

Middle East Technical University
Department of Electrical and Electronics Engineering
EE230: Probability and Random Variables
Spring 2021-2022
Homework 1

Due: 23:59 on March 27, 2022

1. Suppose Ayşe, Bora, and Ceyda take turns in tossing a coin. The probability of getting a head is p for each trial. First Ayşe tosses the coin, then Bora, then Ceyda, then Ayşe, and so on. The first one to get a head wins.
 - a) Define a sample space Ω for this experiment. Clearly explain what is represented by each outcome.
 - b) Identify the following events in terms of Ω , that is, for each one of the following events determine the corresponding subset of Ω :
 - i) Event A: Ayşe wins.
 - ii) Event B: Bora wins.
 - iii) $A \cup B$: Ayşe or Bora wins.
 - iv) $(A \cup B)^c$
 - c) Find $P(A)$ and $P(B)$.
 - d) Using probability axioms, state how to find $P(A \cup B)$ and $P((A \cup B)^c)$. Then find $P(A \cup B)$ and $P((A \cup B)^c)$.
 - e) Verify that $P(A \cap B) \geq P(A) + P(B) - 1$ for the events defined in part b. Also prove using probability axioms that for arbitrary two events C and D, the following is always satisfied: $P(C \cap D) \geq P(C) + P(D) - 1$.
2. Assume a number is selected from the following set $\{1, \dots, 100\}$ uniformly at random. It is observed that this number is divisible by 2. We are interested in whether this number is also divisible by 3 or 5, denoted as event B.
 - a) Write event B in terms of A_2, A_3, A_5 by using the following definitions,
 A_2 : event for the case the randomly selected number is divisible by 2
 A_3 : event for the case the randomly selected number is divisible by 3
 A_5 : event for the case the randomly selected number is divisible by 5
 - b) Determine the probability of event B.
 - c) Let a and b , be two prime numbers less than N . Determine the probability of a randomly selected number from the set $\{1, \dots, N\}$ be divisible by a or b , if the selected number is already observed to be divisible by 2. Find the required probability in terms of $P(A_m)$, $m=1, \dots, N$, where A_m is the particular event for the case that the selected number is divisible by m .
3. Suppose a family has k children with p_k , where $p_1=0.25$, $p_2=0.45$, $p_3=0.2$, $p_4=0.1$. Suppose all children are in different age (i.e. no twins, triples, or such). From this family, a child is randomly chosen.
 - a) Given that the chosen child is the youngest child in the family, find the conditional probability that the family has only 1 child.
 - b) Given that the chosen child is the youngest child in the family, find the conditional probability that the family has 3 children.
 - c) Find the probability that the chosen child is the oldest child in the family.
4. Let Ω be a sample space with three outcomes $\Omega = \{\omega_p, \omega_q, \omega_r\}$ whose singleton event probabilities are equal to p , q , r , respectively.
 - a) Assume 5 independent trials are performed for this experiment. What is the probability of observing 3 times ω_p , 2 times ω_q and no ω_r outcomes as a result of these experiments?
 - b) Assume 5 independent trials are performed for the same experiment. What is the probability of observing outcome ω_p before ω_q during these experiments?
 - c) Now, assume that these experiments are repeated infinitely. Find the probability of outcome ω_p occurs before ω_q during these experiments.

Middle East Technical University
Department of Electrical and Electronics Engineering
EE230: Probability and Random Variables
Spring 2021-2022
Homework 1

Due: 23:59 on March 27, 2022

5. MATLAB. The main goal of this question is to give you an opportunity (and a reason) to gain hands-on experience on writing a report in as a Live Script, which will hopefully serve as a template for your future reports and home works. You will treat this question as a mini, stand-alone homework to write a report, and will mainly be graded based on the presentation of your work rather than your coding skills and theoretical knowledge.
- A Live Script file that guides you through the question is shared with you (file name: *ee230_hw1_MATLAB.mlx*). Feel free to refer to the shared Live Script from the tutorial and use the question Live Script as a template for your own report. If you do use the question Live Script as a template, make sure to delete any previous text from it (aside from, say, the headers), **even if it means you should rewrite what is already written in your own words. The work you submit must be your own, complete work as a stand-alone, coherent report.**
 - The question Live Script contains incomplete code snippets and underlined questions for you to comment. Look for the bullet points to see what you should be coding and what is expected of you. Note that you can choose not to abide by the code pieces left: You can go for different solutions and implementations, as long as they do the job.
 - Present your results well. Whenever you plot anything, make sure to give appropriate titles and axis labels (variable dependent, if appropriate), with legends whenever you have multiple plots in the same figure. In short, make pretty figures. Also: Omitting the semicolon at the end of a line where you produce a result can be a good way of presenting a numerical computation result.
 - Document your code. The main point of using Live Scripts is to have some pretty-looking piece of explanatory text accompany the code. Make comments, explain your procedure, experiment, and show your results. Such a coding format (just like Jupyter Notebooks for Python) is best-suited for exploratory coding: Use it to your advantage.
 - After you complete your report as a Live Script, export it as a PDF and append it to the solutions of the previous questions. Remember that the compiled PDF will not show the interactive controls such as numeric sliders and buttons but only their values at the time of export. While this means there is no way to grade you on using interactive controls, their use should help you in answering the questions that you *can* be graded on. So, you are strongly encouraged to use them.
 - Lastly: Don't forget to start clean and continue so.