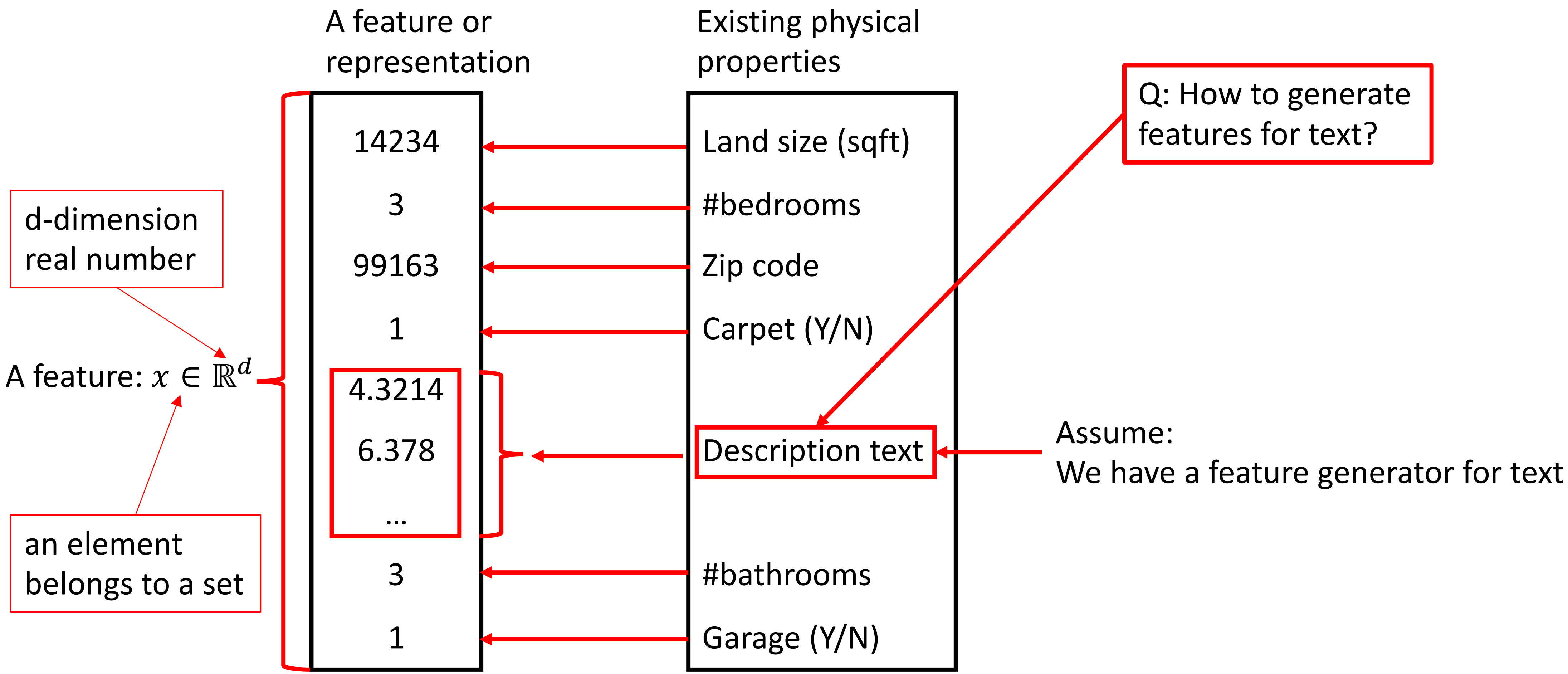
Neural Networks Design And Application

History



The first neural network
1943
1958
Perceptron
BP was invented by different researchers independently
1960's
1986
Hinton and others reinvented BP; it then became popular.
RNN was invented.
Yann LeCunn used convolution to solve image problems; he used BP to learn the filters.
1989
1997
LSTM was invented.
LeNet
1998
Neural networks were considered "dead" by most ML researchers.
AlexNet won the ImageNet Challenge
2012

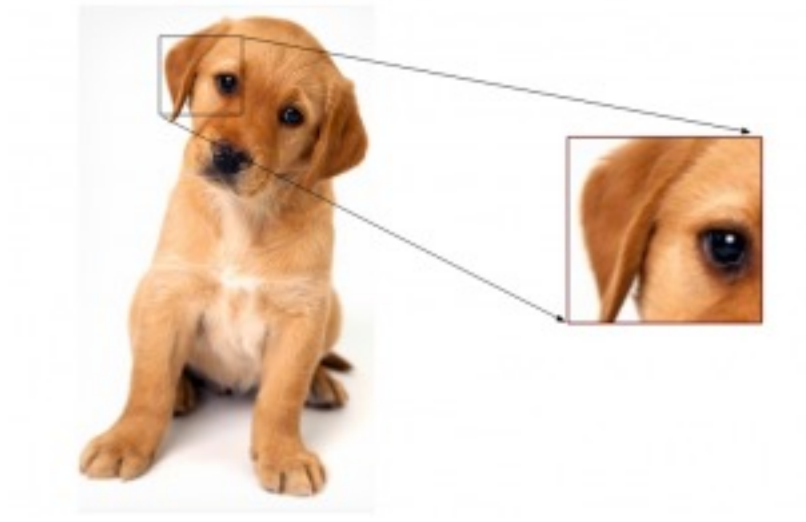
Review: house price prediction



Review: histogram of oriented gradients

- Oriented gradients?
  - Gradients: changes in X and Y directions
  - Oriented:

121	10	78	96	125
48	152	68	125	111
145	78	85	89	65
154	214	56	200	66
214	87	45	102	45



X direction G_x
Subtract the value on the left from the pixel value on the right:
89-78 = 11

Y direction G_y
Subtract the pixel value below from the pixel value above the selected pixel:
68-56=8

Credit for https://www.analyticsvidhya.com/blog/2019/09/feature-engineering-images-introduction-hog-feature-descriptor/

Review: histogram of oriented gradients

- Oriented gradients?
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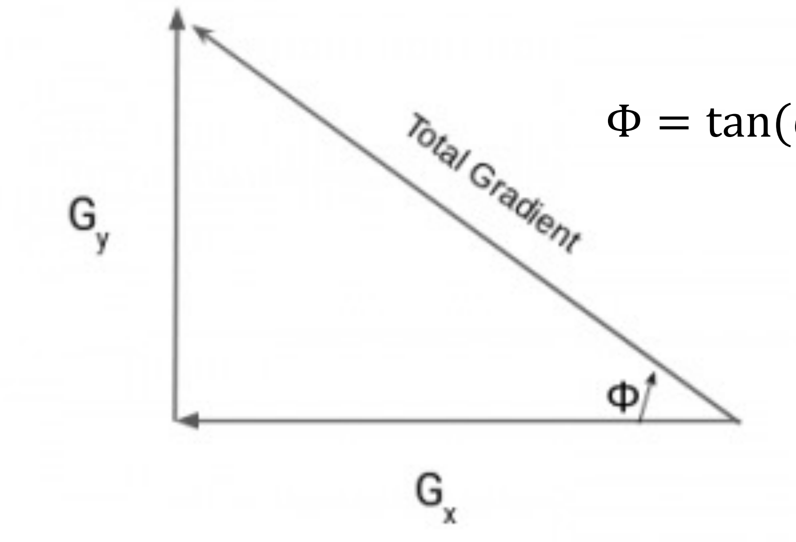

$$\Phi = \tan(G_y/G_x)$$

$$\Phi = \tan^{-1}(G_y/G_x)$$

121	10	78	96	125
48	152	68	125	111
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Review: histogram of oriented gradients

121	10	78	96	125
48	152	68	125	111
145	78	85	89	65
154	214	56	200	66
214	87	45	102	45

Frequency						1									
Angle	1	2	3	4 ...	35	36	37	38	39....	175	176	177	178	179	180

Credit for https://www.analyticsvidhya.com/blog/2019/09/feature-engineering-images-introduction-hog-feature-descriptor/

Review: ImageNet challenge 2012

AlexNet

Task 1

Team name	**Filename**	**Error (5 guesses)**	**Description**
SuperVision	test-preds-141-146.2009-131-137-145-146.2011-145f.	0.15315	Using extra training data from ImageNet Fall 2011 release
SuperVision	test-preds-131-137-145-135-145f.txt	0.16422	Using only supplied training data
ISI	pred_FVs_wLACs_weighted.txt	0.26172	Weighted sum of scores from each classifier with SIFT+FV, LBP+FV, GIST+FV, and CSIFT+FV, respectively.
ISI	pred_FVs_weighted.txt	0.26602	Weighted sum of scores from classifiers using each FV.
ISI	pred_FVs_summed.txt	0.26646	Naive sum of scores from classifiers using each FV.

Review: LeNet-5 in 1999

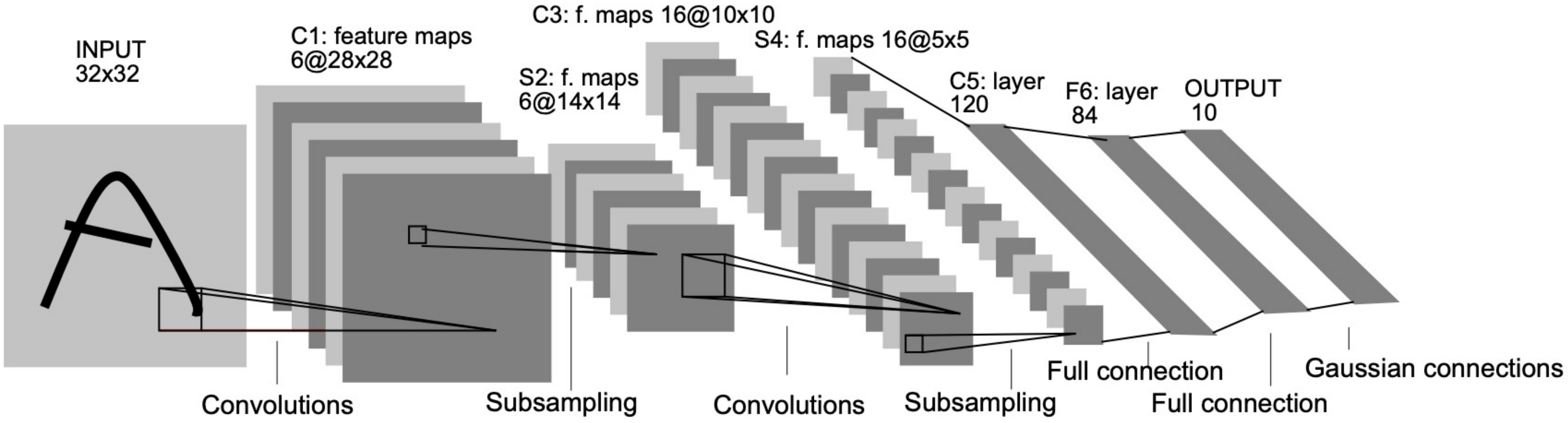
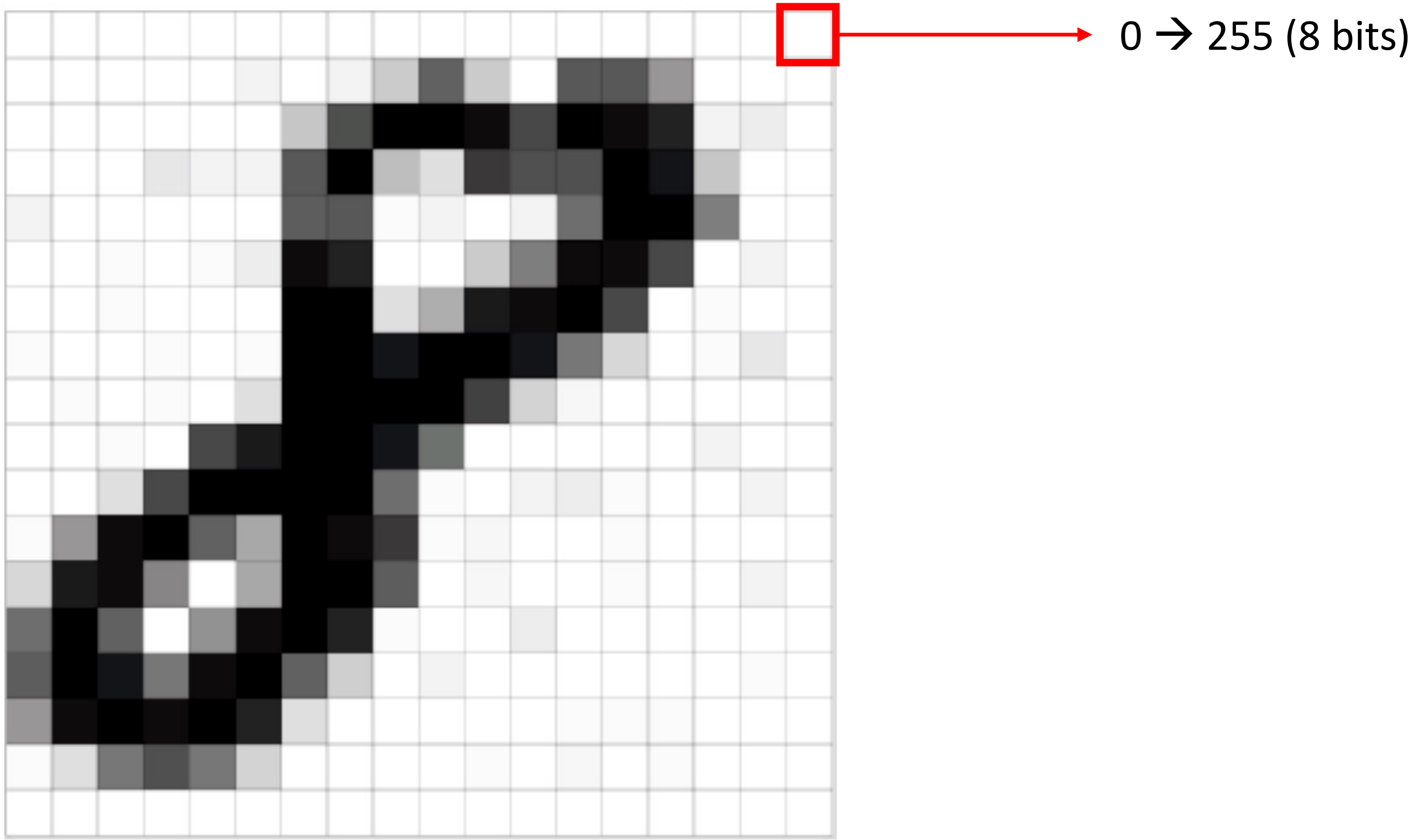


Fig. 1. Architecture of LeNet-5, a Convolutional Neural Network, here for digits recognition. Each plane is a feature map, i.e. a set of units whose weights are constrained to be identical.

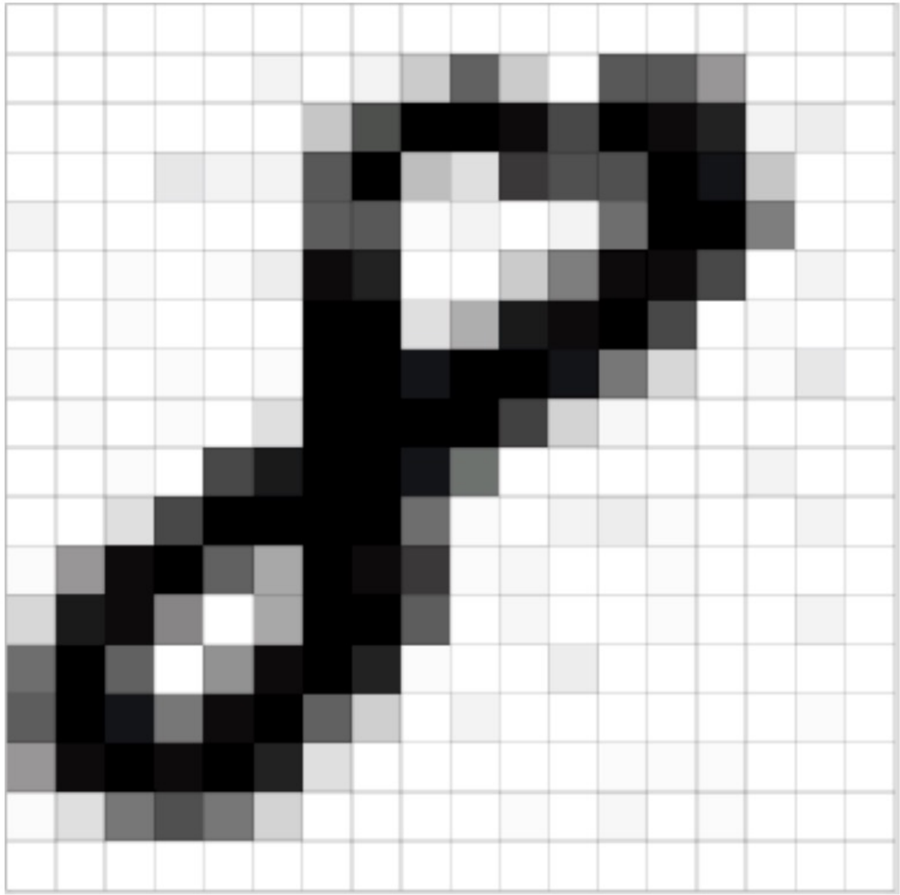
LeCun, Yann, Patrick Haffner, Léon Bottou, and Yoshua Bengio. "Object recognition with gradient-based learning." In *Shape, contour and grouping in computer vision*, pp. 319-345. Springer, Berlin, Heidelberg, 1999.

What is convolutional neural network?



A grayscale image

What is convolutional neural network?



A grayscale image

An image → a matrix

1	1	1	0	0
0	1	1	1	0
0	0	1	1	1
0	0	1	1	0
0	1	1	0	0

Convolution for images (matrices)

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Involving two matrices

Convolution for images (matrices)

Given image

1	1	1	0	0
0	1	1	1	0
0	0	1	1	1
0	0	1	1	0
0	1	1	0	0

*

Filter

1	0	1
0	1	0
1	0	1

Involving two matrices

Convolution for images (matrices)

Larger

Given image

Smaller

Filter

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

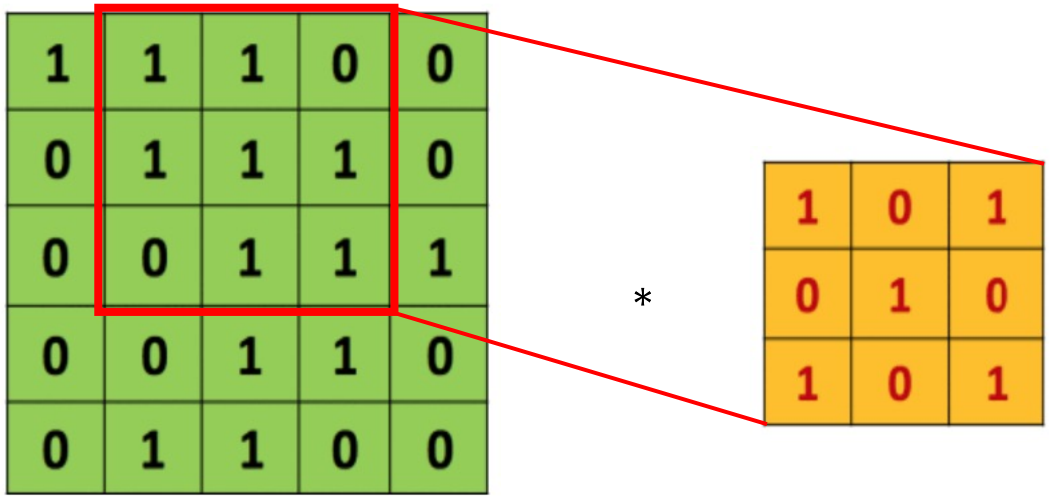
Involving two matrices

Convolution for images (matrices)

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

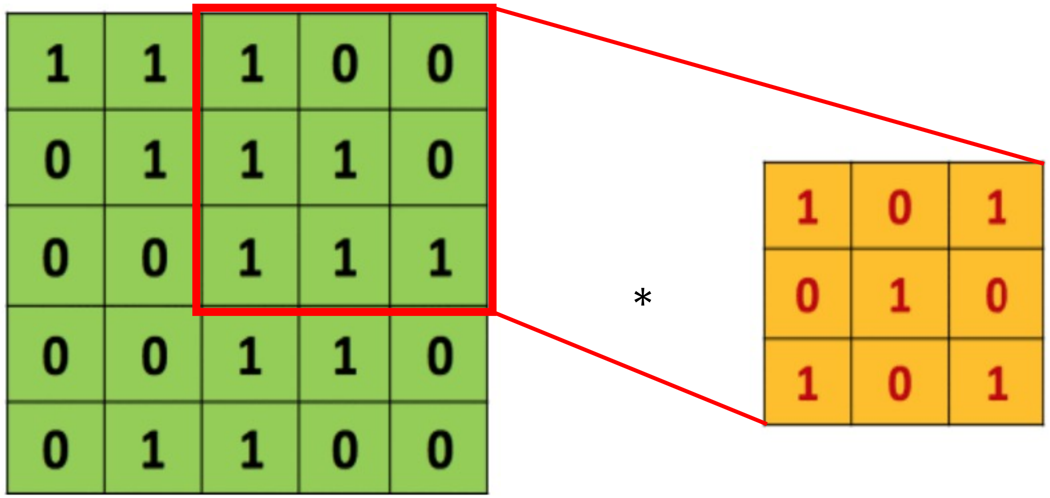
Finding pairs

Convolution for images (matrices)



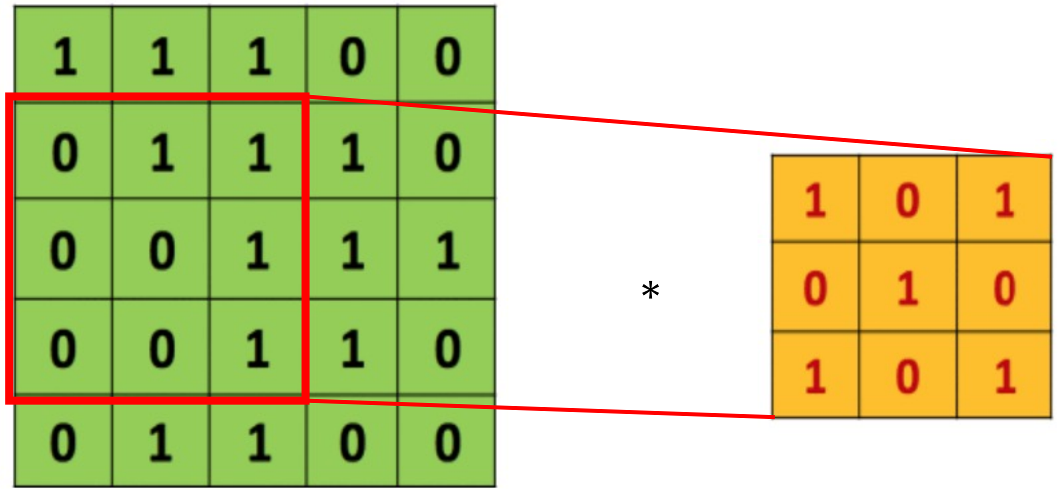
Finding pairs

Convolution for images (matrices)



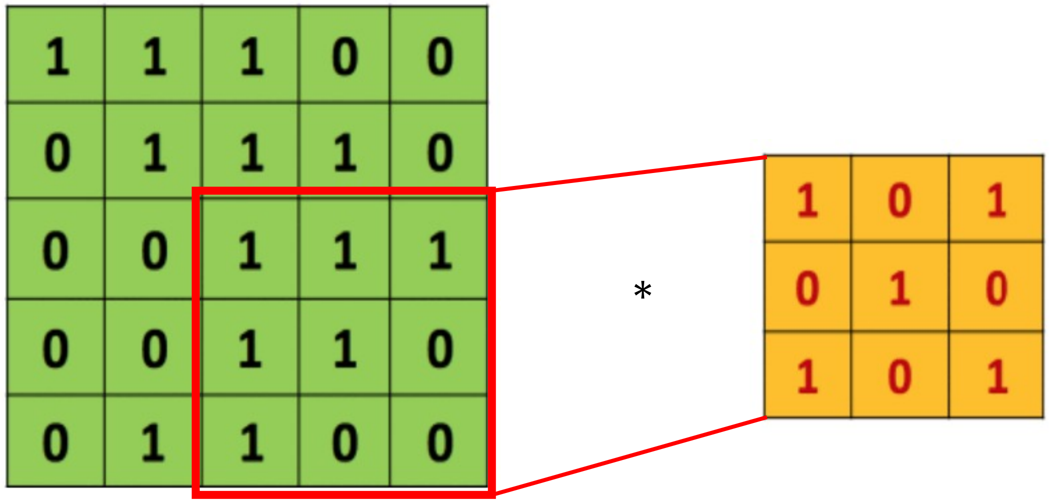
Finding pairs

Convolution for images (matrices)



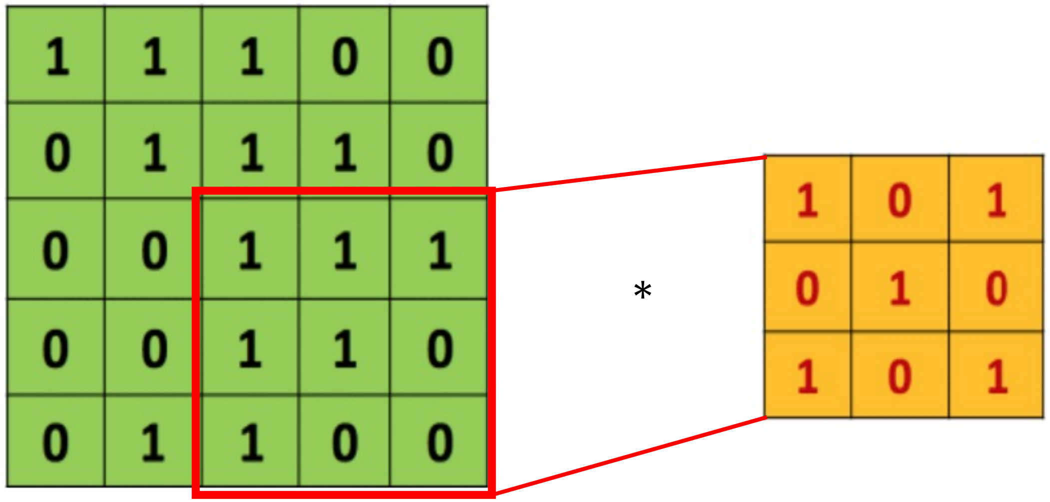
Finding pairs

Convolution for images (matrices)



Finding pairs

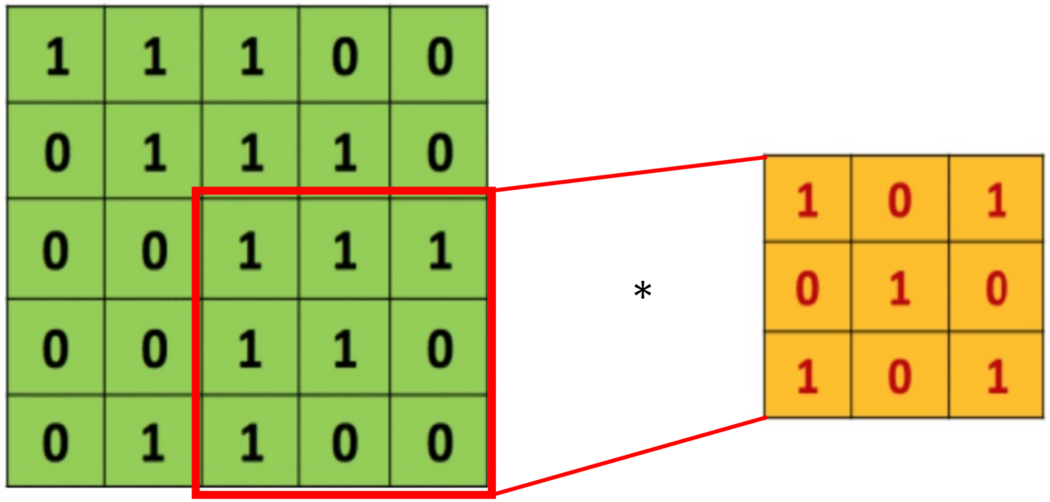
Convolution for images (matrices)



Finding pairs

Q: how many pairs we have?

Convolution for images (matrices)

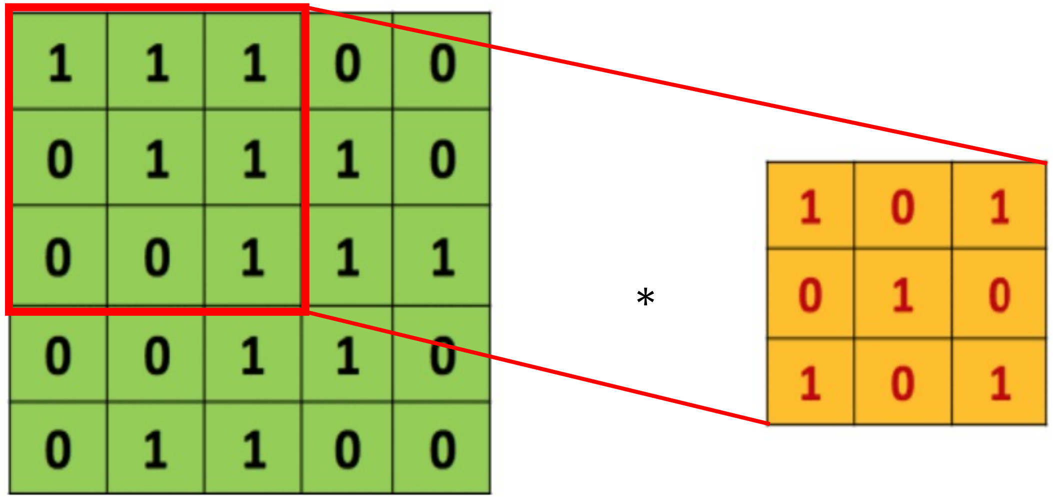


Finding pairs

Q: how many pairs we have?

(5-3+1) * (5-3+1)=9

Convolution for images (matrices)



Inner product of each pair

Convolution for images (matrices)

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Inner product of each pair

Elementwise multiplication + summation

Convolution for images (matrices)

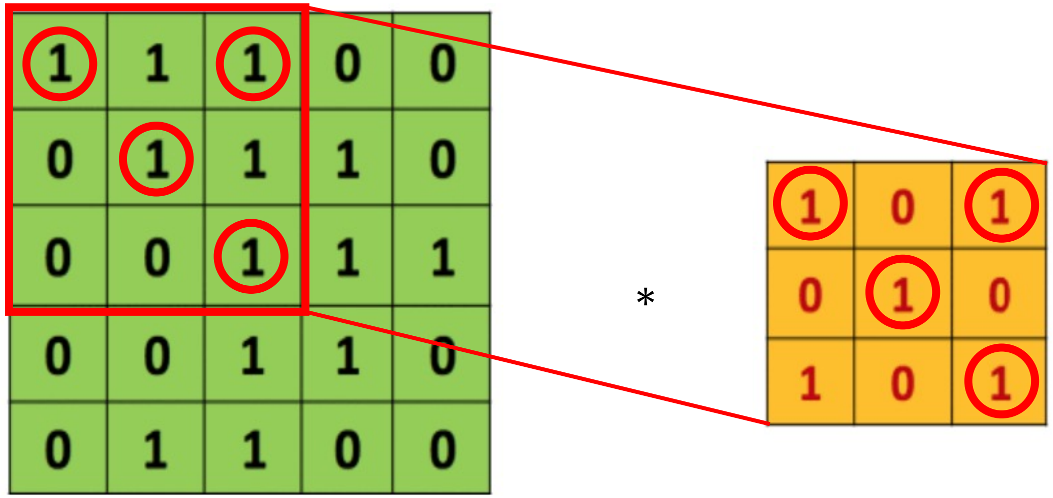
$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Inner product of each pair

Elementwise multiplication + summation

Q: what is your result for the first pair?

Convolution for images (matrices)



Inner product of each pair

Elementwise multiplication + summation

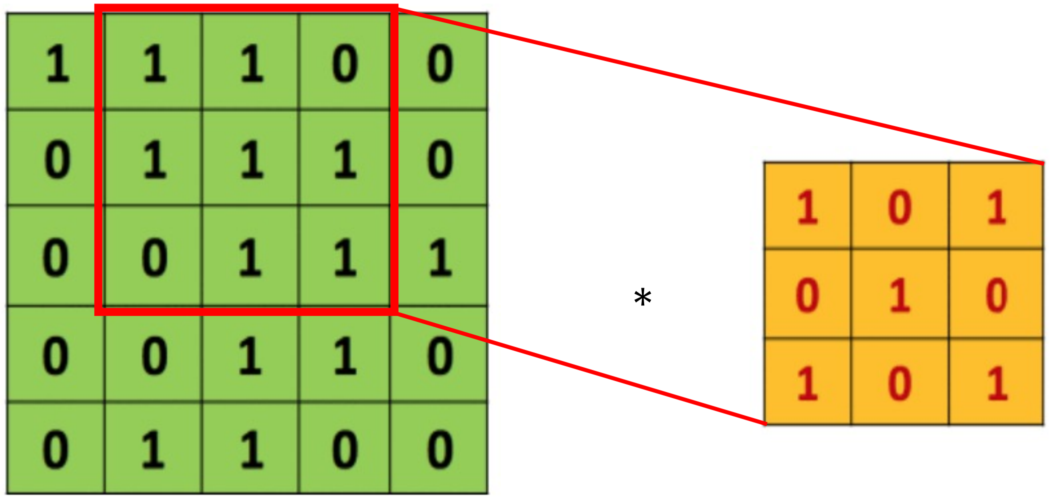
Convolution for images (matrices)

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} = 4$$

Inner product of each pair

Elementwise multiplication + summation

Convolution for images (matrices)

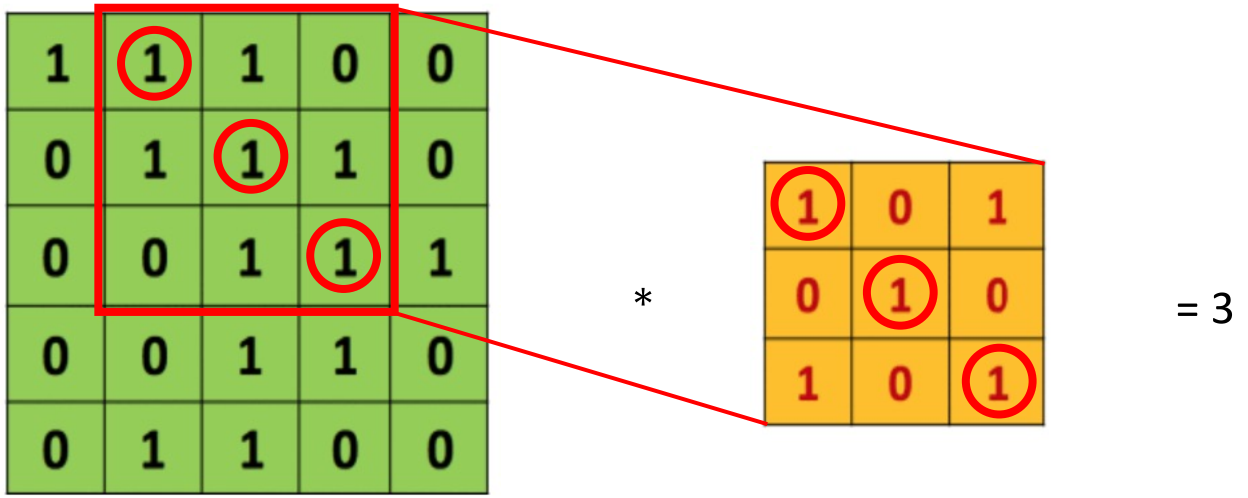


Q: the second pair?

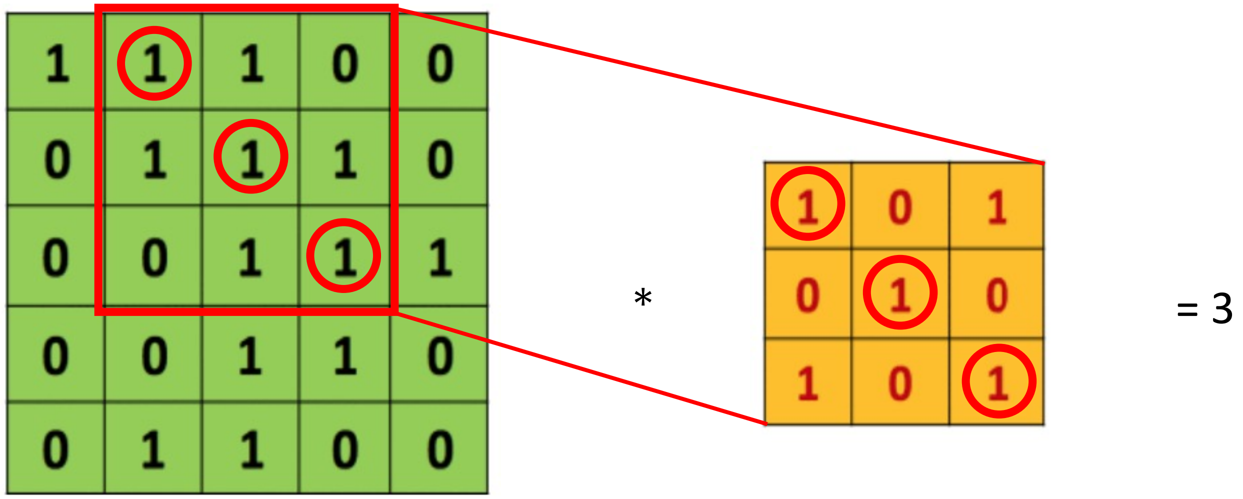
Convolution for images (matrices)

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Convolution for images (matrices)



Convolution for images (matrices)



We can repeat for each pair

Convolution for images (matrices)

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & 3 & 4 \\ 2 & 4 & 3 \\ 2 & 3 & 4 \end{bmatrix}$$

Convolution for images (matrices)

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & 3 & 4 \\ 2 & 4 & 3 \\ 2 & 3 & 4 \end{bmatrix}$$

Place each element
according to their positions

Convolution for images (matrices)

Row: 1
Column: 1

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} \boxed{4} & 3 & 4 \\ 2 & 4 & 3 \\ 2 & 3 & 4 \end{bmatrix}$$

Place each element according to their positions

Convolution for images (matrices)

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & \boxed{3} & 4 \\ 2 & 4 & 3 \\ 2 & 3 & 4 \end{bmatrix}$$

Row: 1
Column: 2

Place each element according to their positions

Convolution for images (matrices)

Row: 3
Column: 3

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & 3 & 4 \\ 2 & 4 & 3 \\ 2 & 3 & \boxed{4} \end{bmatrix}$$

Place each element according to their positions

Convolution for images (matrices)

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & 3 & 4 \\ 2 & 4 & 3 \\ 2 & 3 & 4 \end{bmatrix}$$

n=5 m=3 Q: dimension?

Convolution for images (matrices)

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & 3 & 4 \\ 2 & 4 & 3 \\ 2 & 3 & 4 \end{bmatrix}$$

n=5 m=3 n-m+1=3

Convolution for images (matrices)

$$
\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}
*
\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}
\rightarrow
\begin{bmatrix} 4 & 3 & 4 \\ 2 & 4 & 3 \\ 2 & 3 & 4 \end{bmatrix}
$$

n=5	m=3	n-m+1=3
One matrix	One matrix	One matrix

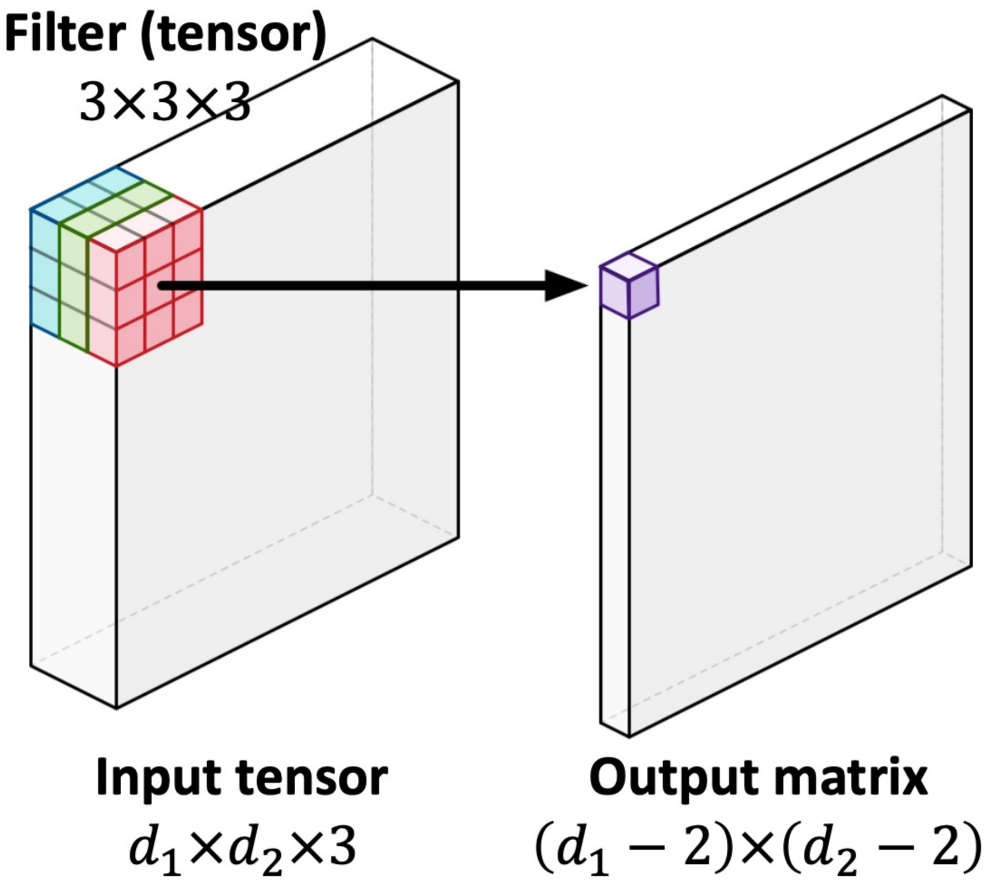
Convolution for images (matrices)

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & 3 & 4 \\ 2 & 4 & 3 \\ 2 & 3 & 4 \end{bmatrix}$$

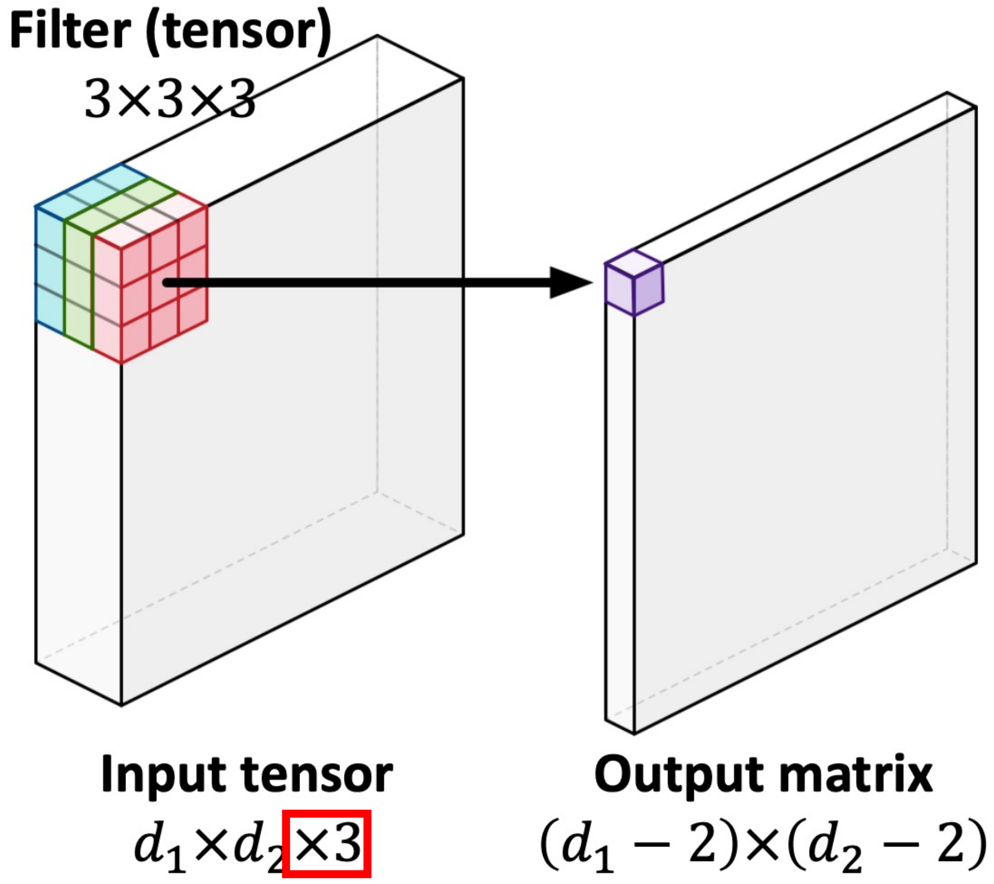
n=5	m=3	n-m+1=3
One matrix	One matrix	One matrix

One input matrix * one filter → one feature matrix

Convolution for images (tensors)



Convolution for images (tensors)



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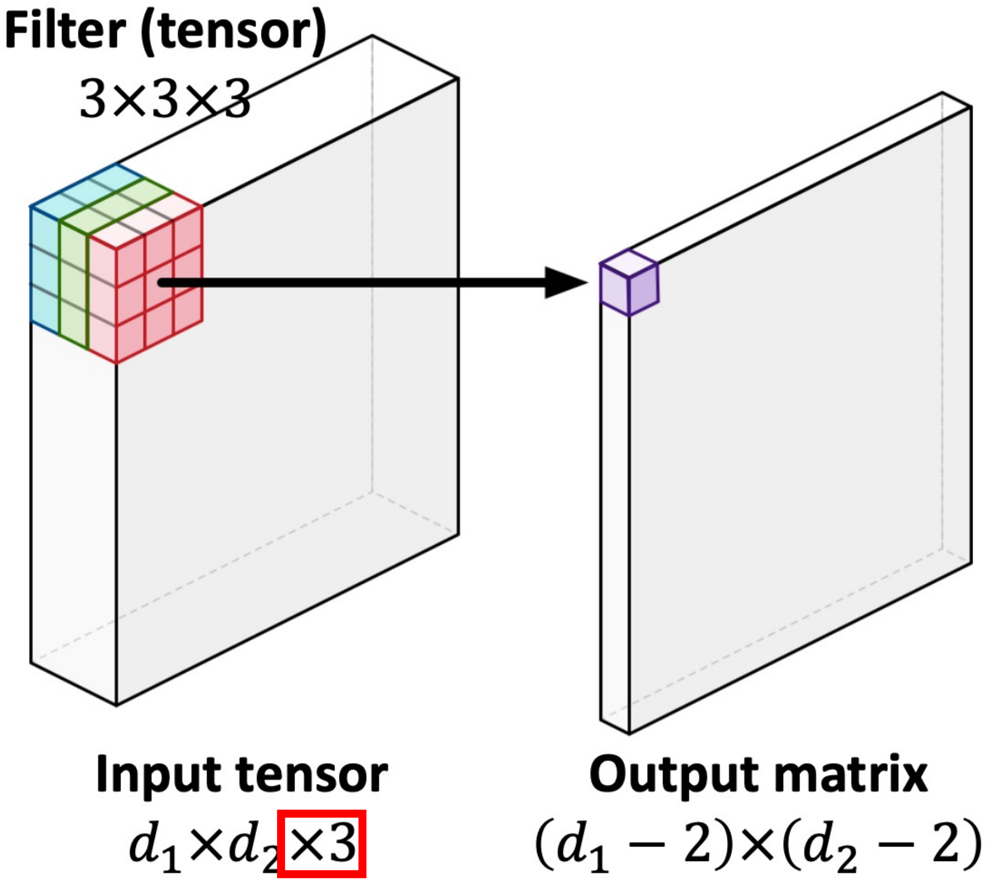
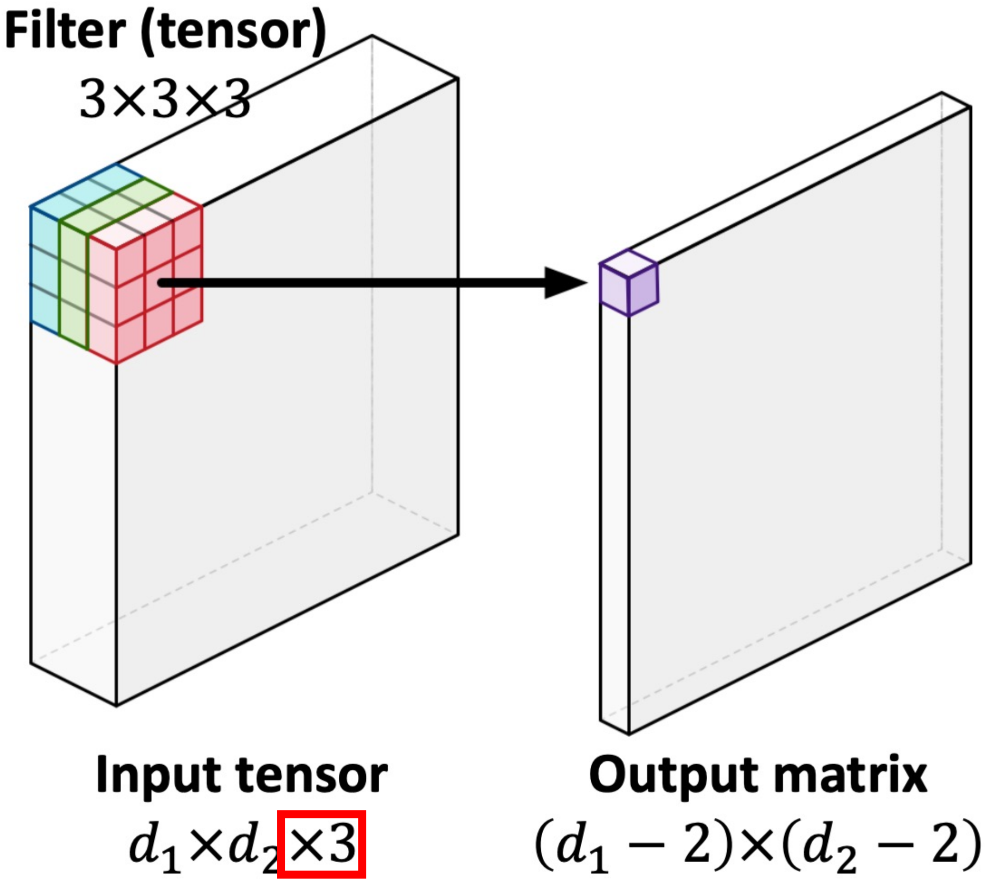


Q: why we care about tensors?

Convolution for images (tensors)



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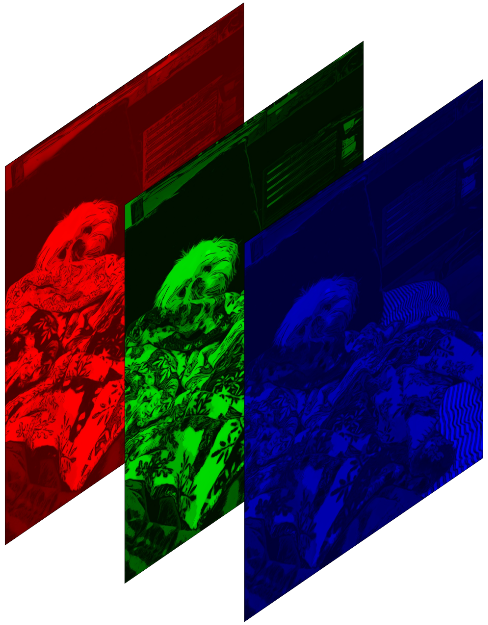
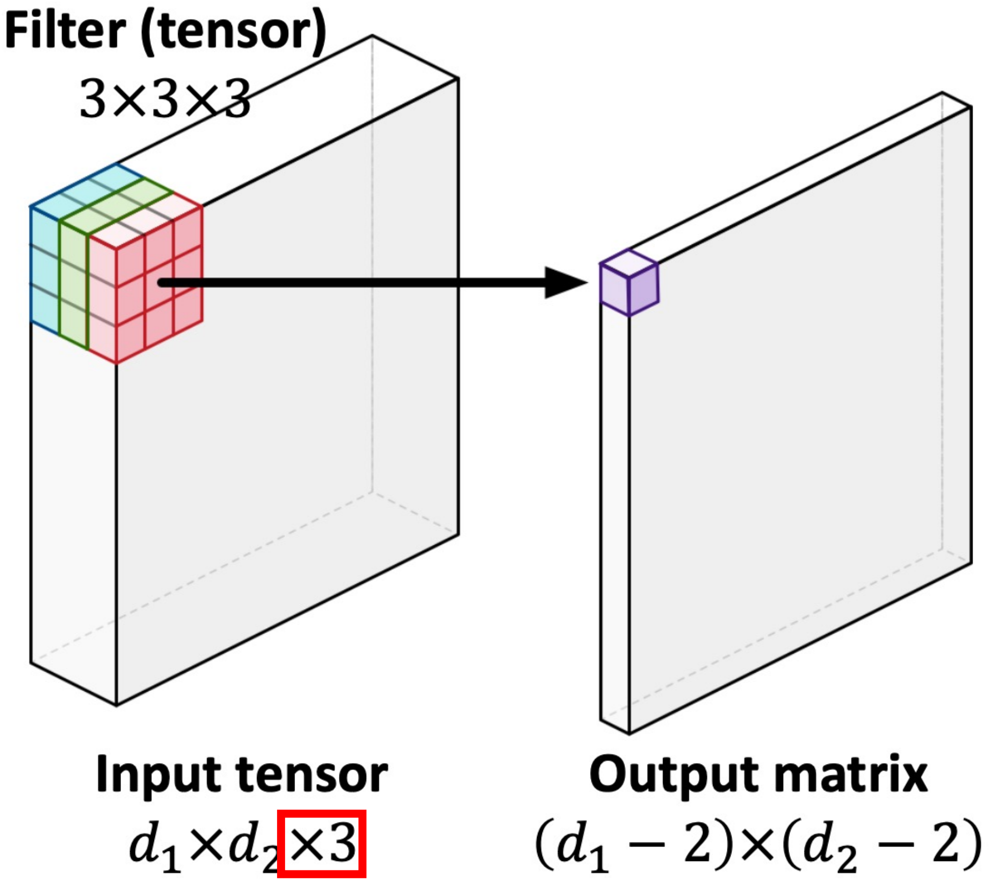


Image from https://e2eml.school/convert_rgb_to_grayscale.html

Convolution for images (tensors)



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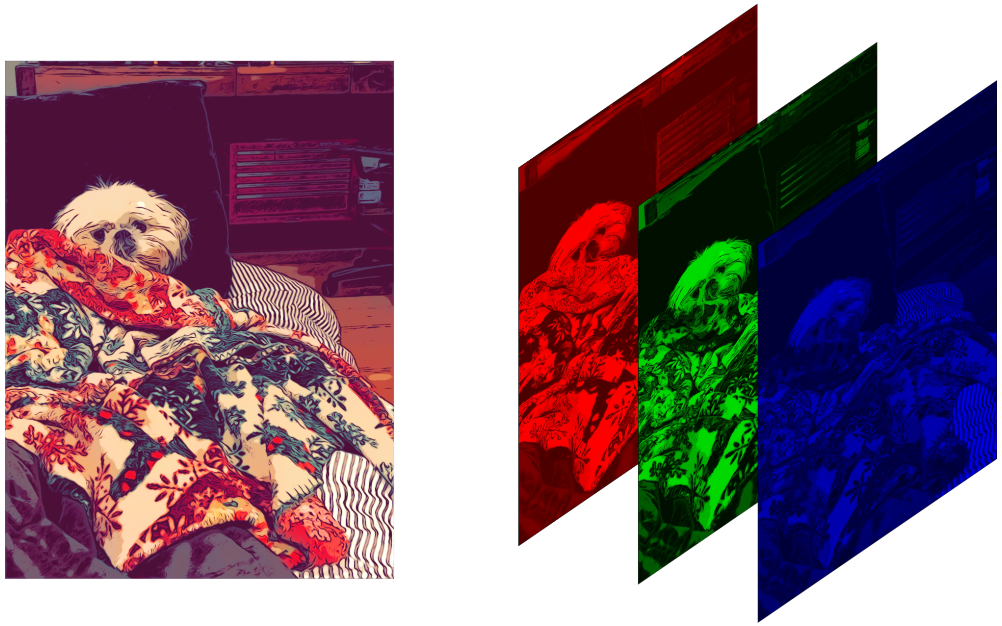
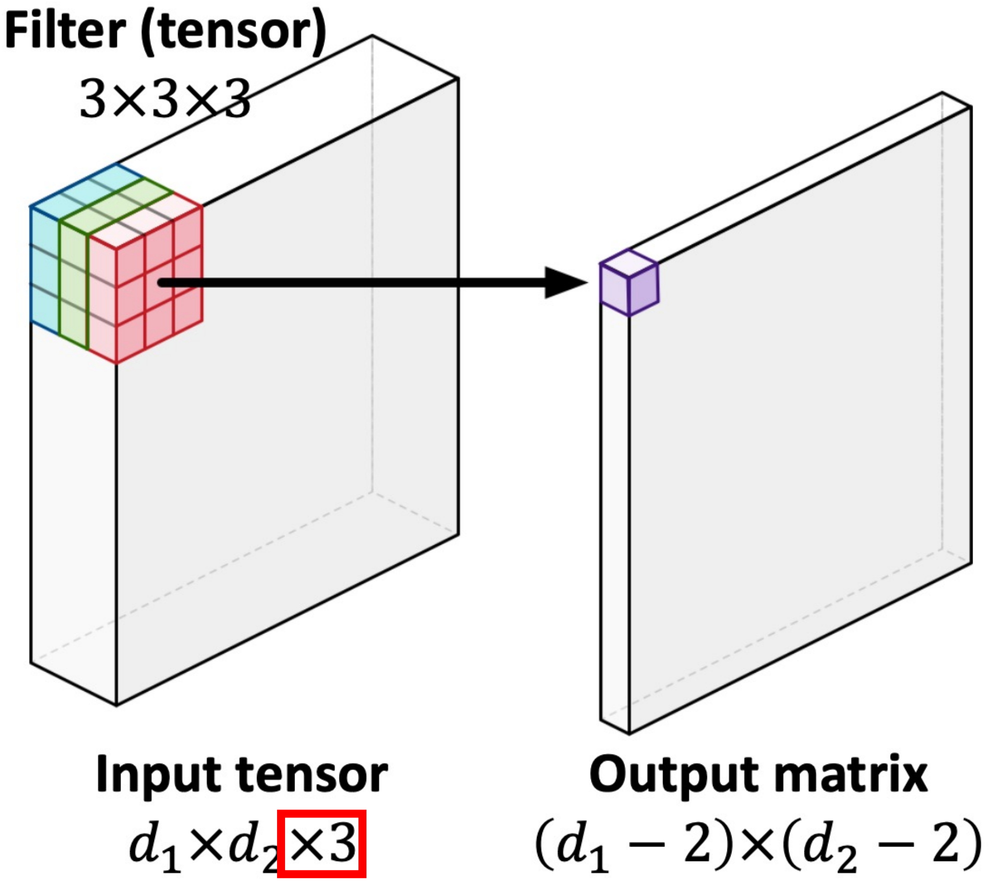


Reason 1:
RGB channels are more common

Image from https://e2eml.school/convert_rgb_to_grayscale.html

Convolution for images (tensors)



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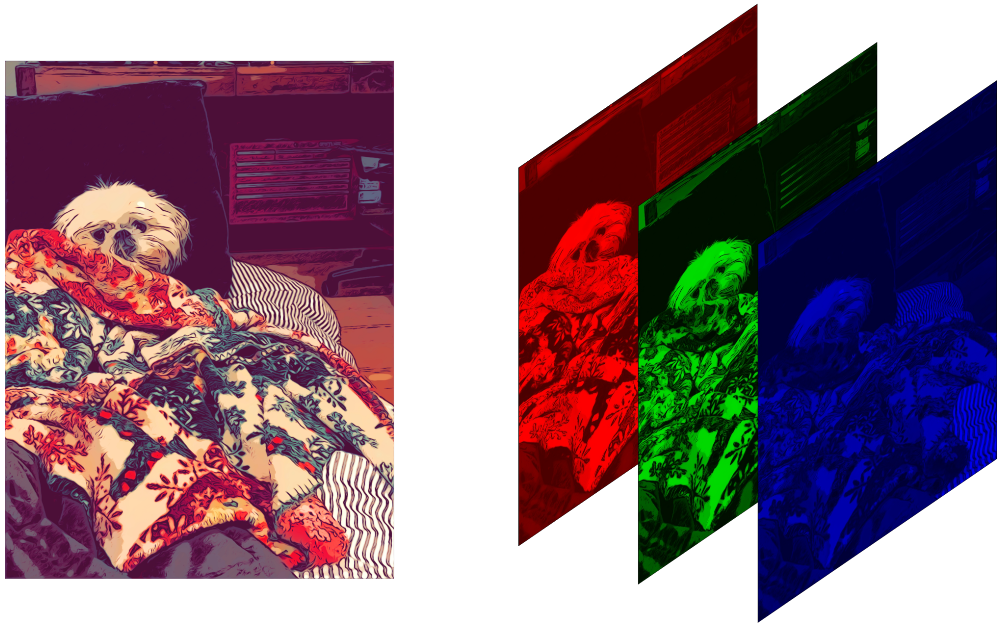

Reason 1:
RGB channels are more common

Each channel → a matrix

Image from https://e2eml.school/convert_rgb_to_grayscale.html

LeNet-5 in 1999

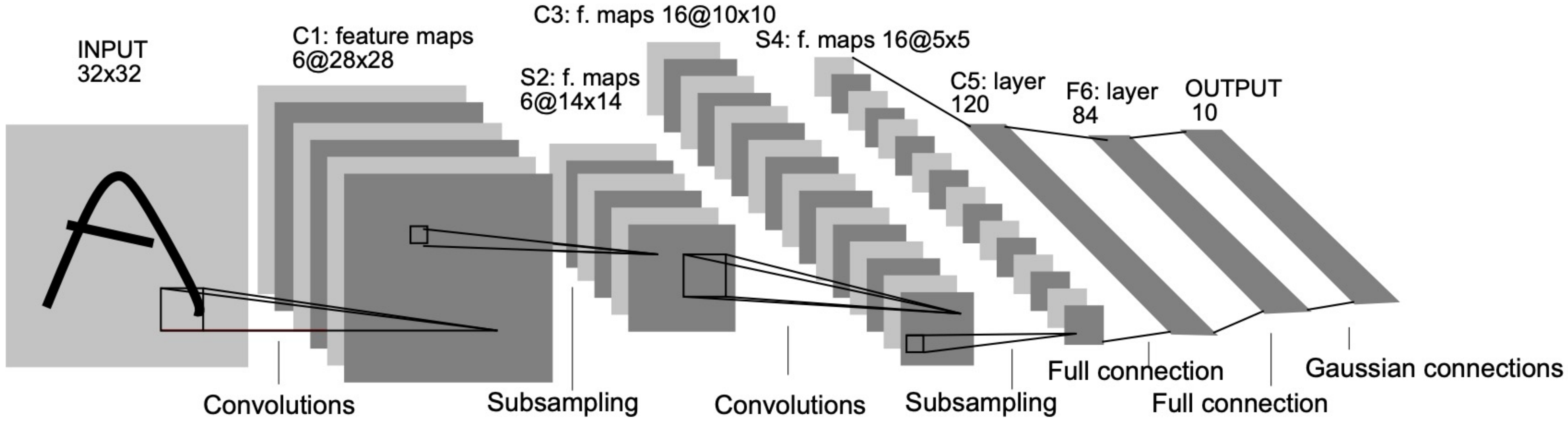

Fig. 1. Architecture of LeNet-5, a Convolutional Neural Network, here for digits recognition. Each plane is a feature map, i.e. a set of units whose weights are constrained to be identical.

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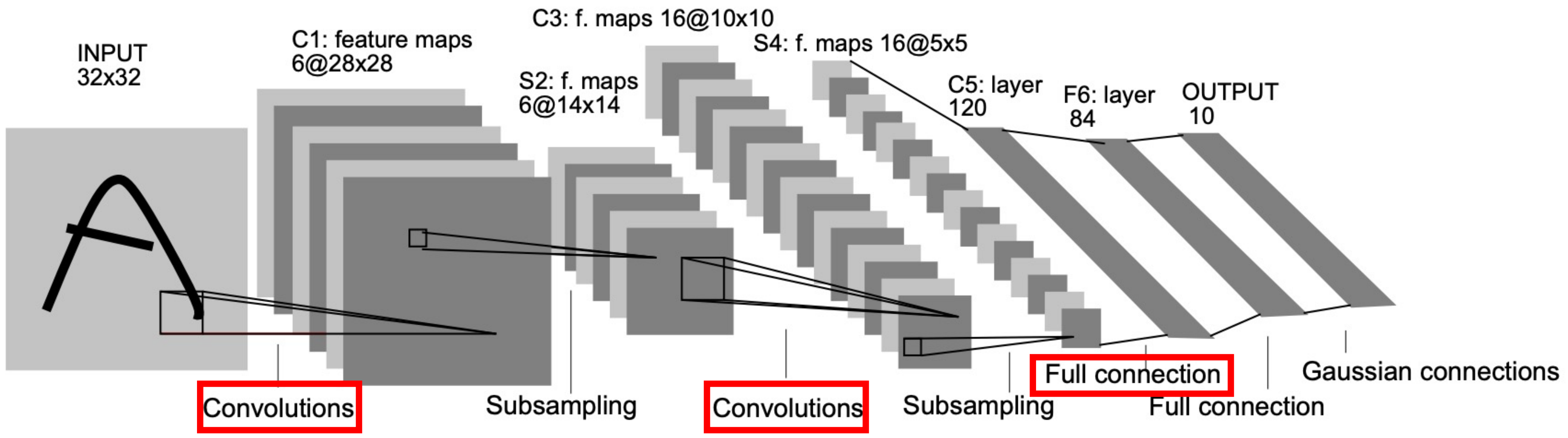
LeNet-5 in 1999



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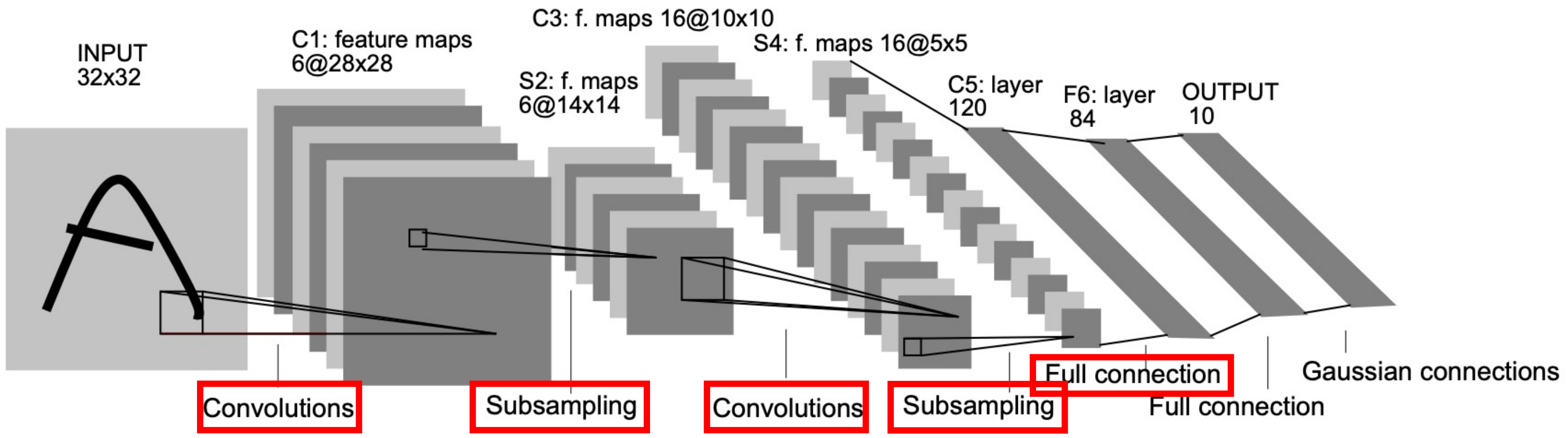
LeNet-5 in 1999



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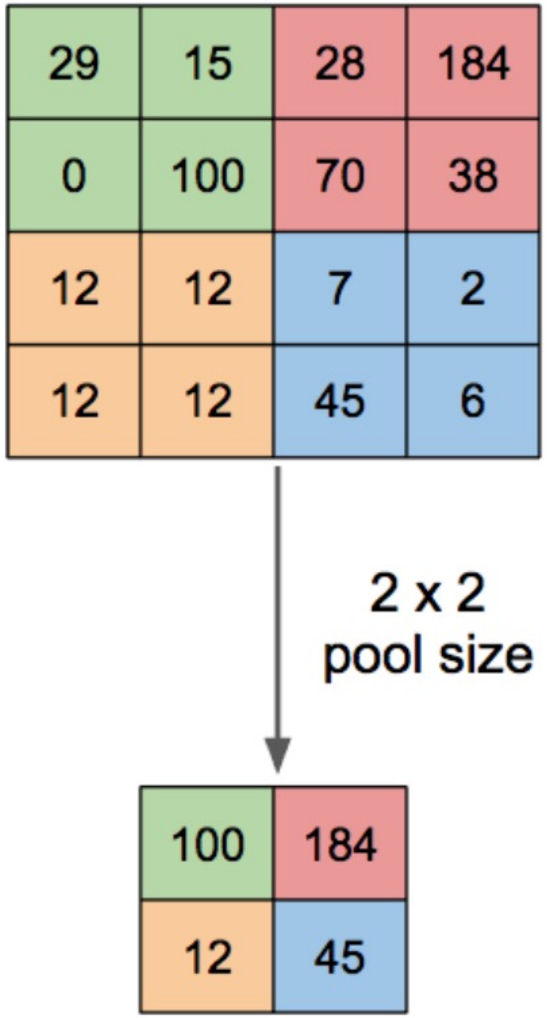
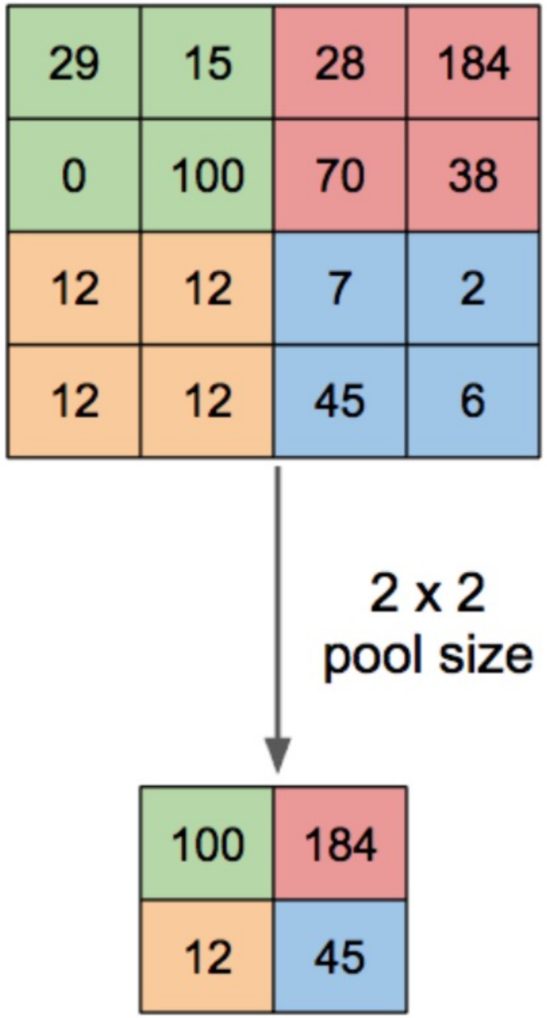
Subsampling operations

- Max pooling



Subsampling operations

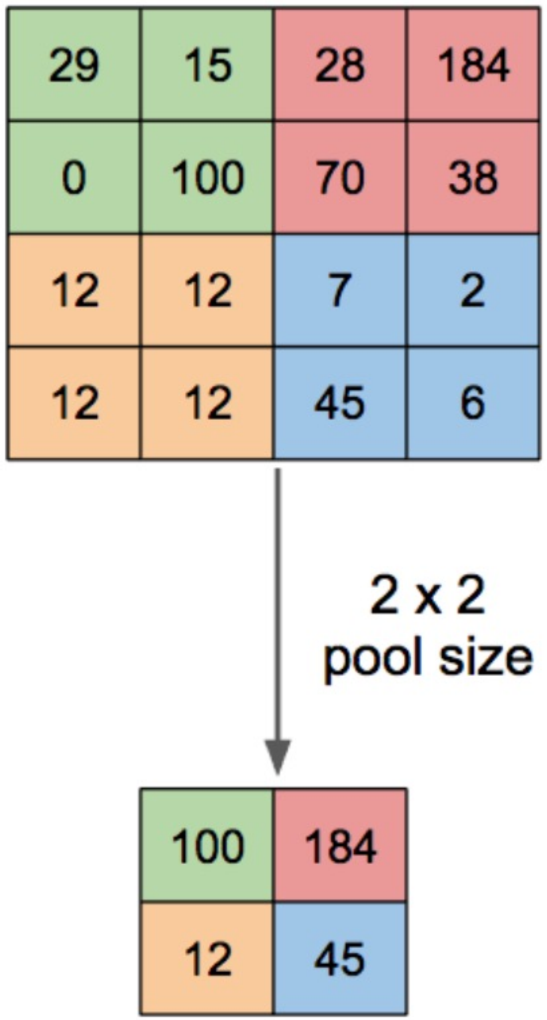
- Max pooling

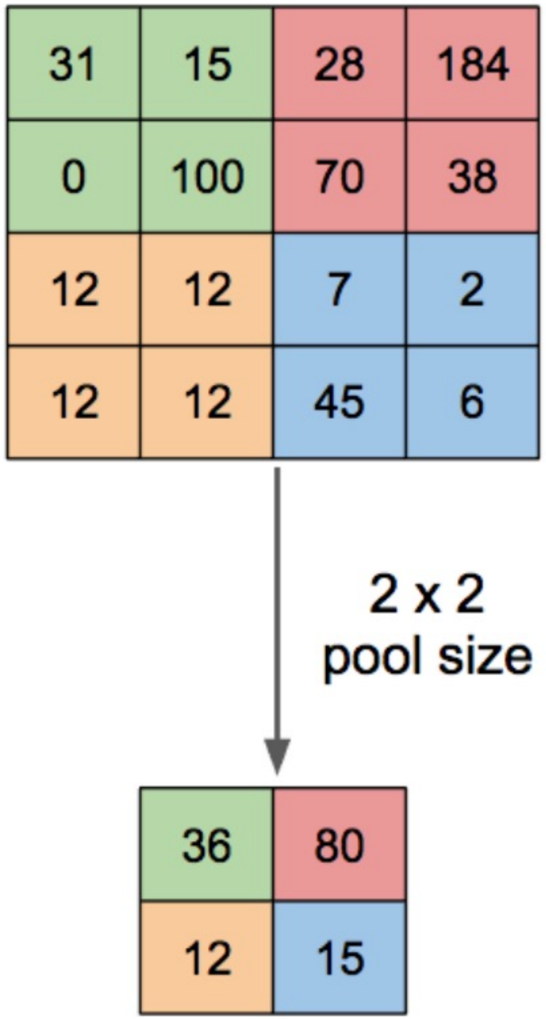


Q: what does max Pooling really do?

Subsampling operations

- Max pooling
- Average pooling

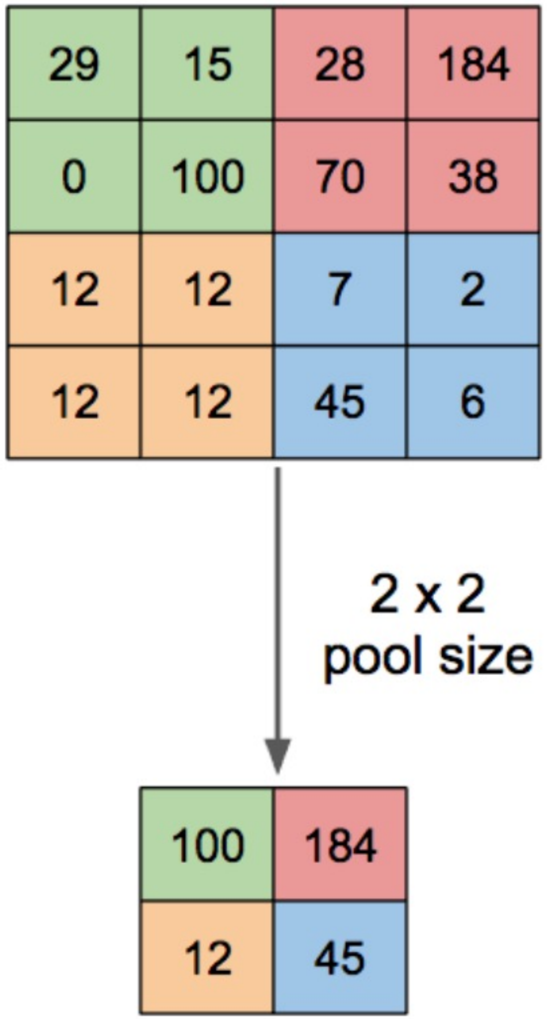


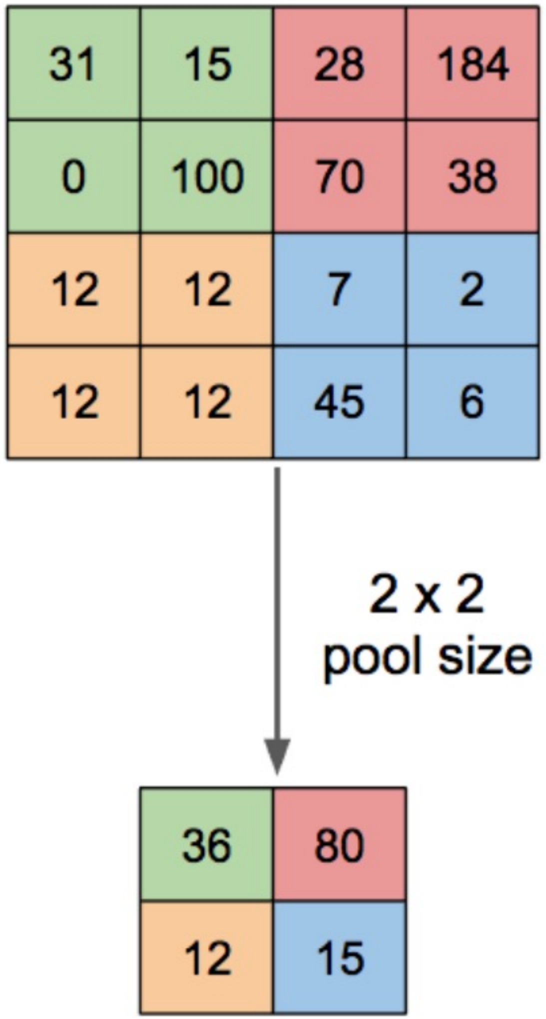


Subsampling operations

Q: what does average Pooling really do?

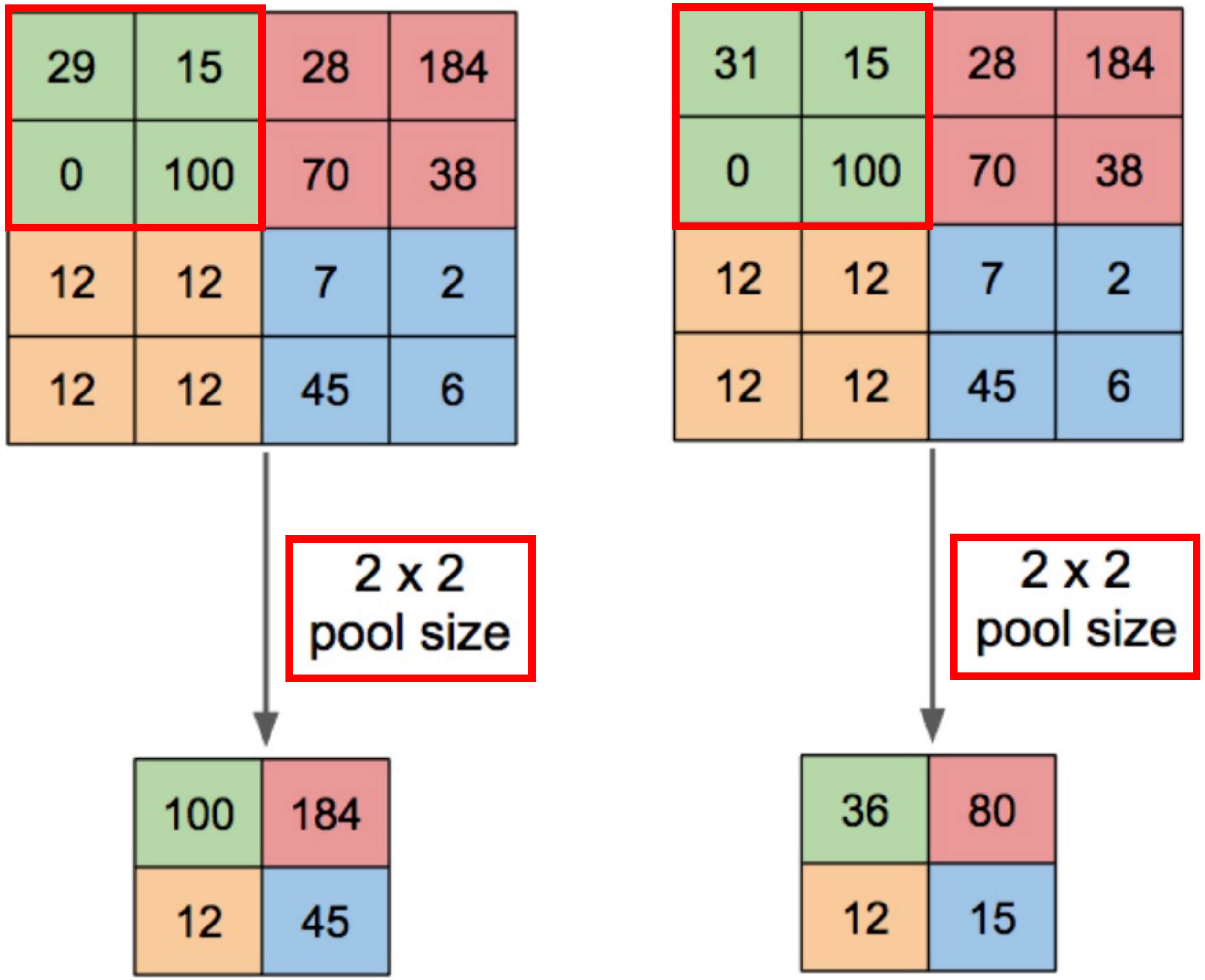
- Max pooling
- Average pooling





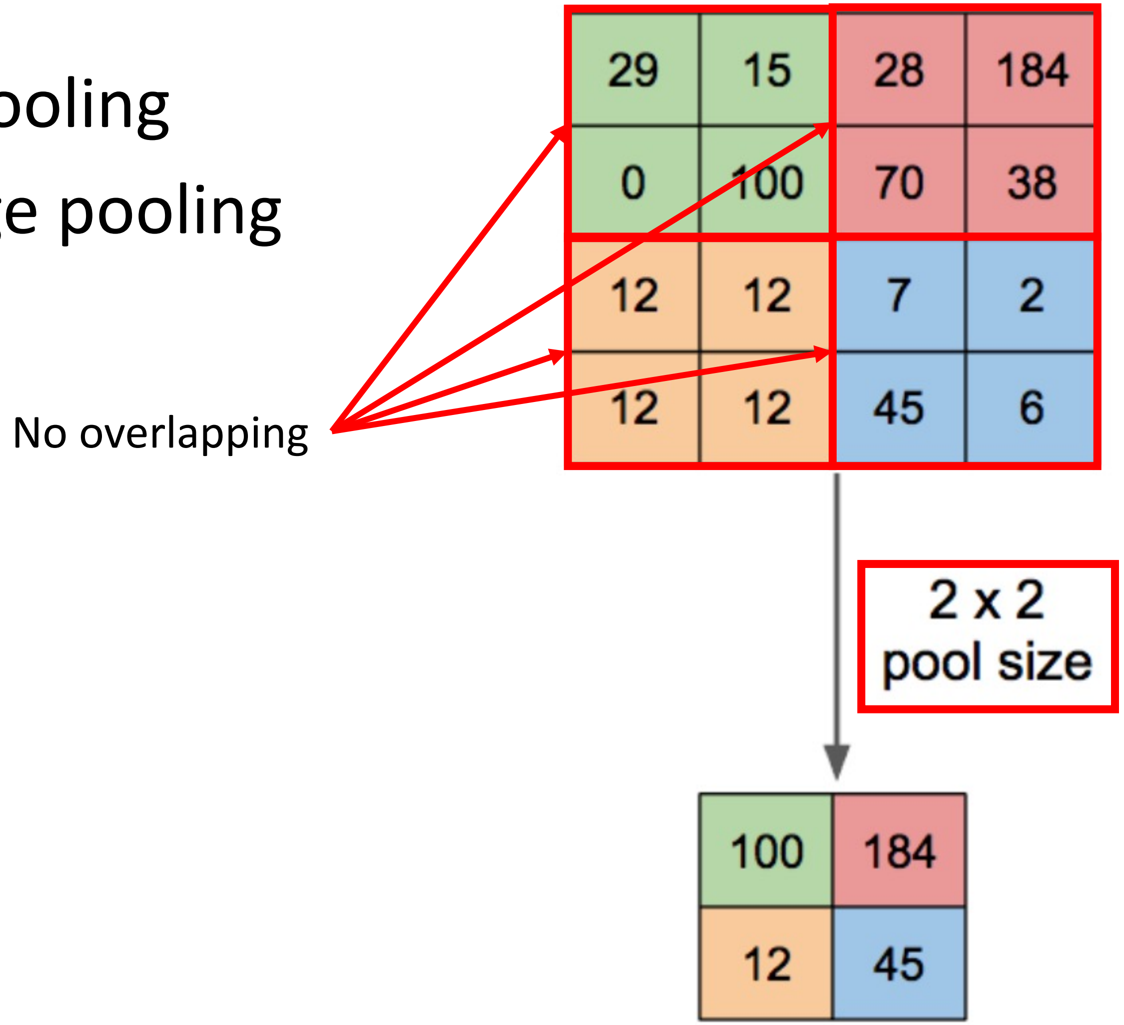
Subsampling operations

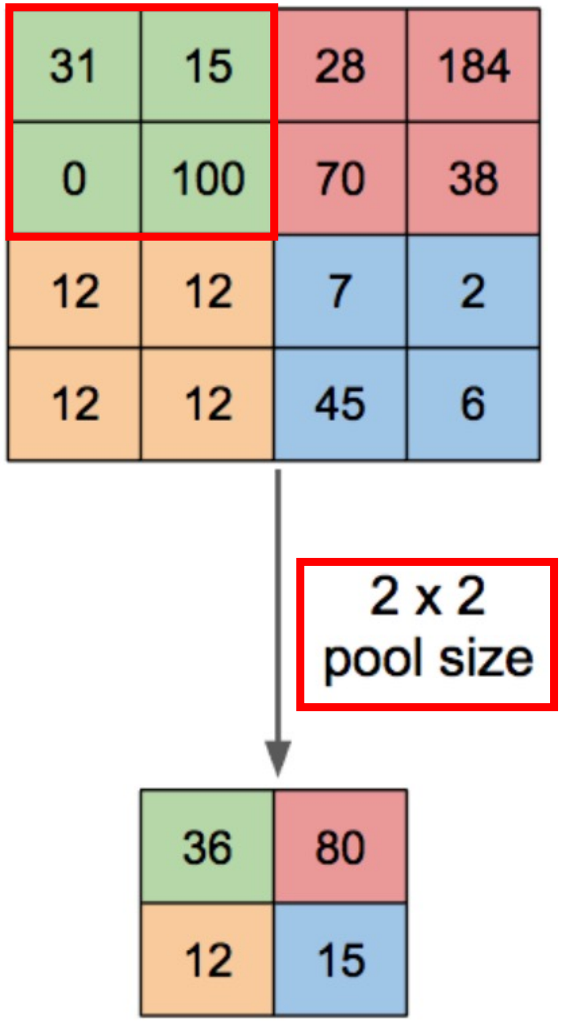
- Max pooling
- Average pooling



Subsampling operations

- Max pooling
- Average pooling

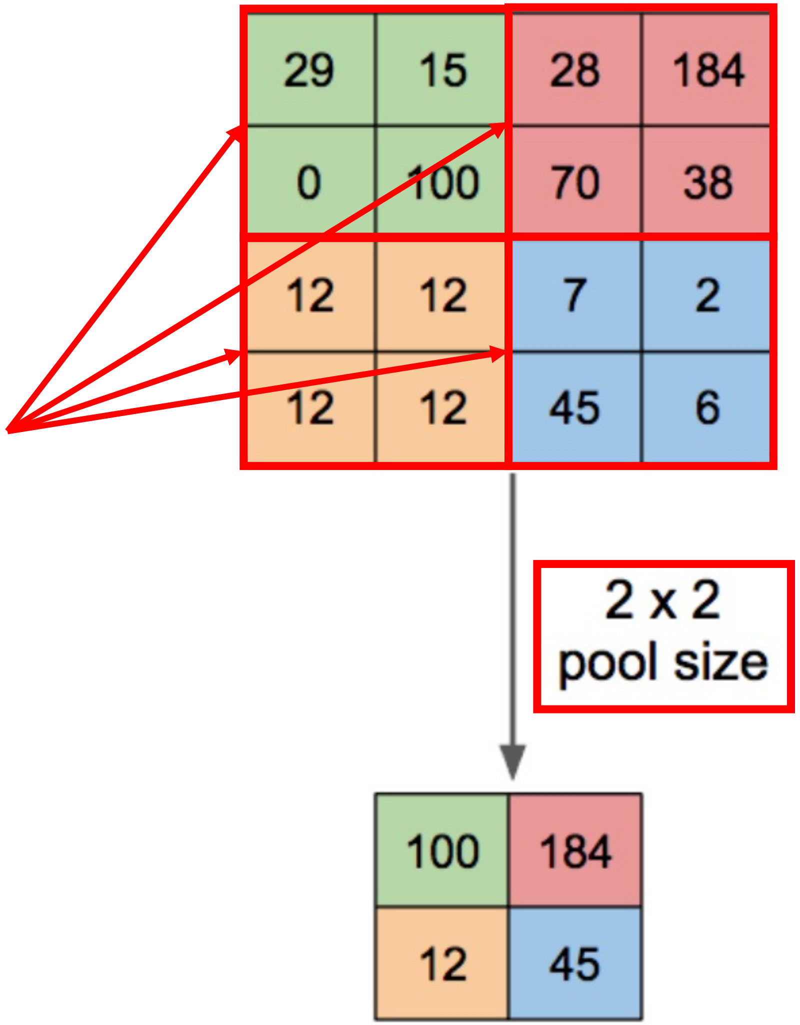


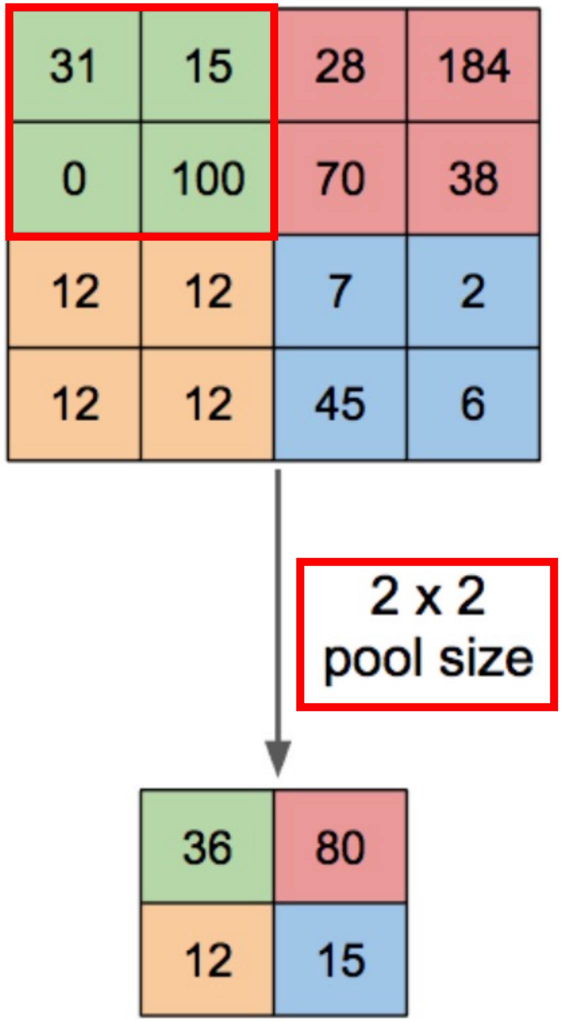


Subsampling operations

- Max pooling
- Average pooling

No overlapping
(stride=2*2)



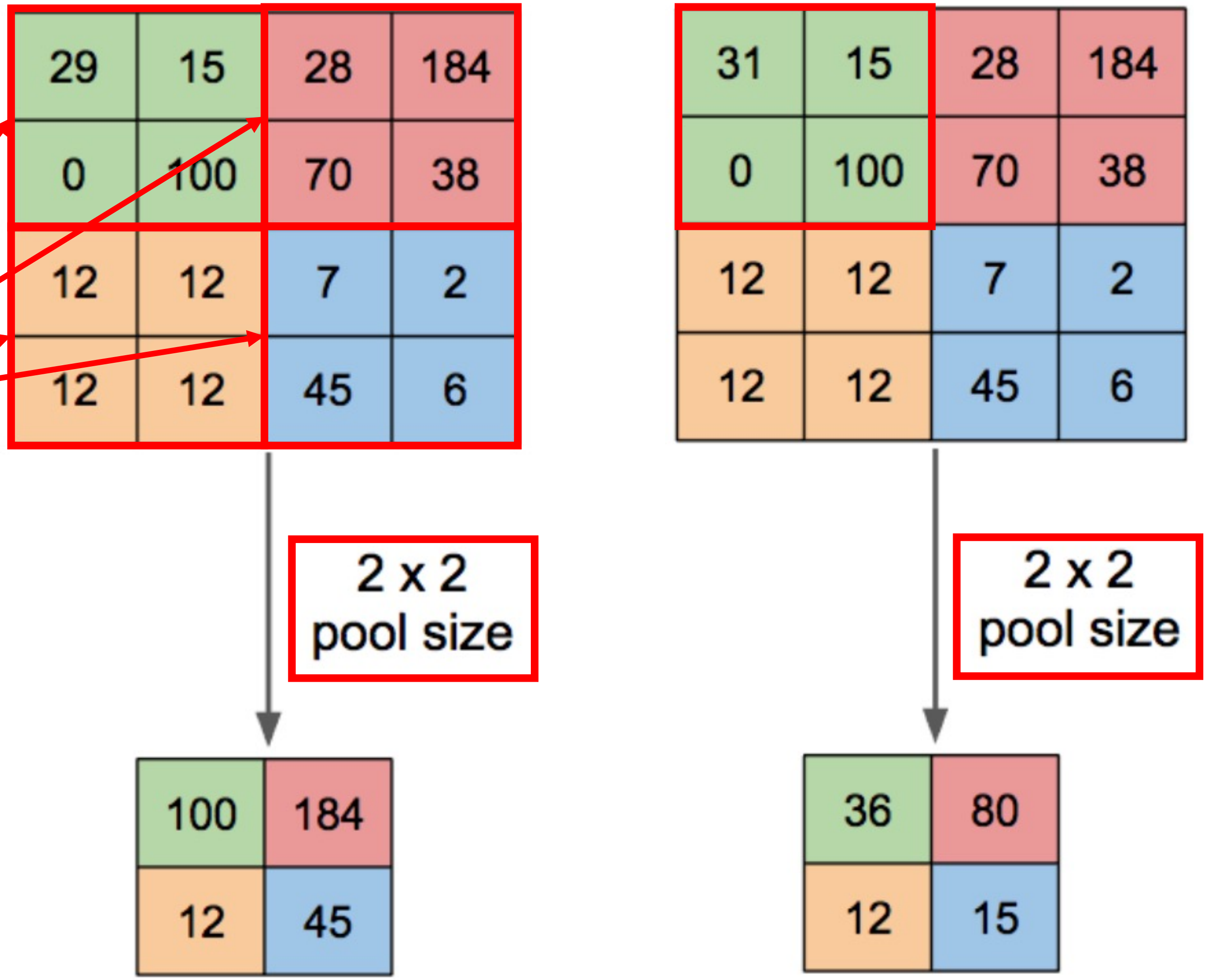


Subsampling operations

- Max pooling
- Average pooling

No overlapping
(stride=2*2)

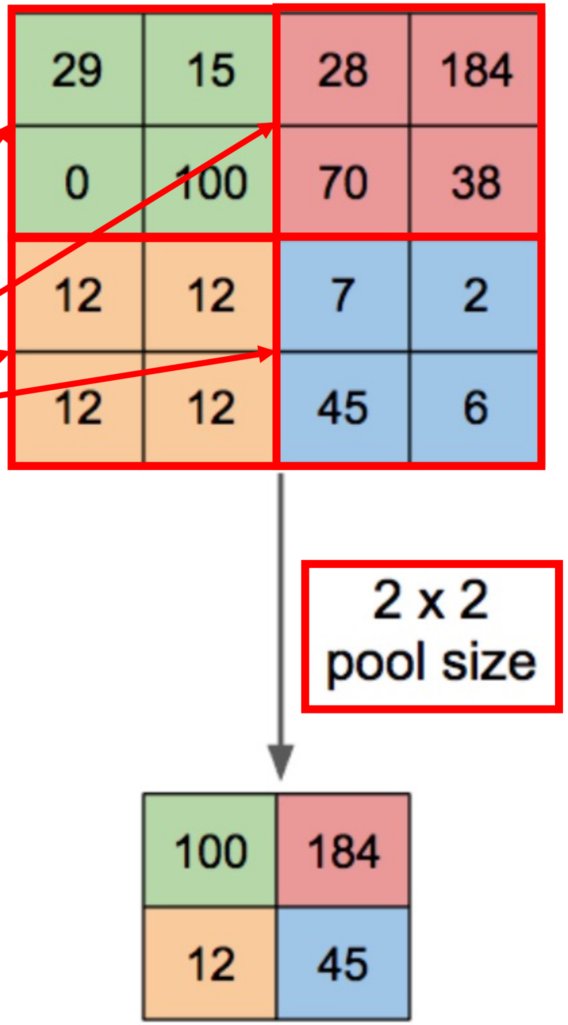
Row stride = 2
Column stride = 2

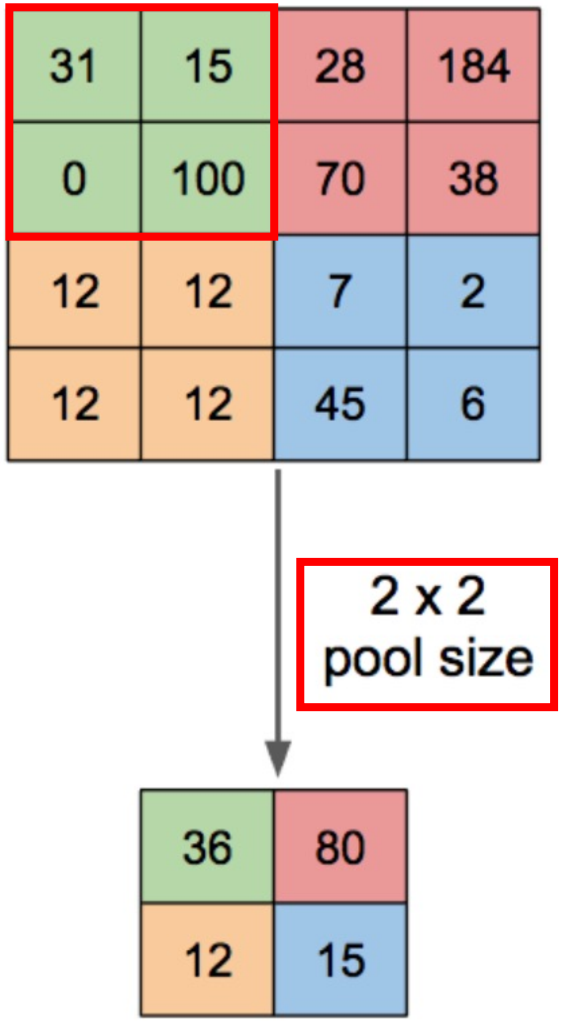


Subsampling operations

- Max pooling
- Average pooling

No overlapping
(stride=2*2)





Row stride = 2
Column stride = 2

Q: Why pooling?
Connection to subsampling?

Subsampling operations

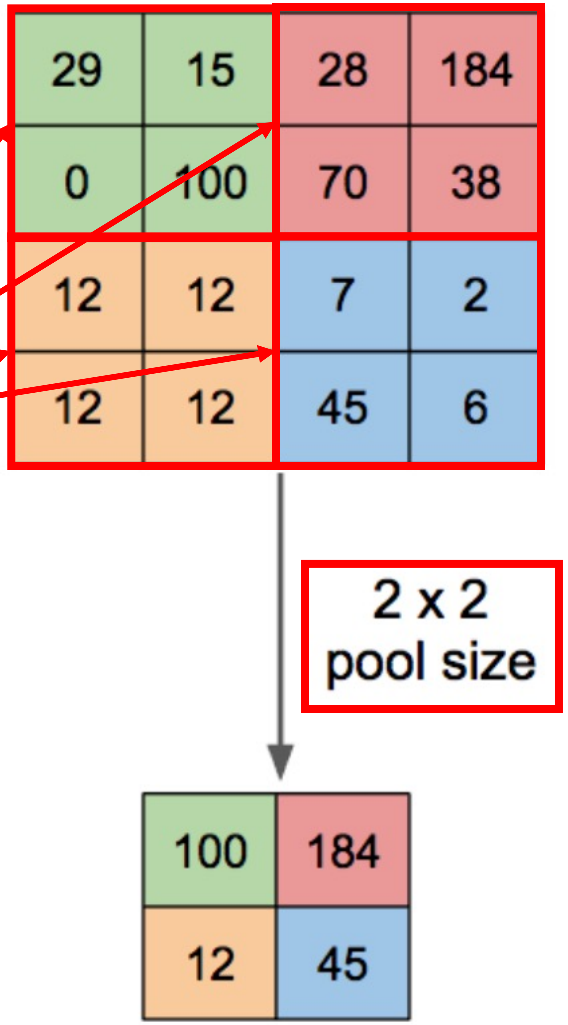
- Max pooling
- Average pooling

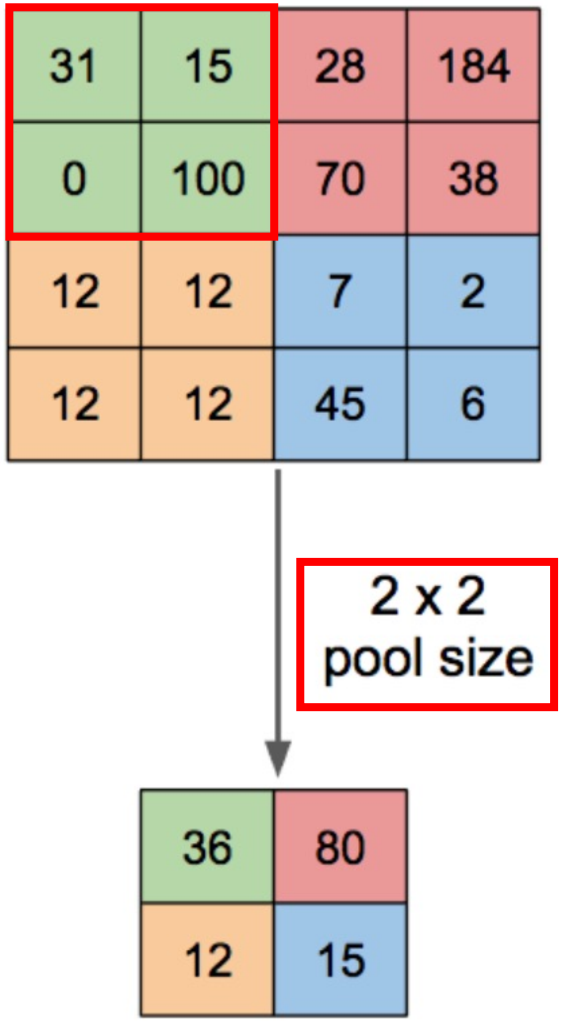
No overlapping
(stride=2*2)

Row stride = 2
Column stride = 2

Q: Why pooling?
Connection to subsampling?

4*4 → 2*2

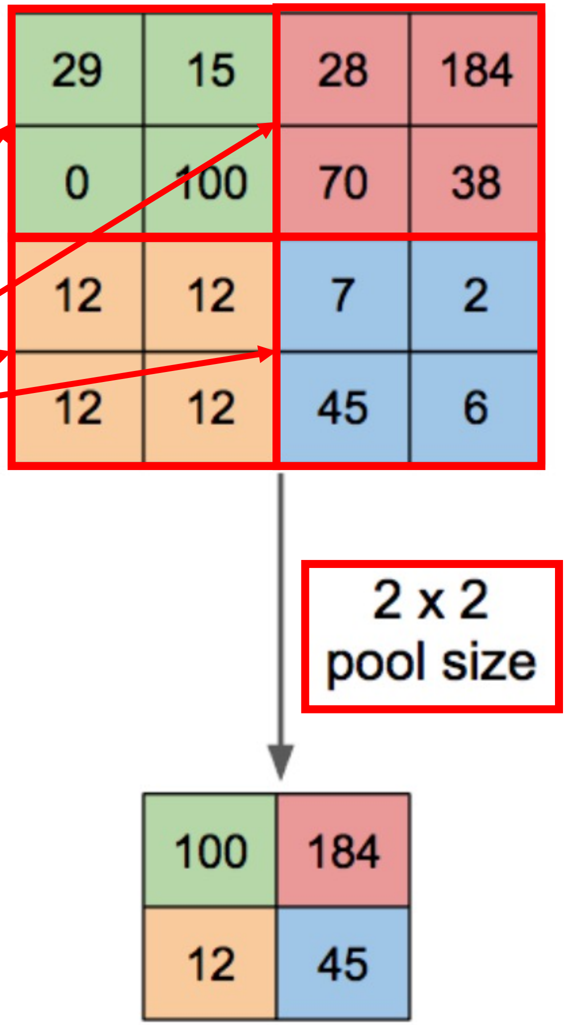


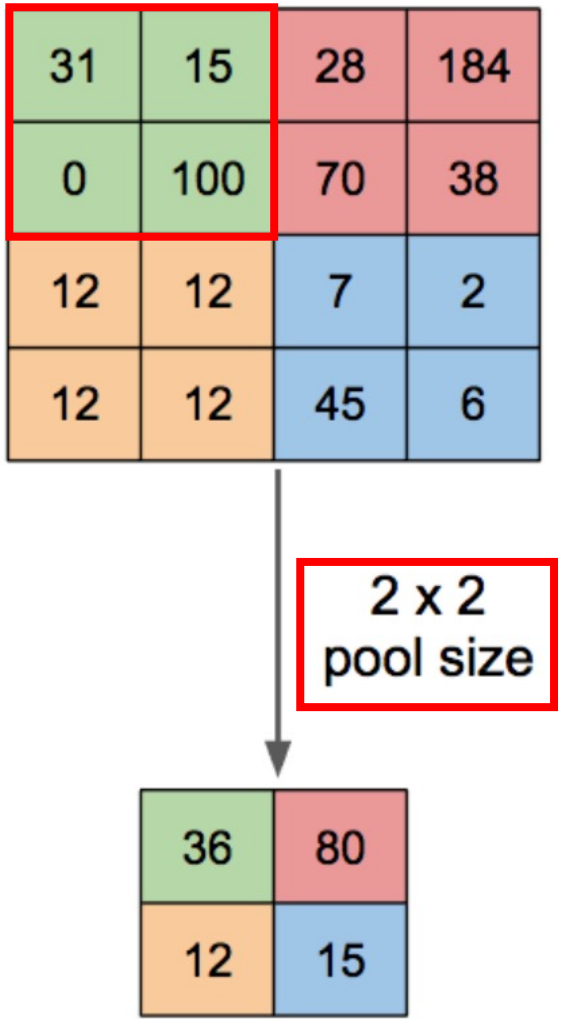


Subsampling operations

- Max pooling
- Average pooling

No overlapping
(stride=2*2)





Row stride = 2
Column stride = 2

Q: Why pooling?
Connection to subsampling?

4*4 → 2*2

Dimension reduced

Subsampling operations

- Max pooling
- Average pooling

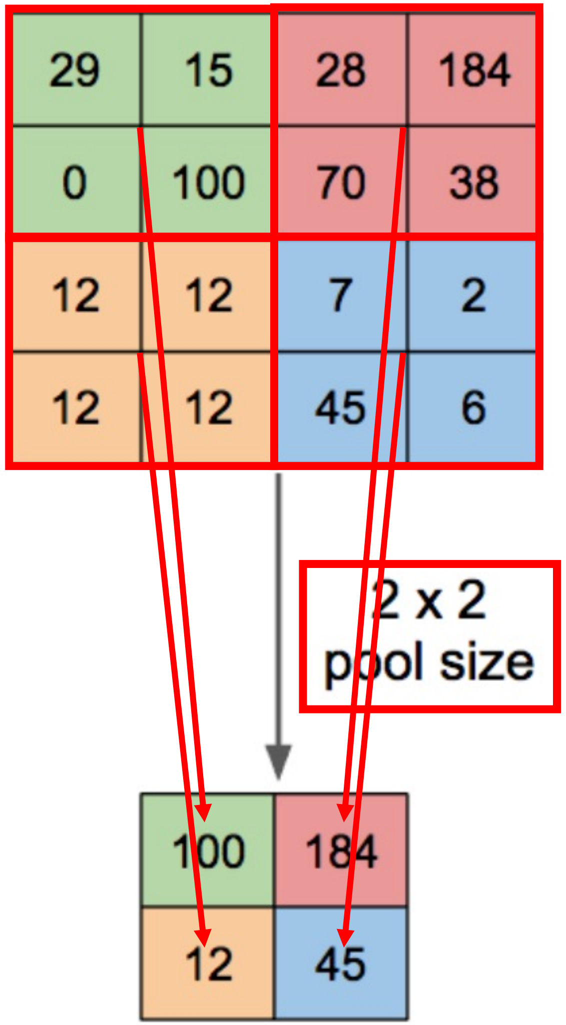
No overlapping
(stride=2*2)

Row stride = 2
Column stride = 2

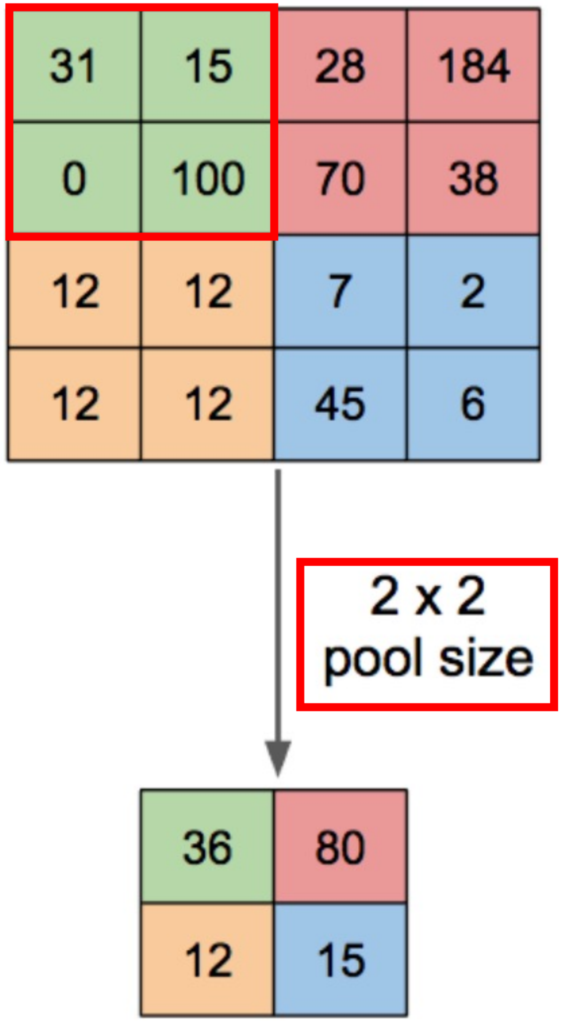
Q: Why pooling?
Connection to subsampling?

4*4 → 2*2

Dimension reduced



Use one to represent all



LeNet-5 in 1999

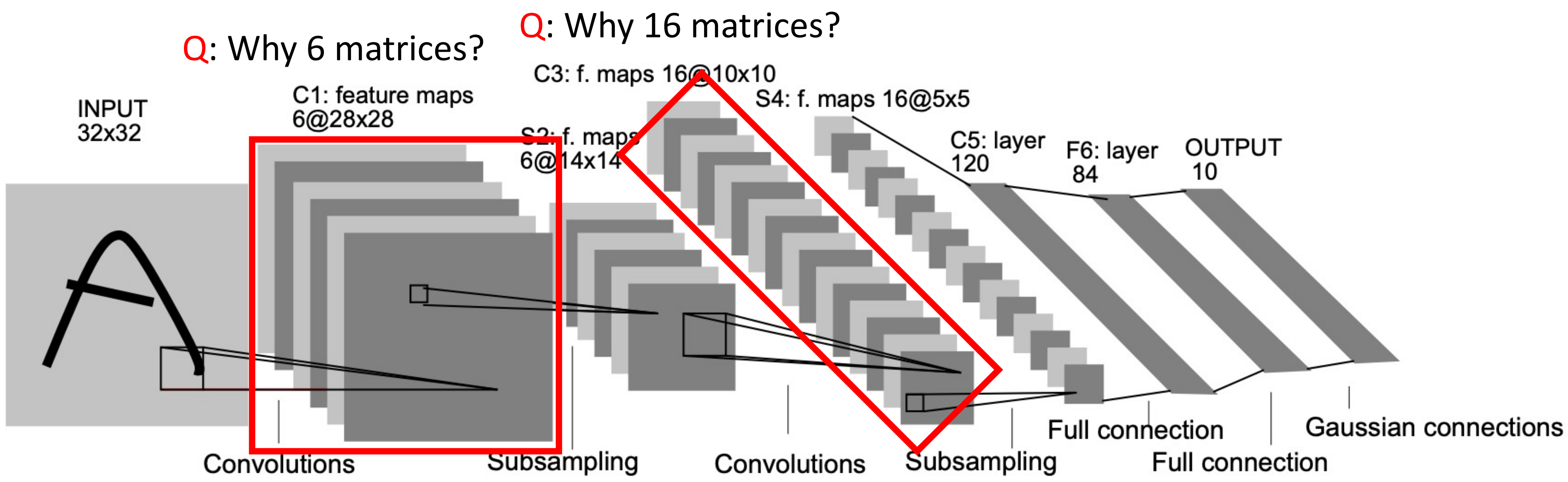
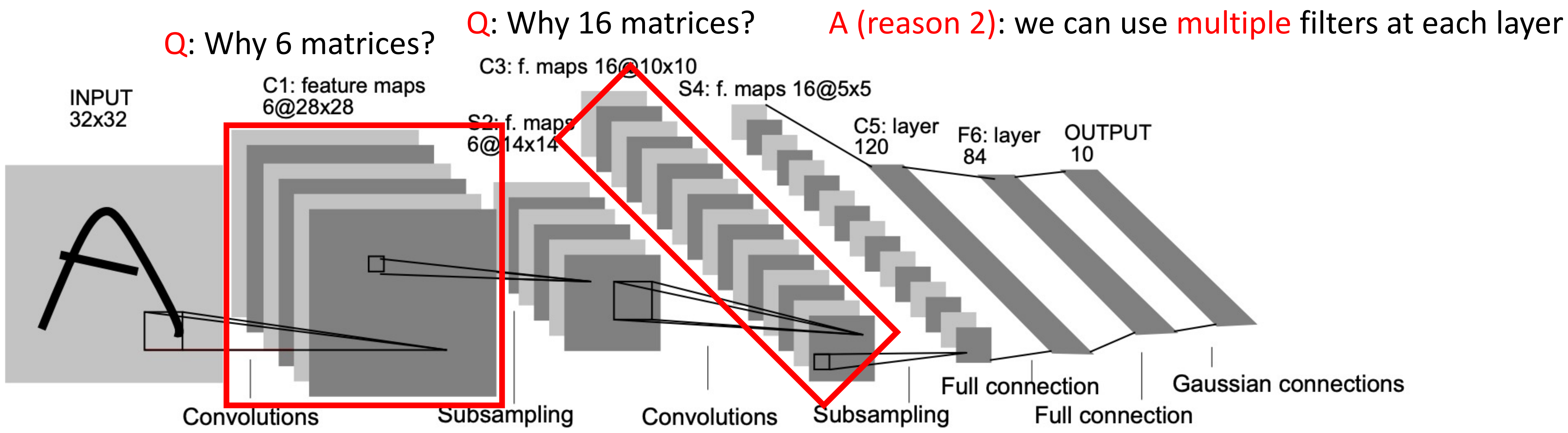


Fig. 1. Architecture of LeNet-5, a Convolutional Neural Network, here for digits recognition. Each plane is a feature map, i.e. a set of units whose weights are constrained to be identical.

LeCun, Yann, Patrick Haffner, Léon Bottou, and Yoshua Bengio. "Object recognition with gradient-based learning." In *Shape, contour and grouping in computer vision*, pp. 319-345. Springer, Berlin, Heidelberg, 1999.

LeNet-5 in 1999



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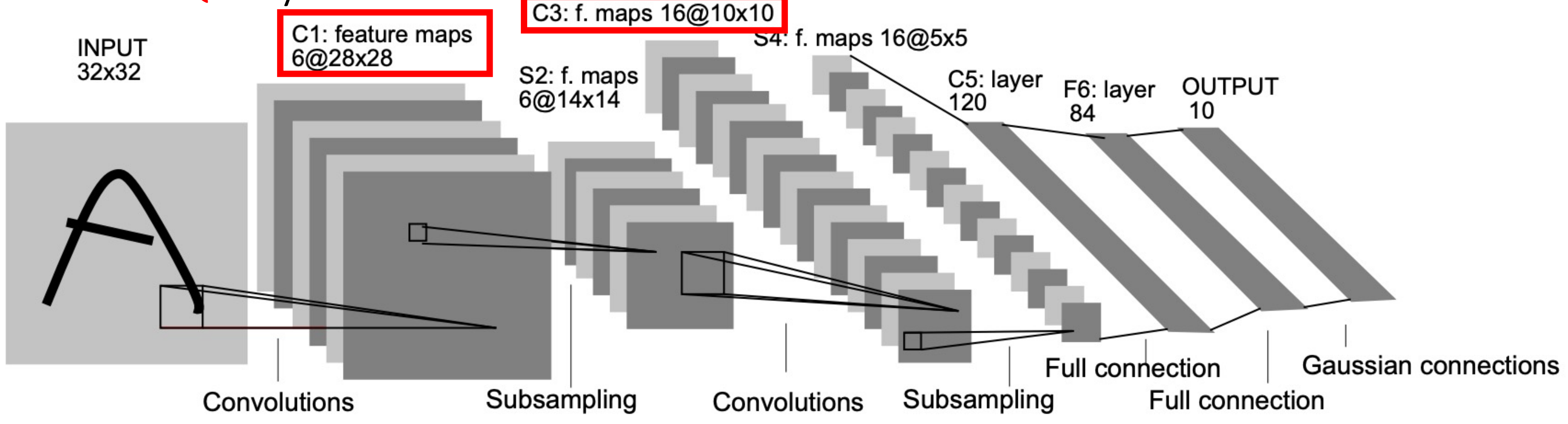
LeNet-5 in 1999

Q: Why 6 matrices?

Q: Why 16 matrices?

A (reason 2): we can use multiple filters at each layer



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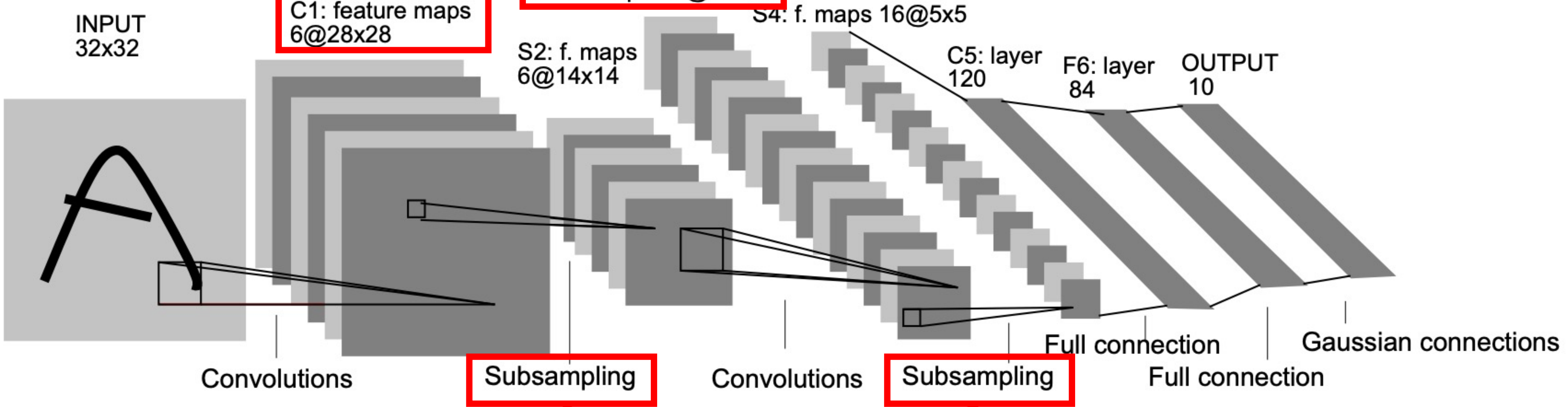

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Subsampling layer: max/average pooling

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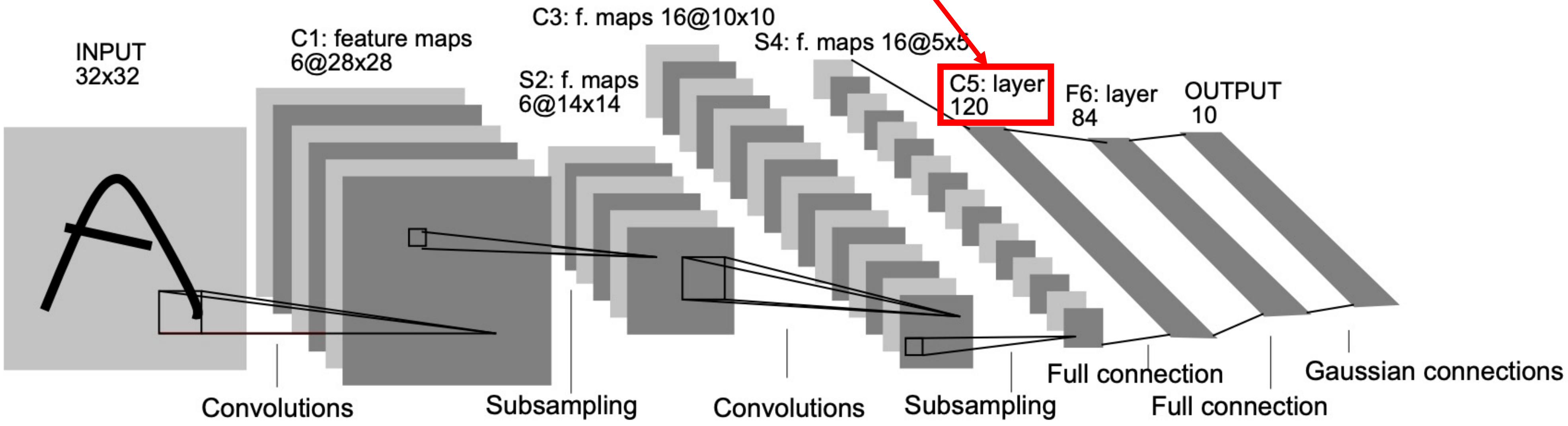
LeNet-5 in 1999

One more question:
How C5 comes from?



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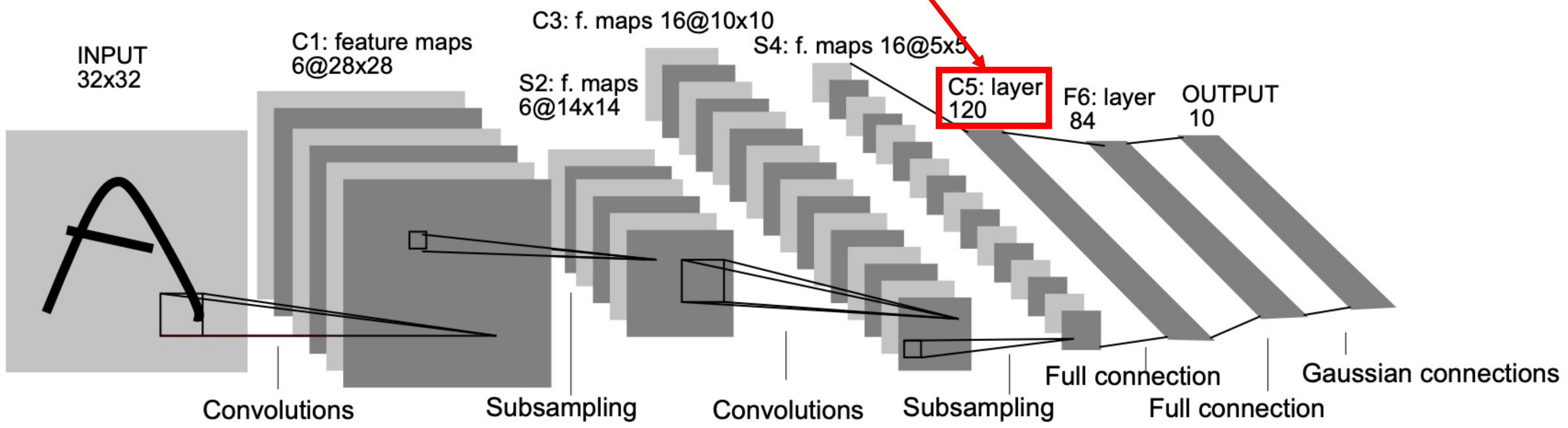
LeNet-5 in 1999

One more question:
How C5 comes from? Matrices → a vector?



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External reading materials

- LeCun, Yann, Patrick Haffner, Léon Bottou, and Yoshua Bengio. "Object recognition with gradient-based learning." In *Shape, contour and grouping in computer vision*, pp. 319-345. Springer, Berlin, Heidelberg, 1999.
  - Online at http://yann.lecun.com/exdb/publis/pdf/lecun-99.pdf
  - Section 2.2
  - Understand architecture of LeNet-5
- LeCun, Yann, Léon Bottou, Yoshua Bengio, and Patrick Haffner. "Gradient-based learning applied to document recognition." *Proceedings of the IEEE* 86, no. 11 (1998): 2278-2324.
  - Online at http://vision.stanford.edu/cs598_spring07/papers/Lecun98.pdf
  - Section II.B