EE301 Homework-3

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Question 1

a)

b)

Question 2

a)

b)

c)

Question 3

a)

b)

Question 4

a)

i)

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} \underbrace{\delta[n]e^{-j\Omega n}}_{\delta[n]e^{-j\Omega 0}} = \sum_{n=-\infty}^{\infty} \delta[n] = 1$$

ii)

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} (2\delta[n-3]-\delta[n-10])e^{-j\Omega n} \overset{\text{(by linearity)}}{=} 2\sum_{n=-\infty}^{\infty} \delta[n-3]e^{-j\Omega n} - \sum_{n=-\infty}^{\infty} \delta[n-10]e^{-j\Omega n}$$

By time-shifting property of DTFT:
$X(e^{j\Omega}) = 2e^{-j3\Omega} - e^{-j10\Omega}$

iii)

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n} = \sum_{n=1}^{4} \frac{1}{n^2}e^{-j\Omega n} = e^{-j\Omega} + \frac{1}{4}e^{-j2\Omega} + \frac{1}{9}e^{-j3\Omega} + \frac{1}{16}e^{-j4\Omega}$$

iv)

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} \left(\left(\frac{1}{2}\right)^n u[n] - 3^n u[-n-1]\right) e^{-j\Omega n}$$

$$\overset{\text{(by linearity)}}{=} \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^n u[n]e^{-j\Omega n} - \sum_{n=-\infty}^{\infty} 3^n u[-n-1]e^{-j\Omega n} = \sum_{n=0}^{\infty} \left(\frac{1}{2}e^{-j\Omega}\right)^n - \sum_{n=-\infty}^{-1} 3^n e^{-j\Omega n}$$

$$\sum_{n=0}^{\infty} \left(\frac{1}{2}e^{-j\Omega}\right)^n = \frac{1}{1-\frac{1}{2}e^{-j\Omega}} \quad \left(\text{since } \left|\frac{1}{2}e^{-j\Omega}\right| = \frac{1}{2} < 1, \text{so the expression is convergent}\right)$$

Let $m = -n$: $\displaystyle\sum_{n=-\infty}^{-1} 3^n e^{-j\Omega n} = \sum_{m=1}^{\infty} 3^{-m} e^{j\Omega m} = \sum_{m=1}^{\infty} \left(\frac{1}{3}e^{j\Omega}\right)^m = \underbrace{\left[\sum_{m=0}^{\infty} \left(\frac{1}{3}e^{j\Omega}\right)^m\right]}_{\frac{1}{1-\frac{1}{3}e^{j\Omega}}} - 1$

$$\Rightarrow X(e^{j\Omega}) = \frac{1}{1-\frac{1}{2}e^{-j\Omega}} - \left(\frac{1}{1-\frac{1}{3}e^{j\Omega}} - 1\right)$$

v)

Say that, $\hat{x}[n] = \left(\frac{1}{2}\right)^n u[n] - 3^n u[-n-1]$ and $\mathscr{F}\{\hat{x}[n]\} = \hat{X}(e^{j\Omega}) = \frac{1}{1-\frac{1}{2}e^{-j\Omega}} - \left(\frac{1}{1-\frac{1}{3}e^{j\Omega}} - 1\right)$
By time-shifting property of DTFT:
$x[n] = \hat{x}[n-7] \longleftrightarrow X(e^{j\Omega}) = \hat{X}(e^{j\Omega})e^{-j7\Omega}$

$$\Rightarrow X(e^{j\Omega}) = \frac{e^{-j7\Omega}}{1-\frac{1}{2}e^{-j\Omega}} - \left(\frac{e^{-j7\Omega}}{1-\frac{1}{3}e^{j\Omega}} - e^{-j7\Omega}\right)$$

vi)

b)

Question 5

a)

b)

c)

d)

Question 6