EE301 Homework-4

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Question 1

a)

$$x_s(t) = x(t)s(t) = \sum_{n=-\infty}^{\infty} x(t)\delta(t - nT_s) = \sum_{n=-\infty}^{\infty} x(nT_s)\delta(t - nT_s)$$

where T_s is called the sampling period.
By the modulation property of CTFT: $X_s(j\omega) = \frac{1}{2\pi} X(j\omega) * S(j\omega)$

s(t) is a periodic signal, therefore we should first find its CTFS representation:

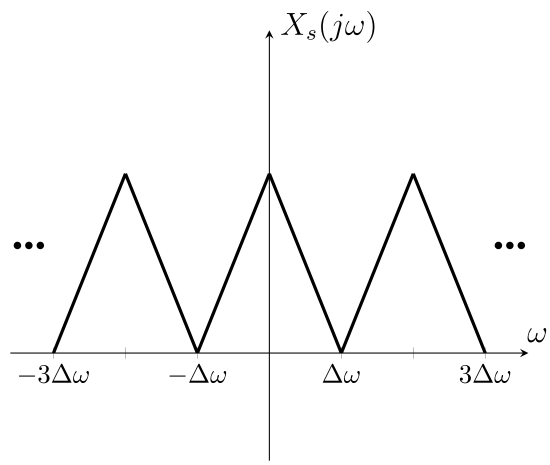
$$s(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_s t}, \omega_s = \frac{2\pi}{T_s}$$

$$a_k = \frac{1}{T_s}\int_{-T_s/2}^{T_s/2} \underbrace{s(t)}_{\delta(t)} e^{-jk\omega_s t} dt \Rightarrow a_k = \frac{1}{T_s}$$

$$s(t) = \sum_{k=-\infty}^{\infty} \frac{1}{T_s} e^{jk\omega_s t} \longrightarrow S(j\omega) = \sum_{k=-\infty}^{\infty} \frac{2\pi}{T_s}\delta(\omega - k\omega_s)$$

$$X_s(j\omega) = \frac{1}{2\pi} X(j\omega) * \sum_{k=-\infty}^{\infty} \frac{2\pi}{T_s}\delta(\omega - k\omega_s) = \sum_{k=-\infty}^{\infty} \frac{1}{T_s} X(j(\omega - k\omega_s))$$

From the Nyquist sampling theorem, the inequality $\omega_s \geq 2\Delta\omega$ should be satisfied so that the shifted replicas of $X(j\omega)$ do not overlap. Thus, it yields perfect reconstruction of the signal $x(t)$ from $x_s(t)$ with no aliasing. So, the minumum sampling rate, $\omega_s = 2\Delta\omega$. The Fourier transform of the sampling system output, $X_s(j\omega)$ can be seen below.

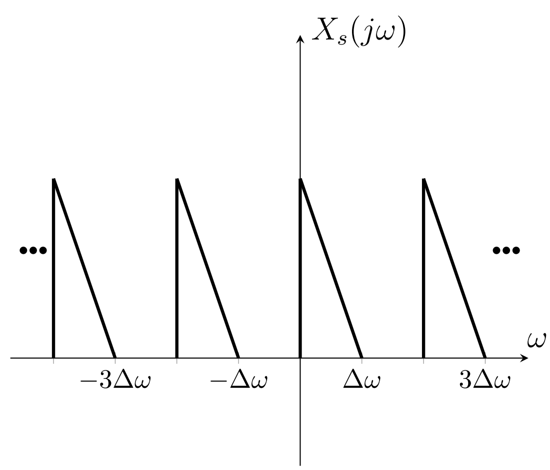


b)

$X(j\omega)$ is not symmetric with respect to y-axis, thus from the symmetry property of CTFT it can be concluded that $x(t)$ is a complex-valued signal.

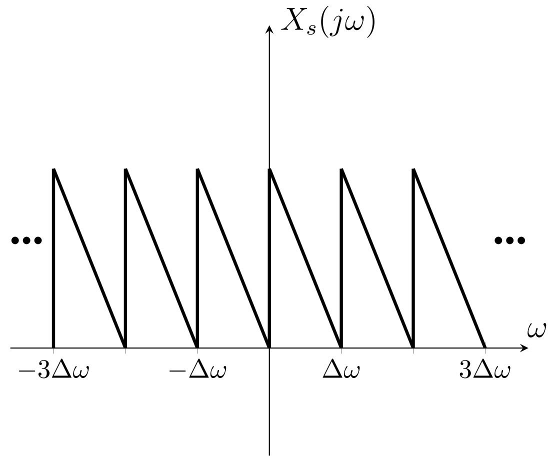
i)

Since the sampling rate of the signal is equal to the Nyquist rate, the aliasing does not occur. The Fourier transform of $X_s(j\omega)$ is plotted below for $T_s = \frac{\pi}{\Delta\omega}$.



ii)

As it can be seen from the graph of the Fourier transform of $x(t)$, the graph is not symmetric with respect to y-axis, i.e., $X(j\omega)$ has no component at negative frequencies. Therefore, as it can be seen from the graph of $X_s(j\omega)$ below, the aliasing can be avoided even when $T_s = \frac{2\pi}{\Delta\omega}$ which below the Nyquist rate.


$X_s(j\omega)$
ω
$-3\Delta\omega$
$-\Delta\omega$
$\Delta\omega$
$3\Delta\omega$

c)

Question 2

i)

ii)

Question 3

a)

b)

i)

ii)

iii)

c)

i)

ii)

d)

i)

ii)

iii)

Question 4

a)

b)

c)

d)