3a)

For given different Y values, the range of X changes for the figures with circle and ellipses. Therefore, we have independent random variables only for the first figure, and the covariance for the figure is zero. For the second and third figures, one variable has no tendency to increase or decrease as other variable increases or decreases, hence they are uncorrelated. For the last figure, one variable has tendency to increase as other variable increases and vice versa, therefore, they are positively correlated.

3b)

$X \sim \text{Uniform}(0,5)$

$f_X(x) = \begin{cases} 1/5, & 0<x<5 \\ 0, & 0 \end{cases}$

$Y = g(X) = \begin{cases} 3X+1, & X<2 \\ X, & \text{o.w.} \end{cases}$

$$E[Y] = E[g(x)] = \int_0^2 (3x+1)\tfrac{1}{5}\,dx + \int_2^5 x\tfrac{1}{5}\,dx$$
$$= \frac{1}{5}\left(\left(\frac{3x^2}{2}+x\right)\Big|_0^2 + \left(\frac{x^2}{2}\right)\Big|_2^5\right)$$
$$= \frac{1}{5}\left(6+\cancel{2}+\frac{25}{2}-\cancel{2}\right)$$
$$\boxed{E[Y] = \frac{37}{10}}$$

$$E[XY] = \int_0^2 x(3x+1)\tfrac{1}{5}\,dx + \int_2^5 x^2\tfrac{1}{5}\,dx$$
$$= \frac{1}{5}\left(\left(x^3+\frac{x^2}{2}\right)\Big|_0^2 + \left(\frac{x^3}{3}\right)\Big|_2^5\right)$$
$$= \frac{1}{5}\left(8+2+\frac{125-8}{3}\right)$$
$$\boxed{E[XY] = \frac{147}{15}}$$

$$E[X] = \int_0^5 x\tfrac{1}{5}\,dx = \left(\frac{x^2}{10}\right)\Big|_0^5 = \frac{5}{2}$$

$$\text{Cov}(X,Y) = E[XY] - E[X]E[Y]$$
$$= \frac{147}{15} - \frac{5}{2}\cdot\frac{37}{10}$$
$$= \frac{588-555}{60}$$
$$\boxed{\text{Cov}(X,Y) = \frac{11}{20}}$$

They have nonzero covariance, so they are dependent.

3c)

i)
$$E[g(X)+Y \mid X=2] = E[g(X) \mid X=2] + E[Y \mid X=2]$$
$$= g(2) + E[Y \mid X=2]$$

ii) Let $E[g(X)Y \mid X] = h_1(X)$ and $g(X)E[Y \mid X] = h_2(X)$

For an arbitrary $X = x_0$

$$h_1(x_0) = E[g(X)Y \mid X=x_0]$$
$$= E[g(x_0)Y \mid X=x_0]$$
$$= g(x_0)E[Y \mid X=x_0] \rightarrow$$

$$h_2(x_0) = g(x_0)E[Y \mid X=x_0]$$

They have same value for any $X = x_0$, hence

$h_1(X) = h_2(X) \Rightarrow E[g(X)Y \mid X] = g(X)E[Y \mid X]$

iii) $f_X(x) = \begin{cases} 1/2, & 0<x<2 \\ 0, & \text{o.w.} \end{cases}$

$$E[X^2] = \int_0^2 x^2\tfrac{1}{2}\,dx = \left(\frac{x^3}{6}\right)\Big|_0^2 = 4/3$$

$$E[X^3] = \int_0^2 x^3\tfrac{1}{2}\,dx = \left(\frac{x^4}{8}\right)\Big|_0^2 = 2$$

$$E[Y \mid X] = X^3$$
$$E[E[Y \mid X]] = E[X^3]$$
$$E[Y] = E[X^3] = 2$$
$$E[X^2+Y] = E[X^2] + E[Y]$$
$$= 4/3 + 2.$$
$$\boxed{E[X^2+Y] = \frac{10}{3}}$$