

4

a

$$h_1[n] = \sum_{k=-\infty}^{n+2} \delta[k] = \begin{cases} 1, & n \geq -2 \\ 0, & n < -2 \end{cases} = u[n+2]$$
$$h_2[n] = \sum_{k=-\infty}^{n-2} \delta[k] = \begin{cases} 1, & n \geq 2 \\ 0, & n < 2 \end{cases} = u[n-2]$$

b

i: Both systems are not memoryless as the output signals depend on past values of input signals.

ii: System 1 is not causal as the output signal depends on the values of the input signal at future time ($n+1, n+2$). System 2, however, is causal since the output signal only depends on the past values of the input signal.

iii: Both systems have unbounded output signals for the input signal chosen as unit step function. Therefore, the systems are not stable.

c

i.

The systems are connected in parallel, so we can calculate the impulse response of the overall system $h[n]$ as:

$$h[n] = h_1[n] - h_2[n]$$

$$h[n] = u[n+2] - u[n-2]$$

ii.

We can calculate the output signal of the overall system $y[n]$ for an arbitrary input signal $x[n]$ as:

$$y[n] = x[n-1] + x[n] + x[n+1] + x[n+2]$$

The system is neither memoryless nor causal since $y[n]$ depends on future values of $x[n]$.

For a bounded input signal, we can say that there exist $M > 0$ such that $|x[n]| < M$. Then $|y[n]| < 4M$ and $y[n]$ is bounded, therefore the system is stable.