The Construction of Nonclassical Polynomials and Associated Quadratures.

We consider the construction of a set of N nonclassical polynomials orthonormal according to

$$\int_a^b w(x)P_n(x)P_m(x)dx = \delta_{nm},$$

where $w(x)$ is the weight function. This is an important aspect for all 4 projects.

The method appears similar to the Gram-Schmidt orthogonalization procedure, See Section 2.2. The method consists of starting with the first monic polynomial, $Q_0(x) = 1$ and its normalization

$$\gamma_1 = \int_a^b w(x)Q_0^2(x)dx = \mu_0,$$

where we denote the n^{th} moment of the weight function with μ_n.

The second monic polynomial is $Q_1(x) = q_{10} + x$. The coefficient q_{10} is determined by making Q_1 orthogonal to Q_0, that is

$$\int_a^b w(x)Q_1(x)Q_0(x)dx = 0,$$

and then determining the normalization

$$\gamma_1 = \int_a^b w(x)Q_1^2 dx.$$

The next monic polynomial is $Q_2(x) = q_{20}x^2 + q_{21}x + q_{20}$ and the cofficients are determined by making $Q_2(x)$ orthogonal to both $Q_1(x)$ and $Q_0(x)$ and then the normalization,

$$\gamma_2 = \int_a^b w(x)Q_2^2 dx.$$

The normalized polynomials are then

$$P_n(x) = \frac{Q_n(x)}{\sqrt{\gamma_n}}.$$

As shown in Section 2.2.1, this method fails, even for the classical polynomials, because of the round-off error that occurs with the calculation of the normalizations, γ_n, See Eq. (2.18).