EE301 Homework-4

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Question 1

a)

$x_s(t) = x(t)s(t) = \sum_{n=-\infty}^{\infty} x(t)\delta(t - nT_s) = \sum_{n=-\infty}^{\infty} x(nT_s)\delta(t - nT_s)$
where T_s is called the sampling period.
By the modulation property of CTFT: $X_s(j\omega) = \frac{1}{2\pi}X(j\omega) * S(j\omega)$

s(t) is a periodic signal, therefore we should first find its CTFS representation:

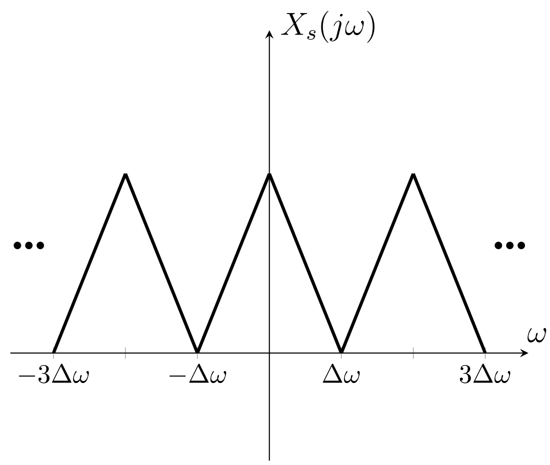
$s(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_s t}, \omega_s = \frac{2\pi}{T_s}$

$a_k = \frac{1}{T_s}\int_{-T_s/2}^{T_s/2} \underbrace{s(t)}_{\delta(t)} e^{-jk\omega_s t}dt \Rightarrow a_k = \frac{1}{T_s}$

$s(t) = \sum_{k=-\infty}^{\infty} \frac{1}{T_s}e^{jk\omega_s t} \longrightarrow S(j\omega) = \sum_{k=-\infty}^{\infty} \frac{2\pi}{T_s}\delta(\omega - k\omega_s)$

$X_s(j\omega) = \frac{1}{2\pi}X(j\omega) * \sum_{k=-\infty}^{\infty} \frac{2\pi}{T_s}\delta(\omega - k\omega_s) = \sum_{k=-\infty}^{\infty} \frac{1}{T_s}X(j(\omega - k\omega_s))$

From the Nyquist sampling theorem, the inequality $\omega_s \geq 2\Delta\omega$ should be satisfied so that the shifted replicas of $X(j\omega)$ do not overlap. Thus, it yields perfect reconstruction of the signal $x(t)$ from $x_s(t)$ with no aliasing. So, the minumum sampling rate, $\omega_s = 2\Delta\omega$. The Fourier transform of the sampling system output, $X_s(j\omega)$ can be seen below.

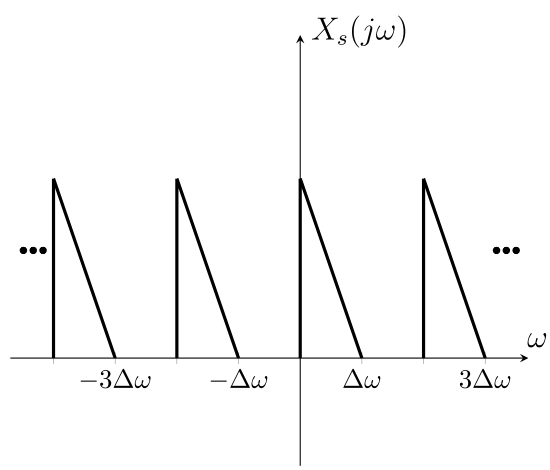


b)

$X(j\omega)$ is not symmetric with respect to y-axis, thus from the symmetry property of CTFT it can be concluded that $x(t)$ is a complex-valued signal.

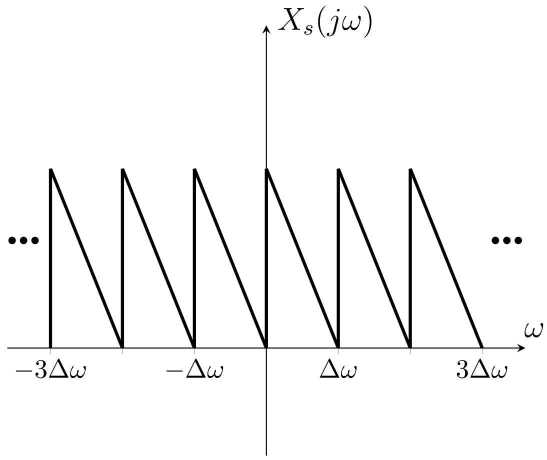
i)

Since the sampling rate of the signal is equal to the Nyquist rate, the aliasing does not occur. The Fourier transform of $X_s(j\omega)$ is plotted below for $T_s = \frac{\pi}{\Delta\omega}$.



ii)

As it can be seen from the graph of the Fourier transform of $x(t)$, the graph is not symmetric with respect to y-axis, i.e., $X(j\omega)$ has no component at negative frequencies. Therefore, as it can be seen from the graph of $X_s(j\omega)$ below, the aliasing can be avoided even when $T_s = \frac{2\pi}{\Delta\omega}$ which below the Nyquist rate.



c)

Question 2

i)

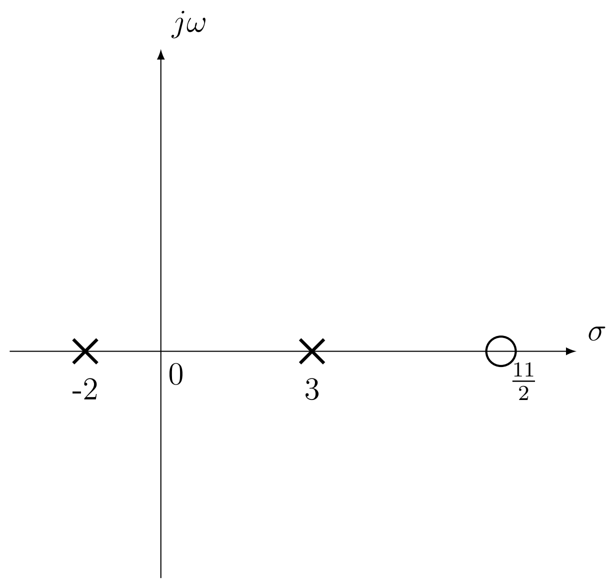
ii)

Question 3

a)

If we take the Laplace transform of the both sides of the equation:
$s^2Y(s) - sY(s) - 6Y(s) = 2sX(s) - 11X(s)$
$Y(s)(s^2 - s - 6) = X(s)(2s - 11)$
$H_1(s) = \frac{Y(s)}{X(s)} = \frac{2s-11}{(s-3)(s+2)} = \frac{-1}{s-3} + \frac{3}{s+2}$

The pole-zero diagram of $H_1(s)$ is drawn below.



We cannot determine $h_1(t)$ unless we know the ROC.

b)

i)

If the system is stable, then ROC includes $j\omega$-axis. Therefore, we can determine ROC as $-2 < \sigma < 3$.
$-2 < \sigma$ implies $\frac{3}{s+2}$ corresponds to right-sided time function which is $3e^{-2t}u(t)$.
$\sigma < 3$ implies $\frac{-1}{s-3}$ corresponds to left-sided time function which is $e^{3t}u(-t)$.
$h_1(t) = e^{3t}u(-t) + 3e^{-2t}u(t)$

ii)

If the system is causal, then ROC is the right side of the right-most pole. Therefore, we can determine ROC as $\sigma > 3$.
$\sigma > -2$ implies $\frac{3}{s+2}$ corresponds to right-sided time function which is $3e^{-2t}u(t)$.
$\sigma > 3$ implies $\frac{-1}{s-3}$ corresponds to right-sided time function which is $-e^{3t}u(t)$.
$h_1(t) = (-e^{3t} + 3e^{-2t})u(t)$

iii)

If the system is anti-causal, then ROC is the left side of the left-most pole. Therefore, we can determine ROC as $\sigma < -2$.
$\sigma < -2$ implies $\frac{3}{s+2}$ corresponds to right-sided time function which is $-3e^{-2t}u(-t)$.

$\sigma < 3$ implies $\frac{-1}{s-3}$ corresponds to right-sided time function which is $e^{3t}u(-t)$.
$h_1(t) = (e^{3t} + -3e^{-2t})u(-t)$

c)

i)

The ROC is the whole s-plane.

ii)

$H(s) = \frac{Y(s)}{X(s)} = s - 3$
$Y(s) = X(s)(s-3) = sX(s) - 3X(s)$
$y(t) = \frac{d}{dt}x(t) - 3x(t)$

d)

i)

Assume $h_1(t)$ is causal. Then it is not stable since the ROC does not include $j\omega$-axis.

ii)

$H(s) = H_1(s)H_2(s) = \frac{2s-11}{(s-3)(s+2)}(s-3)H(s) = \frac{2s-11}{s+2} = 2 - \frac{15}{s+2} \;\; ROC : \sigma > -2$
$h(t) = 2\delta(t) - 15e^{-2t}u(t)$

iii)

The cascaded system has a ROC which includes $j\omega$-axis. Therefore, we can say that the cascaded system is stable although the first system is not stable. The second system is used to make the first system stable.

Question 4

a)

b)

c)

d)