EE301 Homework-2

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Question 1

a)

Let $x[n] = \delta[n]$. Then $y[n] = h[n]$ and equation becomes:
$h[n] - ah[n-1] = \delta[n] - b\delta[n-1]$
Also, we know that $h[n] = 0$ for $n < 0$ since the system is said to be causal.

For $n = 0$, we have:
$h[0] - ah[-1] = \delta[0] - b\delta[-1]$
$h[0] = 1$

For $n = 1$, we have:
$h[1] - ah[0] = \delta[1] - b\delta[0]$
$h[1] - a = -b$
$h[1] = a - b$

For $n = 2$, we have:
$h[2] - ah[1] = \delta[2] - b\delta[1]$
$h[2] - a(a - b) = 0$
$h[2] = a(a - b)$

For $n = 3$, we have:
$h[3] - ah[2] = \delta[3] - b\delta[2]$
$h[3] - a^2(a - b) = 0$
$h[3] = a^2(a - b)$

In general, for $n > 0$, we have $h[n] = a^{n-1}(a-b)$.

$$h[n] = \begin{cases} a^{n-1}(a-b), & n > 0 \\ 1, & n = 0 \\ 0, & n < 0 \end{cases}$$

If the system is stable, then the following expressions must be satisfied:

$$\sum_{k=-\infty}^{\infty} |h[k]| < \infty \iff 1 + \sum_{k=1}^{\infty} |a|^{k-1}|a-b| < \infty \iff 1 + |a-b| \sum_{k=0}^{\infty} |a|^k < \infty$$

Therefore, the system is said to be stable $\iff \begin{cases} a \in \mathbb{R}, & \text{if } a = b \\ -1 < a < 1, & \text{otherwise} \end{cases}$

b)

$y[n] = h[n] * x[n] \quad \text{and} \quad x[n] = e^{j\Omega_1 n} + e^{j\Omega_2 n}$

By distribution property of the convolution over addition:

$$y[n] = h[n] * e^{j\Omega_1 n} + h[n] * e^{j\Omega_2 n} = \sum_{k=-\infty}^{\infty} h[k]e^{j\Omega_1 (n-k)} + \sum_{k=-\infty}^{\infty} h[k]e^{j\Omega_2 (n-k)}$$

$$y[n] = e^{j\Omega_1 n} + \sum_{k=1}^{\infty} a^{k-1}(a - \frac{1}{a})e^{j\Omega_1 (n-k)} + e^{j\Omega_2 n} + \sum_{k=1}^{\infty} a^{k-1}(a - \frac{1}{a})e^{j\Omega_2 (n-k)}$$

$$y[n] = e^{j\Omega_1 n} + \sum_{k=1}^{\infty} a^k \frac{(a^2-1)}{a^2} e^{j\Omega_1 (n-k)} + e^{j\Omega_2 n} + \sum_{k=1}^{\infty} a^k \frac{(a^2-1)}{a^2} e^{j\Omega_2 (n-k)}$$

$$y[n] = e^{j\Omega_1 n} + \frac{(a^2-1)}{a^2} e^{j\Omega_1 n} \sum_{k=1}^{\infty} a^k e^{-j\Omega_1 k} + e^{j\Omega_2 n} + \frac{(a^2-1)}{a^2} e^{j\Omega_2 n} \sum_{k=1}^{\infty} a^k e^{-j\Omega_2 k}$$

$$y[n] = e^{j\Omega_1 n} + \frac{(a^2-1)}{a^2} e^{j\Omega_1 n} \sum_{k=1}^{\infty} (ae^{-j\Omega_1})^k + e^{j\Omega_2 n} + \frac{(a^2-1)}{a^2} e^{j\Omega_2 n} \sum_{k=1}^{\infty} (ae^{-j\Omega_2})^k$$

We know that $-1 \leq a \leq 1$ if $a = b$ for a stable system. Then we have $|ae^{-j\Omega}| \leq 1$ for any Ω.

$$y[n] = e^{j\Omega_1 n} + \tfrac{(a^2-1)}{a^2} e^{j\Omega_1 n} \tfrac{ae^{-j\Omega_1}}{1-ae^{-j\Omega_1}} + e^{j\Omega_2 n} + \tfrac{(a^2-1)}{a^2} e^{j\Omega_2 n} \tfrac{ae^{-j\Omega_2}}{1-ae^{-j\Omega_2}}$$

Question 2

a)

$x(t)$ and $h(t)$ can be written as:

$x(t) = u(t+1) - u(t-1)$

$$h(t) = (1-t)[u(t)-u(t-1)] = \begin{cases} 1-t, & 0<t<1 \\ 0, & otherwise \end{cases}$$

Then, $y(t) = x(t) * h(t)$

Before evaluating the convolution, consider the following:

$\hat{y}(t) = u(t) * h(t) = \int_{-\infty}^{\infty} u(t-\tau)h(\tau)\, d\tau$

$$= \int_{-\infty}^{t} h(\tau)\, d\tau = \begin{cases} 0, & t<0 \\ \int_0^t (1-\tau)\, d\tau, & 0<t<1 \\ \int_0^1 (1-\tau)\, d\tau, & t>1 \end{cases} = \begin{cases} 0, & t<0 \\ t-\frac{t^2}{2}, & 0<t<1 \\ \frac{1}{2}, & t>1 \end{cases}$$

By the properties of the LTI system:

$$\hat{y}(t+1) = u(t+1) * h(t) = \begin{cases} 0, & t+1<0 \\ (t+1)-\frac{(t+1)^2}{2}, & 0<t+1<1 \\ \frac{1}{2}, & t+1>1 \end{cases} = \begin{cases} 0, & t<-1 \\ (t+1)-\frac{(t+1)^2}{2}, & -1<t<0 \\ \frac{1}{2}, & t>0 \end{cases}$$

$$\hat{y}(t-1) = u(t-1) * h(t) = \begin{cases} 0, & t-1<0 \\ (t-1)-\frac{(t-1)^2}{2}, & 0<t-1<1 \\ \frac{1}{2}, & t-1>1 \end{cases} = \begin{cases} 0, & t<1 \\ (t-1)-\frac{(t-1)^2}{2}, & 1<t<2 \\ \frac{1}{2}, & t>2 \end{cases}$$

$$\text{Therefore, } y(t) = \hat{y}(t+1) - \hat{y}(t-1) = \begin{cases} 0, & t<-1 \\ \frac{1-t^2}{2}, & -1<t<0 \\ \frac{1}{2}, & 0<t<1 \\ \frac{1}{2}-(\frac{t^2-4t+3}{2}), & 1<t<2 \\ 0, & t>2 \end{cases}$$

Also, $y(t)$ can be written as:

$$y(t) = (\tfrac{1-t^2}{2}) \cdot u(t+1) + (\tfrac{t^2}{2}) \cdot u(t) - (\tfrac{t^2-4t+3}{2}) \cdot u(t-1) + (\tfrac{t^2-4t+2}{2}) \cdot u(t-2)$$

b)

$w(t) = h(t) * g(t)$

$g(t)$ can be written as: $g(t) = x(t) + x(t-1) - x(t+1)$

Then, $w(t) = h(t) * [x(t) + x(t-1) - x(t+1)]$

By the distributive property of the convolution over addition:

$w(t) = h(t) * x(t) + h(t) * x(t-1) - h(t) * x(t+1)$
$w(t) = y(t) + y(t-1) - y(t+1)$ [by considering part a]

By the time-invarince property of the LTI system:
$y(t-1) = (\frac{2t-t^2}{2}) \cdot u(t) + (\frac{t^2-2t+1}{2}) \cdot u(t-1) - (\frac{t^2-6t+8}{2}) \cdot u(t-2) + (\frac{t^2-6t+7}{2}) \cdot u(t-3)$
$y(t+1) = (\frac{-t^2-2t}{2}) \cdot u(t+2) + (\frac{t^2+2t+1}{2}) \cdot u(t+1) - (\frac{t^2-2t}{2}) \cdot u(t) + (\frac{t^2-2t-1}{2}) \cdot u(t-1)$

As a result:
$w(t) = (\frac{-t^2-2t}{2}) \cdot u(t+2) + (-t^2-t) \cdot u(t+1) + (\frac{t^2}{2}) \cdot u(t) + (\frac{-t^2+4t-1}{2}) \cdot u(t-1) + (t-3) \cdot u(t-2) + (\frac{t^2-6t+7}{2}) \cdot u(t-3)$

Question 3

a)

$x(t) = \cdots + \delta(t+2) - 2\delta(t+1) + \delta(t) - 2\delta(t-1) + \delta(t-2) - 2\delta(t-3) + \cdots$
It is a perodic signal with fundamental period $T_0 = 2$.

$$a_k = \frac{1}{T_0} \int_{t_o}^{t_0+T_0} x(t) e^{-jk\omega_0 t} dt = \frac{1}{2} \int_{\frac{-1}{2}}^{\frac{3}{2}} x(t) e^{-jk\pi t} dt$$

$$a_k = \underbrace{\frac{1}{2} \int_{0^-}^{0^+} \delta(t) e^{-jk\pi t} dt}_{e^{-jk\pi(0)}} + \underbrace{\frac{1}{2} \int_{1^-}^{1^+} -2\delta(t-1) e^{-jk\pi t} dt}_{e^{-jk\pi(1)}} \qquad \boxed{\text{Also, recall that } \int_{-\infty}^{\infty} \delta(t-t_0) x(t) = x(t_0)}$$

$$\Rightarrow a_k = \frac{1}{2} - e^{-jk\pi}$$

$$\Rightarrow x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} = \sum_{k=-\infty}^{\infty} \left(\frac{1}{2} - e^{-jk\pi} \right) e^{jk\pi t}$$

b)

$x(t)$ is a perodic signal with fundamental period, $T_0 = 4$.

$$a_k = \frac{1}{4} \int_{-2}^{2} x(t) e^{-jk\omega_0 t} dt = \frac{1}{4} \int_0^1 cos(\frac{\pi t}{2}) e^{-jk\frac{\pi}{2}t} dt$$

$$a_k = \frac{1}{4} \int_0^1 \frac{1}{2} \left(e^{j\frac{\pi}{2}t} + e^{-j\frac{\pi}{2}t} \right) e^{-jk\frac{\pi}{2}t} dt = \frac{1}{8} \int_0^1 \left(e^{-j\frac{\pi}{2}(k-1)t} + e^{-j\frac{\pi}{2}(k+1)t} \right) dt$$

$$a_k = -\frac{1}{8} \left(\frac{e^{-j\frac{\pi}{2}(k-1)t}}{j(k-1)\frac{\pi}{2}} \Big|_0^1 + \frac{e^{-j\frac{\pi}{2}(k+1)t}}{j(k+1)\frac{\pi}{2}} \Big|_0^1 \right) = \frac{1}{8} \left[\frac{1 - e^{-j\frac{\pi}{2}(k-1)}}{j(k-1)\frac{\pi}{2}} \right] + \frac{1}{8} \left[\frac{1 - e^{-j\frac{\pi}{2}(k+1)}}{j(k+1)\frac{\pi}{2}} \right]$$

$$a_k = \frac{1}{8}\left[\frac{1-e^{-j\frac{\pi}{2}k}e^{j\frac{\pi}{2}}}{j(k-1)\frac{\pi}{2}}\right] + \frac{1}{8}\left[\frac{1-e^{-j\frac{\pi}{2}k}e^{-j\frac{\pi}{2}}}{j(k+1)\frac{\pi}{2}}\right]$$

$$a_k = \frac{1}{8}\left[\frac{1-je^{-j\frac{\pi}{2}k}}{j(k-1)\frac{\pi}{2}}\right] + \frac{1}{8}\left[\frac{1+je^{-j\frac{\pi}{2}k}}{j(k+1)\frac{\pi}{2}}\right] = \frac{1}{4}\left[\frac{k-je^{-j\frac{\pi}{2}k}}{j(k^2-1)\frac{\pi}{2}}\right] = -\frac{1}{4}\left[\frac{e^{-j\frac{\pi}{2}k}+jk}{(k^2-1)\frac{\pi}{2}}\right]$$

$$\Rightarrow x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} = \sum_{k=-\infty}^{\infty} -\frac{1}{4}\left(\frac{e^{-j\frac{\pi}{2}k}+jk}{(k^2-1)\frac{\pi}{2}}\right) e^{jk\frac{\pi}{2}t}$$

c)

$x[n] = (-1)^n + j^n + cos(\frac{2\pi n}{3}) = e^{j\pi n} + e^{j\frac{\pi}{2}n} + cos(\frac{2\pi n}{3})$

Therefore, $x[n]$ can be written as a summation of 3 distinct perodic signals:

$$x[n] = \underbrace{e^{j\pi n}}_{x_1[n]} + \underbrace{e^{j\frac{\pi}{2}n}}_{x_2[n]} + \underbrace{cos\left(\frac{2\pi n}{3}\right)}_{x_3[n]} = x_1[n] + x_2[n] + x_3[n]$$

$x_1[n]$ is a periodic signal with fundamental period $N_0 = 2$.
$x_2[n]$ is a periodic signal with fundamental period $N_0 = 4$.
$x_3[n]$ is a periodic signal with fundamental period $N_0 = 3$.

Then, the fundamental period of $x[n]$ is the least common multiple of the fundamental periods of these three signals, so $N_0 = 12$.

$$a_k = \frac{1}{T_0}\sum_{n=n_0}^{n_0+N_0-1} x[n]e^{-jk\frac{2\pi}{N_0}n} = \frac{1}{12}\sum_{n=-6}^{5} x[n]e^{-jk\frac{\pi}{6}n}$$

d)

$x(t)$ is periodic signal with fundamental period $T_0 = 1$.

$$a_k = \frac{1}{T_0}\int_{t_o}^{t_0+T_0} x(t)e^{-jk\omega_0 t}dt = \int_{\frac{-1}{2}}^{\frac{1}{2}} e^{-t}e^{-jk2\pi t}dt = \int_{\frac{-1}{2}}^{\frac{1}{2}} e^{-t(1+jk2\pi)}dt$$

$$= \frac{-1}{1+jk2\pi}\left[e^{\frac{-1}{2}}\underbrace{e^{-jk\pi}}_{cos(k\pi)} - e^{\frac{1}{2}}\underbrace{e^{jk\pi}}_{cos(k\pi)}\right] = \frac{cos(k\pi)}{1+jk2\pi}\left(e^{\frac{1}{2}} - e^{\frac{-1}{2}}\right)$$

$$\Rightarrow x(t) = \sum_{k=-\infty}^{\infty} \frac{cos(k\pi)}{1+jk2\pi}\left(e^{\frac{1}{2}} - e^{\frac{-1}{2}}\right)e^{jk2\pi t}$$

e)

Recall that: $\boxed{y(t) = h(t) * x(t)}$ $\boxed{x(t) * \delta(t) = x(t)}$ $\boxed{x(t) * \delta(t-t_0) = x(t-t_0)}$

$$\Rightarrow y(t) = x(t) * (\delta(t) - \delta(t-\tfrac{1}{2})) \overset{\text{(by distribution property)}}{=} x(t) * \delta(t) - x(t) * \delta(t-\tfrac{1}{2})$$

$$\Rightarrow y(t) = x(t) - x(t - \tfrac{1}{2})$$

$$= \sum_{k=-\infty}^{\infty} \frac{cos(k\pi)}{1 + jk2\pi} \left(e^{\frac{1}{2}} - e^{\frac{-1}{2}}\right) e^{jk2\pi t} - \sum_{k=-\infty}^{\infty} \frac{cos(k\pi)}{1 + jk2\pi} \left(e^{\frac{1}{2}} - e^{\frac{-1}{2}}\right) e^{jk2\pi(t - \frac{1}{2})}$$

$$= \sum_{k=-\infty}^{\infty} \frac{cos(k\pi)}{1 + jk2\pi} \left(e^{\frac{1}{2}} - e^{\frac{-1}{2}}\right) e^{jk2\pi t} - \sum_{k=-\infty}^{\infty} \frac{cos(k\pi)}{1 + jk2\pi} \left(e^{\frac{1}{2}} - e^{\frac{-1}{2}}\right) e^{jk2\pi t} \underbrace{e^{-jk\pi}}_{cos(k\pi)}$$

$$= \sum_{k=-\infty}^{\infty} \frac{cos(k\pi)}{1 + jk2\pi} \left(e^{\frac{1}{2}} - e^{\frac{-1}{2}}\right) e^{jk2\pi t} \left(1 - cos(k\pi)\right)$$

$$= \sum_{k=-\infty}^{\infty} \frac{cos(k\pi) - \overbrace{cos(k\pi)^2}^{1}}{1 + jk2\pi} \left(e^{\frac{1}{2}} - e^{\frac{-1}{2}}\right) e^{jk2\pi t}$$

$$y(t) = \sum_{k=-\infty}^{\infty} \frac{cos(k\pi) - 1}{1 + jk2\pi} \left(e^{\frac{1}{2}} - e^{\frac{-1}{2}}\right) e^{jk2\pi t}$$

Question 4

a)

Note that, any arbitrary periodic signal x(t) (or, x[n]) with Fourier series coefficients a_k is real-valued if $a_k^* = a_{-k}$.

For $x_1(t)$: $a_k^* = [(\frac{1}{2})^{-k}]^* = (\frac{1}{2})^{-k} \neq (\frac{1}{2})^{-(-k)} = (\frac{1}{2})^{k} = a_{-k}$
For $x_2(t)$: $a_k^* = (cos(k\pi))^* = cos(k\pi) = cos(-k\pi) = a_{-k}$
For $x_3[n]$: $a_k^* = (jsin(\frac{k\pi}{2}))^* = -jsin(\frac{k\pi}{2}) = jsin(\frac{-k\pi}{2}) = a_{-k}$

Thus, $x_2(t)$ and $x_3[n]$ are real-valued signals.

b)

Any arbitrary periodic signal x(t) (or, x[n]) with Fourier series coefficients a_k is even if a_k is a real-valued and even function.
For $x_1(t)$: $a_k = (\frac{1}{2})^{-k} \Rightarrow$ is a real-valued but not even function, i.e., $(\frac{1}{2})^{-k}\Big|_{k=1} \neq (\frac{1}{2})^{-k}\Big|_{k=-1}$
For $x_2(t)$: $a_k = cos(k\pi) \Rightarrow$ is a real-valued and also an even function since consider that $-1 \leq cos(k\pi) \leq 1$ and $cos(k\pi)\Big|_{k=1} = cos(k\pi)\Big|_{k=-1}$
For $x_3[n]$: $a_k = jsin(\frac{k\pi}{2}) \Rightarrow$ neither real-valued nor even function.

Therefore, only the signal $x_2(t)$ is even.

c)

By the time-shifting property of Continuous Time Fourier Series:

$x_2(t-5) = \sum_{k=-100}^{100} cos(k\pi)e^{-jk\frac{\pi}{5}}e^{jk\frac{2\pi}{50}t}$ and, also note that $cos(k\pi) = \dfrac{1}{2}(e^{jk\pi} + e^{-jk\pi})$

$\Rightarrow x_2(t-5) = \sum_{k=-100}^{100} \dfrac{1}{2}(e^{jk\frac{4\pi}{5}} + e^{-jk\frac{6\pi}{5}})e^{jk\frac{2\pi}{50}t}$

Thus, $x_2(t-5)$ is another periodic signal with Fourier series coefficients $b_k = \frac{1}{2}(e^{jk\frac{4\pi}{5}} + e^{-jk\frac{6\pi}{5}})$.
Also, remember the theorem mentioned above, the signal $x_2(t-5)$ is real if $b_k^* = b_{-k}$.
$b_k^* = (\frac{1}{2}(e^{jk\frac{4\pi}{5}} + e^{-jk\frac{6\pi}{5}}))^* = \frac{1}{2}(e^{-jk\frac{4\pi}{5}} + e^{jk\frac{6\pi}{5}}) = b_{-k} \Rightarrow$ the signal $x_2(t-5)$ is real.

To check whether the signal $x_2(t-5)$ is even, b_k can be written as:
$b_k = \frac{1}{2}[cos(k\frac{4\pi}{5}) + jsin(k\frac{4\pi}{5}) + cos(k\frac{6\pi}{5}) - jsin(k\frac{6\pi}{5})]$
$b_k = \frac{1}{2}[cos(k\frac{4\pi}{5}) + cos(k\frac{6\pi}{5})] - \frac{1}{2}j[sin(k\frac{6\pi}{5}) - sin(k\frac{4\pi}{5})]$

Also, note that $\boxed{sina - sinb = 2cos(\dfrac{a+b}{2})sin(\dfrac{a-b}{2})}$

Thus, $b_k = \frac{1}{2}[cos(k\frac{4\pi}{5}) + cos(k\frac{6\pi}{5})] - j[cos(2\pi k)sin(k\frac{2\pi}{5})]$
$\Rightarrow b_k = \frac{1}{2}(cos(k\frac{4\pi}{5}) + cos(k\frac{6\pi}{5})) - jsin(k\frac{2\pi}{5})$
As a result, it can be seen that b_k is not real-valued, so the signal $x_2(t-5)$ is not even.

d)

By differentiation property of Continuous Time Fourier Series:

$\frac{d}{dt}x_2(t) = \sum_{k=-\infty}^{\infty} cos(k\pi)jk\dfrac{2\pi}{50}e^{jk\frac{2\pi}{50}} \Rightarrow$ is a periodic signal with Fourier series coefficients c_k such that $c_k = cos(k\pi)jk\dfrac{2\pi}{50}$.

$c_k^* = -jk\dfrac{2\pi}{50}cos(k\pi)$ and $c_{-k} = j(-k)\dfrac{2\pi}{50}cos(-k\pi) = -jk\dfrac{2\pi}{50}cos(k\pi)$

$c_k^* = c_{-k} \Rightarrow \dfrac{d}{dt}x_2(t)$is real-valued.

Note that, $cos(k\pi)$ take only the values $+1$ or -1, so it is always real-valued. However, $-jk\frac{2\pi}{50}$ is purely imaginary. Therefore, $\frac{d}{dt}x_2(t)$ is not even.

e)

Parseval's Identity: For any continuous-time periodic signal $x(t)$ with the Fourier series coefficients a_k,

$$\frac{1}{T_0}\int_{t_0}^{t_0+T_0} x(t)x^*(t)dt = \sum_{k=-\infty}^{\infty} a_k a_k^* \Rightarrow \frac{1}{T_0}\int_{t_0}^{t_0+T_0} |x(t)|^2 dt = \sum_{k=-\infty}^{\infty} |a_k|^2$$

In this question, $x_1(t)$ is a periodic signal whose fundamental period, $T_0 = 50$ and its Fourier series coefficients $a_k = (\frac{1}{2})^{-k}$. Please also note that, for this signal, the coefficients are defined only for $0 \leq k \leq 100$ since outside this domain all of the coefficients are equal to 0. So, by using Parseval's identity, the average power of $x_1(t)$ in one period:

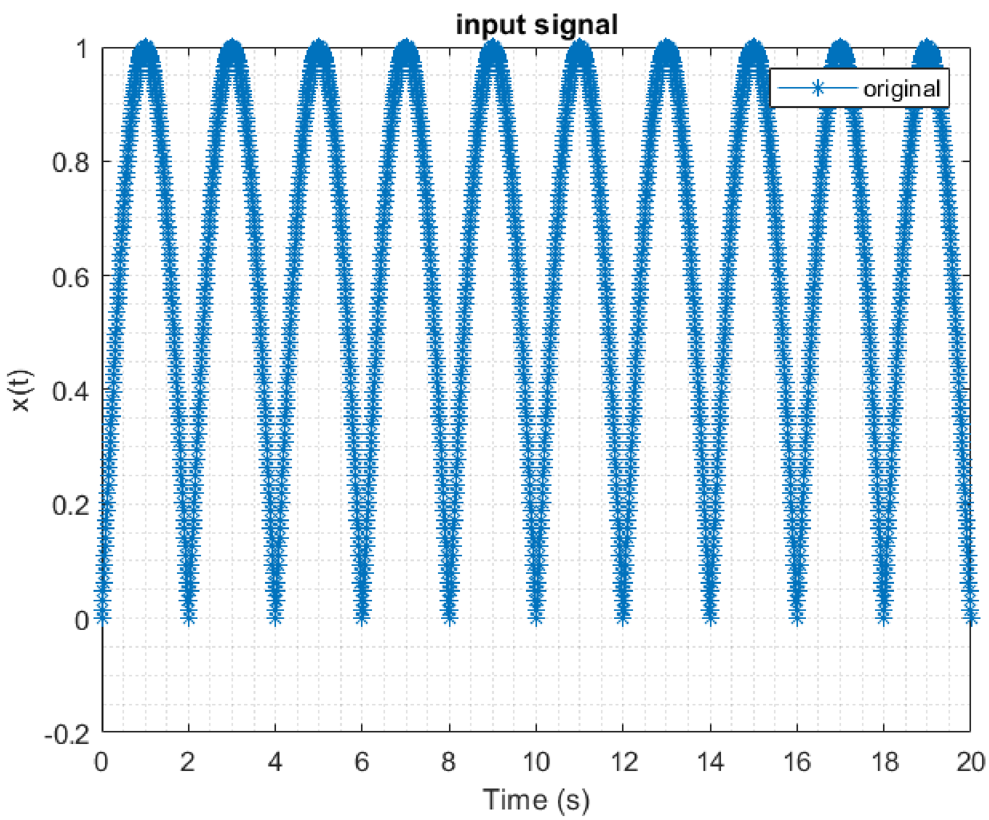
$$\frac{1}{50}\int_{0}^{50} |x_1(t)|^2 dt = \sum_{k=0}^{100} |a_k|^2 = \sum_{k=0}^{100} \left(\frac{1}{2}\right)^{-2k} = \sum_{k=0}^{100} 4^k = 1 + 4^1 + 4^2 + 4^3 + \text{..........} + 4^{100}$$

Question 5

Part 1

Step 1: Generate the original signal x[n]

```
t_original = 0:0.01:4;
x_original(1,1:200) = sin(pi*0.5*t_original(1:200));
x_original(1,201:401) = -sin(pi*0.5*t_original(201:end));

t_w5 = 0:0.01:20; % time of 5 period
x_original_w5 = [x_original x_original(2:end) x_original(2:end) x_original(2:end) x_original(2:

figure;
plot(t_w5,x_original_w5,'-*'); grid minor;
xlabel('Time (s)'); ylabel('x(t)'); title('Input signal x(t) over 5 period');
legend('original');
```



Step 2: Generate fourier series a_k by [7 9 11 21 51] elements and calculate x(t) signals respectively

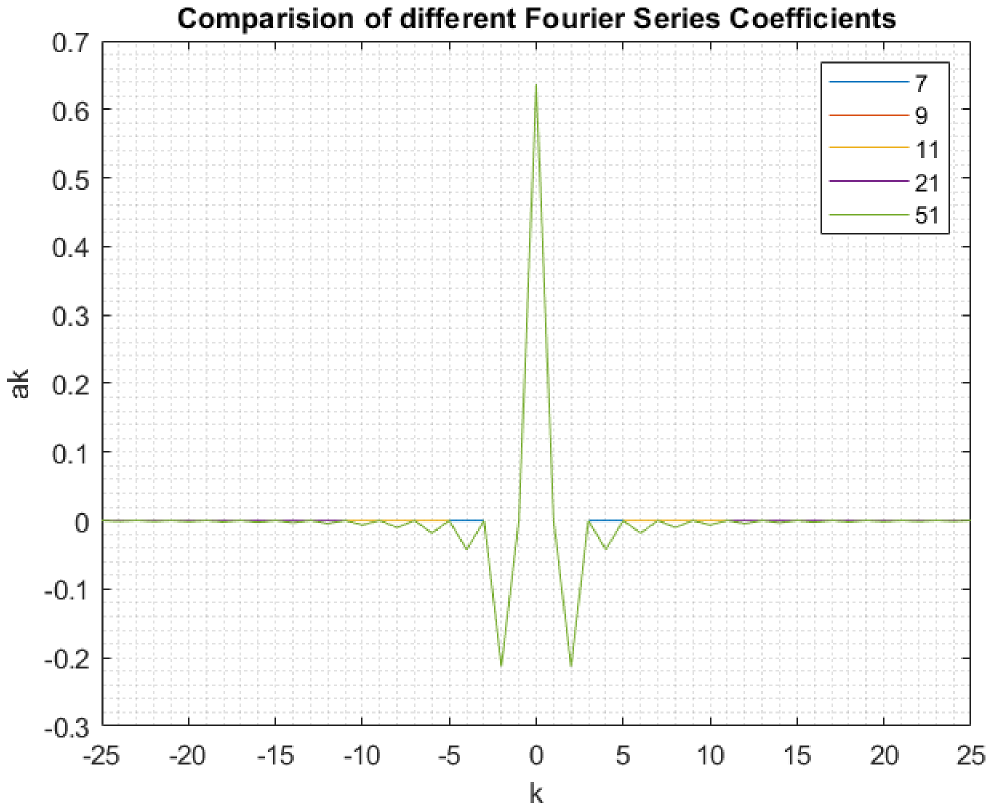
```
num = [7 9 11 21 51];
k_range = (num - 1)./2;
x = zeros(length(k_range),length(t_original));
% first row of x will be in 7 range
% second row of x will be in 9 range and so on and on...
```

```
coeff1 = zeros(1,num(1));
coeff2 = zeros(1,num(2));
coeff3 = zeros(1,num(3));
coeff4 = zeros(1,num(4));
coeff5 = zeros(1,num(5));


t = 0;
for m = 1:length(k_range)
    for i = 1:length(t_original)

        for k = -k_range(m):1:k_range(m)
            if k==1 || k==-1
                ak = 0;
            else
                ak = (-0.5/pi)*( (cos(pi*(k+1))-1)/(k+1) + (cos(pi*(1-k))-1)/(1-k) );
            end
            x(m,i) = real(x(m,i) + ak*exp(1j*k*pi*0.5*t));
            if m==1
                coeff1(k+1+k_range(m)) = ak;
            elseif m==2
                coeff2(k+1+k_range(m)) = ak;
            elseif m==3
                coeff3(k+1+k_range(m)) = ak;
            elseif m==4
                coeff4(k+1+k_range(m)) = ak;
            else
                coeff5(k+1+k_range(m)) = ak;
            end

        end

        t = t + 0.01;

    end
    t = 0;
end
```

Step 3: Gather the different coefficient in one matrix and plot them.

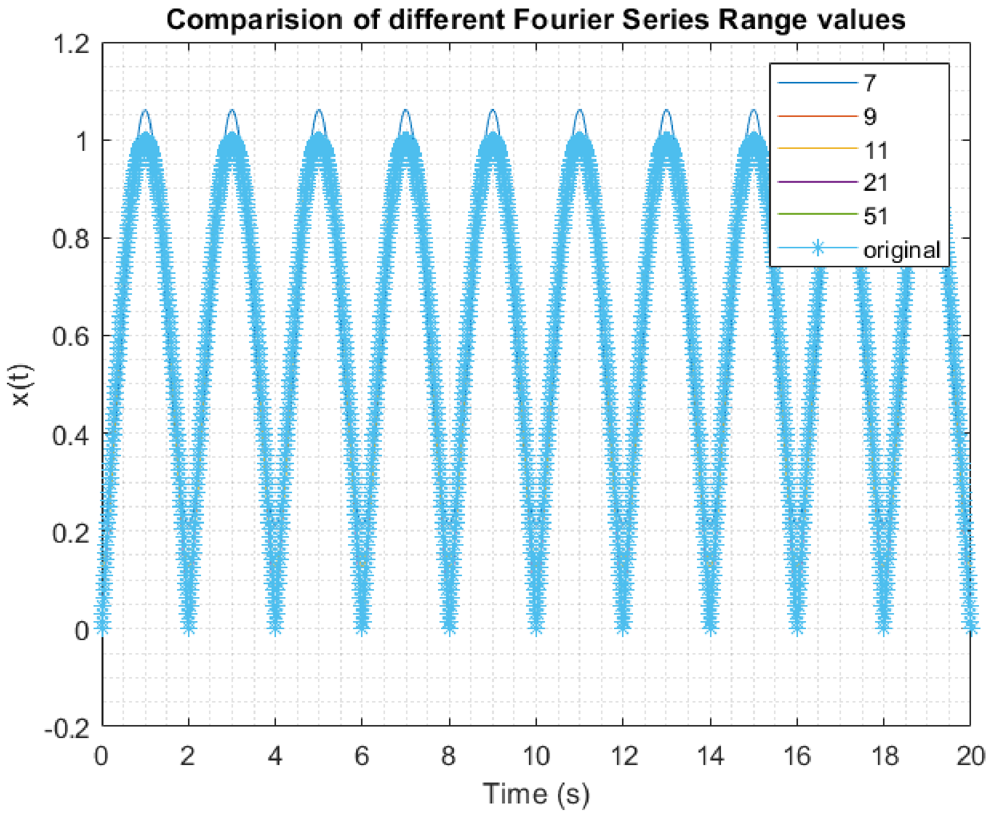
```
coeffs = [zeros(1,22),coeff1,zeros(1,22);
          zeros(1,21),coeff2,zeros(1,21);
          zeros(1,20),coeff3,zeros(1,20);
          zeros(1,15),coeff4,zeros(1,15);
                    coeff5];

k_ak = -25:1:25;
figure;
plot(k_ak,coeffs(1,:));
hold on;
```

```
plot(k_ak,coeffs(2,:));
plot(k_ak,coeffs(3,:));
plot(k_ak,coeffs(4,:));
plot(k_ak,coeffs(end,:)); grid minor;
xlabel('k'); ylabel('ak'); title('Comparison of different Fourier Series Coefficients');
legend('7','9','11','21','51');
```



Step 4: Compare the plots of input signals which are calculated from different fourier series ranges. (in 5 period scale)

```
x_w5 = zeros(5,length(t_w5));
for u=1:5
    x_w5(u,:) = [x(u,:) x(u,2:end) x(u,2:end) x(u,2:end) x(u,2:end)];
end

figure;
plot(t_w5,x_w5(1,:));
hold on;
plot(t_w5,x_w5(2,:));
plot(t_w5,x_w5(3,:));
plot(t_w5,x_w5(4,:));
plot(t_w5,x_w5(end,:));
plot(t_w5,x_original_w5,'-*'); grid minor;
xlabel('Time (s)'); ylabel('x(t)'); title('Comparison of different Fourier Series Range values
legend('7','9','11','21','51','original');
```



Comment: From the last plot of the question, it can be seen that how far we expand the range of fourier series coefficients (ideally infinity), it converges to the original input signal.

Part 2

Step 1: Adding delay to the coefficients.

Since the x(t) = Σ(a_k).exp(j.k.w.t) and x(t - t_0) = Σ(a_k).exp(j.k.w.(t - t_0))

which can be also written as x(t - t_0)= Σ(a_k*exp(j.k.w.(t_0))).exp(j.k.w.t).

So our new coefficient b_k = a_k*exp(j.k.w.(t_0))

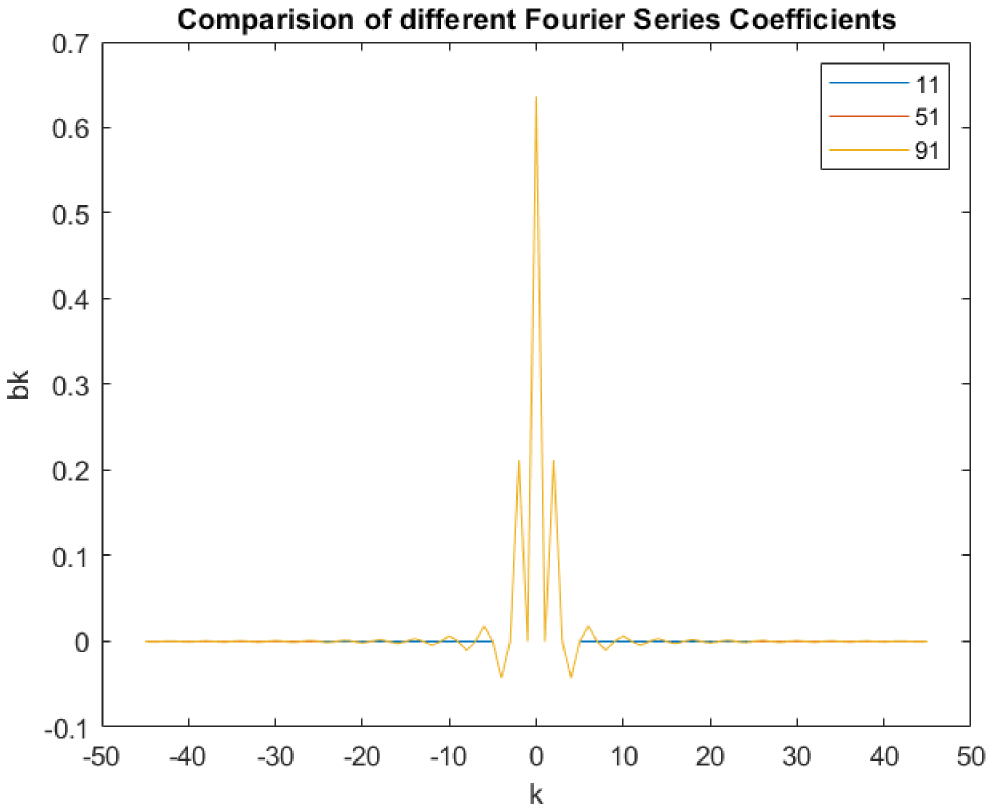
The delay that is indicated in the question t_0 = 1.

```
t_0 = 1;

num_D = [11,51,91];
k_range_D = (num_D - 1)./2;
x_D = zeros(length(k_range_D),length(t_original));
coeff1_D = zeros(1,num(1));
coeff2_D = zeros(1,num(2));
coeff3_D = zeros(1,num(3));

t = 0;
for m = 1:length(k_range_D)
    for i = 1:length(t_original)
```

```
        for k = -k_range_D(m):1:k_range_D(m)

            if k==1 || k==-1
                bk = 0;
            else
                bk = (-0.5/pi)*( (cos(pi*(k+1))-1)/(k+1) + (cos(pi*(1-k))-1)/(1-k) );
            end

            delay = exp(1j*k*pi*0.5*(t_0));
            bk = bk*delay;

            x_D(m,i) = real(x_D(m,i) + bk*exp(1j*k*pi*0.5*t));

            if m==1
                coeff1_D(k+1+k_range_D(m)) = bk;
            elseif m==2
                coeff2_D(k+1+k_range_D(m)) = bk;
            else
                coeff3_D(k+1+k_range_D(m)) = bk;
            end

        end

        t = t + 0.01;

    end
    t = 0;
end
```

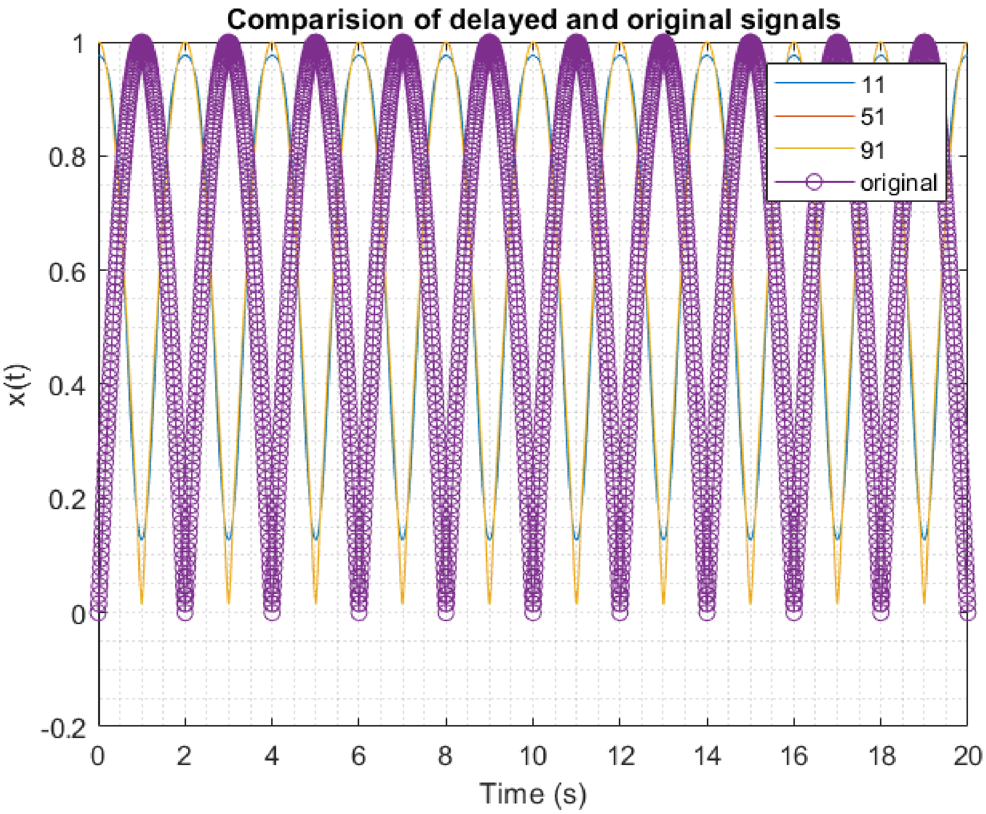
Step 2: Plot the delayed coefficients

```
coeffs_D = [zeros(1,40),coeff1_D,zeros(1,40);
            zeros(1,20),coeff2_D,zeros(1,20);
                        coeff3_D];

coeffs_D = real(coeffs_D);
k_bk = -45:1:45;
figure;
plot(k_bk,coeffs_D(1,:));
hold on;
plot(k_bk,coeffs_D(2,:));
plot(k_bk,coeffs_D(3,:));
xlabel('k'); ylabel('bk'); title('Comparison of different Fourier Series Coefficients');
legend('11','51','91');
```



Step 3: Plot the delayed and original waveforms

```
x_D_w5 = zeros(5,length(t_w5));
for u=1:3
    x_D_w5(u,:) = [x_D(u,:) x_D(u,2:end) x_D(u,2:end) x_D(u,2:end) x_D(u,2:end)];
end

figure;
plot(t_w5,x_D_w5(1,:));
hold on;
plot(t_w5,x_D_w5(2,:));
plot(t_w5,x_D_w5(3,:));
plot(t_w5,x_original_w5,'-o'); grid minor;
xlabel('Time (s)'); ylabel('x(t)'); title('Comparison of delayed and original signals');
legend('11','51','91','original');
```



Comparision of delayed and original signals
x(t)
Time (s)
11
51
91
original
1
0.8
0.6
0.4
0.2
0
-0.2
0
2
4
6
8
10
12
14
16
18
20