1

System 1

1 - The system is memoryless since the value of the output $y(t)$ depends on only the value of the input $x(t)$ at present time instant.
2 - To determine the linearity of the system, apply the input $\alpha x(t)$ and let the output $\hat{y}(t) = \alpha x(t) cos(2\pi f t) = \alpha y(t) \longrightarrow$ the system is linear.
3 - The system is causal since the output $y(t)$ does not depend on the future value of the input $x(t)$.
4 - The system is time-variant. To see this, apply the input $x(t - t_0)$ and let the output $\hat{y}(t) = x(t - t_0) cos(2\pi f t) \neq y(t - t_0) = x(t - t_0) cos(2\pi f (t - t_0))$.
5 - A system is said to be stable if any bounded input creates a bounded output. Let $x(t)$ be a bounded. That is, there exist $M > 0$ such that $|x(t)| \leq M$. Also, $-1 \leq cos(2\pi f t) \leq 1$. Thus, $|y(t)| = |x(t) cos(2\pi f t)| \leq |x(t)| \, |cos(2\pi f t)| \leq M \longrightarrow$ so the system is stable.

System 2

1 - The system is memoryless.
2 - The system is non-linear since if we apply the input $\alpha x(t)$ then the output becomes $\hat{y}(t) = c_1 \alpha x(t) + c_2 (\alpha x(t))^2 = \alpha c_1 x(t) + \alpha^2 c_2 x^2(t) \neq \alpha y(t) = \alpha c_1 x(t) + \alpha c_2 x^2(t)$
3 - The system is causal.
4 - The system is time-invariant consider that if we apply the input $x(t - t_0)$ and let the output $\hat{y}(t) = c_1 x(t - t_0) + c_2 x^2(t - t_0) = y(t - t_0)$
5 - Apply a bounded input $x(t)$ such that $|x(t)| \leq M$ then $|y(t)| = |c_1 x(t) + c_2 x^2(t)| \leq |c_1| \, |x(t)| + |c_2| \, |x^2(t)| \leq |c_1| \, M + |c_2| \, M^2 < \infty \longrightarrow$ the system is stable.

System 3

1 - The system is memoryless.
2 - The system is non-linear. Apply the input $\alpha x(t)$ and let the output $\hat{y}(t) = \alpha x(t) + 4 \neq \alpha y(t) = \alpha x(t) + \alpha 4$
3 - The system is causal.
4 - Apply the input $x(t - t_0)$:
$\hat{y}(t) = x(t - t_0) + 4 = y(t - t_0) \longrightarrow$ the system is time-invariant.
5 - Let $x(t)$ be bounded input such that $|x(t)| \leq M$. Then,
$|y(t)| = |x(t) + 4| \leq |x(t)| + 4 \leq M + 4$ (which is a finite number). So, the system is stable.

System 4

1 - The system is not memoryless as it is not causal.
2 - The system is linear.
3 - For $t < 0$, $y(t)$ is calculated by future values of $x(t)$, therefore, the system is not causal.
4 - Apply the input $x(t - t_0)$:
$\hat{y}(t) = x((t - t_0)/3) = y(t - t_0) \longrightarrow$ the system is time-invariant.
5 - Let $x(t)$ be bounded input such that $|x(t)| \leq M$. Then, $|y(t)| = |x(t/3)|$ (the expanded version of $x(t)$) $\leq M \longrightarrow$ the system is stable.

System 5

1 - The system has a memory since the value of the output $y(t)$ depends on the future values of the input $x(t)$.
2 - The system is linear.
3 - The system is not causal because the output $y(t)$ does not depend only on present and past values of the input.
4 - Apply $x(t - t_0)$:
$\hat{y}(t) = tx(t - t_0 + 5) \neq y(t - t_0) = (t - t_0)x(t - t_0 + 5) \longrightarrow$ the system is time-variant.
5 - The system is not stable. To see this, consider the following example:
$x(t) = u(t) \Rightarrow x(t + 5) = u(t + 5)$ also $|x(t + 5)| = |u(t + 5)| \leq 1$ which means that the input is bounded. However, the output $y(t) = tu(t + 5)$ is not bounded. Therefore, the system is not stable.

System 6

$$y(t) = u(x(t)) = \begin{cases} 1, & x(t) \geq 0 \\ 0, & x(t) < 0 \end{cases}$$

1 - The system is memoryless.
2 - The system is non-linear, apply $\alpha x(t)$:
$\hat{y}(t) = u(\alpha x(t))$ may not be equal to $\alpha y(t) = \alpha u(x(t))$. For example, let the input $x(t) = u(t)$ then for $t \geq 0$, the $x(t) = 1$. Also, choose $\alpha = -1$. As a result, for $t \geq 0$:
$u(\alpha x(t)) = 0 \neq \alpha y(t) = \alpha u(x(t)) = -1$.
3 - The system is causal.
4 - Apply $x(t - t_0)$:
$\hat{y}(t) = u(x(t - t_0)) = y(t - t_0) \longrightarrow$ the system is time-invariant.
5 - Remember that $|u(t)| \leq 1$ so it is bounded. In this system, whatever the input $x(t)$ is,

the output $y(t) = u(x(t)) = \begin{cases} 1, & x(t) \geq 0 \\ 0, & x(t) < 0 \end{cases}$. Hence, the system is stable.

2

a

$y_1(t) = x_1(t) * h(t) = \int_{-\infty}^{t} \alpha^{t-\tau}\, d\tau = \frac{-1}{ln\alpha}$
$y_2(t) = x_2(t) * h(t) = \int_{-\infty}^{\infty} x_2(\tau)h(t-\tau)\, d\tau$
For $t < 0$, $y_2(t) = 0$
For $t > 0$, $y_2(t) = \int_{0}^{t} \alpha^{t-\tau}\, d\tau = \frac{\alpha^t - 1}{ln\alpha}$
$\mathrm{y}_2(t) = \frac{\alpha^t - 1}{ln\alpha}u(t)$
$y_3(t) = x_3(t) * h(t) = \int_{-\infty}^{\infty} x_3(\tau)h(t-\tau)\, d\tau$
For $t < -1$, $y_3(t) = \int_{-\infty}^{t} \alpha^{t-\tau}\, d\tau = \frac{-1}{ln\alpha}$
For $t > -1$, $y_3(t) = \int_{-\infty}^{-1} \alpha^{t-\tau}\, d\tau = \frac{-\alpha^{t+1}}{ln\alpha}$
$\mathrm{y}_3(t) = \frac{-1}{ln\alpha} + (\frac{-\alpha^{t+1}+1}{ln\alpha})u(t+1)$

b

Let $x_5(t) = u(t+1)$. Then $x_4(t) = x_5(t) - x_2(t)$.
$y_5(t) = x_5(t) * h(t) = y_2(t+1) = \frac{\alpha^{t+1}-1}{ln\alpha}u(t+1)$
$y_4(t) = x_4(t) * h(t) = (x_5(t) - x_2(t)) * h(t)$
$= (\mathrm{x}_5(t) * h(t)) - (x_5(t) * h(t))$
$= \mathrm{y}_5(t) - y_2(t)$
$\mathrm{y}_4(t) = \frac{\alpha^{t+1}-1}{ln\alpha}u(t+1) - \frac{\alpha^t-1}{ln\alpha}u(t)$

3

a

Recall that in continuous-time systems one can apply the derivative operation to any arbitrary input signal. For example,
$\Delta t = \frac{d}{dt}u(t) \longrightarrow \int_{\infty}^{-\infty} \Delta t\, dt$
Also, it can be applied by using the formal definition of the derivative:
$\lim_{h \to 0} \frac{x(t+h)-x(t)}{h}$
However, in discrete-time systems h cannot go to zero and the minimum value for h can be one. So, in discrete time the derivative expression becomes

$\lim_{h\to 1} \frac{x(n+h)-x(n)}{h} = x[n+1] - x[n]$
Thus, the derivative operation in continuous-time systems is analogous of the difference operation in discrete-time. Hence, we can obtain the impulse response of the difference operation as: $h[n] = \delta[n] - \delta[n-1]$

b

By convolution, $y[n] = x[n] * h[n] = x[n] * (\delta[n] - \delta[n-1])$
By distributive property of the convolution operation,
$x[n] * (\delta[n]–\delta[n-1]) = (x[n] * \delta[n])–(x[n] * \delta[n-1])$
$y[n] = x[n]–x[n-1]$

c

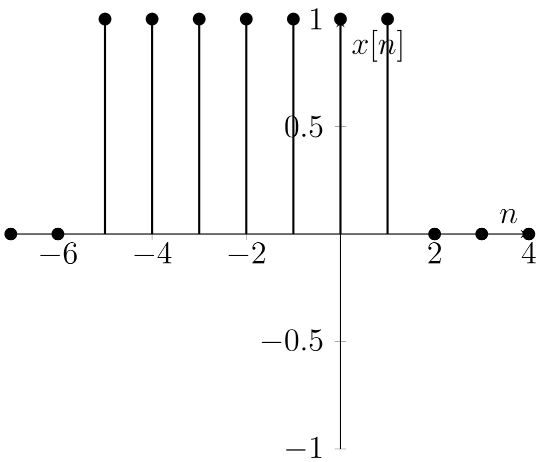
$e^{j\Omega_0 n}(1 - e^{-j\Omega_0}) = (cos(\Omega_0 n) + jsin(\Omega_0 n))(1 - e^{-j\Omega_0})$
$=\cos(\Omega_0 n) + jsin(\Omega_0 n) - (cos(\Omega_0 n) + jsin(\Omega_0 n))(cos(\Omega_0 n) - jsin(\Omega_0 n))$
$=\cos(\Omega_0 n) + jsin(\Omega_0 n) - cos(\Omega_0 n)cos(\Omega_0) + jcos(\Omega_0 n)sin(\Omega_0) - jsin(\Omega_0 n)cos(\Omega_0) - sin(\Omega_0 n)sin(\Omega_0)$
$=\cos(\Omega_0 n) + jsin(\Omega_0 n) - (cos(\Omega_0 n)cos(\Omega_0) + sin(\Omega_0 n)sin(\Omega_0)) - j(sin(\Omega_0 n)cos(\Omega_0) - cos(\Omega_0 n)sin(\Omega_0))$
$=\cos(\Omega_0 n) + jsin(\Omega_0 n) - cos(\Omega_0 n - \Omega_0) - jsin(\Omega_0 n - \Omega_0)$
$=\cos(\Omega_0 n) - cos(\Omega_0(n-1)) + j(sin(\Omega_0 n) - sin(\Omega_0(n-1)))$
$|y[n]| = ((cos(\Omega_0 n) - cos(\Omega_0(n-1)))^2 + (sin(\Omega_0 n) - sin(\Omega_0(n-1)))^2)^{0.5}$
$= (\cos^2(\Omega_0 n) - 2cos(\Omega_0 n)cos(\Omega_o(n-1)) + cos^2(\Omega_0(n-1)) + sin^2(\Omega_0 n) - 2sin(\Omega_0 n)sin(\Omega_o(n-1)) + sin^2(\Omega_0(n-1)))^{0.5}$
$= (2 - 2(\cos(\Omega_0 n)cos(\Omega_0(n-1)) + sin(\Omega_0 n)sin(\Omega_0(n-1))))^{0.5}$
$= (2 - 2\cos(\Omega_0 n - \Omega_0(n-1)))^{0.5}$
$= (2 - 2\cos(\Omega_0))^{0.5}$

d

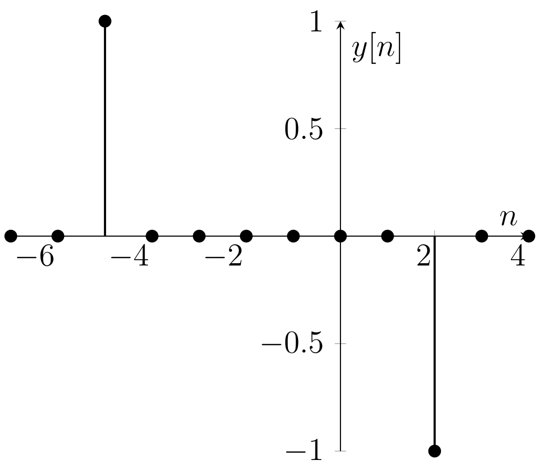
$\Omega_0 = 0 \longrightarrow |y[n]| = \sqrt{2 - 2cos(0)} = 0$
$\Omega_0 = \frac{2\pi}{8} \longrightarrow |y[n]| = \sqrt{2 - 2cos(\frac{2\pi}{8})} = \sqrt{2 - \sqrt{2}}$
$\Omega_0 = \frac{2\pi}{4} \longrightarrow |y[n]| = \sqrt{2 - 2cos(\frac{2\pi}{4})} = \sqrt{2}$
$\Omega_0 = \frac{2\pi}{2} \longrightarrow |y[n]| = \sqrt{2 - 2cos(\frac{2\pi}{2})} = 2$

e

$x[n] = u[n+5] - u[n-2]$ can be plotted as:



We can calculate $y[n]$ for $x[n] = u[n+5] - u[n-2]$ by using the answer in (b) as:
$y[n] = u[n+5] - u[n+2] - (u[n+4] + u[n-3])$
$y[n]$ can be plotted as:



From the graph of $y[n]$, we can say that $x[n]$ has edges at $n = -5$ and $n = 2$.

f

From (d), we can say that magnitude of the output signal of the system increases as the frequency increases. In (f), $x[n]$ has rapid changes at $n = -5$ and $n = 2$, and as a result, $y[n]$ has magnitude 1 at $n = -5$ and $n = 2$, and 0 elsewhere.

4

a

$h_1[n] = \sum_{k=-\infty}^{n+2} \delta[k] = \begin{cases} 1, & n \geq -2 \\ 0, & n < -2 \end{cases} = u[n+2]$

$\mathrm{h}_2[n] = \sum_{k=-\infty}^{n-2} \delta[k] = \begin{cases} 1, & n \geq 2 \\ 0, & n < 2 \end{cases} = u[n-2]$

b

i: Both systems are not memoryless as the output signals depend on past values of input signals.
ii: System 1 is not causal as the output signal depends on the values of the input signal at future time $(n+1, n+2)$. System 2, however, is causal since the output singal only depends the past values of the input signal.
iii: Both systems have unbounded output signals for the input signal chosen as unit step function. Therefore, the systems are not stable.

c

i.

The systems are connected in parallel, so we can calculate the impulse response of the overall system $h[n]$ as:
$h[n] = h_1[n] - h_2[n]$
h[n] = u[n+2] - u[n-2]

ii.

We can calculate the output signal of the overall system $y[n]$ for an arbitrary input signal $x[n]$ as:
$y[n] = x[n-1] + x[n] + x[n+1] + x[n+2]$
The system is neither memoryless nor causal since $y[n]$ depends future values of $x[n]$.
For a bounded input signal, we can say that there exist $M > 0$ such that $|x[n]| < M$. Then $|y[n]| < 4M$ and $y[n]$ is bounded, therefore the system is stable.