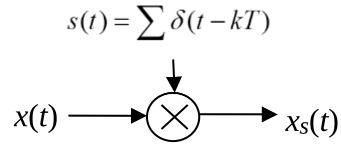
METU EE 301 - Signals & Systems I

Homework 4

(Due: January 06, 2023)



Q1. The input $x(t)$ of the sampling system shown on right has three various CTFT, $X(j\omega)$, plots, as shown in Figures (a), (b) and (c).

a) Find the minimum sampling rate, $1/T$, that yields perfect reconstruction with no aliasing for the signal having CTFT $X(j\omega)$, given in Fig.(a). Plot the Fourier transform $X_S(j\omega)$ of the sampling system output.

b) If the signal $x(t)$ with the Fourier transform given in Fig.(b) is sampled at the sampling periods:
(*i*) $T=\pi/\Delta\omega$ (the Nyquist rate), (*ii*) $T=2\pi/\Delta\omega$ (below the Nyquist rate!),
can aliasing be avoided in both cases? Plot the Fourier transform $X_S(j\omega)$ for each case. Indicate whether the time domain signal $x(t)$ is real or complex.

c) For the signal whose CTFT is in Fig.(c), find the minimum sampling rate, $1/T$, which results in no aliasing. Indicate whether the corresponding time domain signal is bandlimited or bandpass. Plot the CTFT for $X_S(j\omega)$.

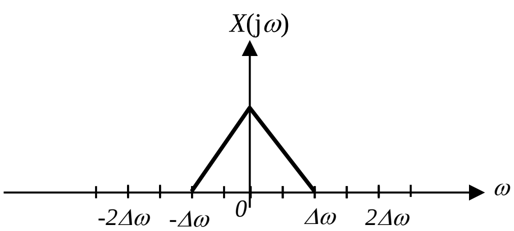


Fig. (a)

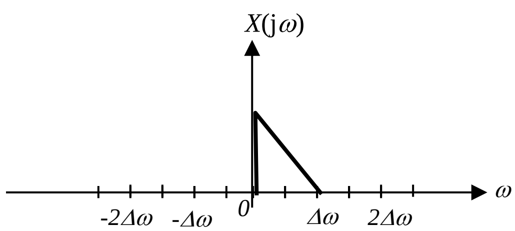


Fig. (b)

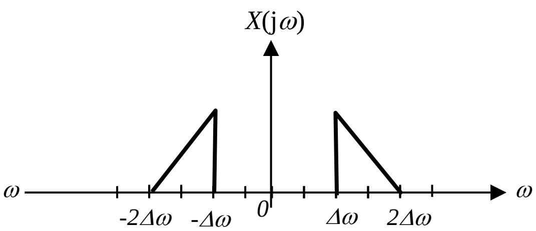
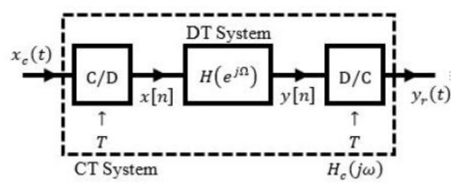


Fig. (c)

Q2. Assume continuous-time (CT) system below processes a CT signal by using a discrete-time (DT) system. All the components in this system are assumed to be ideal.



The input-output relation of the DT System is given as follows:

$$y[n]-\frac{1}{3}y[n-1]=\frac{2}{3}\,x[n]-2x[n-1]$$

If the input for this system is equal to $x_c(t)=\sin(1000\pi t)$, determine the output $y_r(t)$, for the sampling rates, $1/T$, being equal to

(i) 2kHz
(ii) 1kHz

Q3. a) The input-output relation of a continuous-time LTI system with impulse response $h_1(t)$ is given as follows:

$$\frac{d^2y(t)}{dt^2} - \frac{dy(t)}{dt} - 6\,y(t) = 2\frac{dx(t)}{dt} - 11\,x(t)$$

Find the transfer function $H_1(s)$ of this system and draw its pole-zero diagram. Discuss whether it is possible to find $h_1(t)$ if no other information is available about the system.

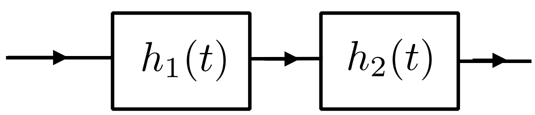
b) For the system in part (a), determine the region of convergence (ROC) of $H_1(s)$ and the corresponding impulse response $h_1(t)$ for each one of the cases below:

i. The system is stable
ii. The system is causal
iii. The system is anti-causal (An anti-causal system is a system such that $h_1(t) = 0$ for $t > 0$.)

c) Next, consider another system with transfer function $H_2(s) = s - 3$.

i. Determine the ROC of $H_2(s)$.
ii. Find the input-output relation of this system.

d) Now consider that the system with impulse response $h_1(t)$ in part (a) is connected in cascade to the system in part (c) as illustrated in the figure below. Assume that the system with impulse response $h_1(t)$ is causal as in part (b).ii.



i. Discuss whether the first system with impulse response $h_1(t)$ is stable.
ii. Find the transfer function $H(s)$ and the impulse response $h(t)$ of the overall cascaded system.
iii. Discuss whether the overall cascaded system is stable. Compare your answer to that of part (i). Comparing the ROCs for $H_1(s)$ and $H(s)$, comment on the result.

Q4 (Z-Transform) Consider a *causal* discrete-time (DT) linear time invariant (LTI) system whose system function is given as

$$H(z) = \frac{z(z-1)}{z^2 - a\sqrt{2}z + a^2}$$

where a is a *positive real* number:

a. Show that the impulse response $h[n]$ is always real-valued.
b. Find the range of a for which the Discrete Time Fourier Transform (DTFT) of $h[n]$ exists.

For the rest of the problem, set $a = \sqrt{2}$:

c. Find the output of that system, namely $y[n]$, for all n, when $u[n-1]$ is applied as an input.
d. A continuous-time (CT) periodic signal $\cos(2000\pi t)$ is first sampled at a sampling rate of *2 kHz*, then applied as the input to another DT system whose impulse response is $\left(\frac{1}{2}\right)^n h[n]$. Find the corresponding output signal.