Selected Answers for Contemporary Abstract Algebra

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These are write-ups for the questions left to the reader in Joseph A. Gallian's 8th edition of *Contemporary Abstract Algebra*. The questions were selected because they were the most interesting.

Contents

1 Chapter 0: Preliminaries

5. Show that if a and b are positive integers, then $ab = \text{lcm}(a, b) \bullet \text{gcd}(a, b)$.

Proof. By the Fundamental Theorem of Arithmetic (Theorem 0.3 in the book) that a is the product of primes, and by definition the greatest common divisor is a product of a subset of these primes

$$a = \text{gcd}(a, b) \prod_{i=1}^{n} p_{a,i}$$

Since by this definition

$$\{\emptyset\} = \{p_{a,i}\} \cap \{p_{b,i}\}$$

Thus

$$\text{lcm}(a, b) = \text{gcd}(a, b) \prod_{i=1}^{n} p_{a,i} \prod_{j=1}^{m} p_{b,j}$$

Therefore

$$\begin{aligned} \text{lcm}(a, b)\text{gcd}(a, b) &= \text{gcd}(a, b)\text{gcd}(a, b) \prod_{i=1}^{n} p_{a,i} \prod_{j=1}^{m} p_{b,j} && (1) \\ &= \big(\text{gcd}(a, b) \prod_{i=1}^{n} p_{a,i}\big)\big(\text{gcd}(a, b) \prod_{j=1}^{m} p_{b,j}\big) && (2) \\ &= ab && (3) \end{aligned}$$

□

2 Notation

Throughout the paper, we will reference the notation listed in this section.

$$\begin{aligned} \mathbb{Z} &= \text{integers} && (4) \\ \mathbb{Z}^+ &= \text{positive integers} && (5) \\ & && (6) \end{aligned}$$