CLRS 3-ed, Page 306, Problem 12-4b

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Problem Statement
Show that $B(x) = xB(x)^2 + 1$

Solution

$$
\begin{aligned}
B(X) &= \sum_{n=0}^{\infty} b_n x^n \\
&= b_0 + \sum_{n=1}^{\infty} b_n x^n \\
&= b_0 + x \sum_{n=1}^{\infty} \sum_{k=0}^{n-1} b_k b_{n-k-1} x^{n-1} \\
&= b_0 + x \sum_{n=0}^{\infty} \sum_{k=0}^{n} b_k b_{n-k} x^n \\
&= b_0 + x \sum_{n=0}^{\infty} b_n x_n \cdot \sum_{n=0}^{\infty} b_n x_n \text{ (check this by computing the coefficient for every term } x^i\text{)} \\
&= b_0 + xB(X)^2 \\
&= 1 + xB(X)^2
\end{aligned}
$$