4) $p_N(k) = e^{-\lambda}\frac{\lambda^k}{k!}$ $\quad M_N(s) = E[e^{sN}] = \sum_{k=0}^{\infty} e^{sk} e^{-\lambda}\frac{\lambda^k}{k!} = e^{-\lambda}\sum_{k=0}^{\infty}\frac{(e^s\lambda)^k}{k!}$

$$= e^{-\lambda} e^{(\lambda e^s)}$$

$$\boxed{M_N(s) = e^{\lambda(e^s-1)}}$$

$f_{X_i}(x) = \lambda e^{-\lambda x}$

$M_{X_i}(s) = \frac{\lambda}{\lambda - s}$

$$M_Y(s) = M_N(\log M_X(s))$$
$$= e^{\lambda(e^{\log M_X(s)}-1)}$$
$$= e^{\lambda(M_X(s)-1)}$$
$$\boxed{M_Y(s) = e^{\frac{\lambda s}{\lambda - s}}}$$

$p_{X_i}(x) = \begin{cases} p, & x=1 \\ 1-p, & x=0 \\ 0, & \text{o.w.} \end{cases}$

$M_{X_i}(s) = \sum_{k=0}^{1} e^{sk} p_{X_i}(k)$

$= 1-p+pe^s$

$$M_Y(s) = M_N(\log M_X(s))$$
$$= e^{\lambda(e^{\log M_X(s)}-1)}$$
$$= e^{\lambda(M_X(s)-1)}$$
$$\boxed{M_Y(s) = e^{\lambda p(-1+e^s)}}$$

$\longrightarrow$ Y is a poisson r.v with mean λp

$$\boxed{p_Y(x) = e^{-\lambda p}\frac{(\lambda p)^x}{x!}}$$