

# COMBINATORICS

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The `itertools` module contains a few functions for generating

permutations combinations

It also has a function to generate the Cartesian product of multiple iterables

All these functions return lazy iterators



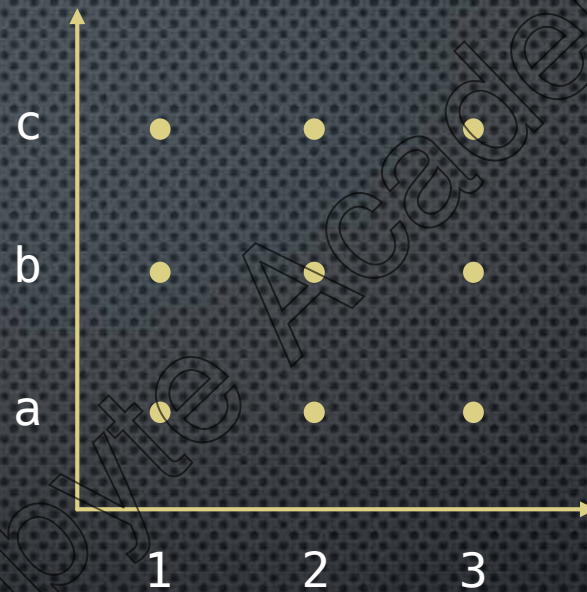
## Cartesian Product

$$\{1, 2, 3\} \times \{a, b, c\}$$

(1, a)  
(2, a)  
(3, a)

(1, b)  
(2, b)  
(3, b)

(1, c)  
(2, c)  
(3, c)



2-dimensional:

{ | }

n-dimensional:

{ | }



## Cartesian Product

Let's say we wanted to generate the Cartesian product of two lists:

`l1 = [1, 2, 3]`      `l2 = ['a', 'b', 'c', 'd']`      → notice not same length

```
def cartesian_product(l1, l2):  
    for x in l1:  
        for y in l2:  
            yield (x, y)
```

```
cartesian_product(l1, l2)
```

→ (1, 'a'), (1, 'b'), (1, 'c'), (1, 'd'), ..., (3, 'd')



`itertools.product(*args)` → lazy iterator

`l1 = [1, 2, 3]`      `l2 = ['a', 'b', 'c', 'd']`

`product(l1, l2)` → `(1, 'a'), (1, 'b'), (1, 'c'), (1, 'd'), ..., (3, 'd')`

`l3 = [100, 200]`

`product(l1, l2, l3)` → `(1, 'a', 100), (1, 'a', 200),`  
`(1, 'b', 100), (1, 'b', 200),`  
`(1, 'c', 100), (1, 'c', 200),`  
`...`  
`(3, 'd', 100), (3, 'd', 200)`



## Permutations

This function will produce all the possible permutations of a given iterable

In addition, we can specify the length of each permutation

→ maxes out at the length of the iterable

```
itertools.permutations(iterable, r=None)
```

→ `r` is the size of the permutation

→ `r = None` means length of each permutation is the length of the iterable

Elements of the iterable are considered **unique** based on their **position**, not their value

→ if iterable produces repeat values  
then permutations will have repeat values too



## Combinations

Unlike permutations, the order of elements in a combination is not considered

→ OK to always sort the elements of a combination

Combinations of length  $r$ , can be picked from a set

- **without** replacement → once an element has been picked from the set it **cannot** be picked again
- **with** replacement → once an element has been picked from the set it **can** be picked again



```
itertools.combinations(iterable, r)
```

```
itertools.combinations_with_replacement(iterable, r)
```

Just like for permutations:

the elements of an iterable are **unique** based on their **position**, not their value

The different combinations produced by these functions are **sorted** based on the **original ordering** in the **iterable**



# Code Exercises