

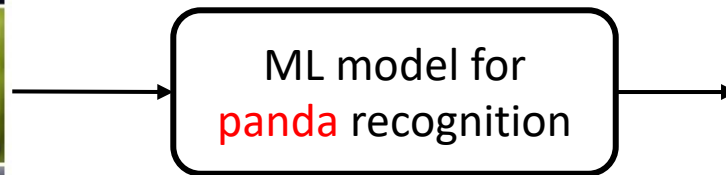
# Generative models

Neural Networks Design And Application

# Machine learning paradigm



Training data



Testing data

# Machine learning paradigm



$x$

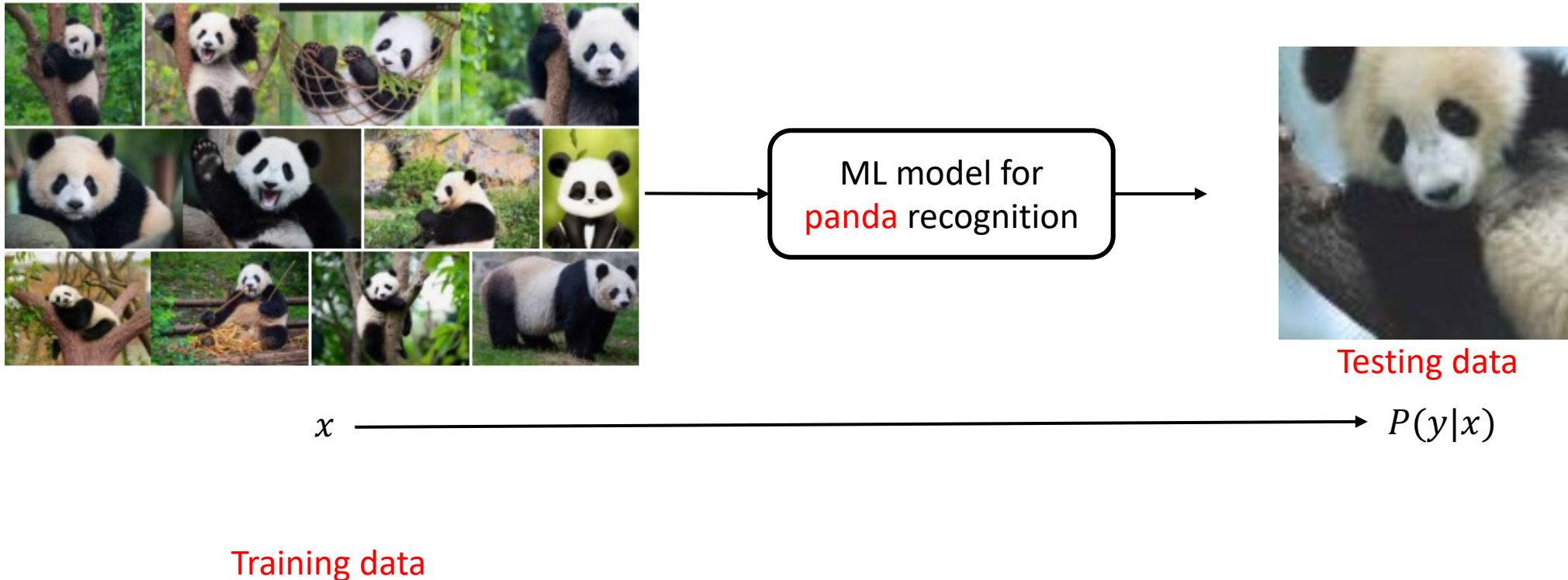
Training data

ML model for  
**panda** recognition

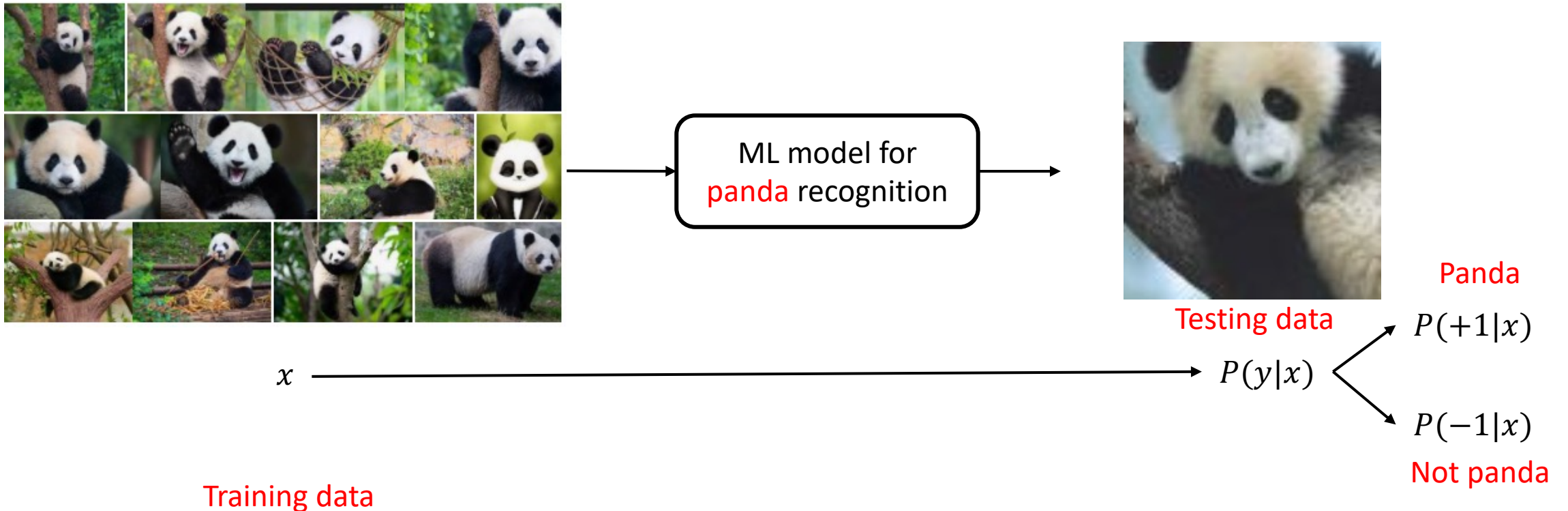


Testing data

# Machine learning paradigm



# Machine learning paradigm



# Machine learning paradigm



ML model for  
**panda** recognition



**Panda**

Testing data

$x$

mapping

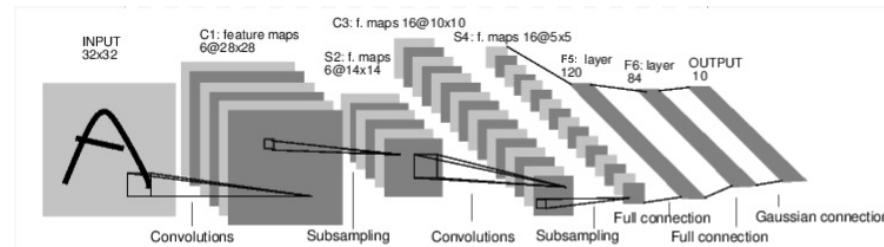
$P(y|x)$

$P(+1|x)$

$P(-1|x)$

**Not panda**

Training data





# Machine learning paradigm



ML model for  
**panda** recognition



**Panda**

Testing data

$x$

mapping

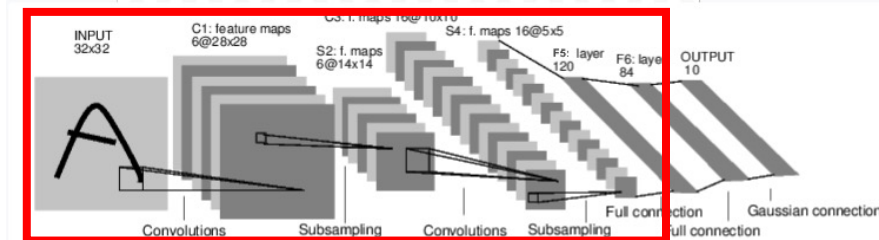
$P(y|x)$

$P(+1|x)$

$P(-1|x)$

**Not panda**

Training data



# Machine learning paradigm



ML model for  
**panda** recognition



**Panda**

Testing data

$x$

mapping

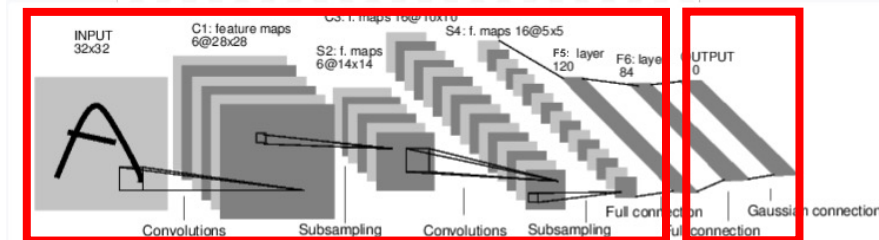
$P(y|x)$

$P(+1|x)$

$P(-1|x)$

**Not panda**

Training data





# Machine learning paradigm



ML model for  
**panda** recognition



**Panda**

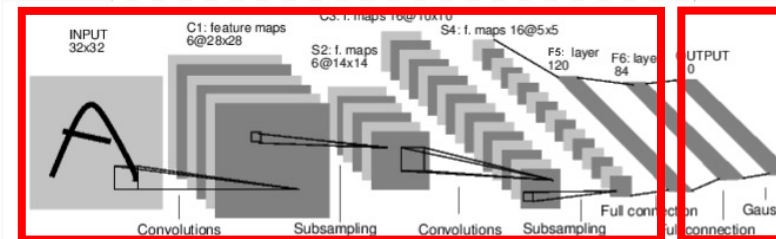
Testing data

$x$

mapping

$P(y|x)$

Training data



$$P(+1|x) = 0.97$$

$$P(-1|x) = 0.03$$

$$P(+1|x)$$

$$P(-1|x)$$

**Not panda**

# Machine learning paradigm



Discriminative model

ML model for  
**panda** recognition



**Panda**

Testing data

$x$

mapping

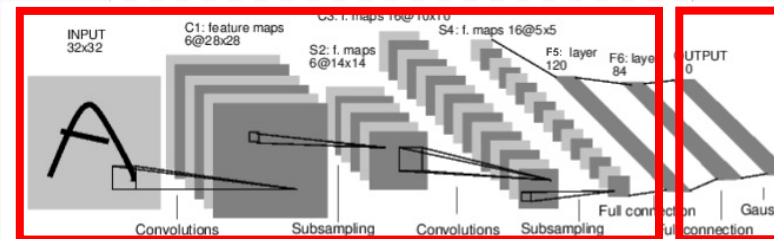
$P(y|x)$

$P(+1|x)$

$P(-1|x)$

**Not panda**

Training data



$$P(+1|x) = 0.97$$

$$P(-1|x) = 0.03$$

# Machine learning paradigm

Q: other way for classification?



Discriminative model

ML model for  
**panda** recognition



Panda

Testing data

$x$

mapping

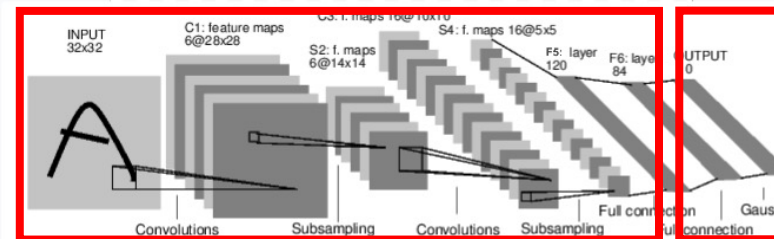
$P(y|x)$

$P(+1|x)$

$P(-1|x)$

Not panda

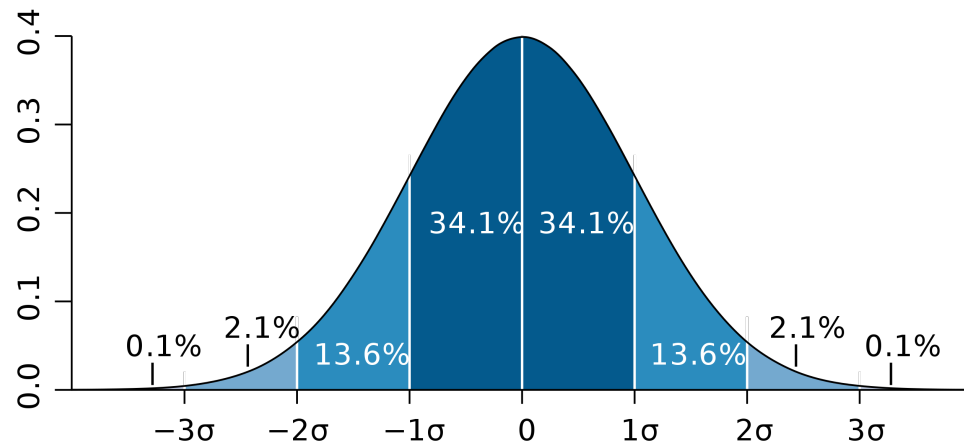
Training data



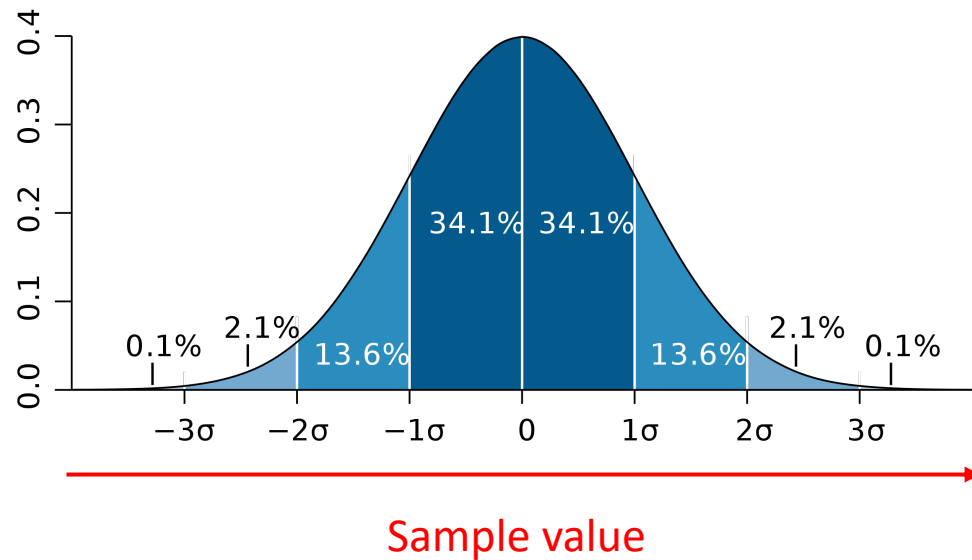
$P(+1|x) = 0.97$

$P(-1|x) = 0.03$

# What is a probability distribution?

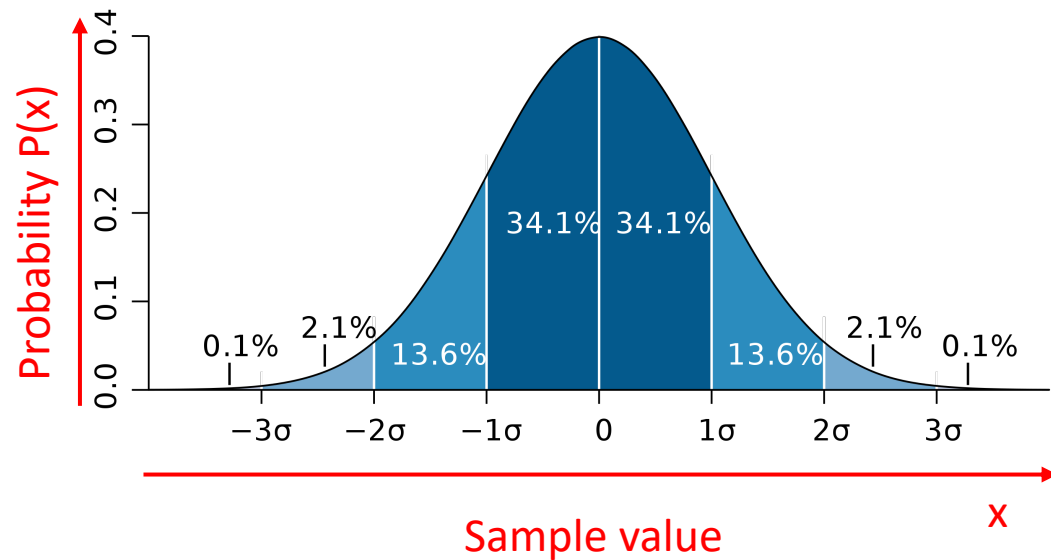


# What is a probability distribution?

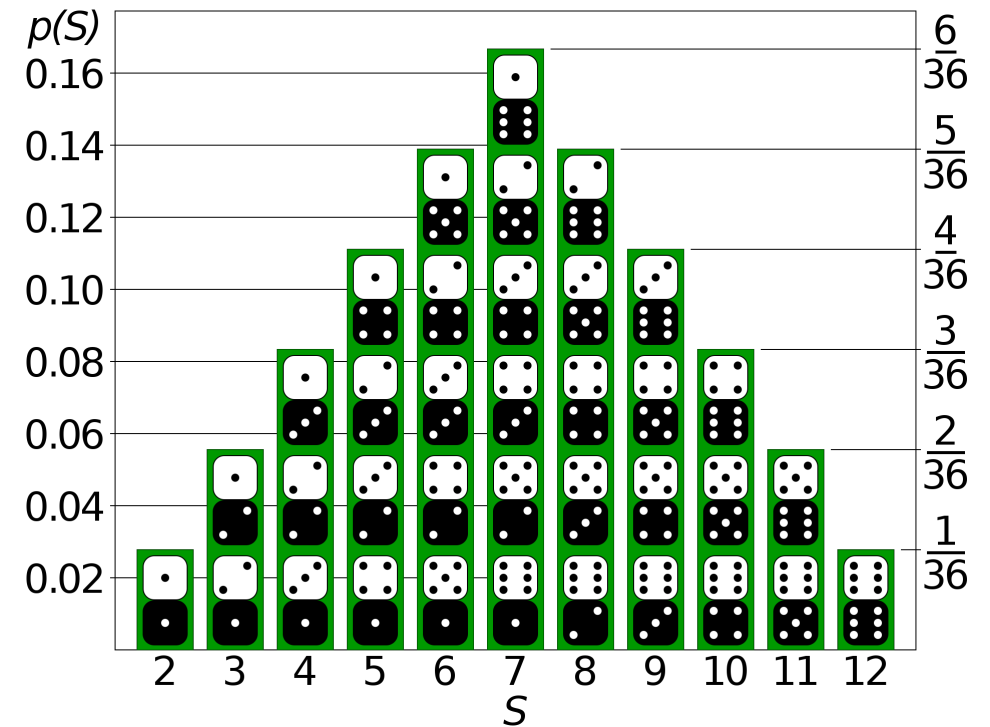
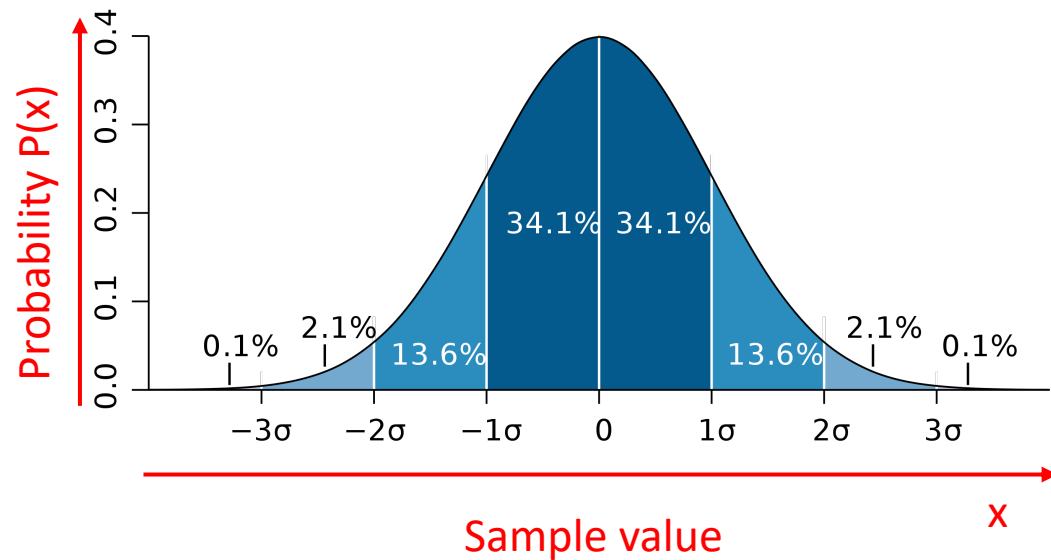




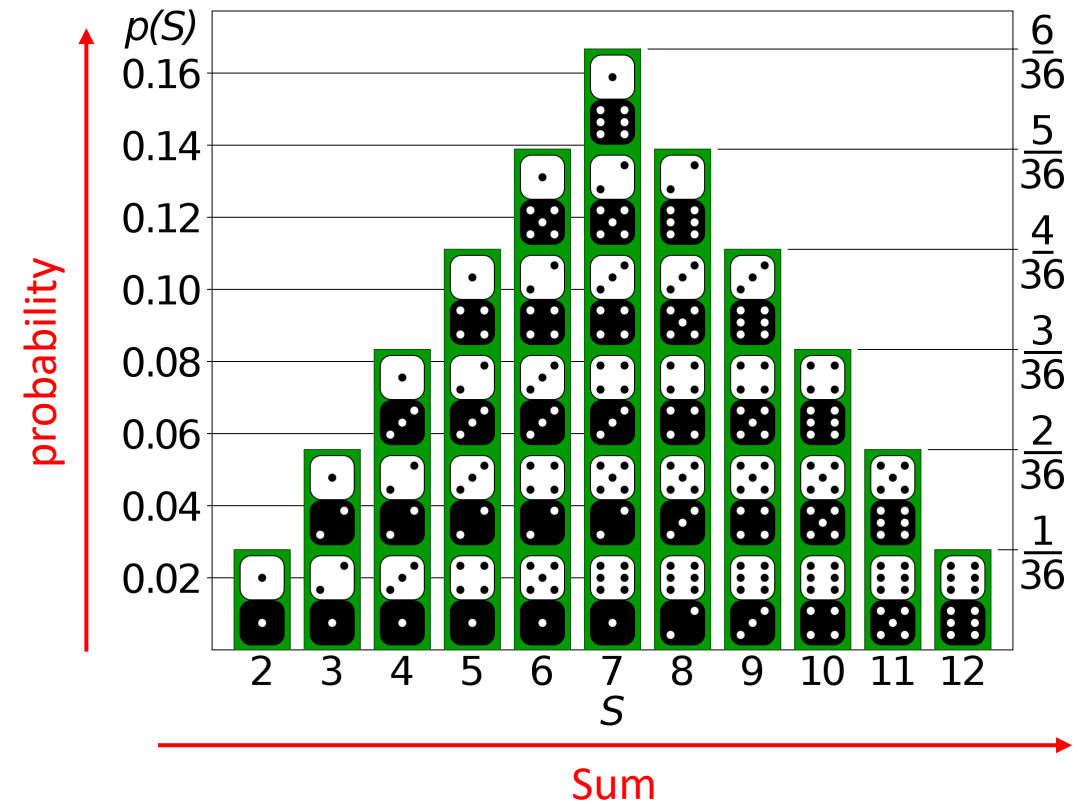
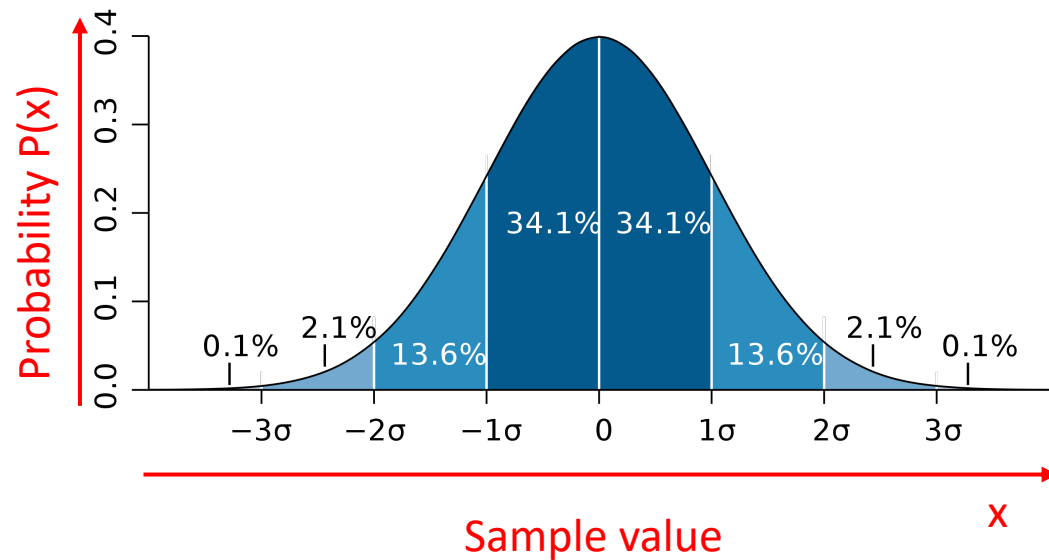
# What is a probability distribution?



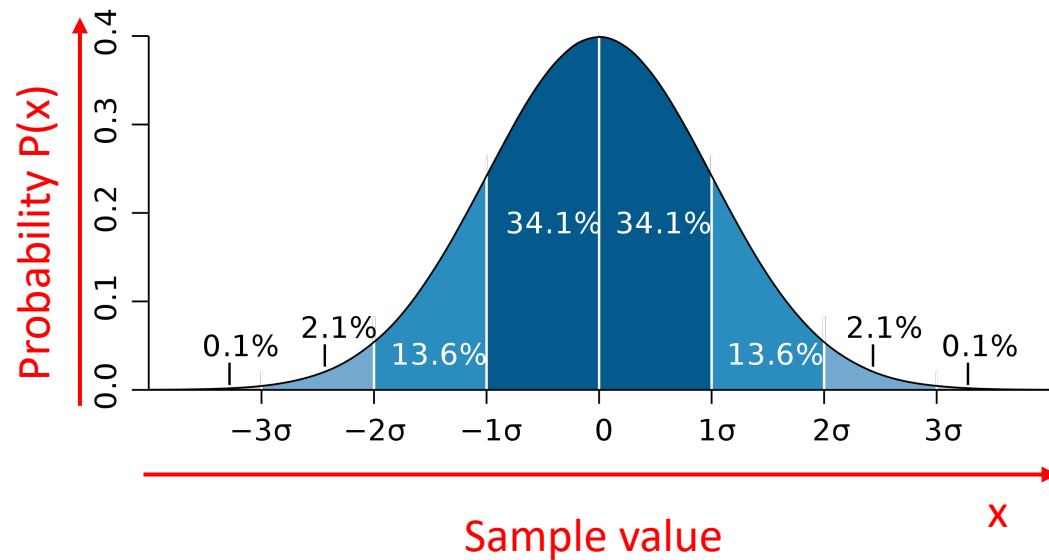
# What is a probability distribution?



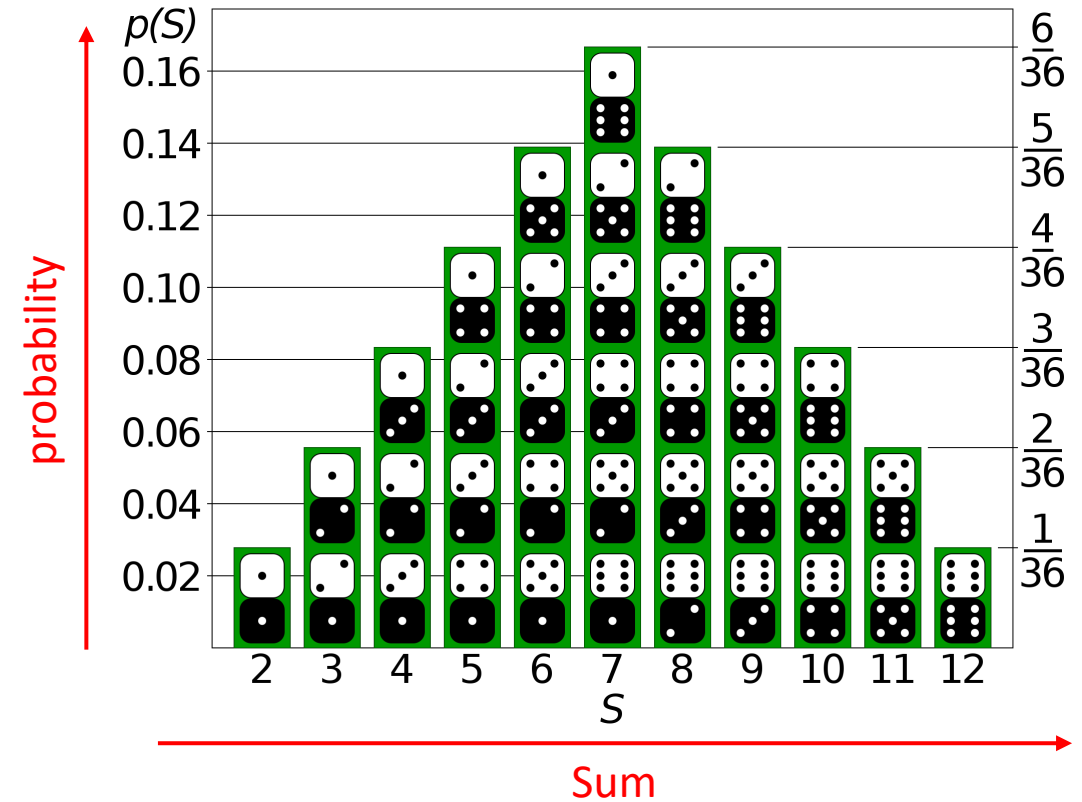
# What is a probability distribution?



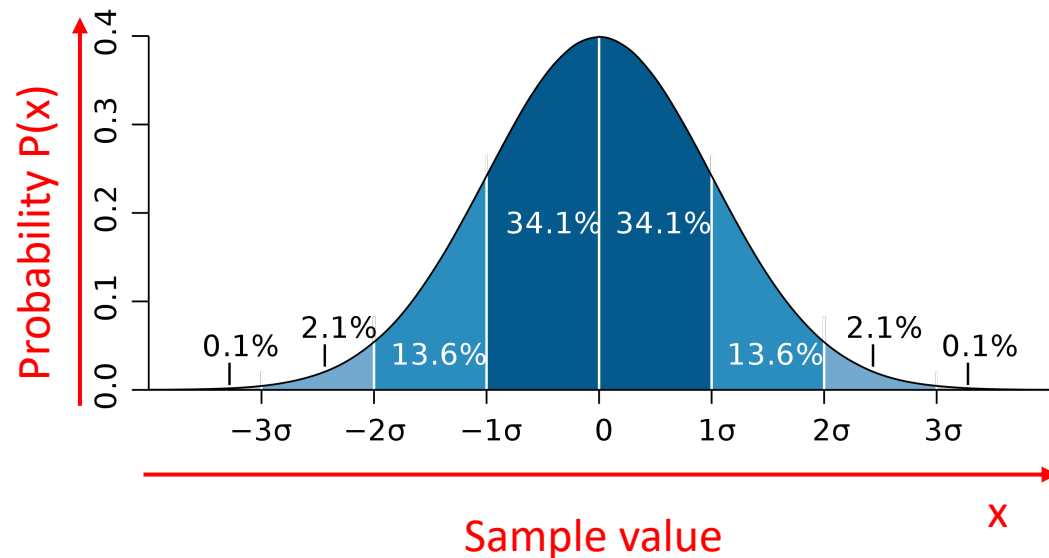
# What is a probability distribution?



$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

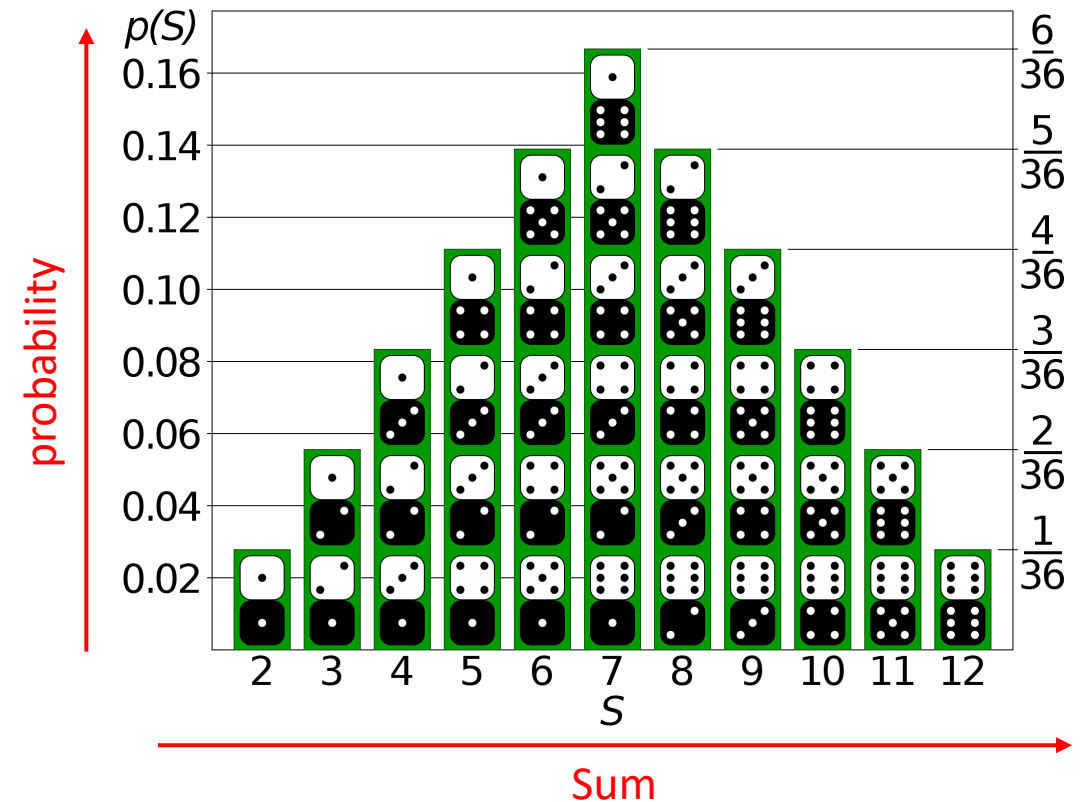


# What is a probability distribution?



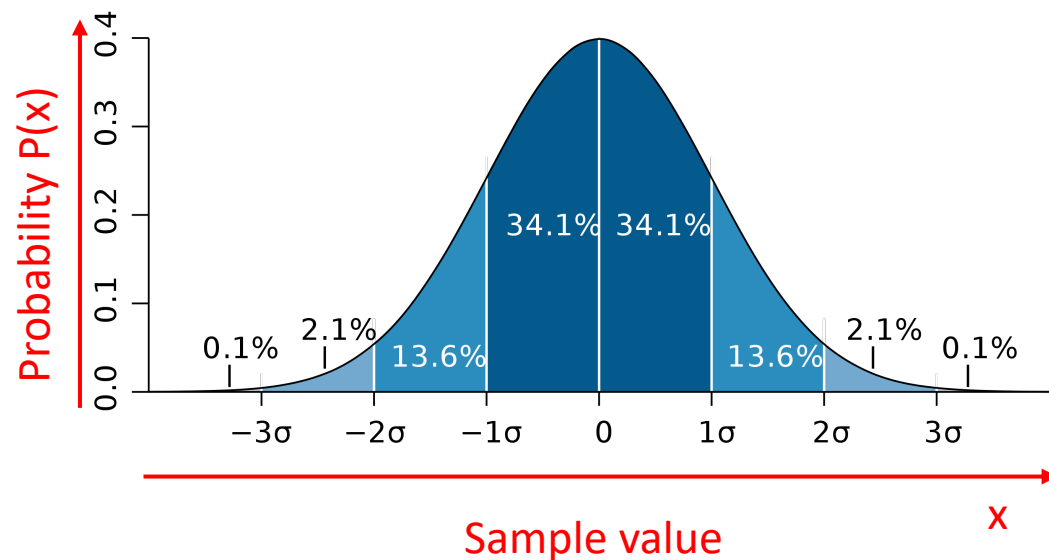
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \longrightarrow f^{-1}$$

Inverse distribution function



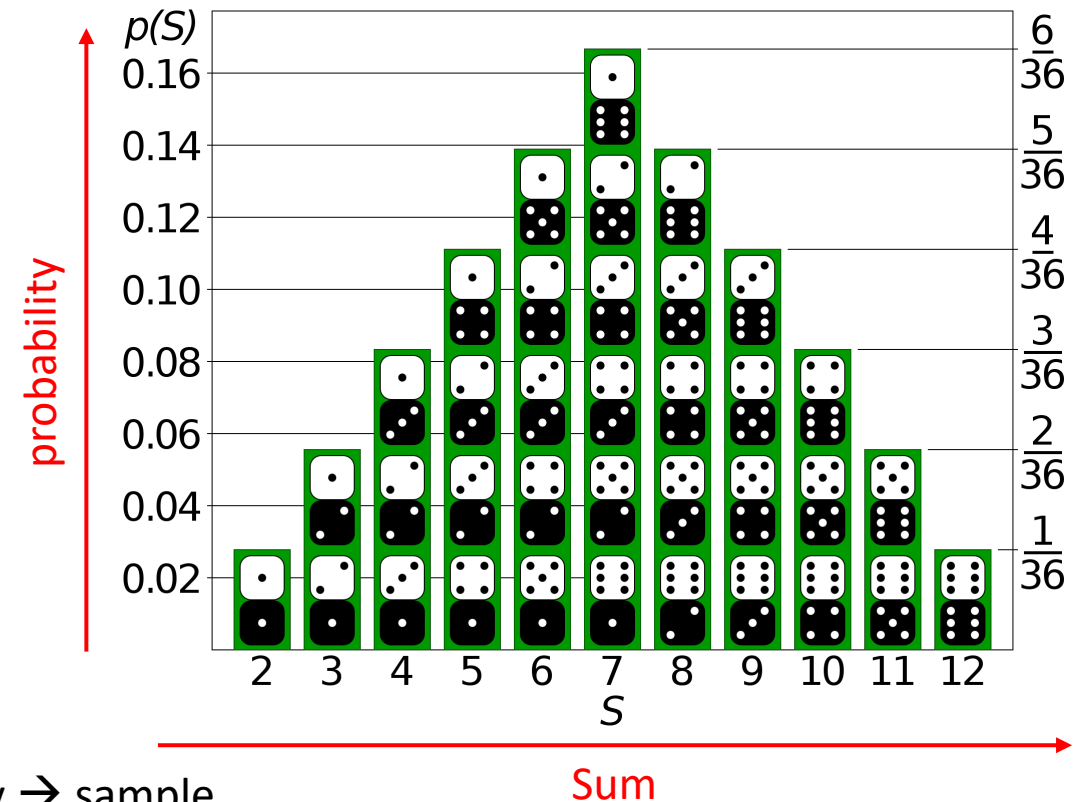


# What is a probability distribution?

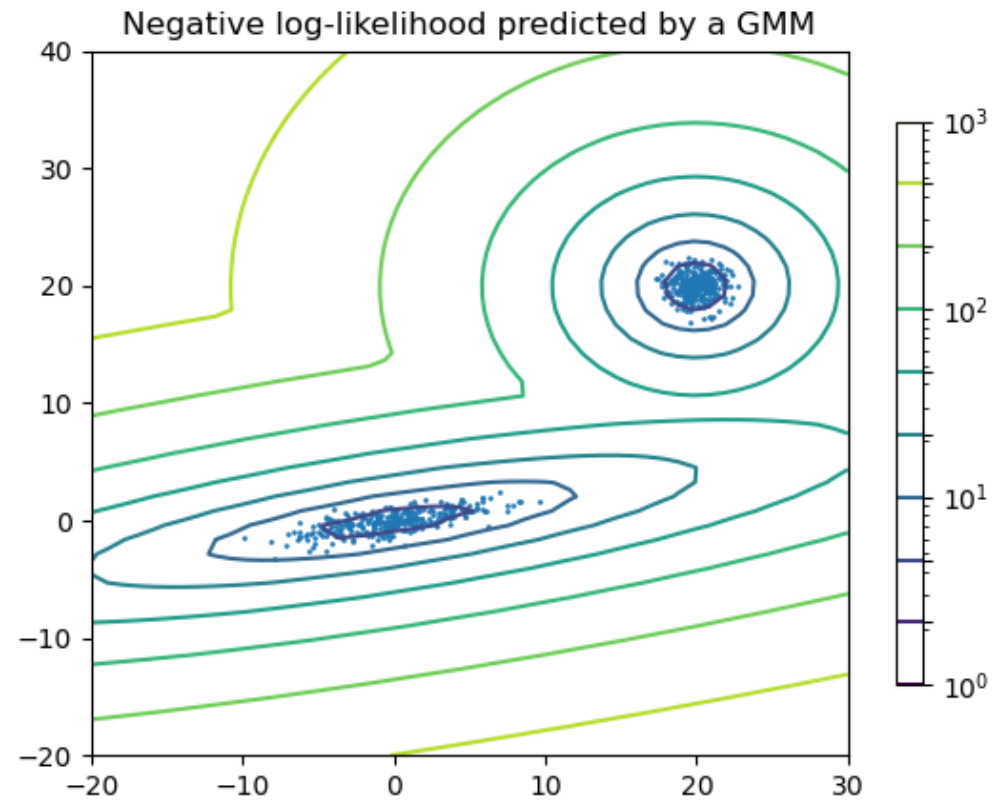


$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \longrightarrow f^{-1} \longrightarrow \text{Probability} \rightarrow \text{sample}$$

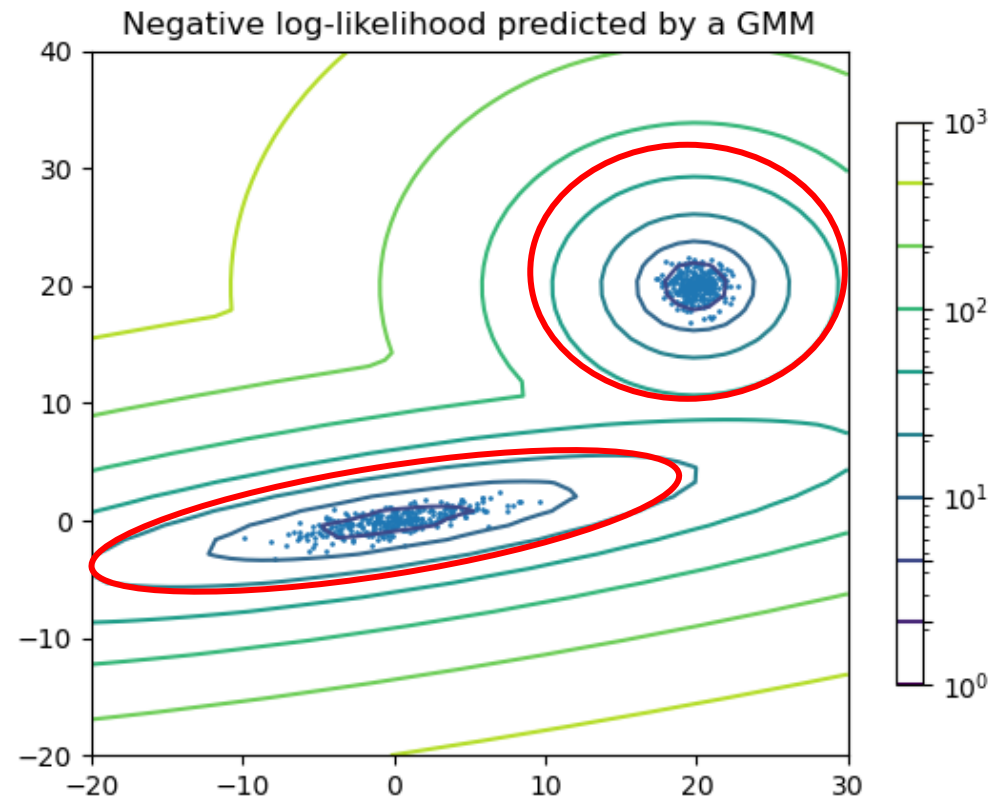
Inverse distribution function



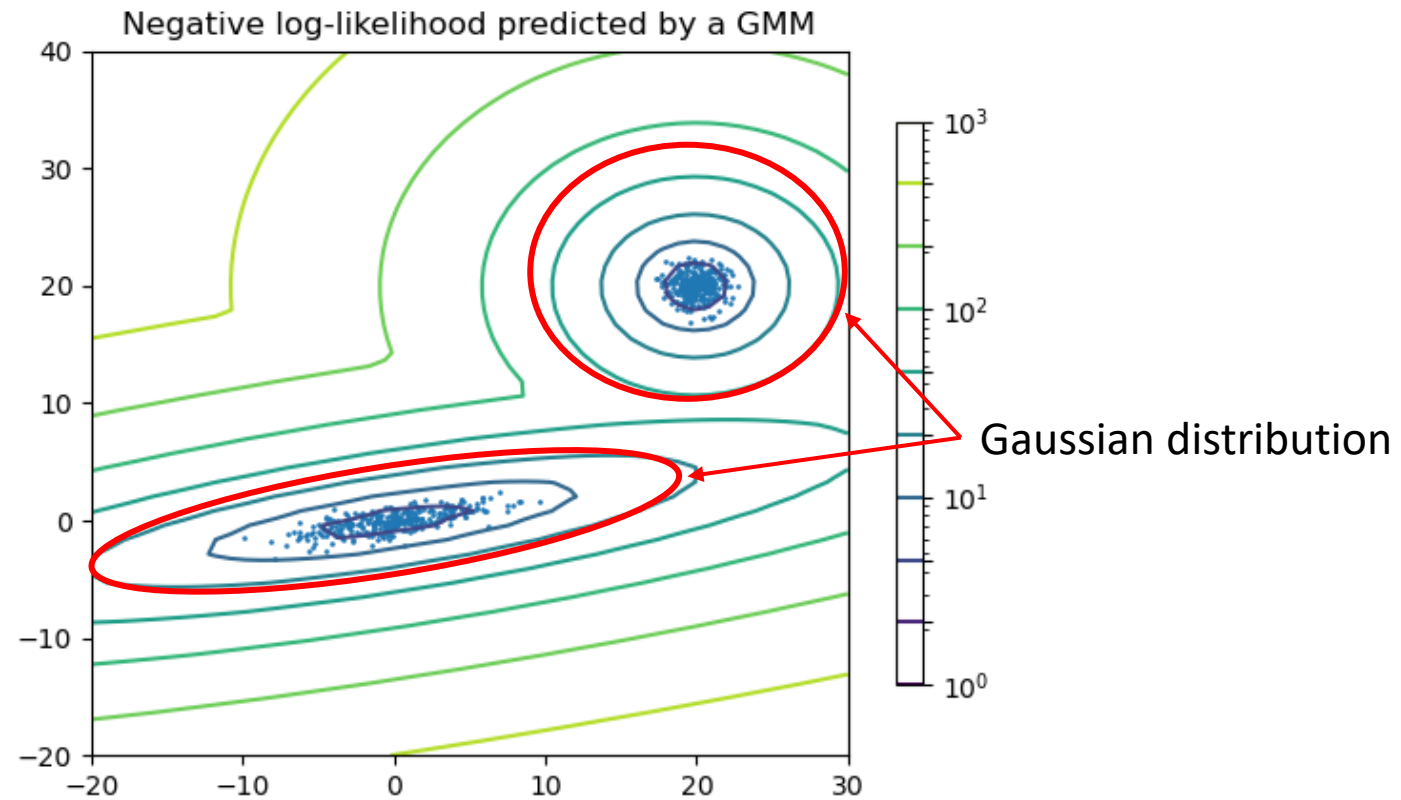
# Gaussian mixture model



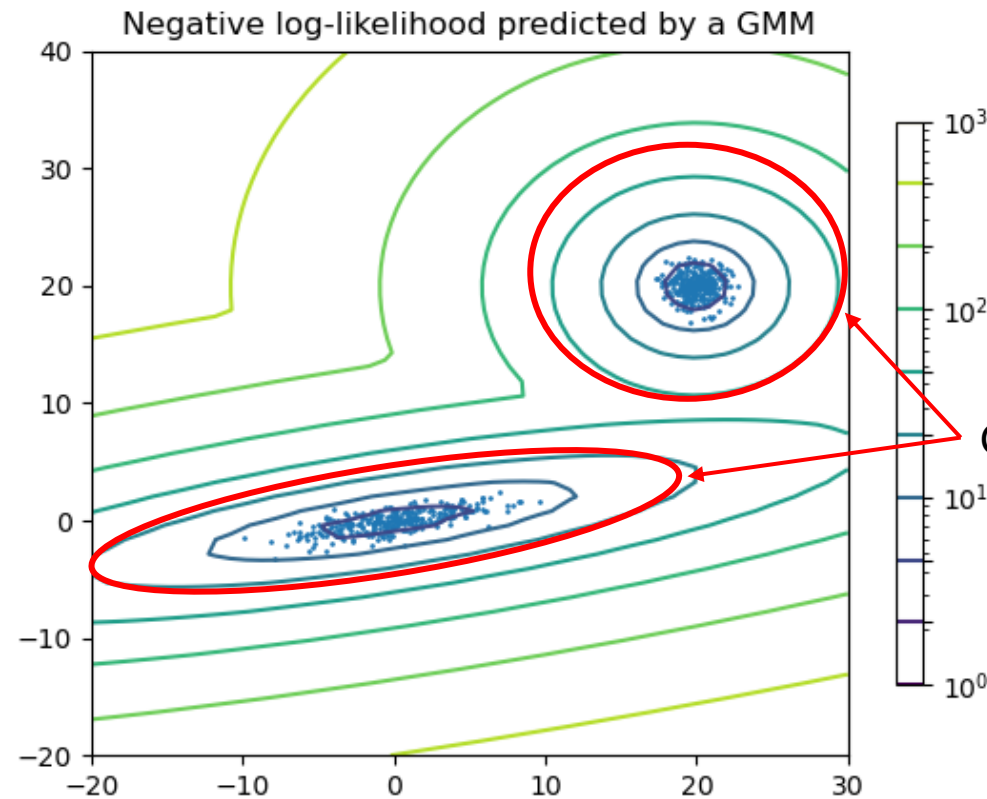
# Gaussian mixture model



# Gaussian mixture model



# Gaussian mixture model

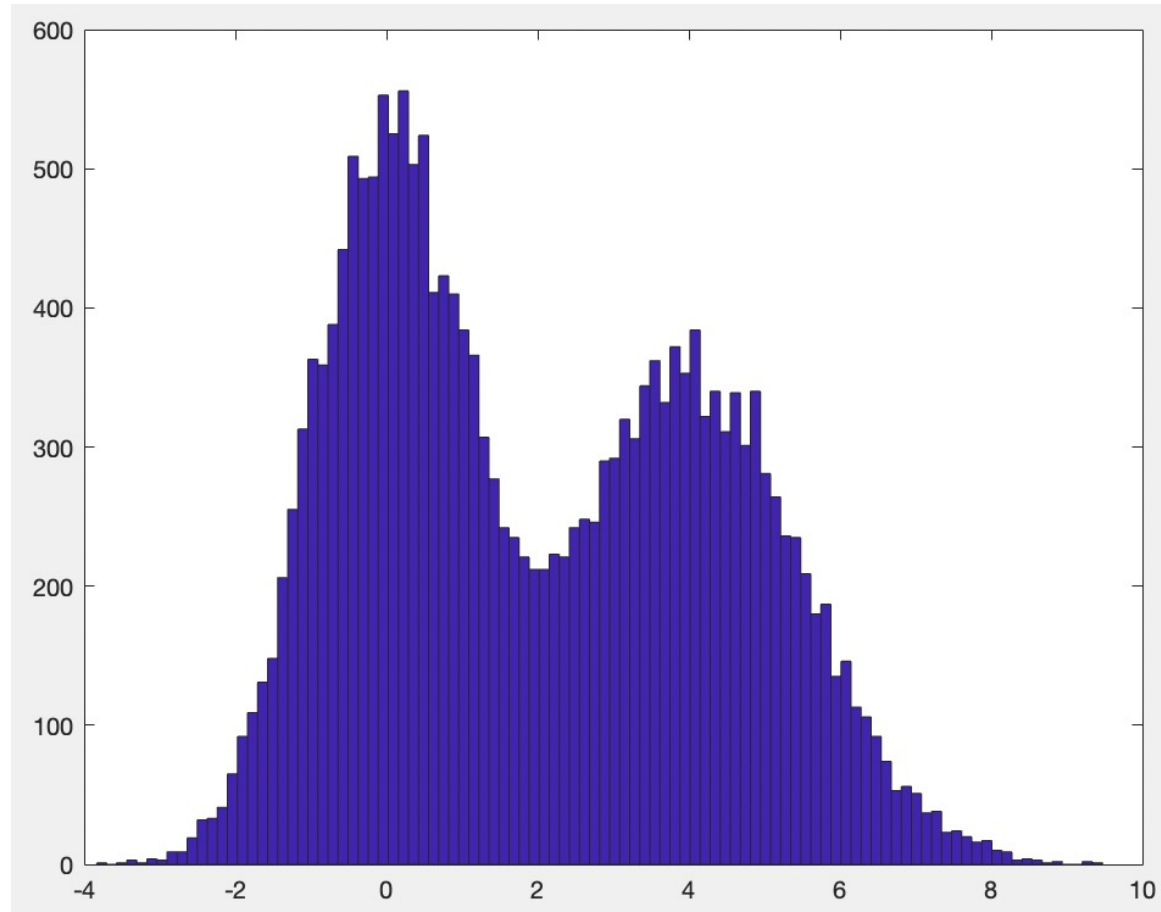


$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Gaussian distribution

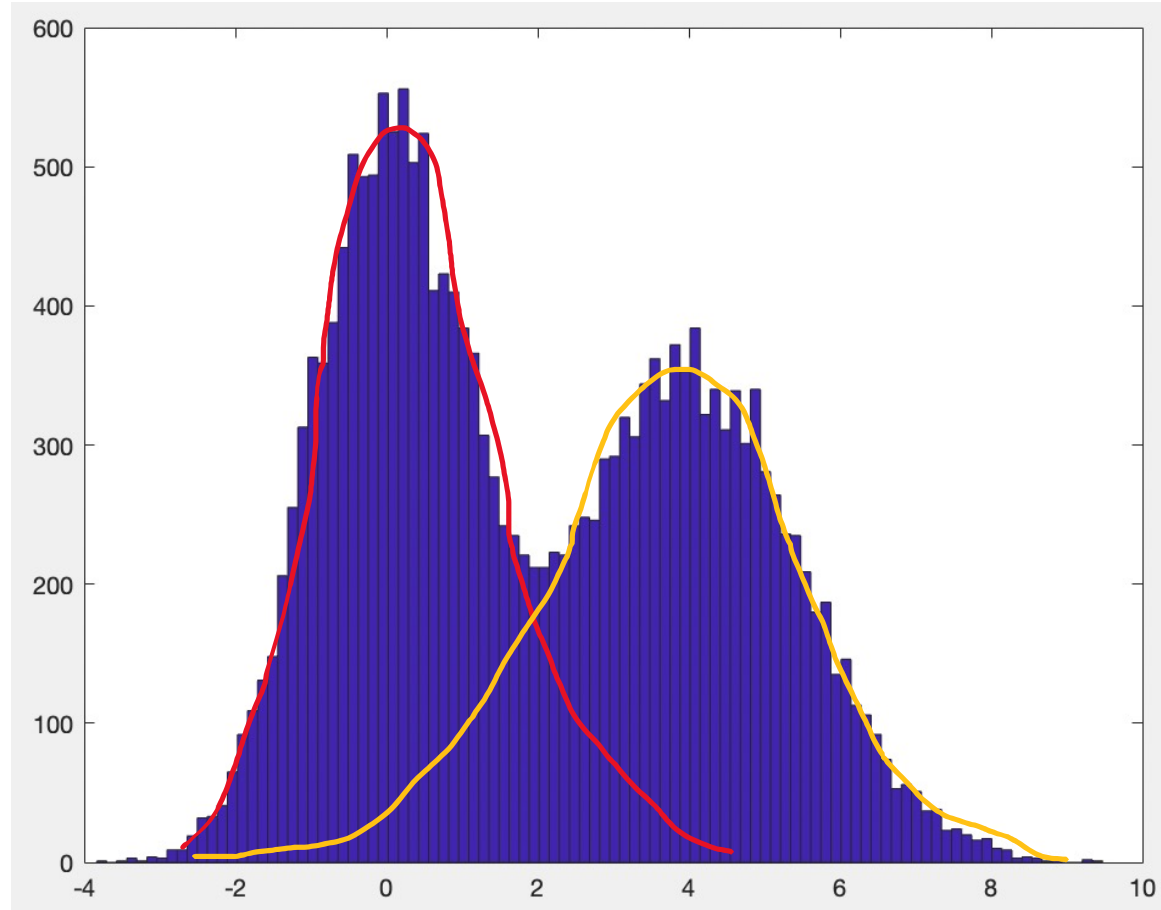


# Gaussian mixture model



Q: how many Gaussian distributions?

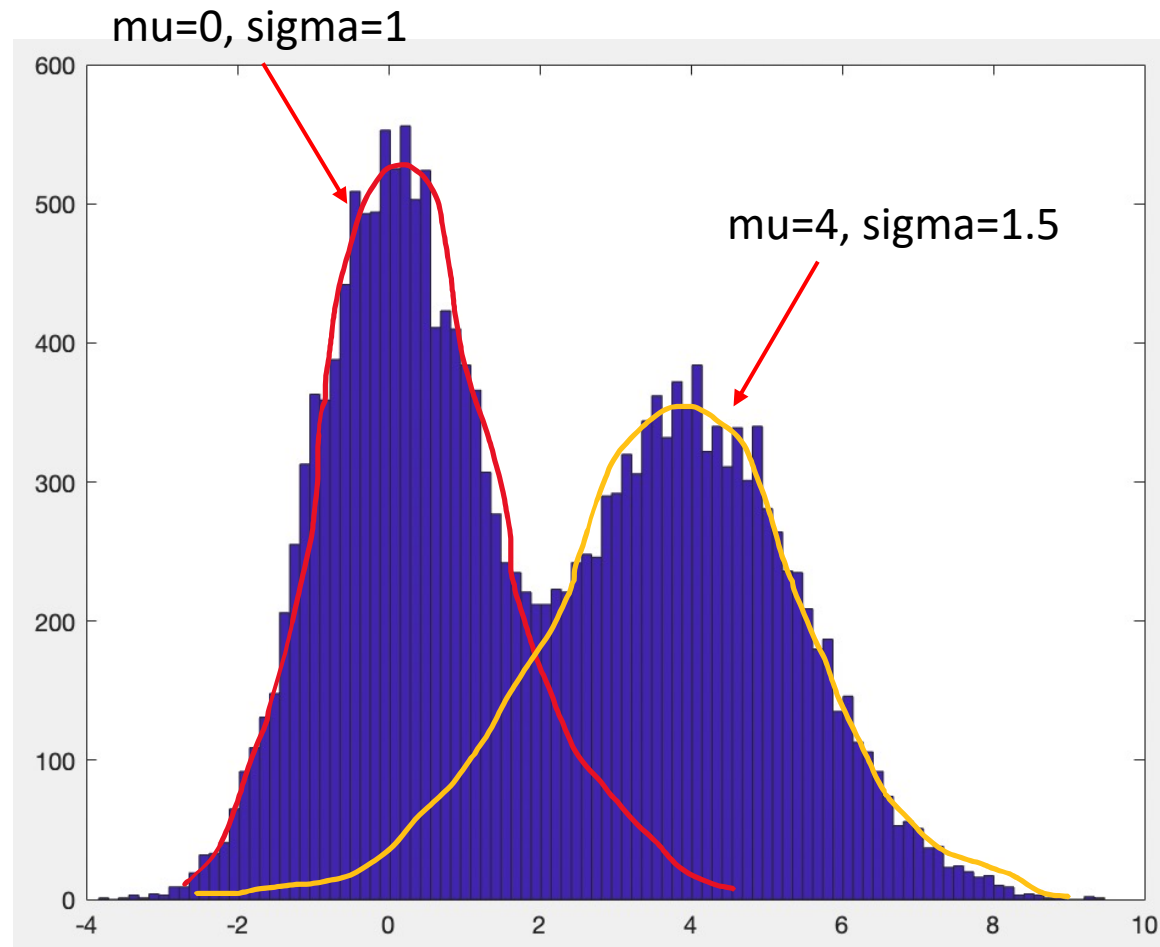
# Gaussian mixture model



$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Q: how many Gaussian distributions?

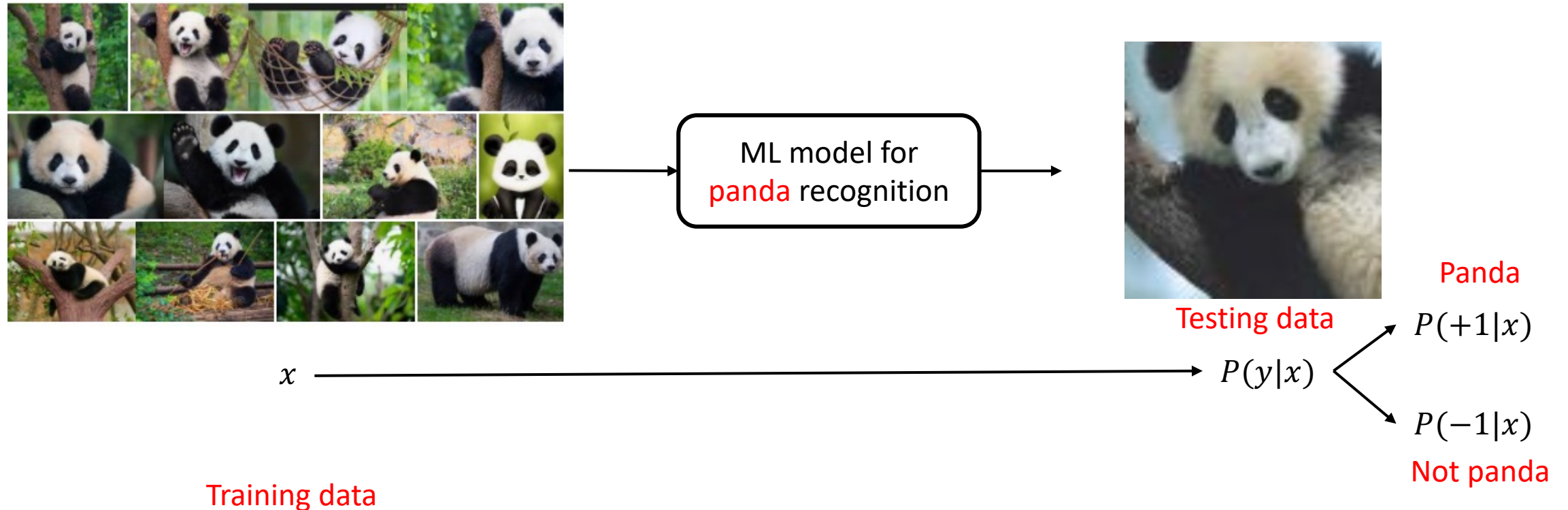
# Gaussian mixture model



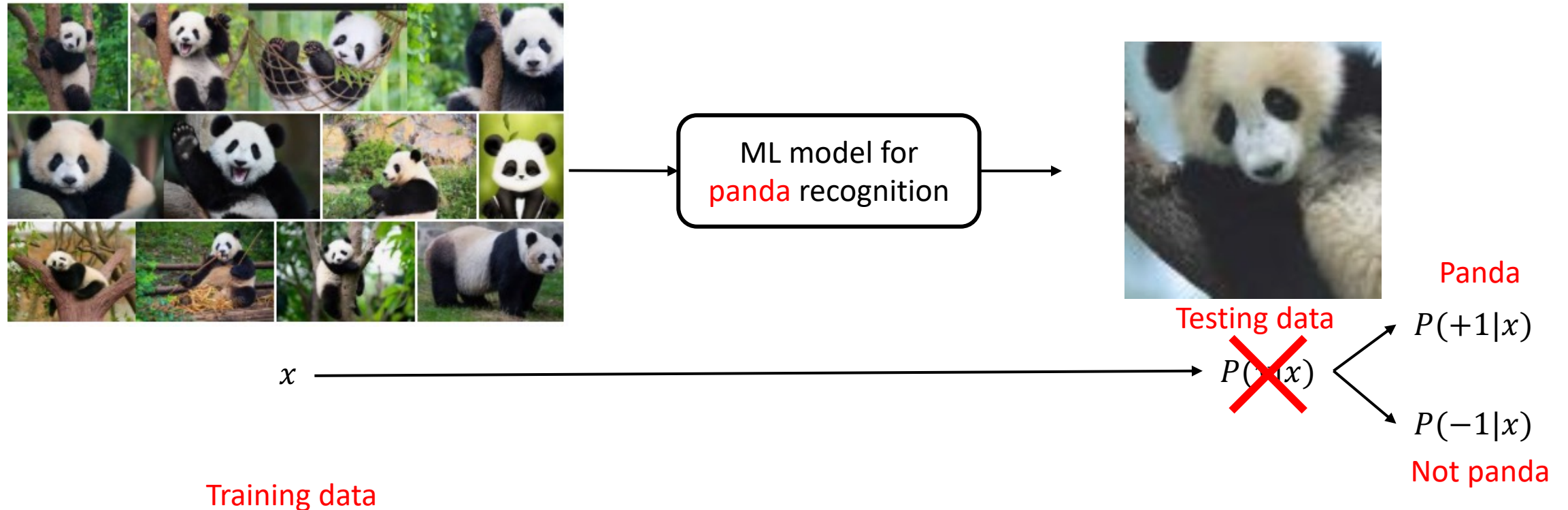
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Q: how many Gaussian distributions?

# Machine learning paradigm

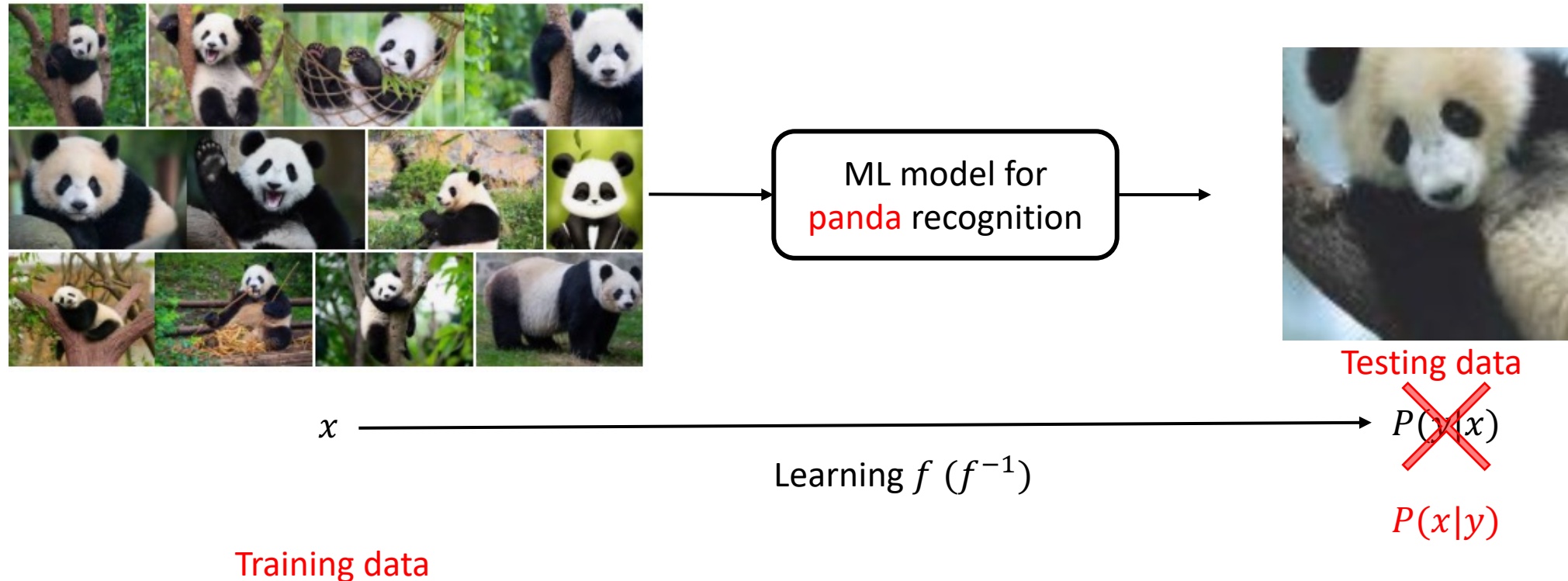


# Machine learning paradigm

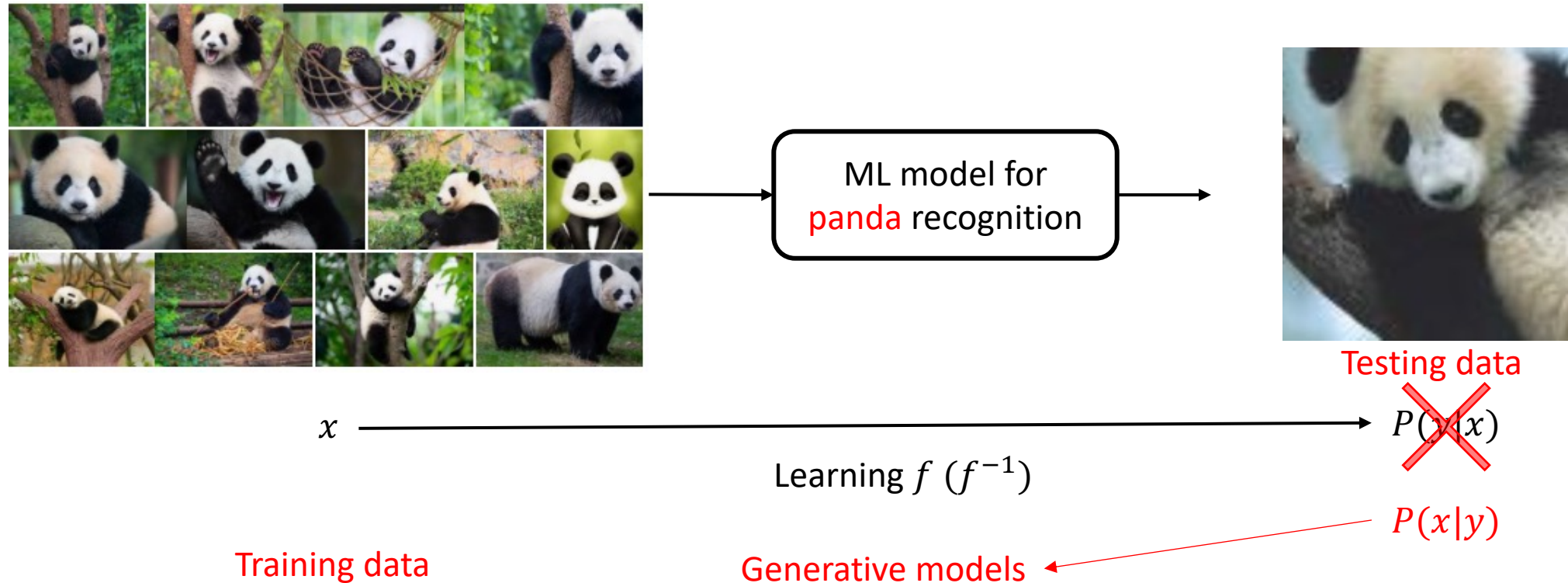




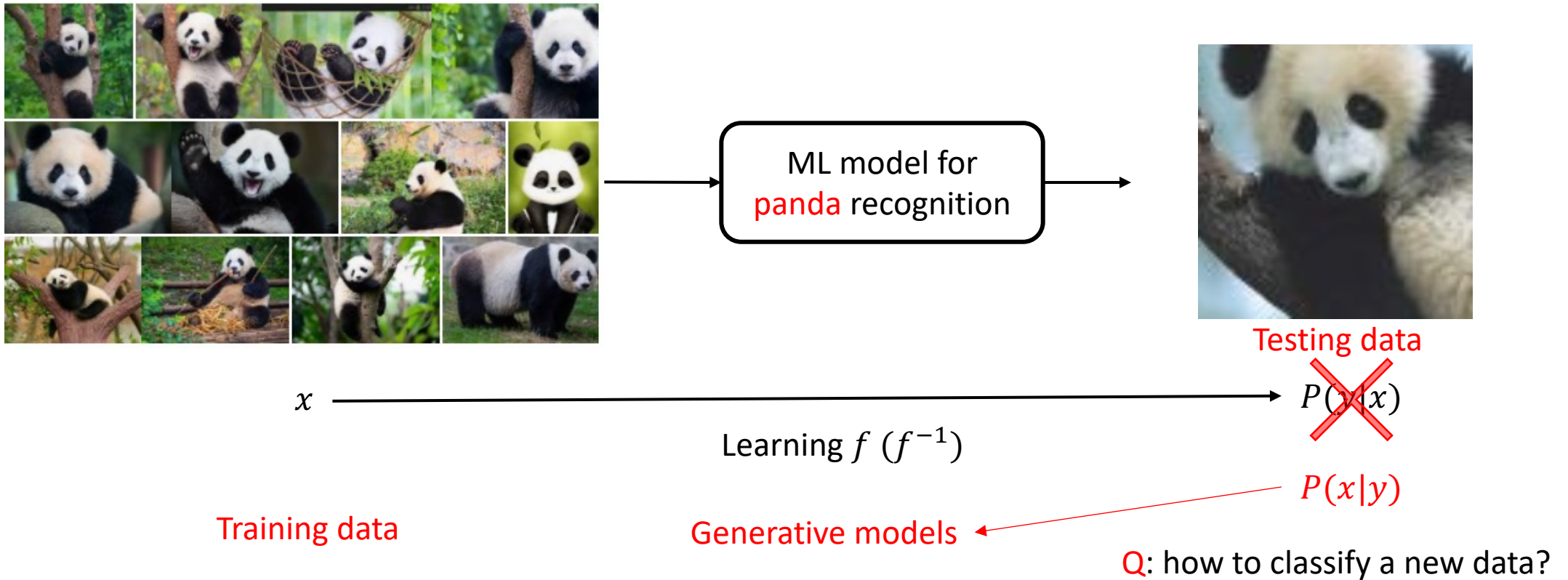
# Machine learning paradigm



# Machine learning paradigm



# Machine learning paradigm



# Machine learning paradigm

$$P(x', y) = P(x'|y)P(y)$$



ML model for  
**panda** recognition



Testing data

$x$

~~$P(y|x)$~~

Learning  $f (f^{-1})$

Training data

Generative models

$P(x|y)$

Q: how to classify a new data?

# Machine learning paradigm

$$P(x', y) = P(x'|y)P(y)$$

Pick the label  $y$  with largest  $P(x', y)$



ML model for  
**panda** recognition



Testing data

$x$

~~$P(y|x)$~~

Learning  $f (f^{-1})$

Training data

Generative models

$P(x|y)$

Q: how to classify a new data?



# Machine learning paradigm

$$P(x', +1) = P(x'|+1)P(+1)$$

$$P(x', -1) = P(x'|-1)P(-1)$$

$$P(x', y) = P(x'|y)P(y)$$

Pick the label  $y$  with largest  $P(x', y)$



ML model for  
**panda** recognition



Testing data

$x$

~~$P(y|x)$~~

Learning  $f$  ( $f^{-1}$ )

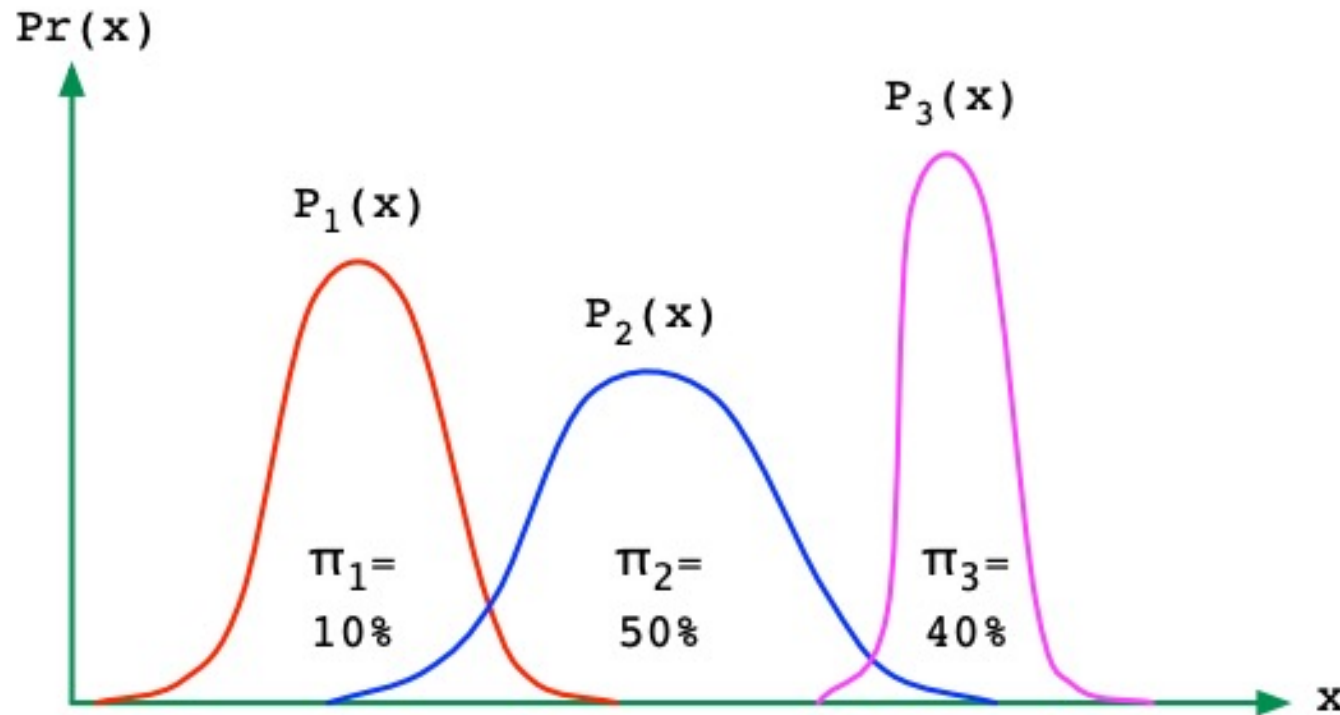
$P(x|y)$

Training data

Generative models

Q: how to classify a new data?

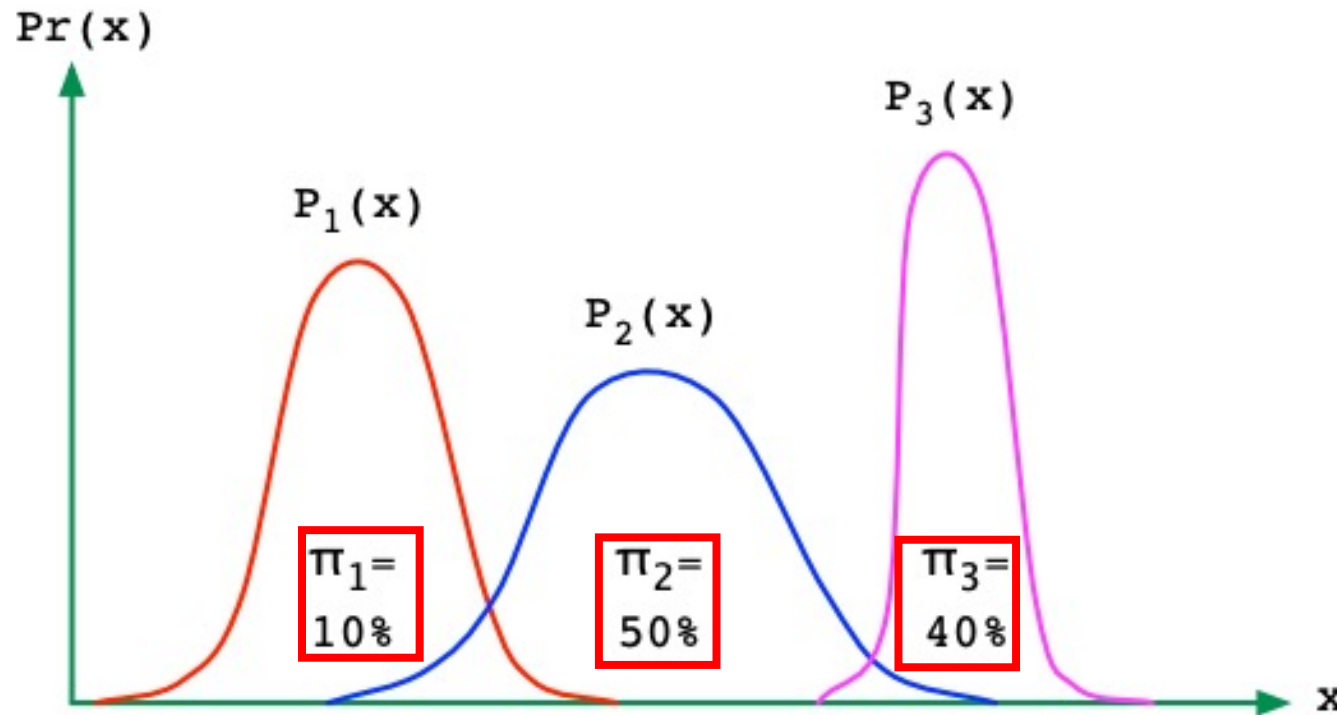
# Generative models



$$\text{Pr}(x, y) = \text{Pr}(y)\text{Pr}(x|y) = \pi_y P_y(x).$$

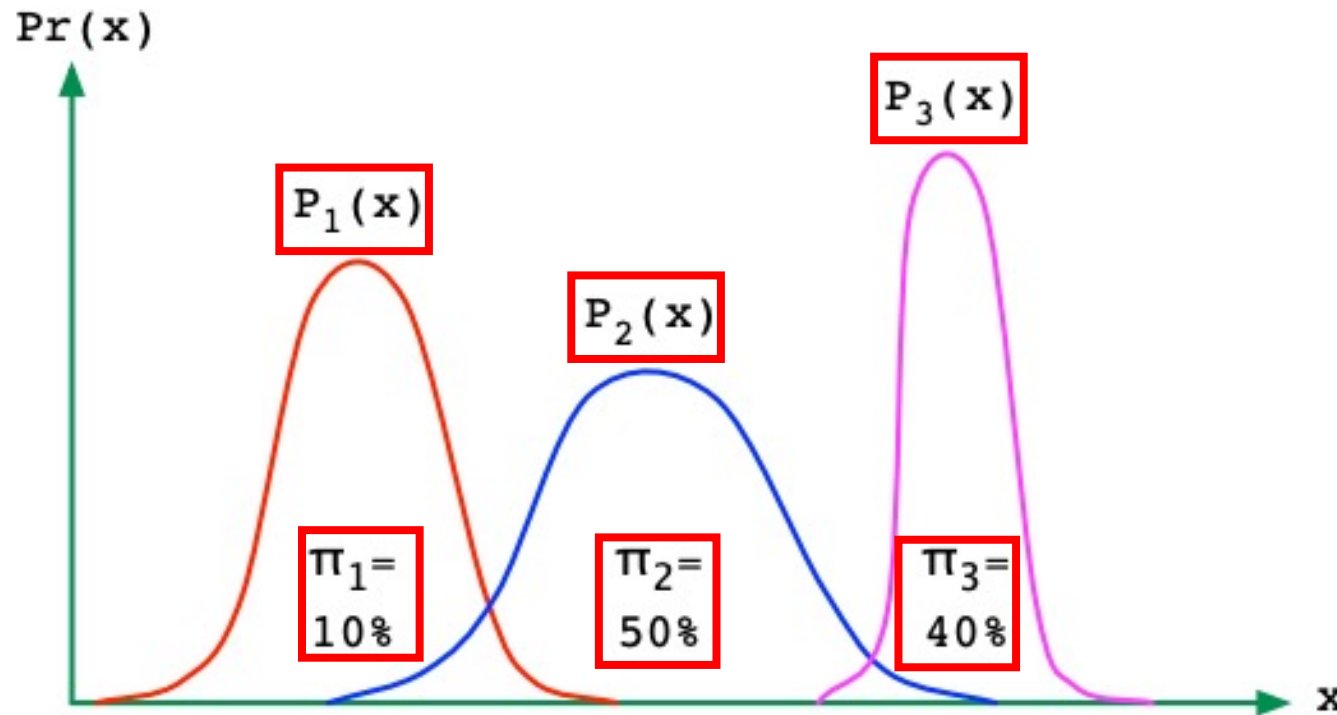


# Generative models



$$\text{Pr}(x, y) = \boxed{\text{Pr}(y)} \text{Pr}(x|y) = \boxed{\pi_y} P_y(x).$$

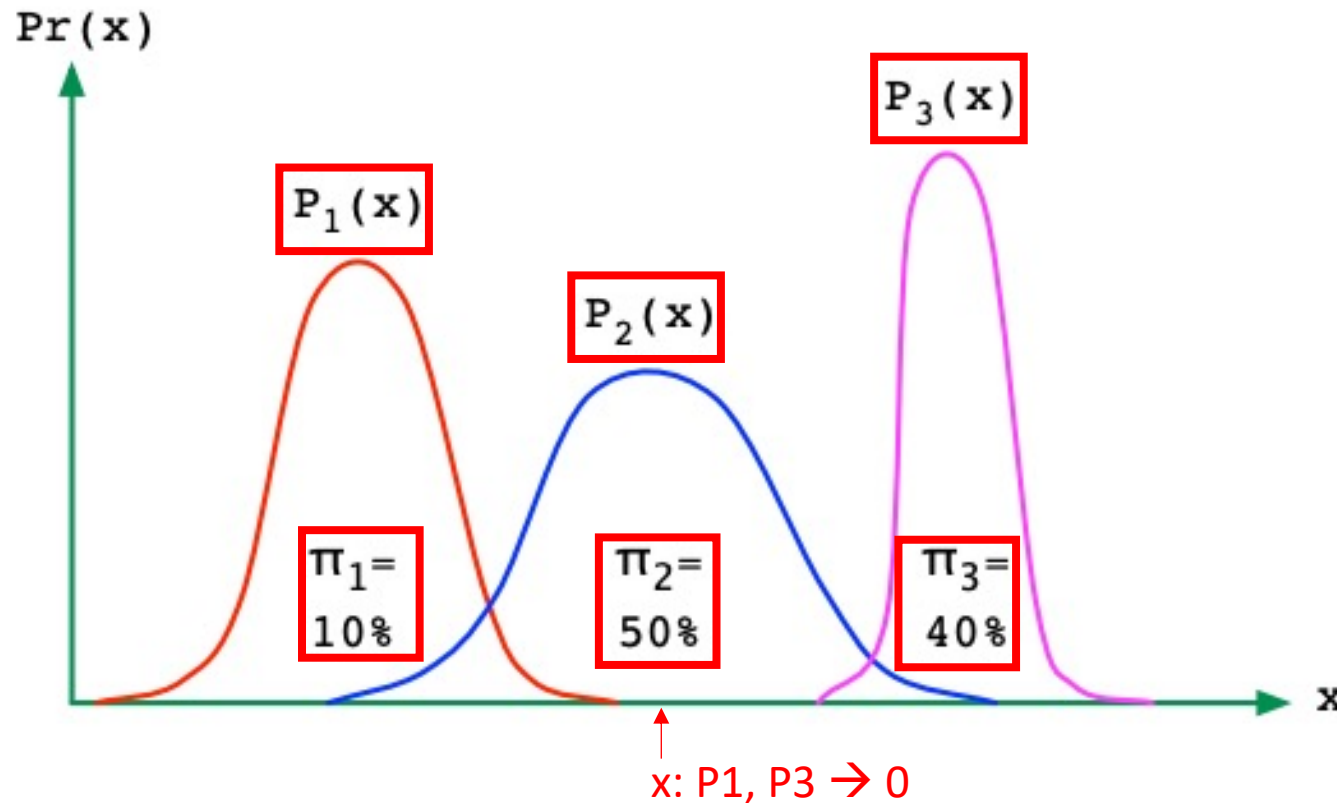
# Generative models



$$\text{Pr}(x, y) = \text{Pr}(y) \text{Pr}(x|y) = \pi_y P_y(x).$$

Pick the label  $y$  making  $P(x, y)$  largest

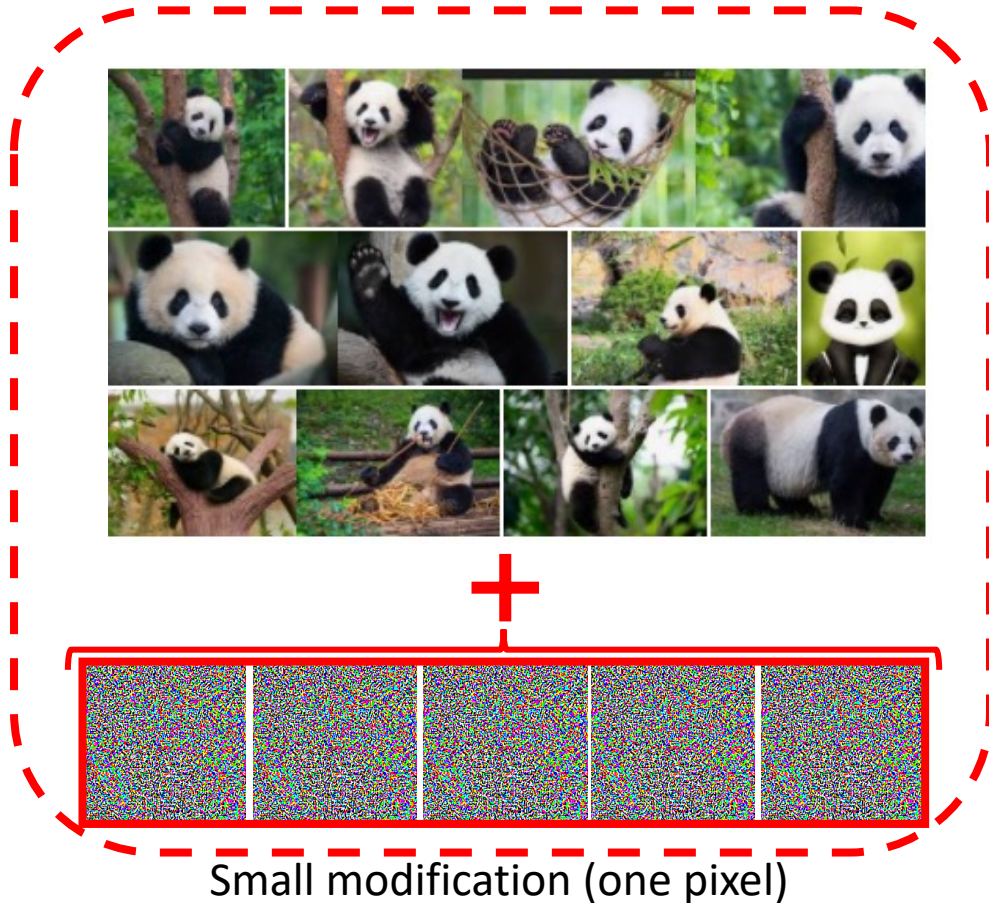
# Generative models



$$\text{Pr}(x, y) = \text{Pr}(y) \text{Pr}(x|y) = \pi_y P_y(x).$$

Pick the label  $y$  making  $P(x, y)$  largest

# Adversarial learning and generative models



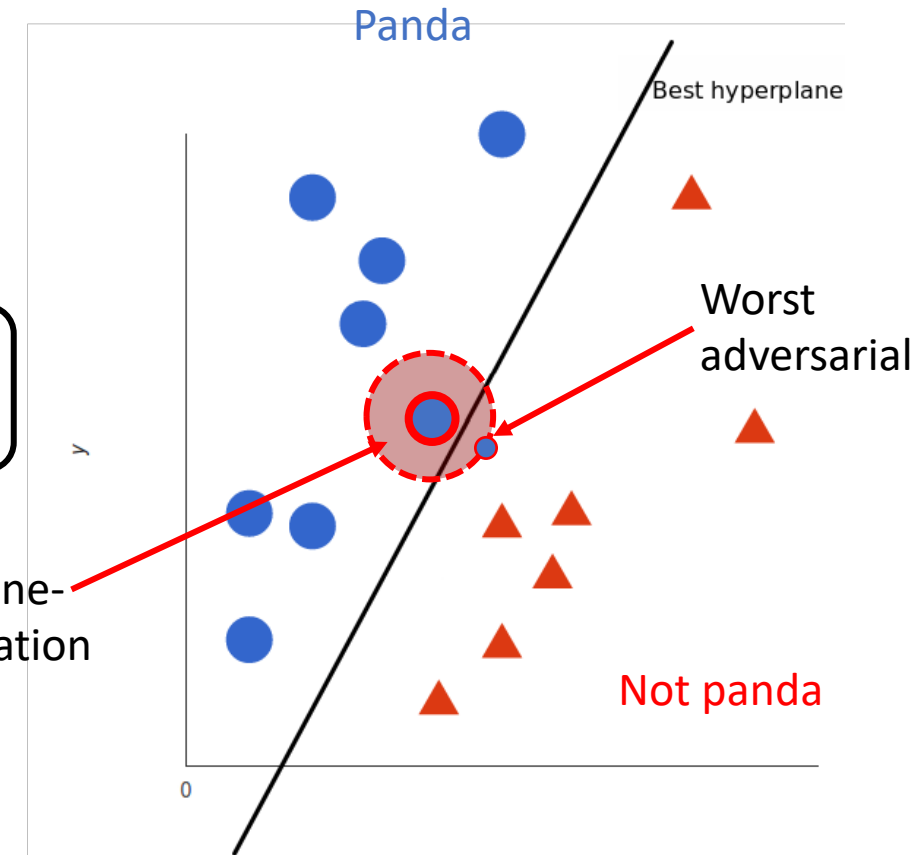
ML model for  
**panda** recognition

All possible one-  
pixel modification

0~255  
↑

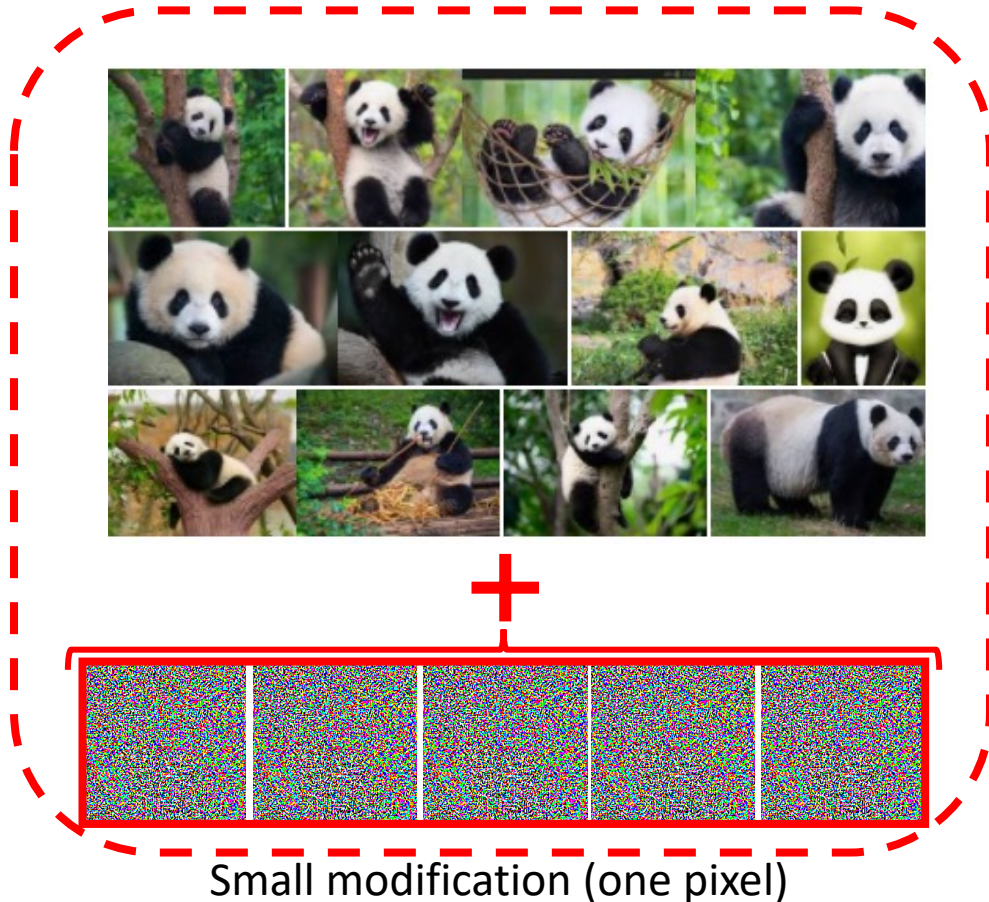
Q: for a **grey scale** images of size **28x28**, how many possible one-pixel changes can we have?

$255 \times 28 \times 28 = 199920$



# Adversarial learning and generative models

**Limit:** additive noise is only one way to generate adversarial data



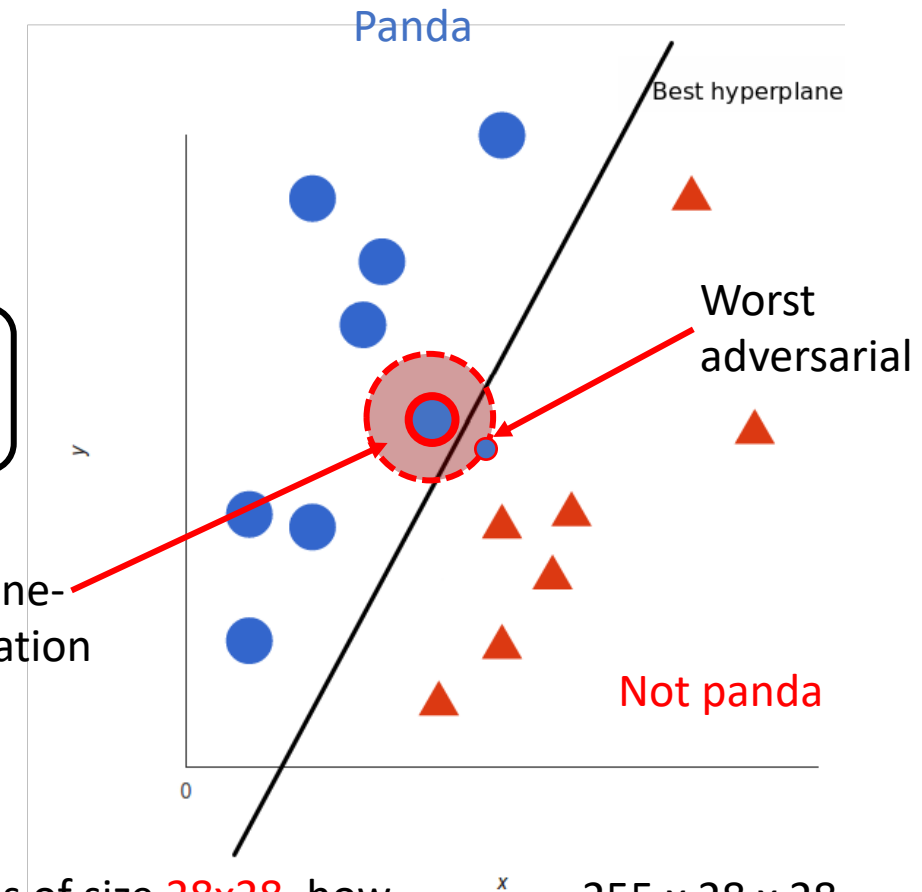
ML model for  
**panda** recognition

All possible one-  
pixel modification

0~255  
↑

Q: for a **grey scale** images of size **28x28**, how many possible one-pixel changes can we have?

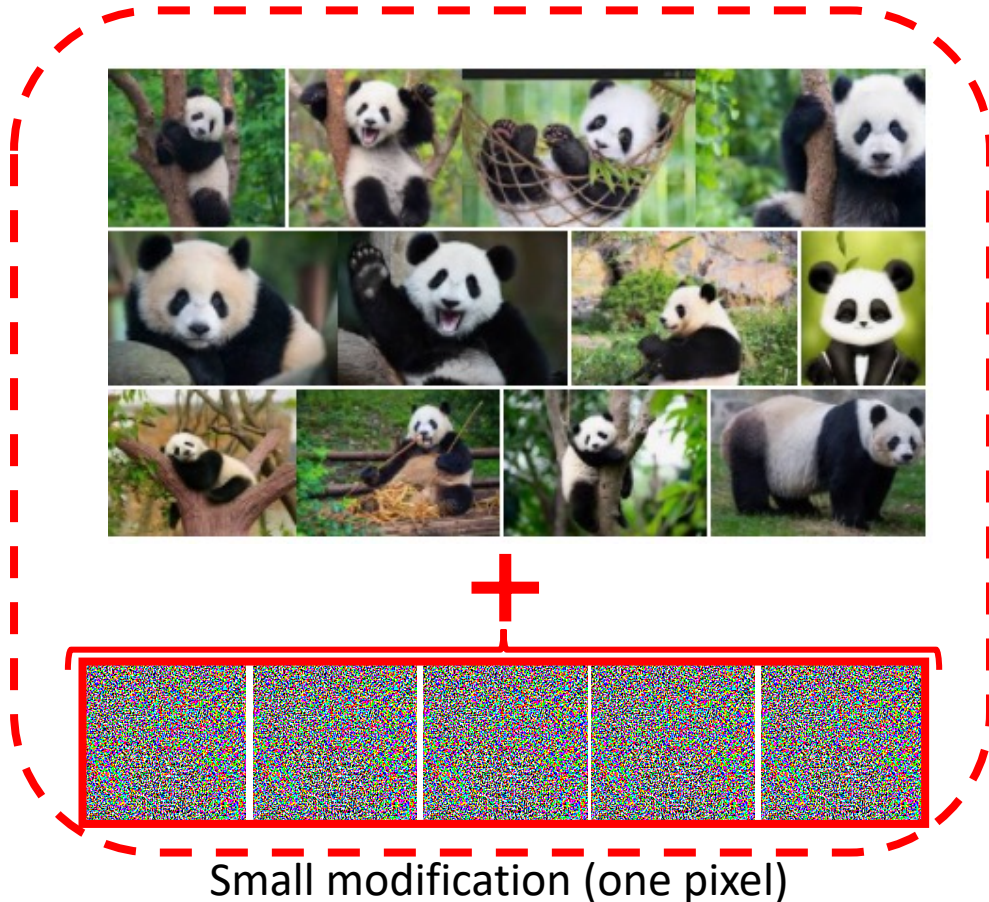
255 x 28 x 28  
= **19920**



# Adversarial learning and generative models

**Limit:** additive noise is only one way to generate adversarial data

**Q:** can generative models help?



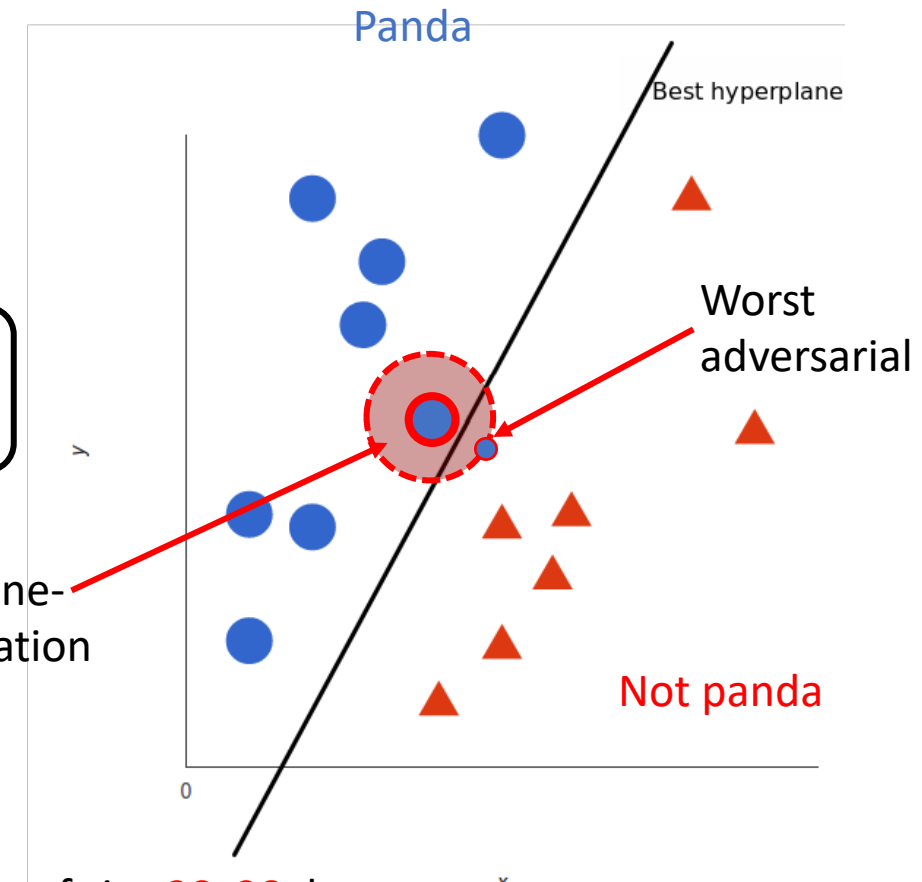
ML model for  
**panda** recognition

All possible one-  
pixel modification

0~255

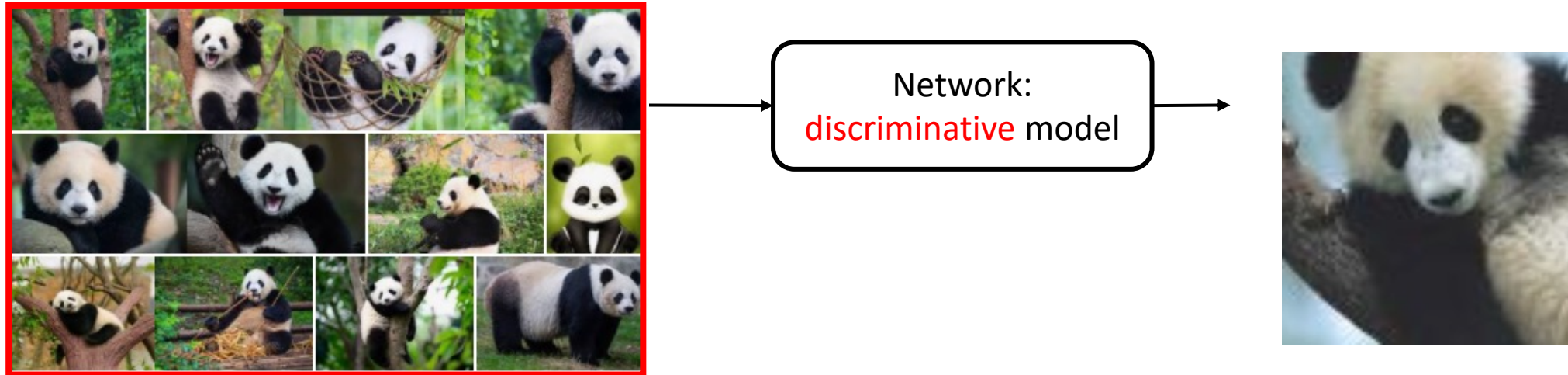
**Q:** for a **grey scale** images of size **28x28**, how many possible one-pixel changes can we have?

$255 \times 28 \times 28 = 199920$

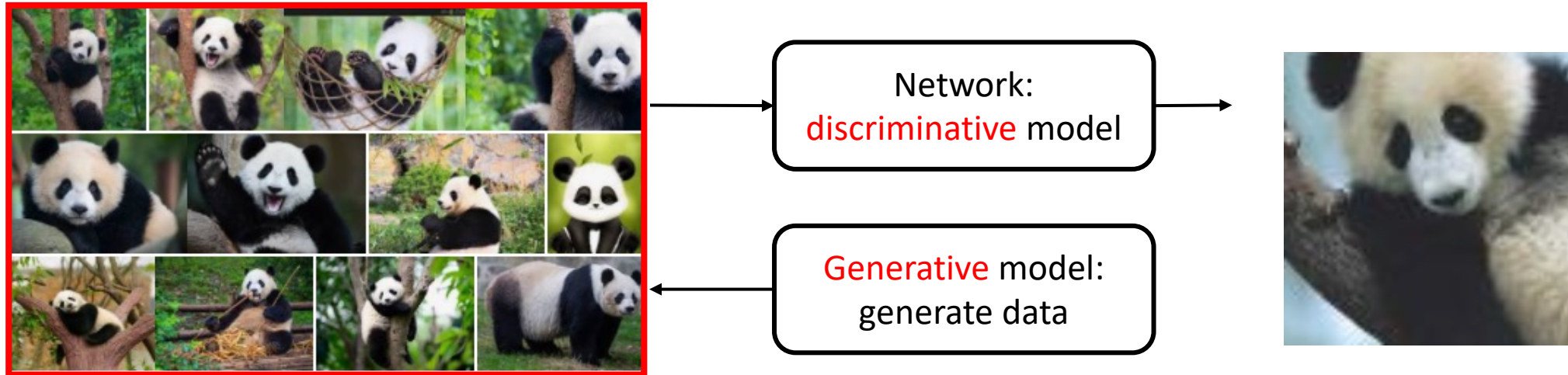




# Generative adversarial networks

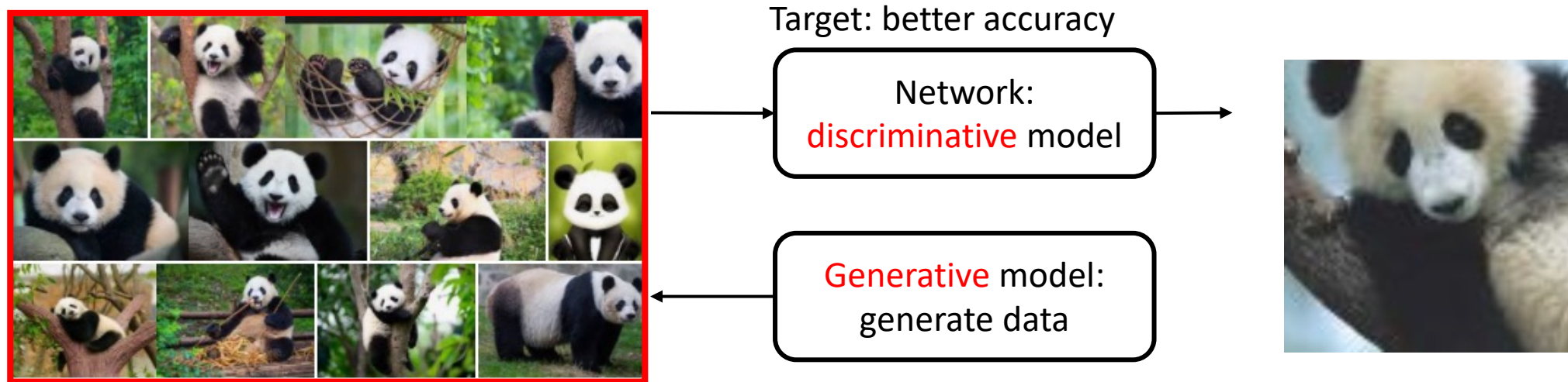


# Generative adversarial networks

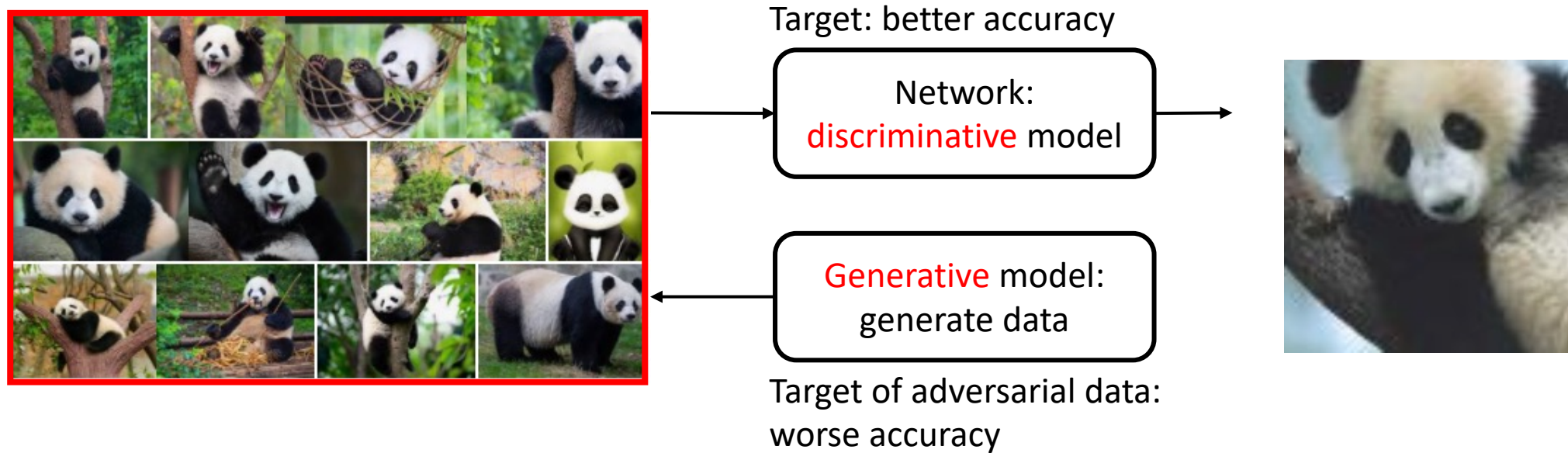




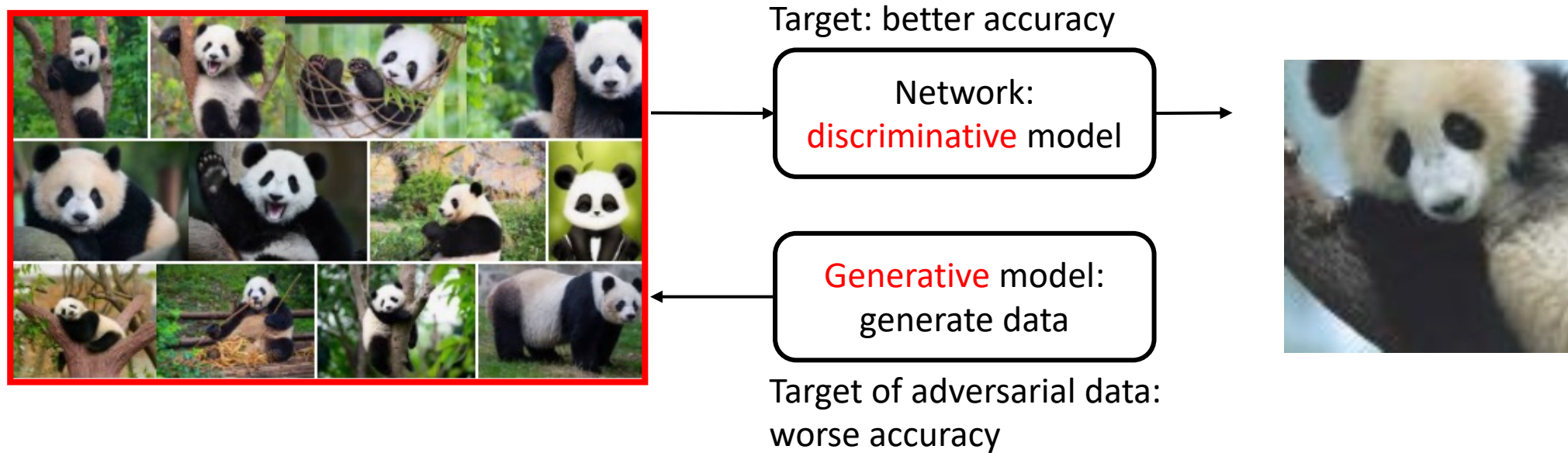
# Generative adversarial networks



# Generative adversarial networks

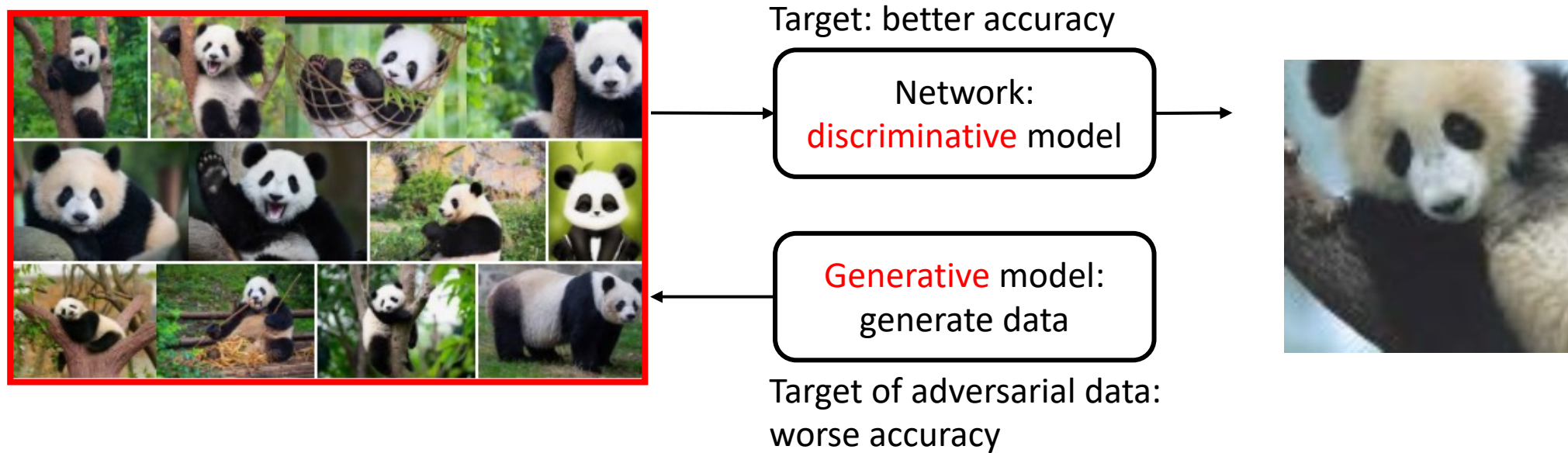


# Generative adversarial networks



$$\min_G \max_D V(D, G) = \mathbb{E}_{\mathbf{x} \sim p_{\text{data}}(\mathbf{x})} [\log D(\mathbf{x})] + \mathbb{E}_{\mathbf{z} \sim p_z(\mathbf{z})} [\log(1 - D(G(\mathbf{z})))].$$

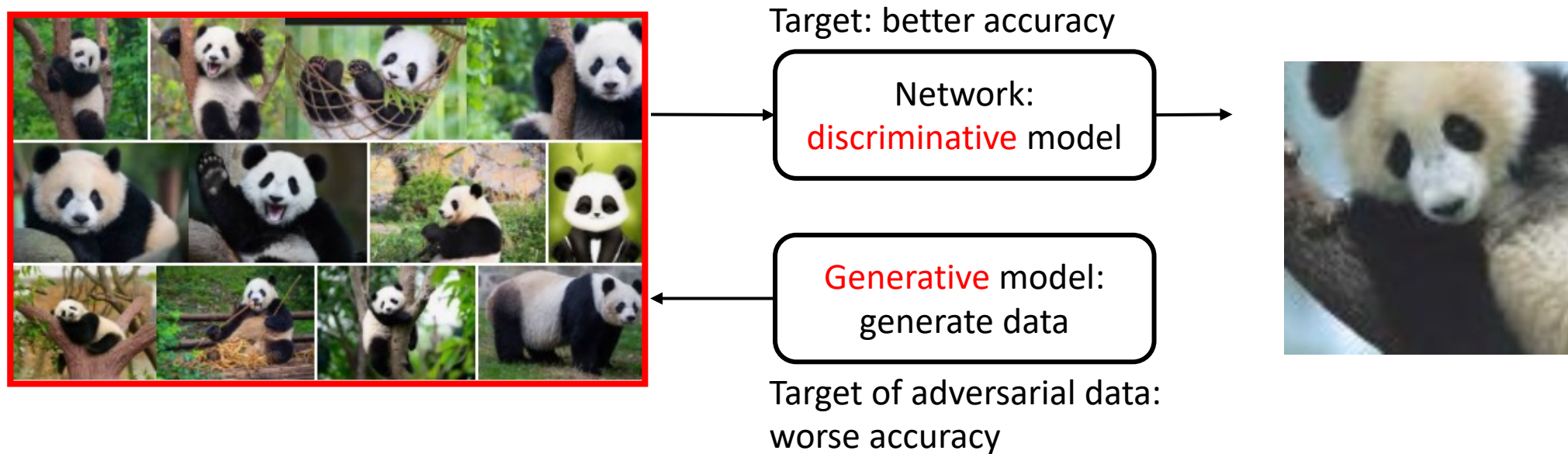
# Generative adversarial networks



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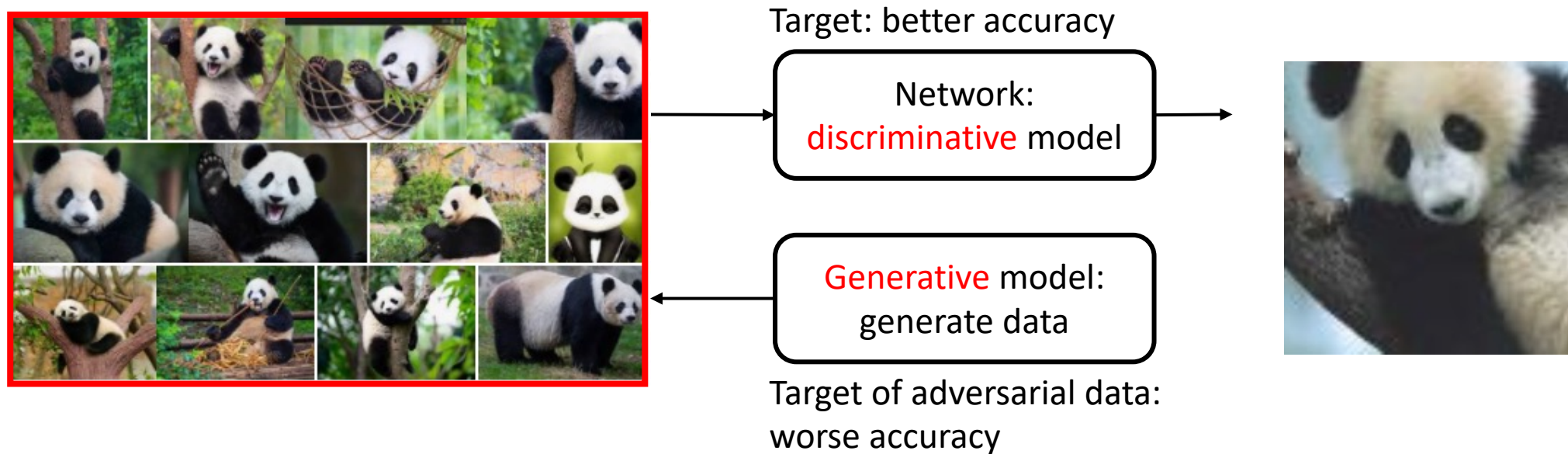


# Generative adversarial networks



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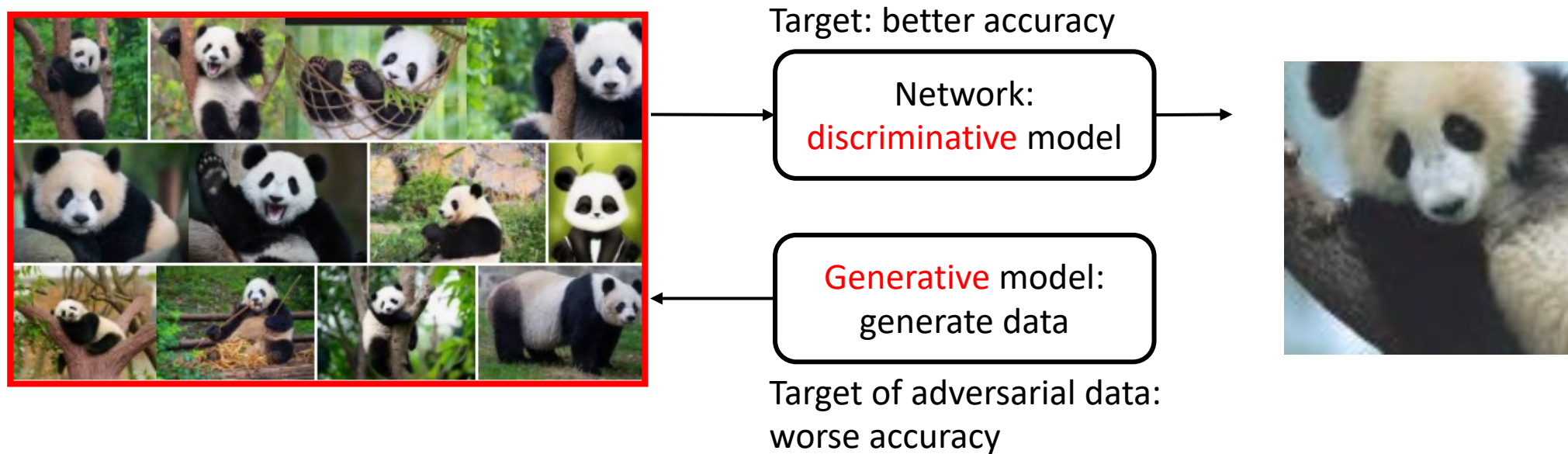
# Generative adversarial networks



$$\min_G \max_D V(D, G) = \mathbb{E}_{\mathbf{x} \sim p_{\text{data}}(\mathbf{x})} [\log D(\mathbf{x})] + \mathbb{E}_{\mathbf{z} \sim p_z(\mathbf{z})} [\log(1 - D(G(\mathbf{z})))]$$

Use original training data

# Generative adversarial networks

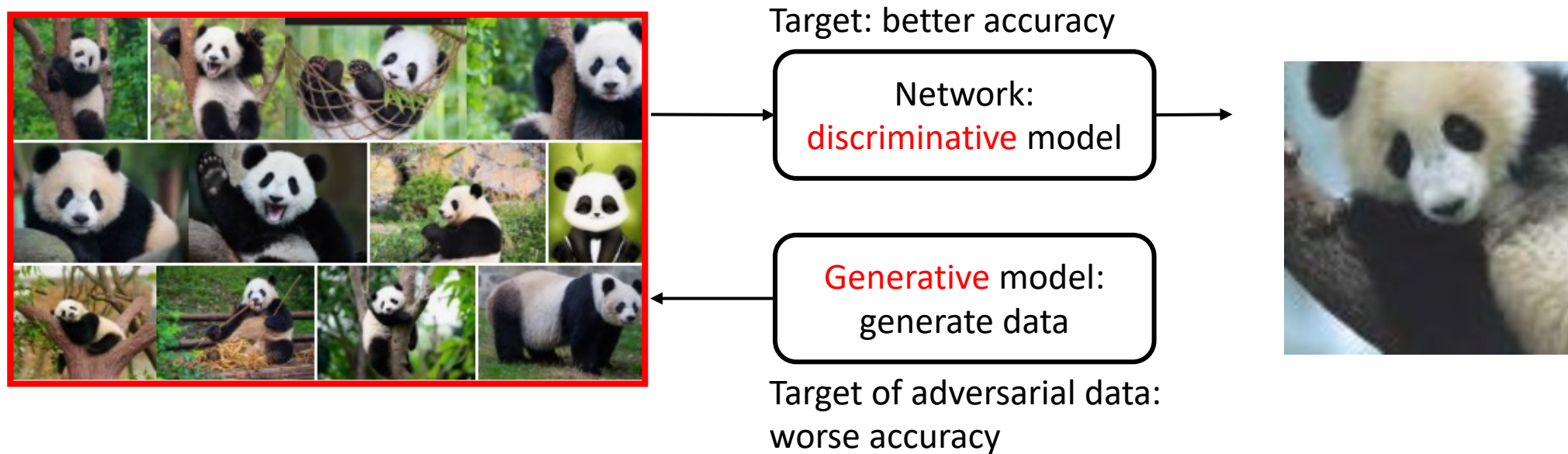


$$\min_G \max_D V(D, G) = \mathbb{E}_{\mathbf{x} \sim p_{\text{data}}(\mathbf{x})} [\log D(\mathbf{x})] + \mathbb{E}_{\mathbf{z} \sim p_z(\mathbf{z})} [\log(1 - D(G(\mathbf{z})))]$$

Use original training data

Maximum likelihood estimation

# Generative adversarial networks



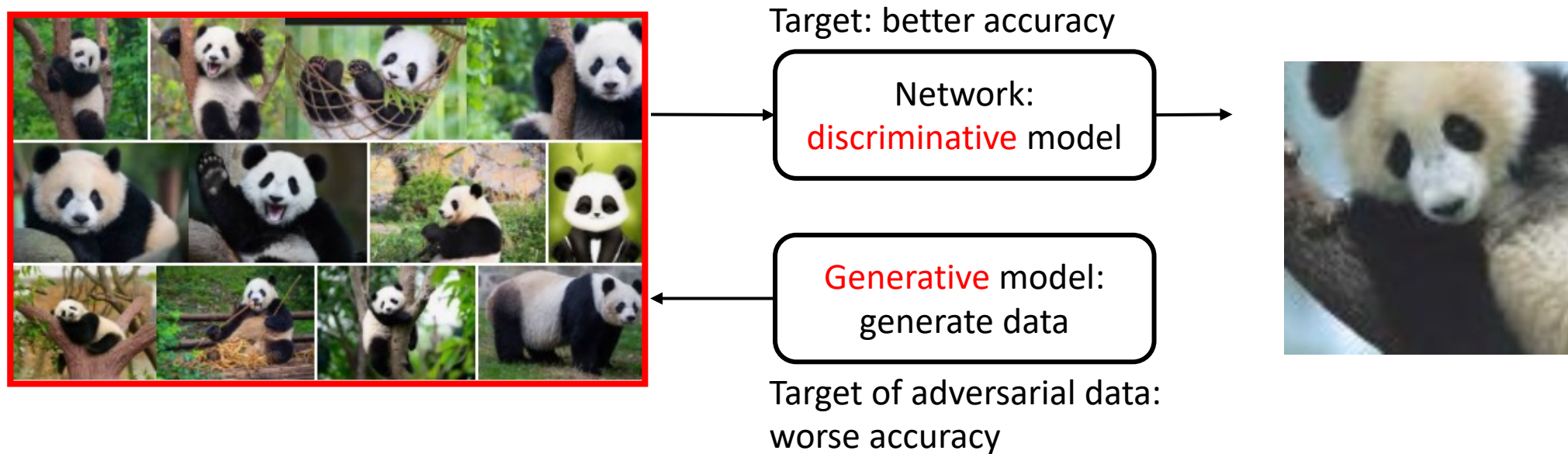
$$\min_G \max_D V(D, G) = \mathbb{E}_{\mathbf{x} \sim p_{\text{data}}(\mathbf{x})} [\log D(\mathbf{x})] + \mathbb{E}_{\mathbf{z} \sim p_z(\mathbf{z})} [\log(1 - D(G(\mathbf{z})))].$$

Use original training data  
Maximum likelihood estimation

Input noise variable



# Generative adversarial networks



$$\min_G \max_D V(D, G) = \mathbb{E}_{\mathbf{x} \sim p_{\text{data}}(\mathbf{x})} [\log D(\mathbf{x})] + \mathbb{E}_{\mathbf{z} \sim p_{\mathbf{z}}(\mathbf{z})} [\log(1 - D(G(\mathbf{z})))]$$

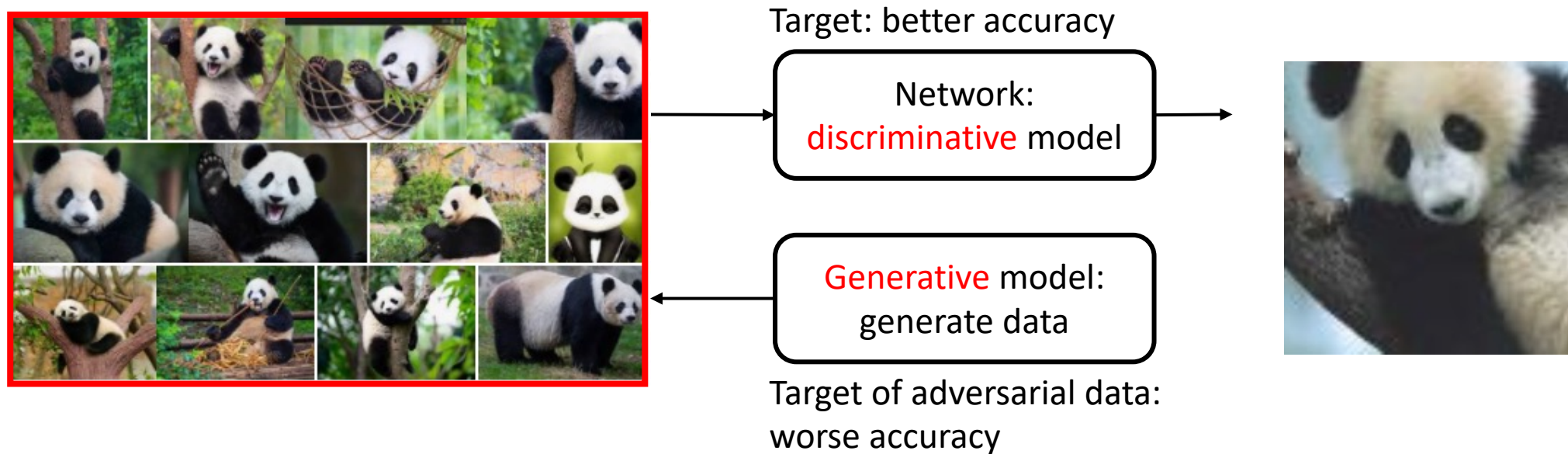
Use original training data

Maximum likelihood estimation

Input noise variable

Use **generated data** (from noise)

# Generative adversarial networks



$$\min_G \max_D V(D, G) = \mathbb{E}_{\mathbf{x} \sim p_{\text{data}}(\mathbf{x})} [\log D(\mathbf{x})] + \mathbb{E}_{\mathbf{z} \sim p_z(\mathbf{z})} [\log(1 - D(G(\mathbf{z})))]$$

Use original training data

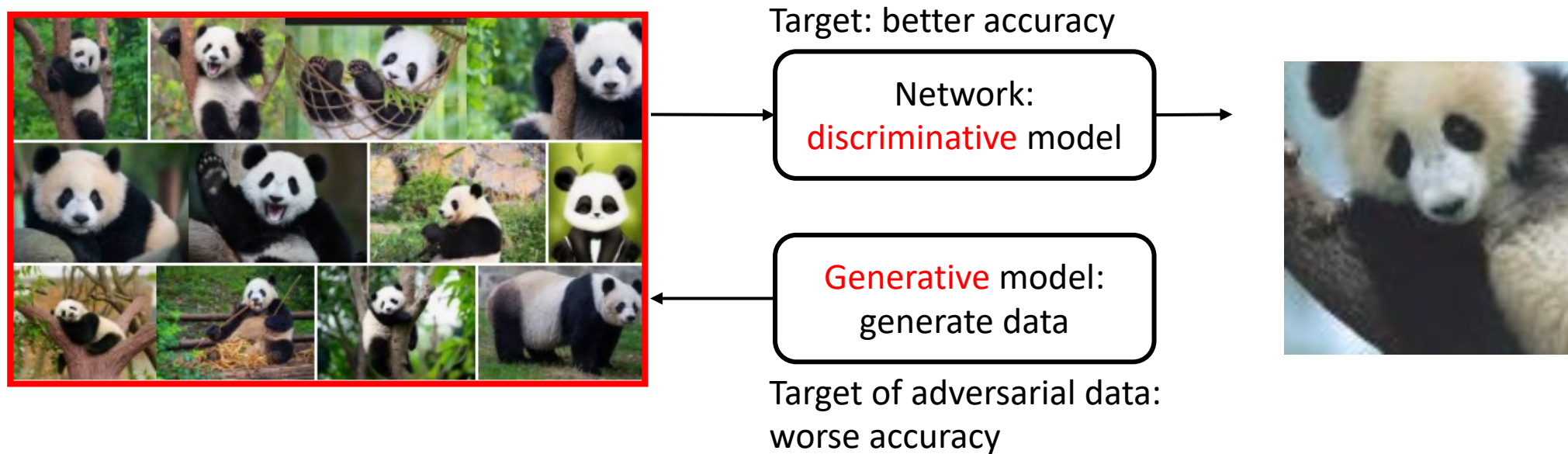
Maximum likelihood estimation

Input noise variable

Use generated data (from noise)

**Generated data** → **discriminator**

# Generative adversarial networks



$$\min_G \max_D V(D, G) = \mathbb{E}_{\mathbf{x} \sim p_{\text{data}}(\mathbf{x})} [\log D(\mathbf{x})] + \mathbb{E}_{\mathbf{z} \sim p_{\mathbf{z}}(\mathbf{z})} [\log(1 - D(G(\mathbf{z})))].$$

MLE of  $-D(\text{adv.})$

Use original training data

Maximum likelihood estimation

Input noise variable

Use generated data (from noise)

Generated data  $\rightarrow$  discriminator

# Generative adversarial networks

