

# Neural Architecture Search

Neural Networks Design And Application

# A framework for NAS

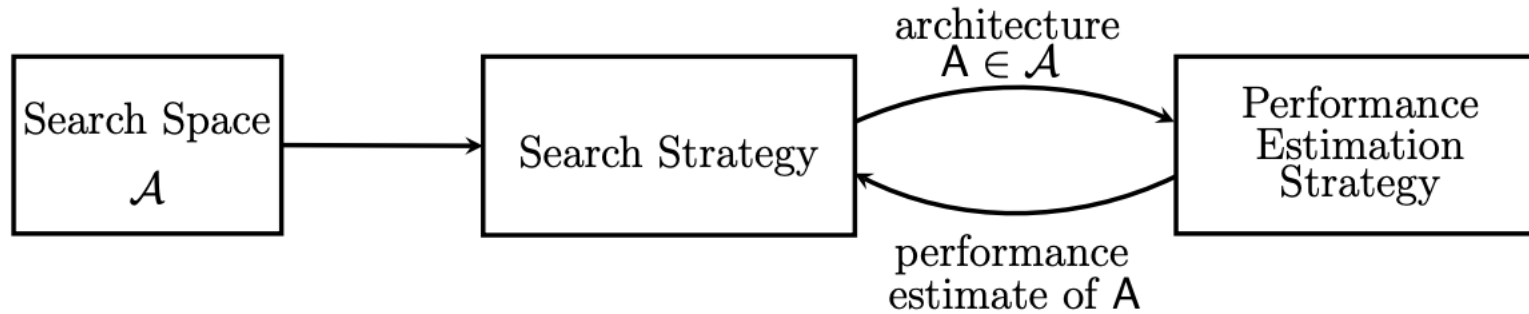
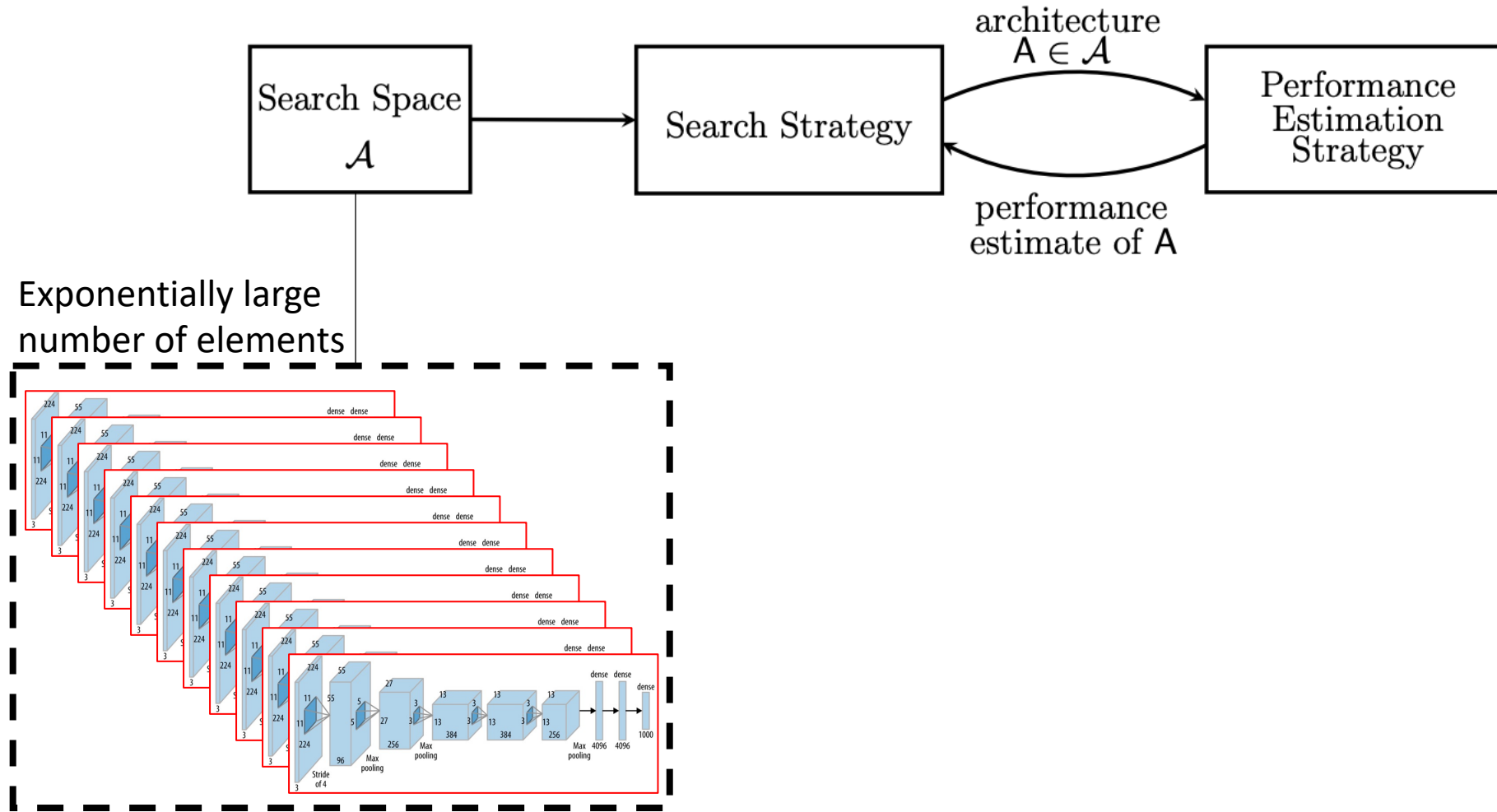


Image credit:

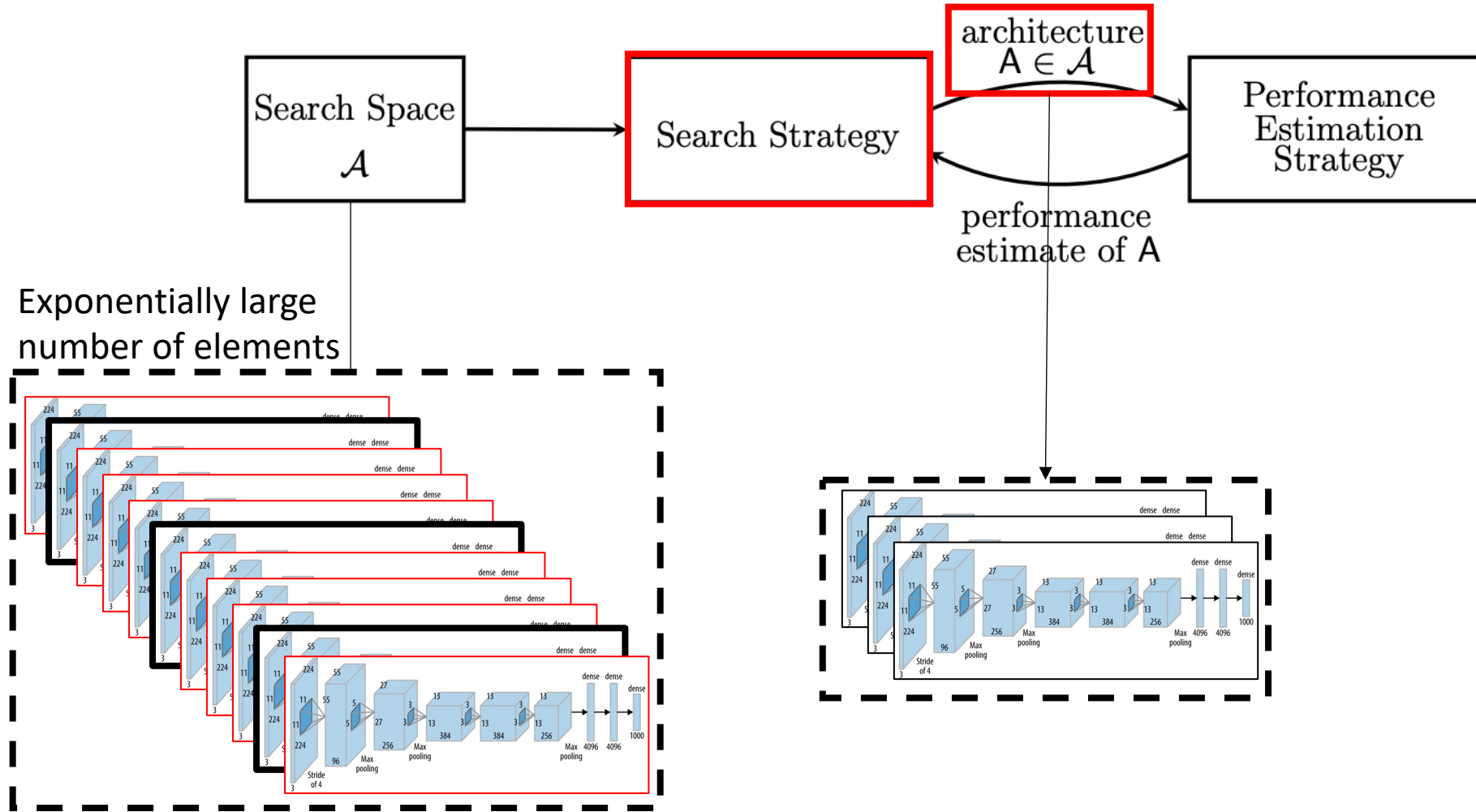
Elsken, Thomas, Jan Hendrik Metzen, and Frank Hutter. "Neural architecture search: A survey." *J. Mach. Learn. Res.* 20, no. 55 (2019): 1-21.

<https://arxiv.org/pdf/1808.05377.pdf>

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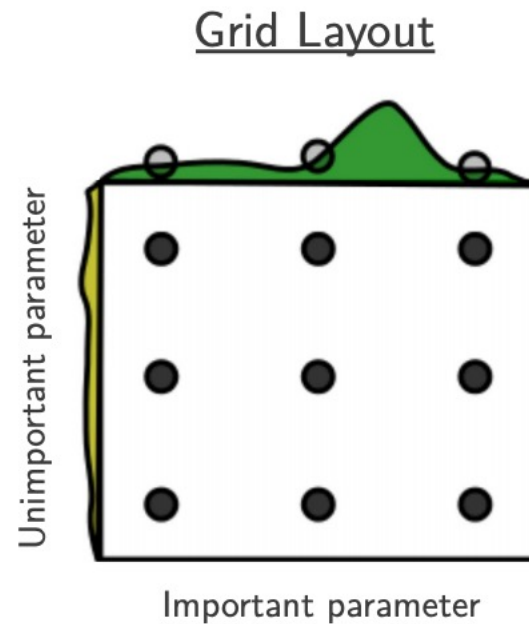


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- Weight decay (L2 regularization): {0.0001, 0.001, 0.01, 0.1, 1, 10}

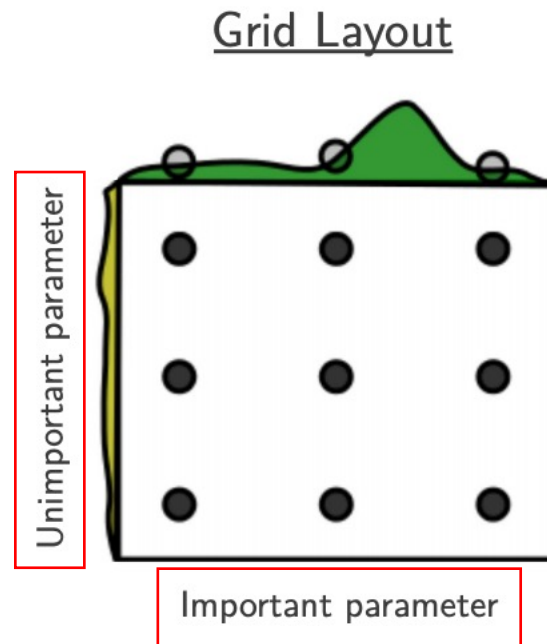
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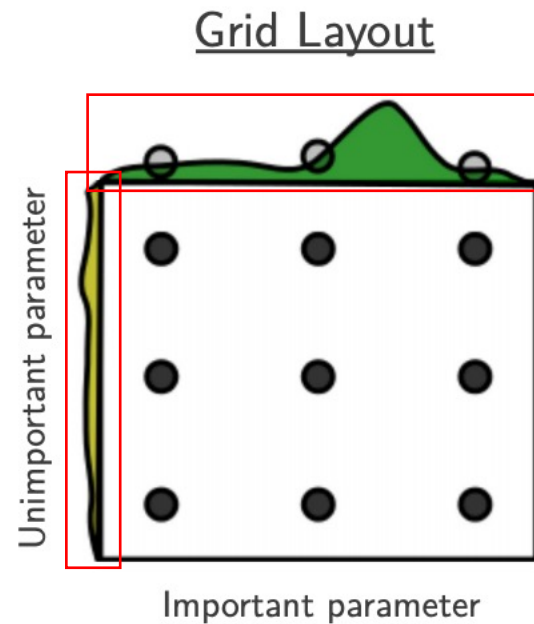
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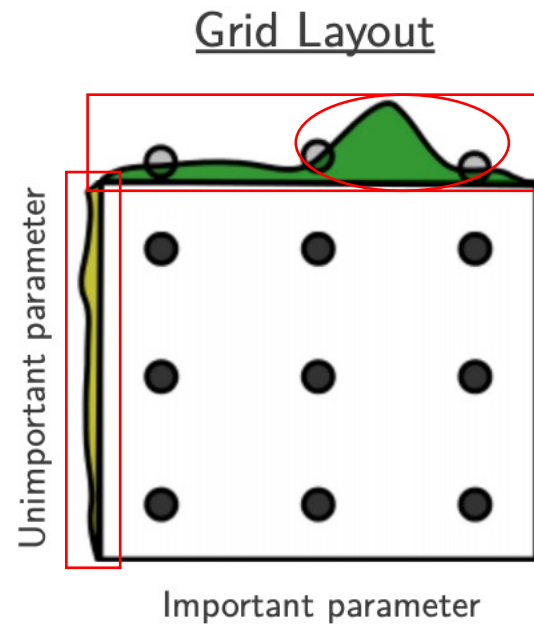
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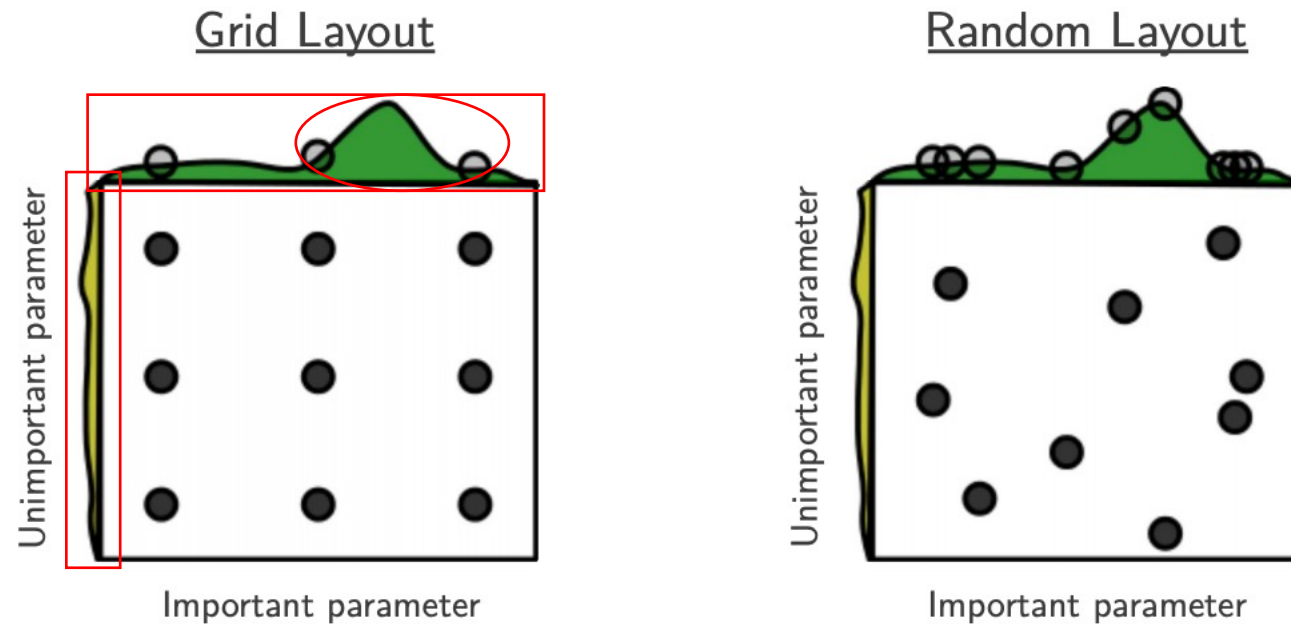
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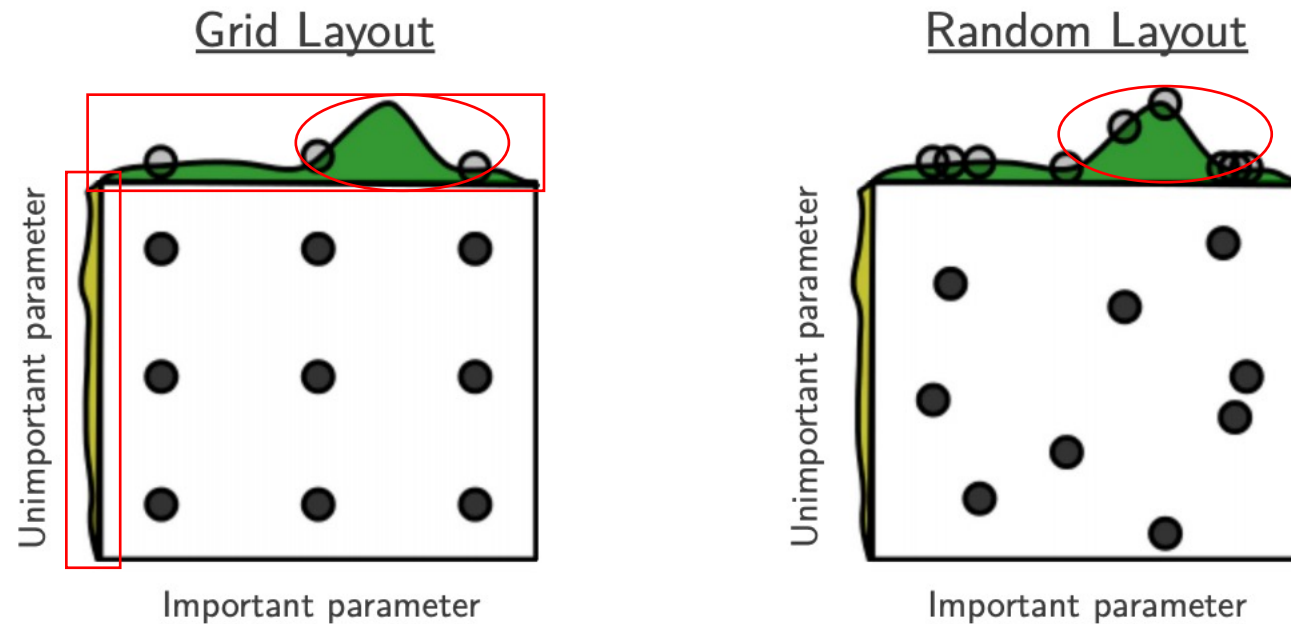
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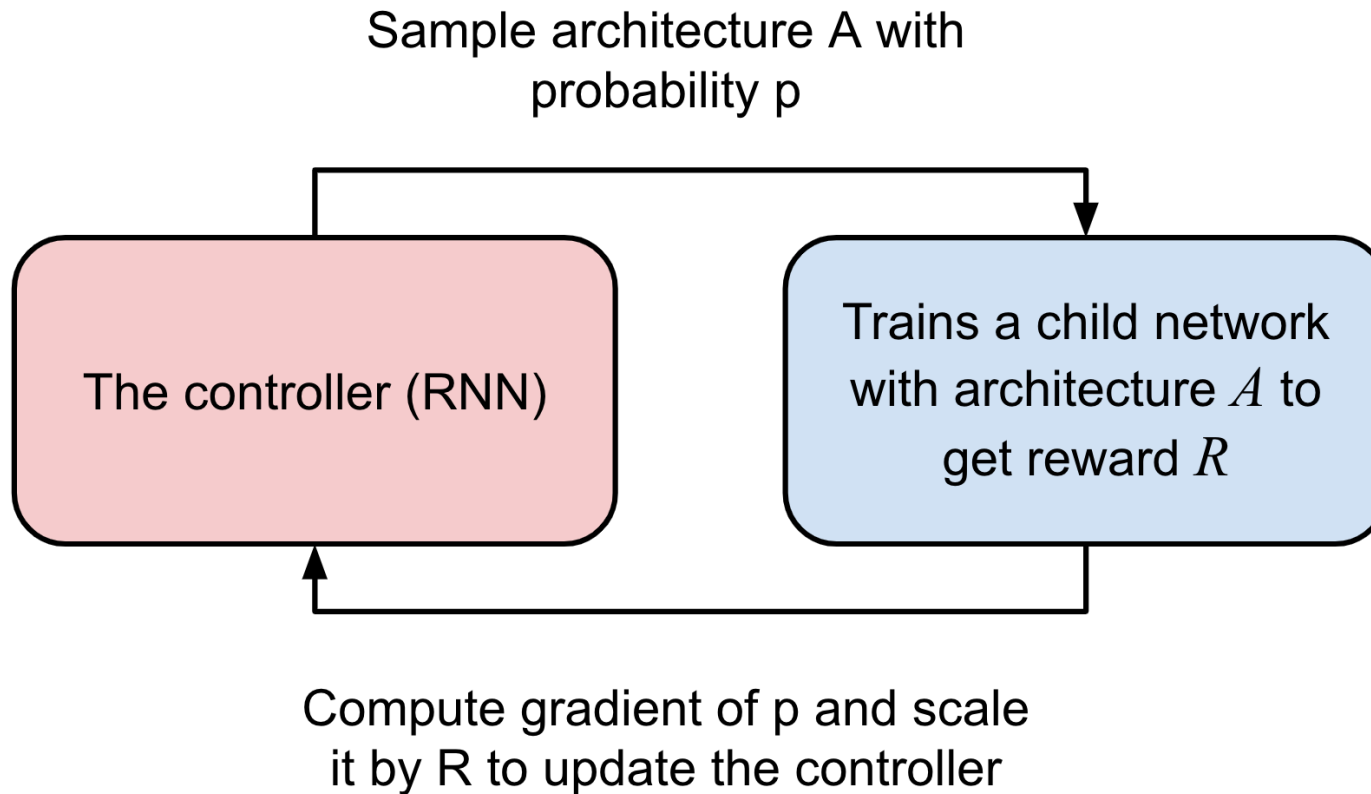


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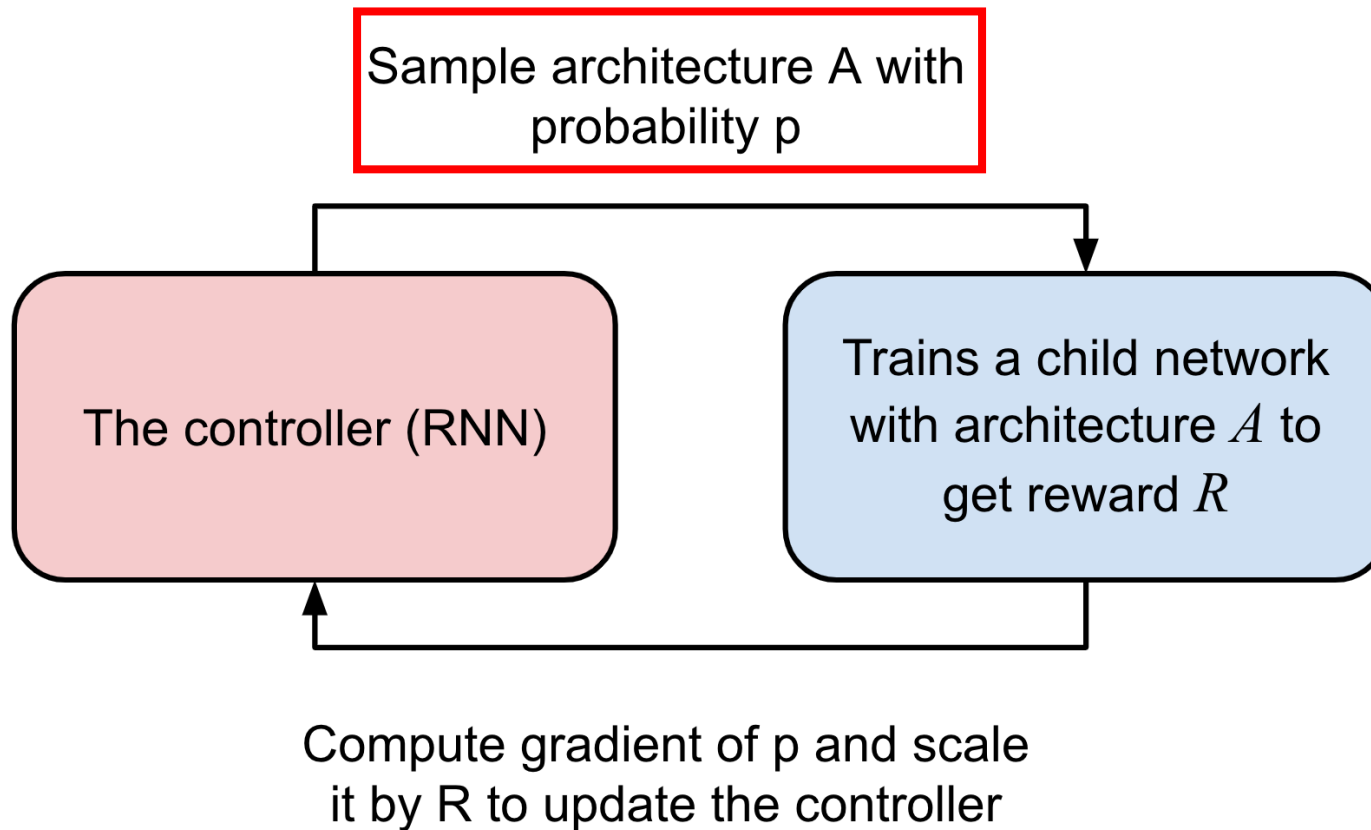
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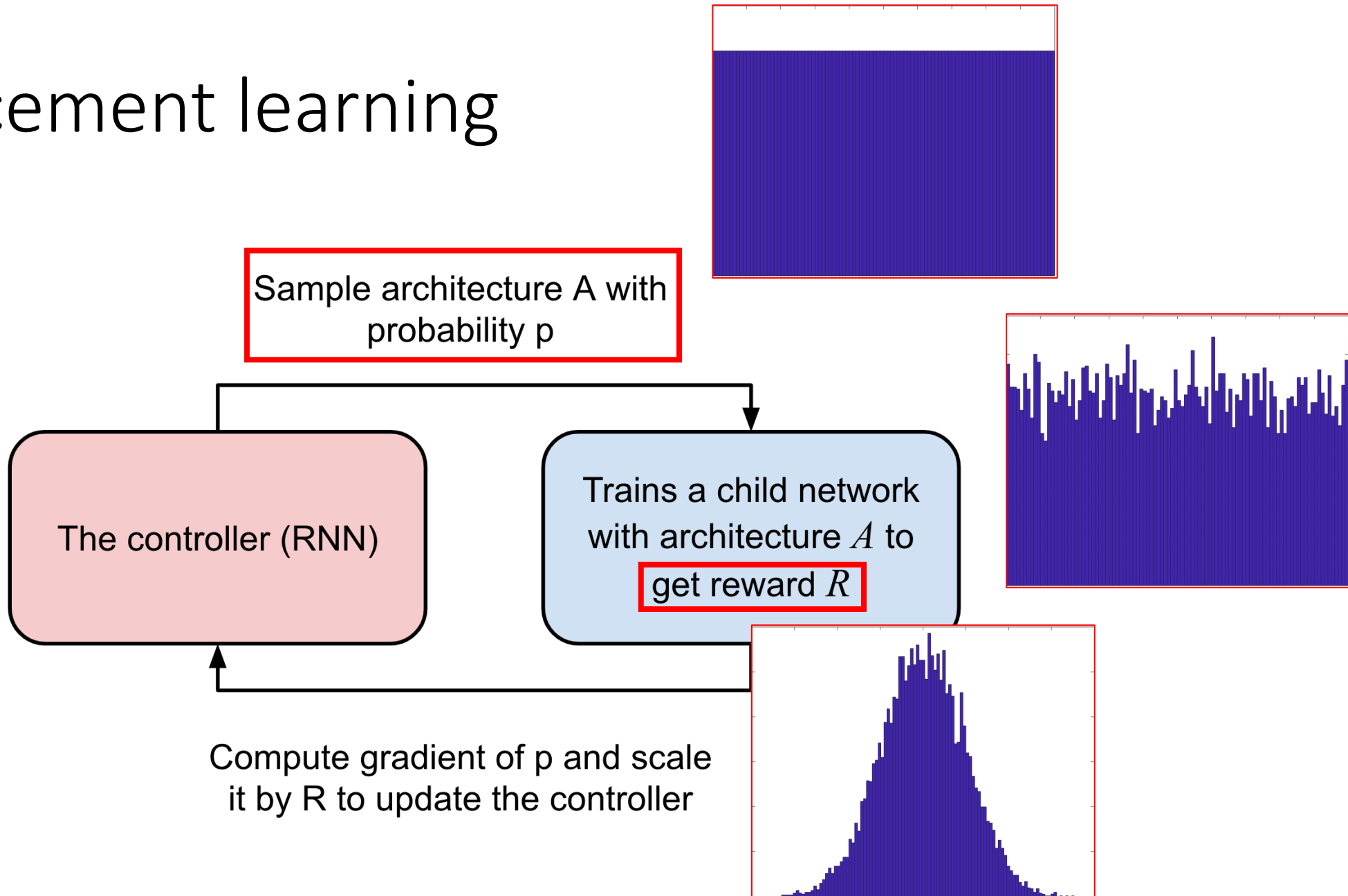
# Reinforcement learning



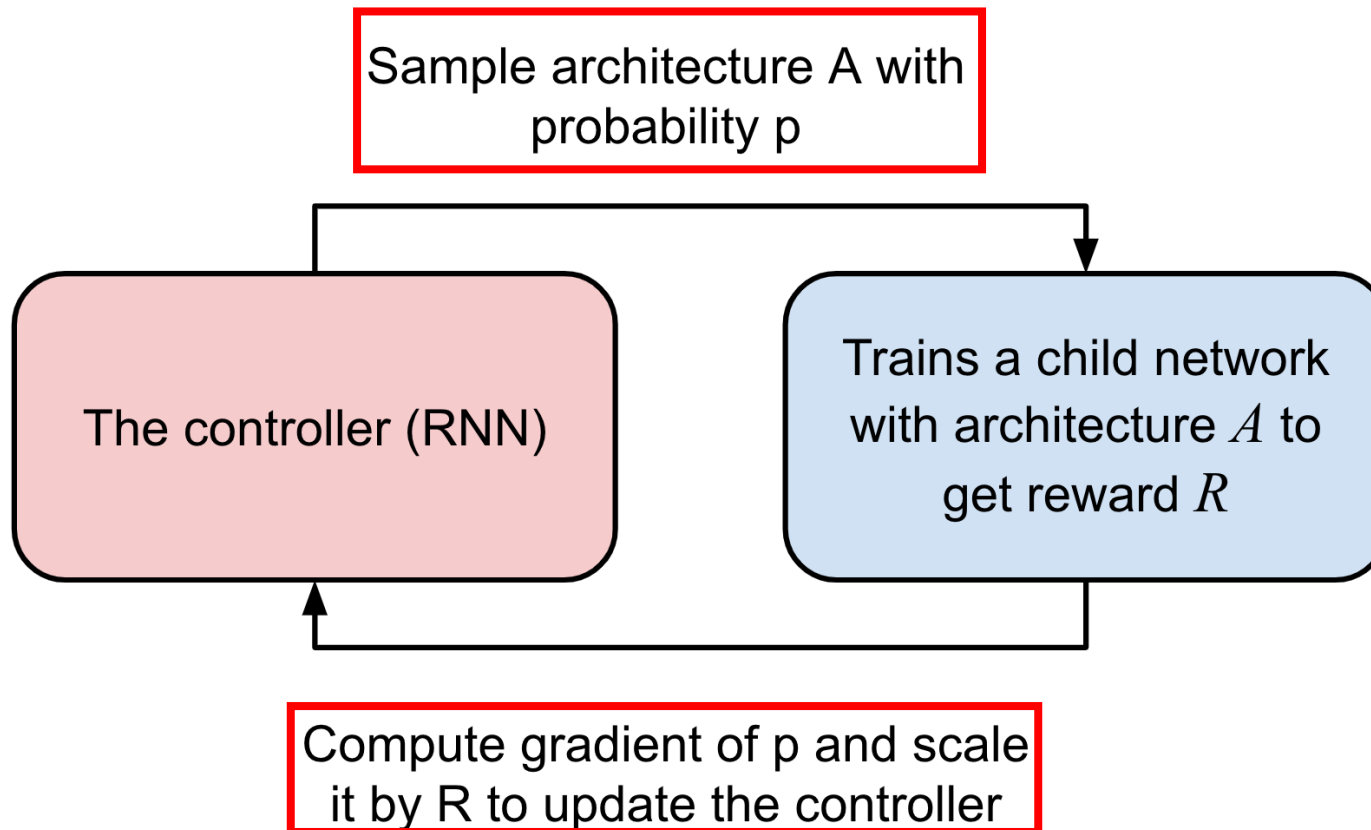
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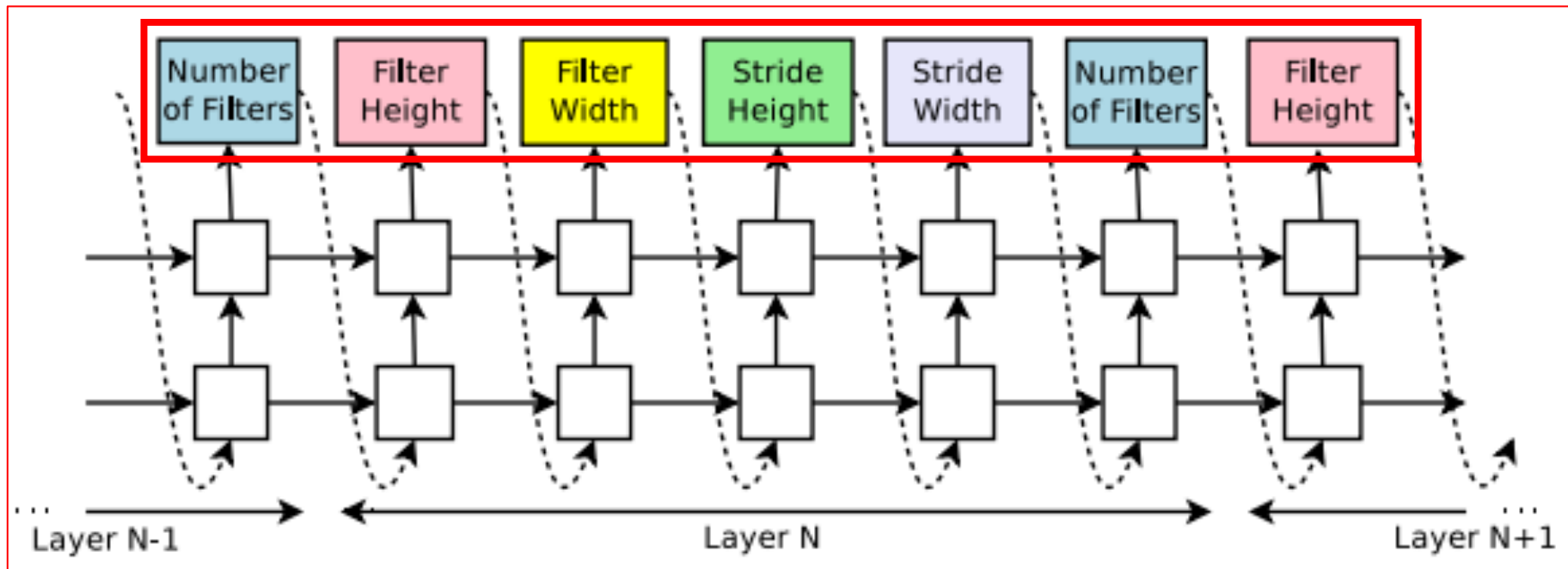


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# Defining search space



A RNN network

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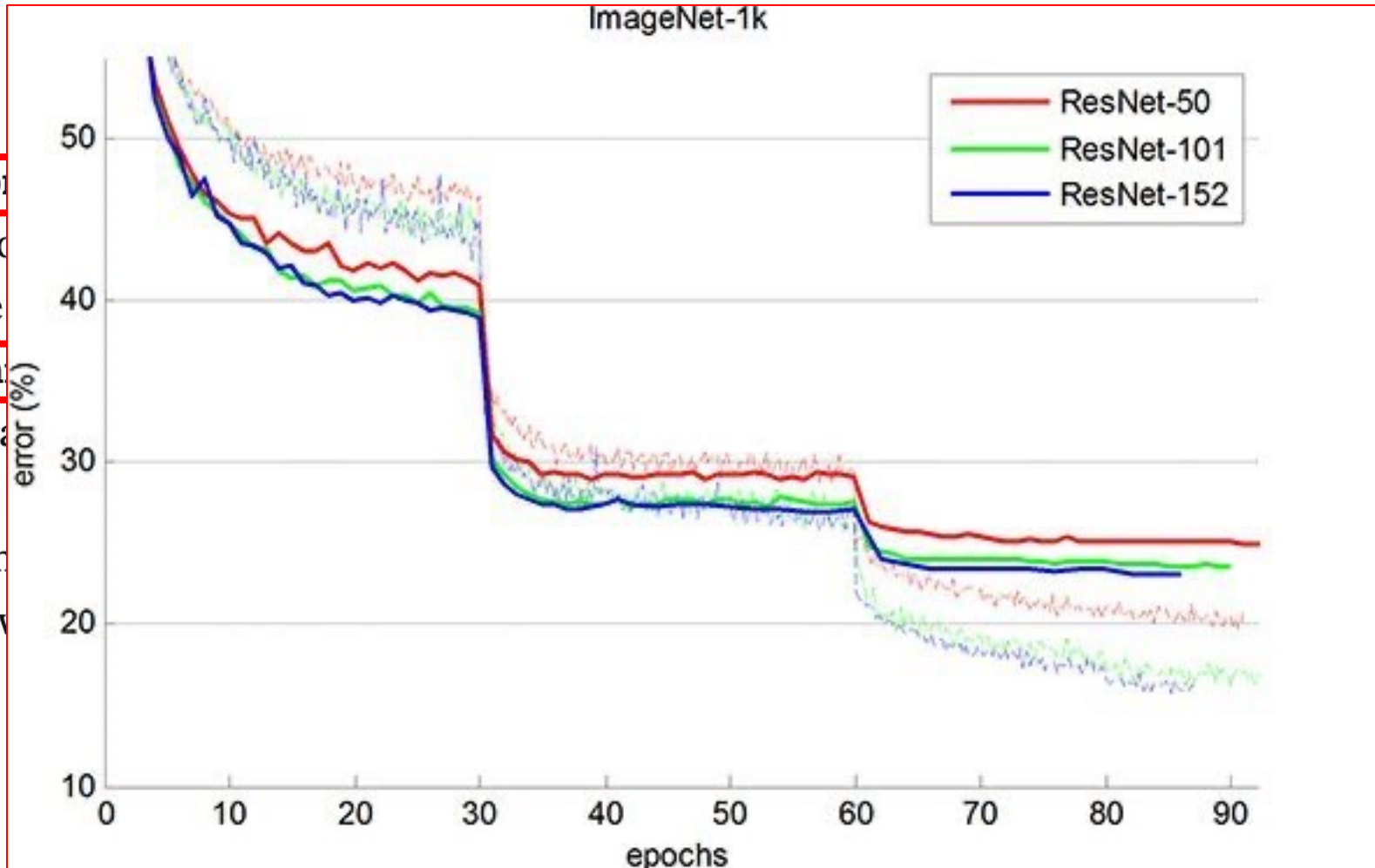
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# Reinforcement learning

- **Action**: the action taken by the model
- **Reward**: the reward received by the model
- **Loss**: the loss function used to train the model



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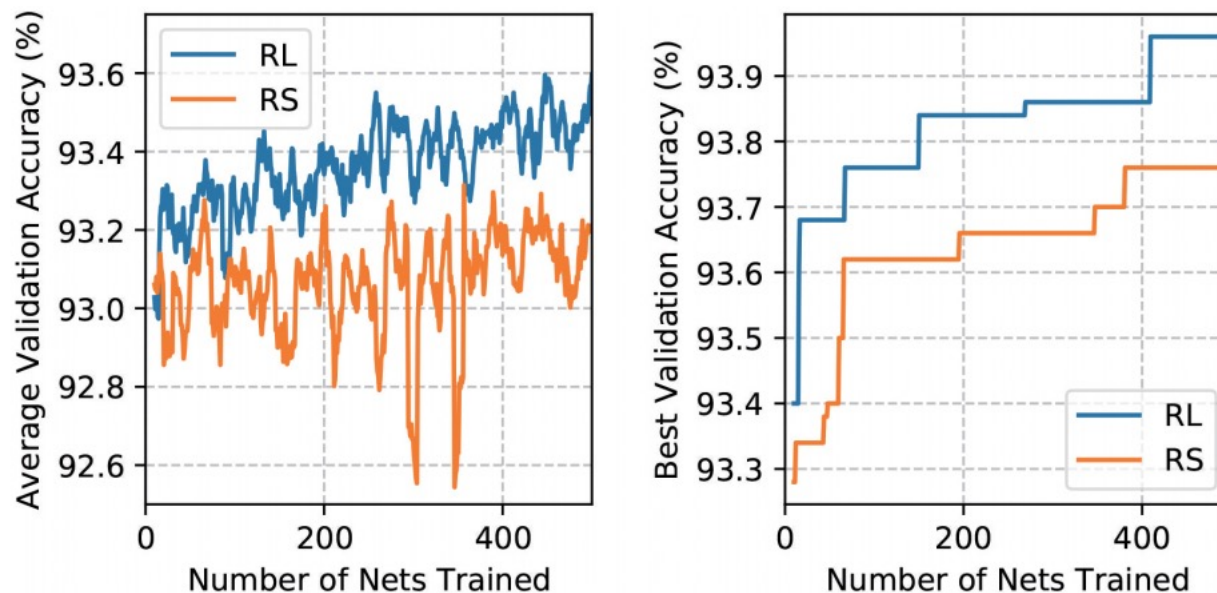
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$$\mathbb{E}[X] = \sum_{i=1}^k x_i p_i = x_1 p_1 + x_2 p_2 + \cdots + x_k p_k.$$

$$L(\theta | x) = \prod_{i=1}^n p_X(x_i | \theta)$$

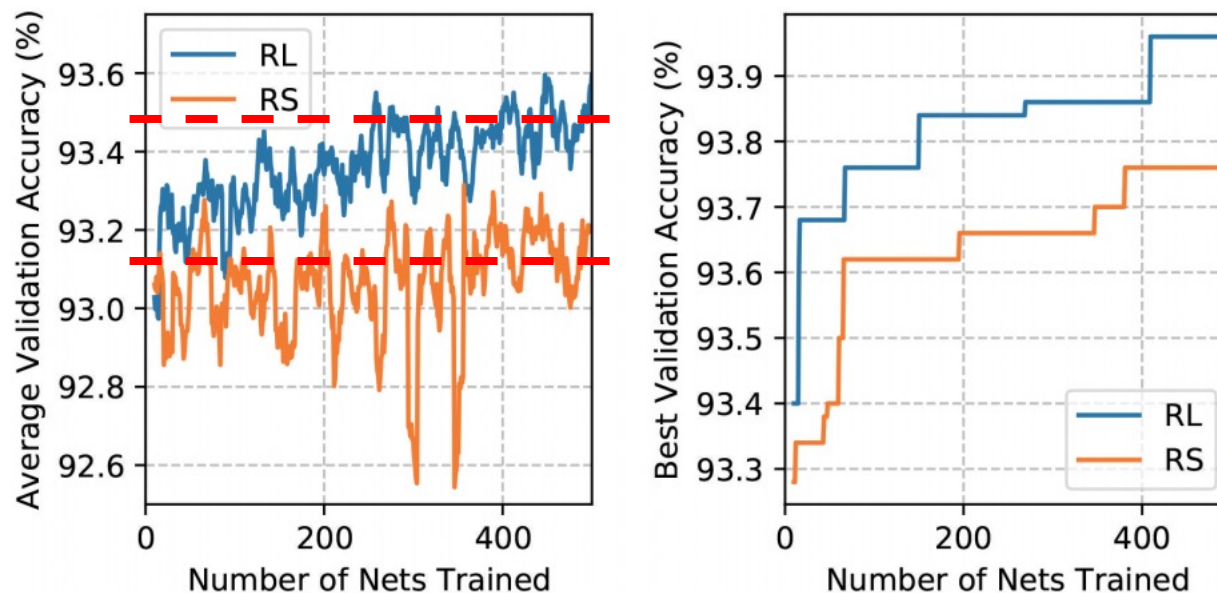
# RS vs RL



Cai, Han, Jiacheng Yang, Weinan Zhang, Song Han, and Yong Yu. "Path-level network transformation for efficient architecture search." In *International Conference on Machine Learning*, pp. 678-687. PMLR, 2018.

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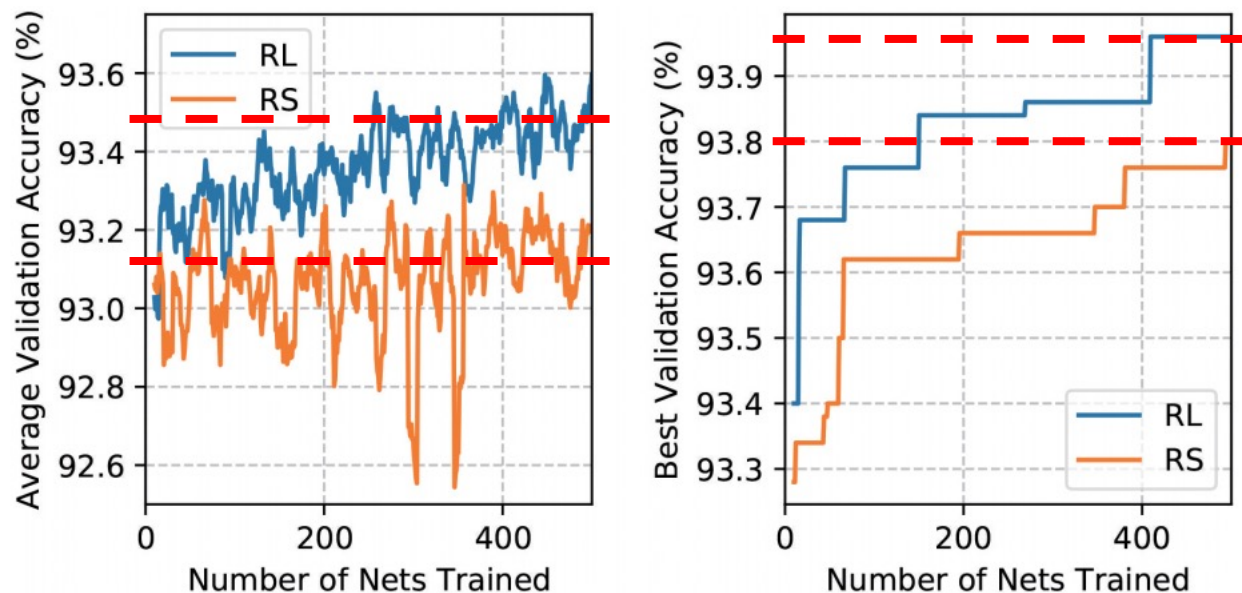
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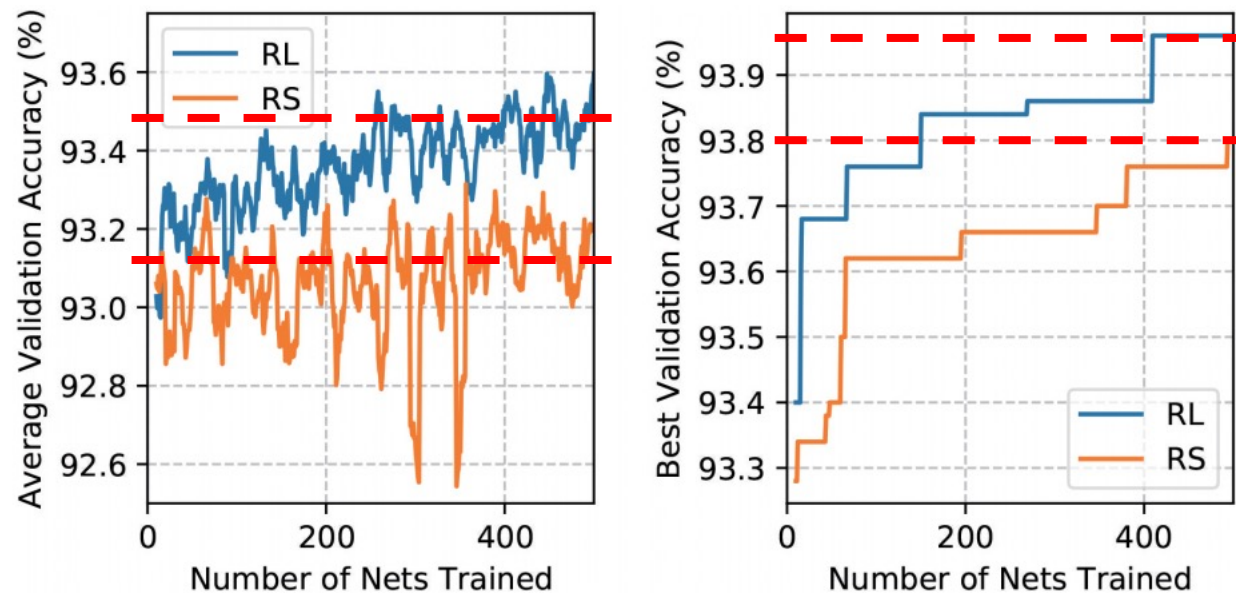


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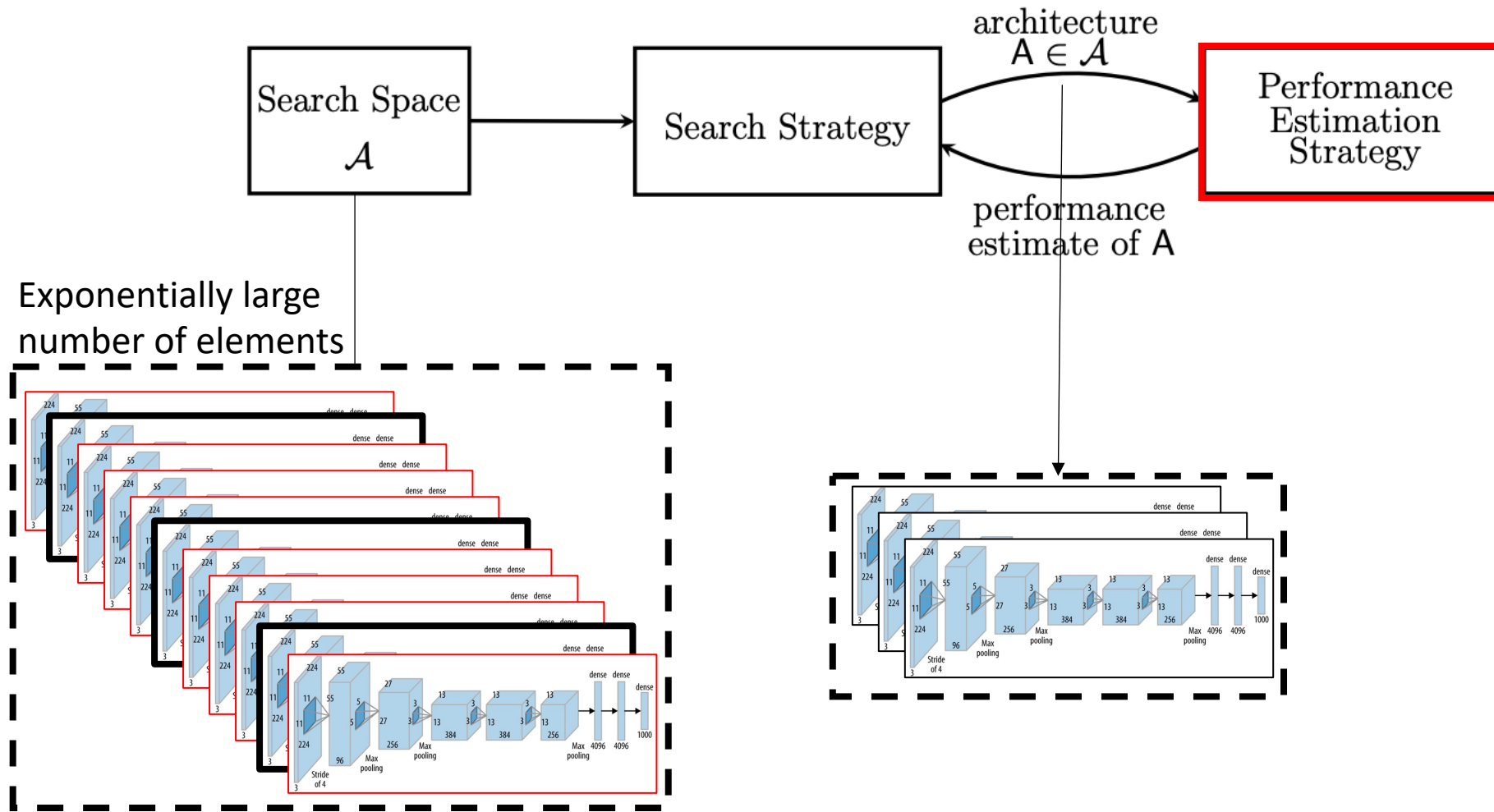
Q: what conclusion can we make?



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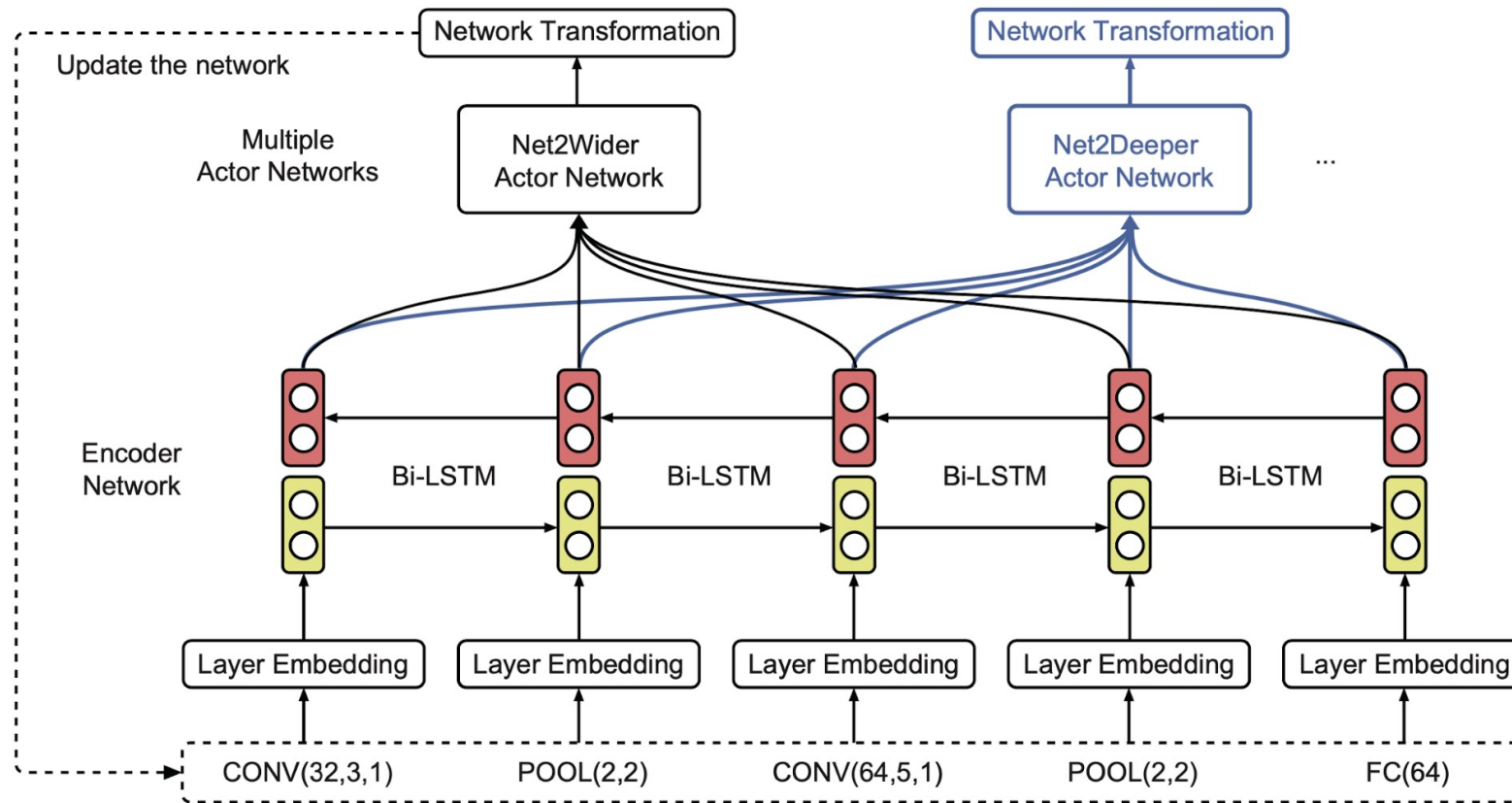
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    - E.g., once a cell structure is learned, the final architecture is determined by selecting a number of cell repeats <https://arxiv.org/pdf/1707.07012.pdf>

putational budget. After having learned the convolutional cells, several hyper-parameters may be explored to build a final network for a given task: (1) the number of cell repeats  $N$  and (2) the number of filters in the initial convolutional cell. After selecting the number of initial filters, we use a

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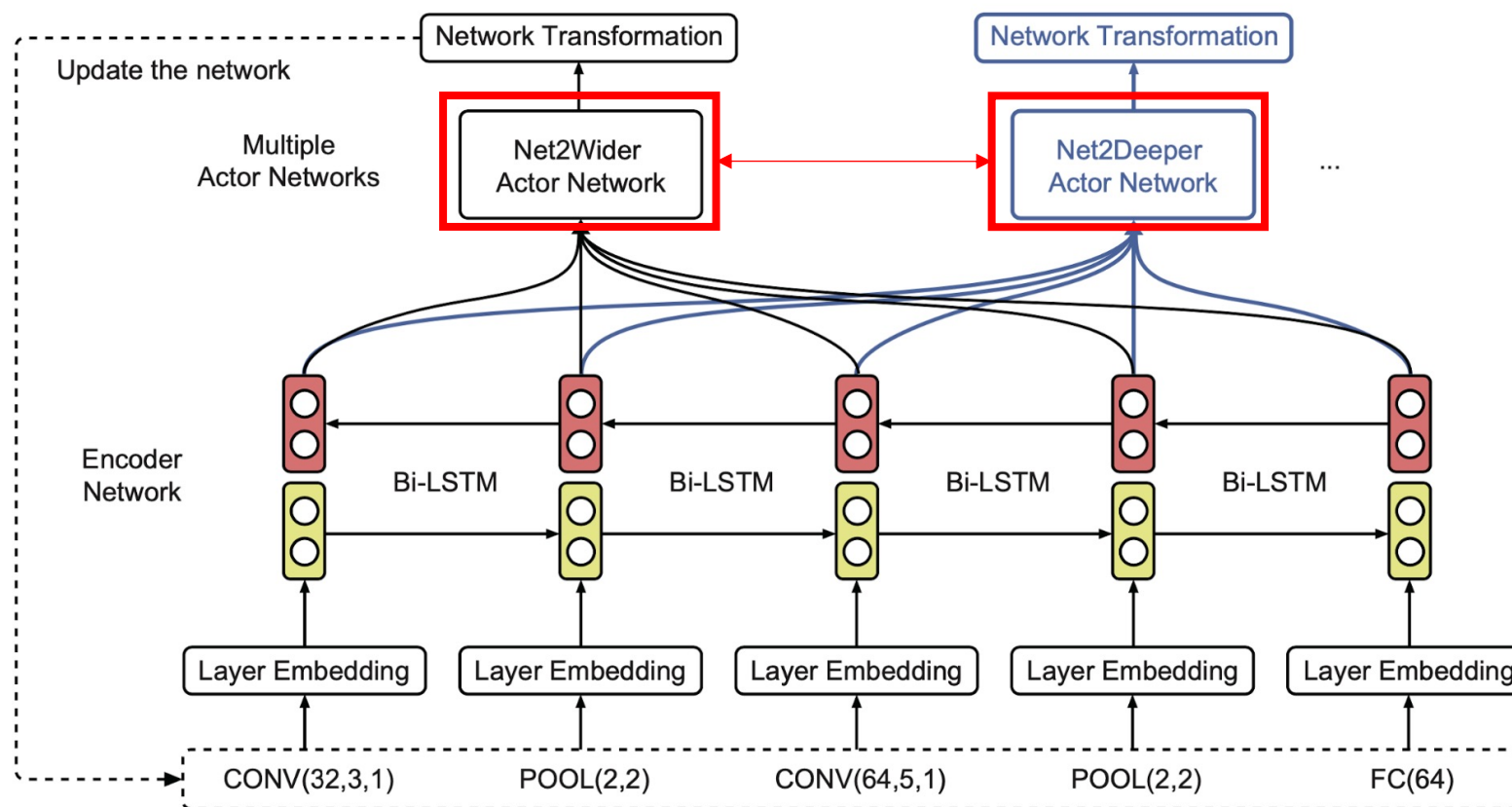
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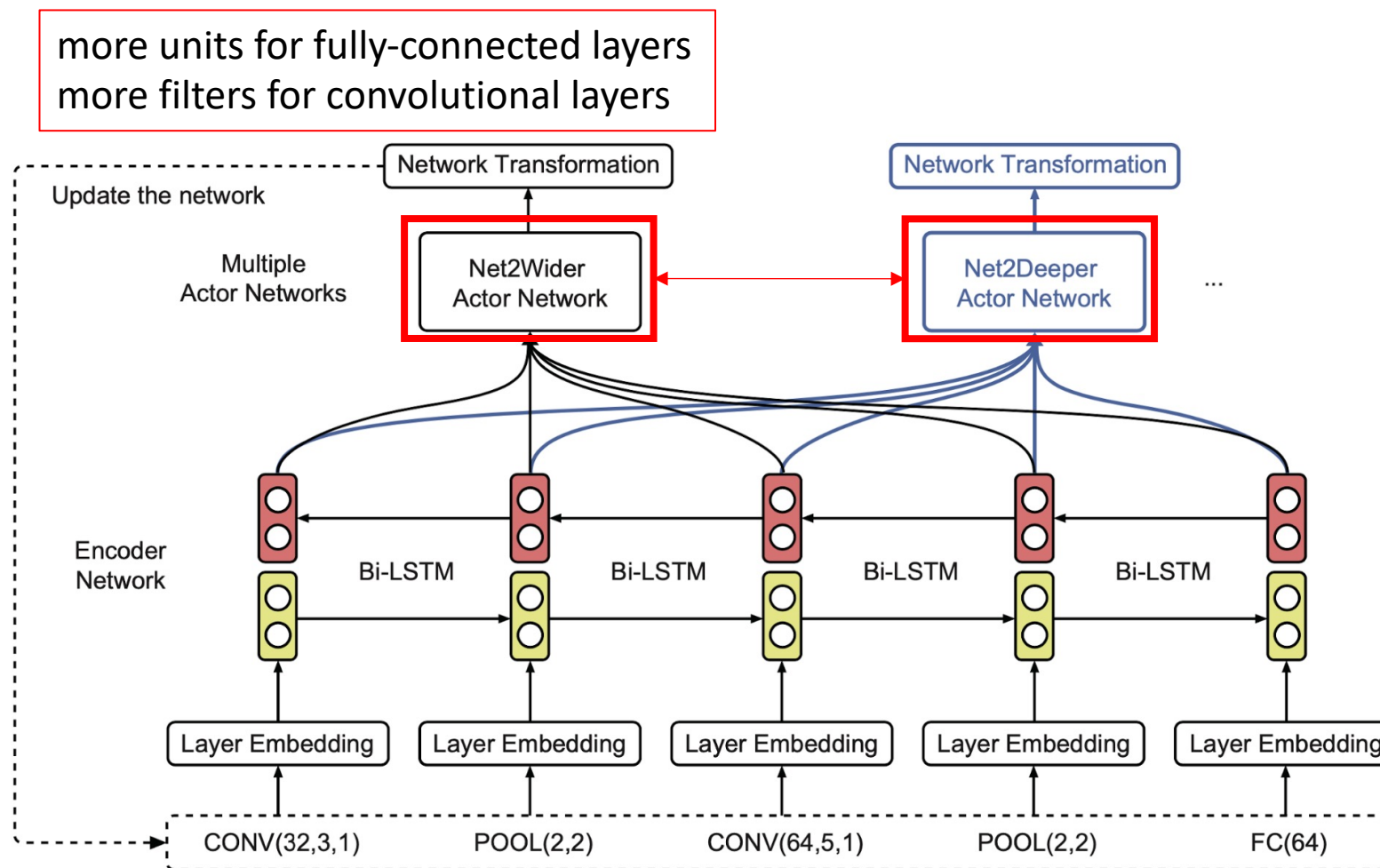


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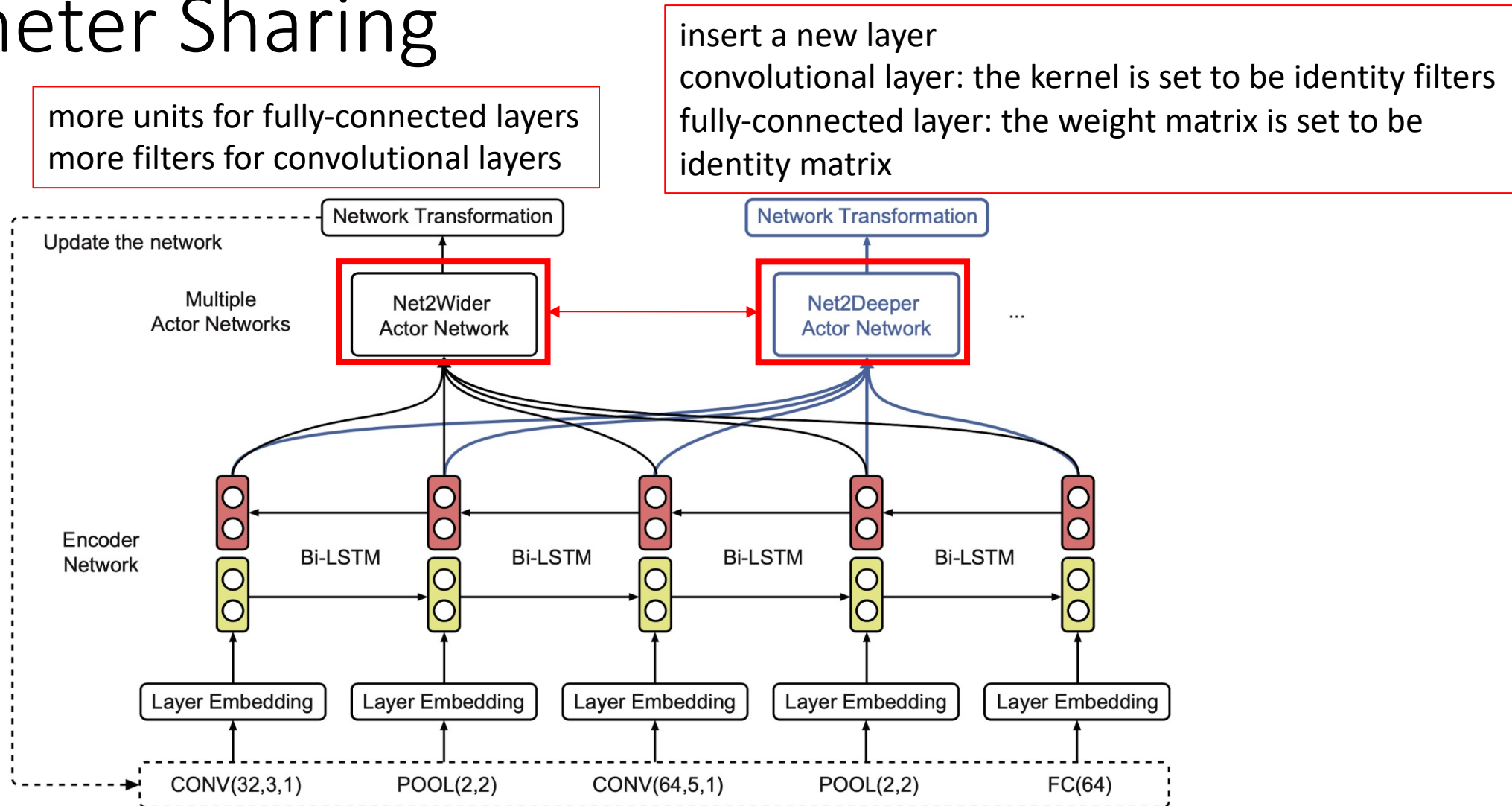
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# DARTS: differentiable architecture search

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**Algorithm 1:** DARTS – Differentiable Architecture Search

---

Create a mixed operation  $\bar{o}^{(i,j)}$  parametrized by  $\alpha^{(i,j)}$  for each edge  $(i, j)$

**while** *not converged* **do**

- 1. Update weights  $w$  by descending  $\nabla_w \mathcal{L}_{train}(w, \alpha)$
- 2. Update architecture  $\alpha$  by descending  $\nabla_\alpha \mathcal{L}_{val}(w - \xi \nabla_w \mathcal{L}_{train}(w, \alpha), \alpha)$

Replace  $\bar{o}^{(i,j)}$  with  $o^{(i,j)} = \operatorname{argmax}_{o \in \mathcal{O}} \alpha_o^{(i,j)}$  for each edge  $(i, j)$

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$$\begin{aligned} & \min_{\alpha} \mathcal{L}_{\text{validate}}(w^*(\alpha), \alpha) \\ & \text{s.t. } w^*(\alpha) = \arg \min_w \mathcal{L}_{\text{train}}(w, \alpha) \end{aligned}$$

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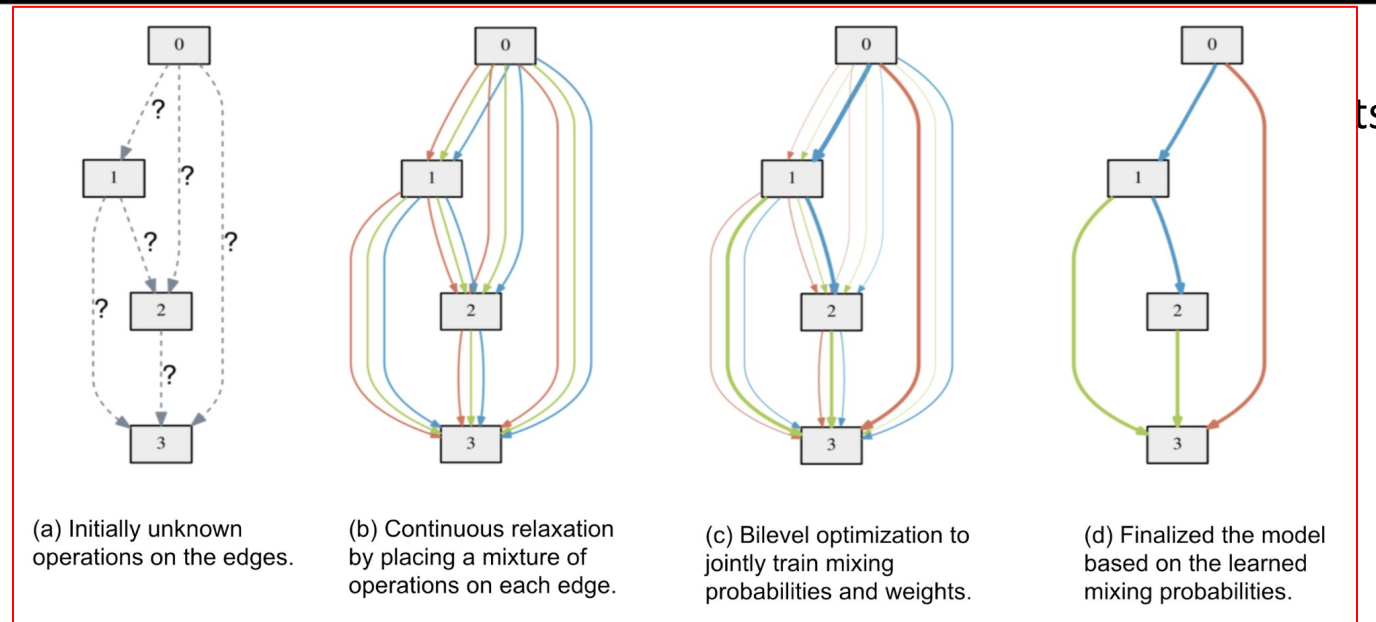
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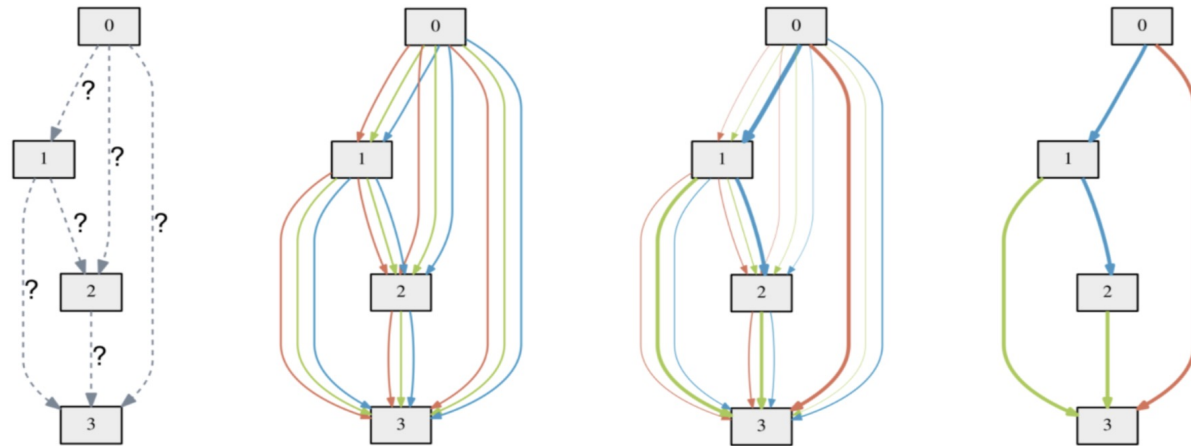
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Replace  $\bar{o}^{(i,j)}$  with  $o^{(i,j)} = \operatorname{argmax}_{o \in \mathcal{O}} \alpha_o^{(i,j)}$  for each edge  $(i, j)$

---

$$x_i = \sum_{j < i} o^{(i,j)}(x_j)$$

$$\bar{o}^{(i,j)}(x) = \sum_{o \in \mathcal{O}} \frac{\exp(\alpha_{ij}^o)}{\sum_{o' \in \mathcal{O}} \exp(\alpha_{ij}^{o'})} o(x)$$



(a) Initially unknown operations on the edges.

(b) Continuous relaxation operations by placing a mixture of operations on each edge.

(c) Bilevel optimization to jointly train mixing probabilities and weights.

(d) Finalized the model based on the learned mixing probabilities.

# DARTS: differentiable architecture search

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**Algorithm 1:** DARTS – Differentiable Architecture Search

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Create a mixed operation  $\bar{o}^{(i,j)}$  parametrized by  $\alpha^{(i,j)}$  for each edge  $(i, j)$

**while not converged do**

1. Update weights  $w$  by descending  $\nabla_w \mathcal{L}_{train}(w, \alpha)$

2. Update architecture  $\alpha$  by descending  $\nabla_{\alpha} \mathcal{L}_{val}(w - \xi \nabla_w \mathcal{L}_{train}(w, \alpha), \alpha)$

Replace  $\bar{o}^{(i,j)}$  with  $o^{(i,j)} = \operatorname{argmax}_{o \in \mathcal{O}} \alpha_o^{(i,j)}$  for each edge  $(i, j)$

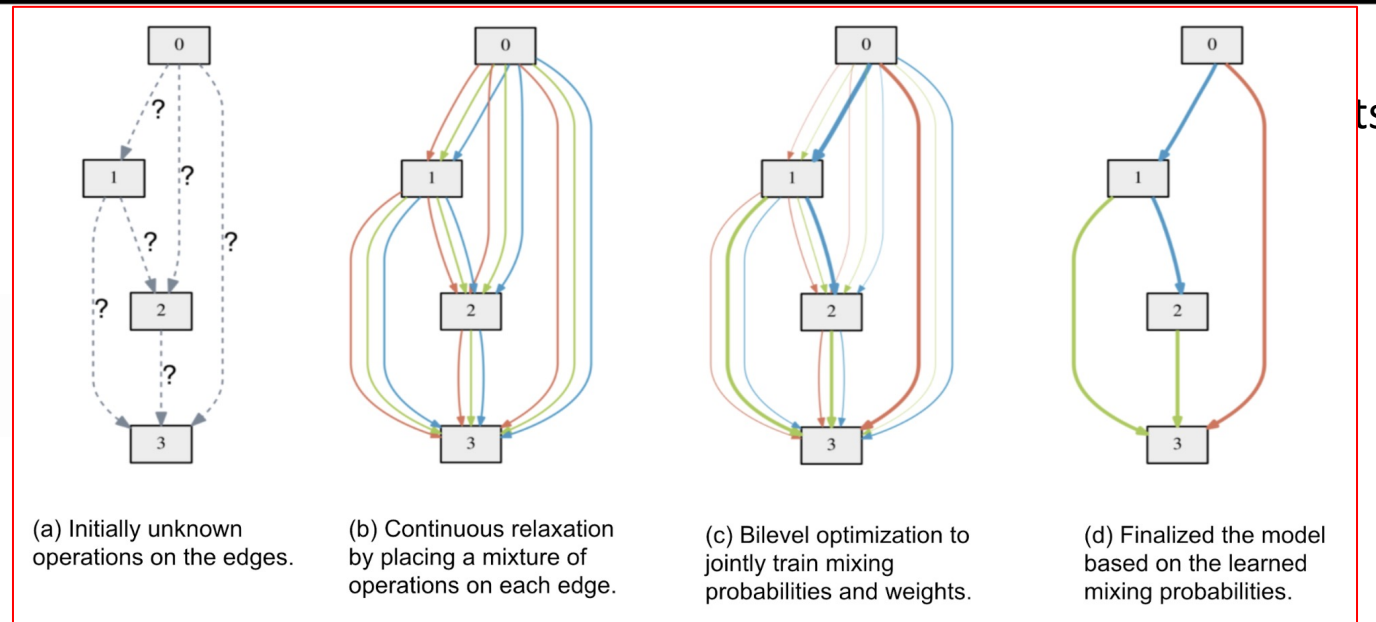
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Mix of operations

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differentiable



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