Externalities and Public Goods

Simple Bilateral Externality

Definition: An externality is present whenever the well-being of a consumer or the production possibilities of a firm are **directly** affected by the actions of another agent in the economy.

"Directly" exclude any effects that are mediated by prices.

That is, an externality is present if, say, a fishery's productivity is affected by the emissions from a nearby oil refinery, but not simply because the fishery's profitability is affected by the price of oil (which, in turn, is to some degree affected by the oil refinery's output of oil).

The latter type of effect [pecuniary externality] is present in any competitive market but creates no inefficiency.

Indeed, with price-taking behavior, the market is precisely the mechanism that guarantees a Pareto optimal outcome.

This suggests that the presence of an externality is not merely a technological phenomenon **but also a function of the set of markets in existence**.

Consider implications of external effects for competitive equilibria and public policy in the context of a simple two-agent, partial equilibrium model.

Two consumers, $i = 1,2$, who constitute a small part of the overall economy.

The actions of these consumers do not affect the prices $p \in \mathbb{R}^L$ of the L traded goods in the economy.

At these prices, consumer i 's wealth is w_i.

In contrast with the standard competitive model, assume that each consumer has preferences not only over her consumption of the L traded goods $(x_{1i}, \dots, x_{Li})$ **but also over some action $h \in \mathbb{R}_+$ taken by consumer 1**.

Consumer i 's (differentiable) utility function takes the form $u_i(x_{1i}, \dots, x_{Li}, h)$

> Assume $\partial u_2(x_{12}, \dots, x_{L.2}, h)/\ \partial h \neq 0$.

Because consumer 1's choice of h affects consumer 2's well-being, it generates an externality.

For example, the two consumers may live next door to each other, and h may be a measure of how loudly consumer 1 plays music.

Or the consumers may live on a river, with consumer 1 further upstream. In this case, h could represent the amount of pollution put into the river by consumer 1; more pollution lowers consumer 2's enjoyment of the river.

We should hasten to add that external effects need not be detrimental to those affected by them.

> Action h could, for example, be consumer 1's beautification of her property, which her neighbor, consumer 2, also gets to enjoy.
>
> Externalities may be positive or negative.

Define for each consumer i a derived utility function over the level of h, assuming optimal commodity purchases by consumer i at prices $p \in \mathbb{R}^L$ and wealth w_i :

$$v_i(p, w_i, h) = \text{Max}_{x_i \geq 0} \quad u_i(x_i, h)$$
$$\text{s.t. } p \cdot x_i \leq w_i$$

Assume that the consumers' utility functions take a quasilinear form with respect to a numeraire commodity.

Derived utility function $v_i(\cdot)$ as $v_i(p, w_i, h) = \phi_i(p, h) + w_i$

Since prices of the L traded goods are assumed to be unaffected by any of the changes we are considering, we shall suppress the price vector p and simply write $\phi_i(h)$.

We assume that $\phi_i(\cdot)$ is twice differentiable with $\phi_i''(\cdot) < 0$.

Although we shall speak in terms of this consumer interpretation, everything we do here applies equally well to the case in which the two agents are firms (or one firm and one consumer).

Nonoptimality of the Competitive Outcome

Consider a competitive equilibrium in which commodity prices are p.

> That is, at the equilibrium position, each of the two consumers maximizes her utility limited only by her wealth and the prices p of the traded goods.

It must therefore be the case that consumer 1 chooses her level of $h \geq 0$ to maximize $\phi_1(h)$.

Assume throughout interior solutions: now, $h^* > 0$.

The **equilibrium level** of h, h^*, satisfies the necessary and sufficient first-order condition

$$\phi_1'(h^*) = 0$$

In contrast, in any Pareto optimal allocation, the optimal level of h, h°, must maximize the joint surplus of the two consumers, and so must solve

$$\text{Max}_{h \geq 0} \ \phi_1(h) + \phi_2(h)$$

This problem gives us the necessary and sufficient first-order condition for h°:

$$\phi_1'(h^\circ) = -\phi_2'(h^\circ)$$

When external effects are present, so that $\phi_2'(h) \neq 0$ at all h, the equilibrium level of h is not optimal (unless $h^\circ = h^* = 0$).

Consider, for example, the case in which we have interior solutions, that is, where $(h^*, h^\circ) \gg 0$.

$$\text{If } \phi_2'(\cdot) < 0 \text{ (negative externality), then } \phi_1'(h^\circ) = -\phi_2'(h^\circ) > 0$$

ϕ_1' decreasing and $\phi_1'(h^*) = 0$ implies $h^* > h^\circ$.

Analogously, $\phi_2'(\cdot) > 0$ implies $h^* < h^\circ$.

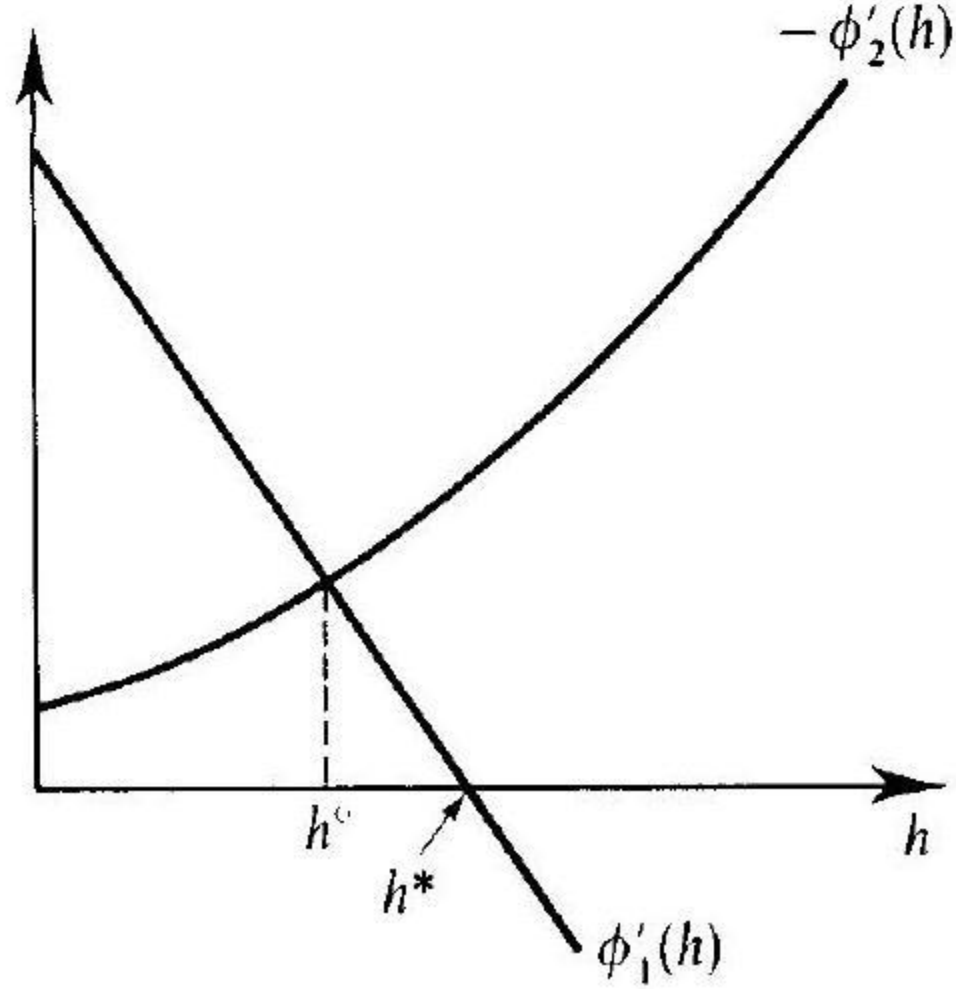


Figure 11.B.1: The equilibrium (h^*) *and optimal* (h°) *levels of a negative externality.*

Figure 11.B.1 depicts the solution for a case in which h constitutes a negative external effect, so that $\phi_2'(h) < 0$ at all h.

In the figure, we graph $\phi_1'(\cdot)$ and $-\phi_2'(\cdot)$.

The competitive equilibrium level of the externality h^* occurs at the point where the graph of $\phi_1'(\cdot)$ crosses the horizontal axis.

In contrast, the optimal externality level h° corresponds to the point of intersection between the graphs of the two functions.

Optimality does not usually entail the complete elimination of a negative externality.

Rather, the externality's level is adjusted to the point where the marginal benefit to consumer 1 of an additional unit of the externality-generating activity, $\phi_1'(h^\circ)$, equals its marginal cost to consumer 2, $-\phi_2'(h^\circ)$.

In the current example, quasilinear utilities lead the optimal level of the externality to be independent of the consumers' wealth levels.

In the absence of quasi-linearity, wealth effects for the consumption of the externality make its optimal level depend on the consumers' wealth levels.

When the agents under consideration are firms, wealth effects are always absent.

Traditional Solutions to the Externality Problem

Quotas and taxes

To fix ideas, suppose that h generates a negative external effect, so that $h^\circ < h^*$.

The government can simply **mandate** that h be no larger than h°, its optimal level.

With this constraint, consumer 1 will indeed fix the level of the externality at h°.

A second option is for the government to attempt to restore optimality by imposing a tax on the externality-generating activity.

This solution is known as Pigouvian taxation.

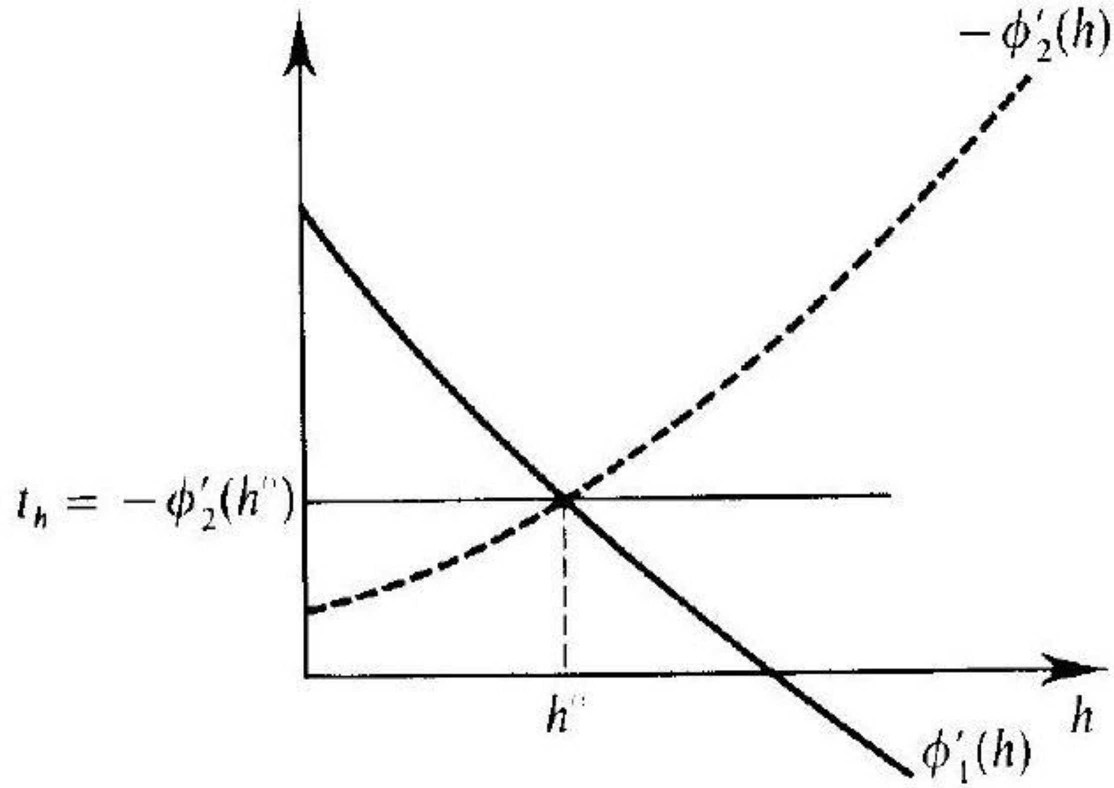


Figure 11.B.2: The optimality restoring Pigouvian tax.

To this effect, suppose that consumer 1 is made to pay a tax of t_h per unit of h.

Define $t_h = -\phi_2'(h^\circ) > 0$

This will implement the optimal level of the externality.

Indeed, consumer 1 will then choose the level of h that solves

$$\text{Max}_{h \geq 0}\ \phi_1(h) - t_h h$$

which has the necessary and sufficient first-order condition

$$\phi_1'(h) = t_h$$

Given $t_h = -\phi_2'(h^\circ), h = h^\circ$ satisfies optimality.

Recall that h° is defined by the condition: $\phi_1'(h'') = -\phi_2'(h'')$.

Moreover, given $\phi_1''(\cdot) < 0$, h° must be the unique solution to problem.

Figure 11.B.2 illustrates this solution for a case in which $h^\circ > 0$.

The optimality-restoring tax is exactly equal to the marginal externality at the optimal solution.

That is, it is exactly equal to the amount that consumer 2 would be willing to pay to reduce h slightly from its optimal level h'.

When faced with this tax, consumer 1 is effectively led to carry out an individual cost benefit computation that internalizes the externality that she imposes on consumer 2.

The principles for the case of a positive externality are exactly the same, only now when we set $t_h = -\phi_2'(h') < 0, t_h$ takes the form of a per-unit subsidy.

Several additional points are worth noting about this Pigouvian solution.

First, we can actually achieve optimality either by taxing the externality or by subsidizing its reduction.

Consider, for example, the case of a negative externality.

Suppose the government pays a subsidy of $s_h = -\phi_2'(h^\circ) > 0$ for every unit that consumer 1's choice of h is below h^*, its level in the competitive equilibrium.

If so, then consumer 1 will maximize $\phi_1(h) + s_h(h^* - h) = \phi_1(h) - t_h h + t_h h^*$.

But this is equivalent to a tax of t_h per unit on h combined with a lump-sum payment of $t_h h^*$.

Hence, a subsidy for the reduction of the externality combined with a lump-sum transfer can exactly replicate the outcome of the tax.

Second, in general, it is essential to tax the externality-producing activity directly.

For instance, suppose that, in the example of consumer 1 playing loud music, we tax purchases of music equipment instead of taxing the playing of loud music itself.

In general, this will not restore optimality.

Consumer 1 will be led to lower her consumption of music equipment (perhaps she will purchase only a CD player, rather than a CD player and a tape player) but may nevertheless play whatever equipment she does purchase too loudly.

A common example of this sort arises when a firm pollutes in the process of producing output.

A tax on its output leads the firm to reduce its output level but may not have any effect (or, more generally, may have too little effect) on its pollution emissions.

Taxing output achieves optimality only in the special case in which emissions bear a fixed monotonic relationship to the level of output.

In this special case, emissions can be measured by the level of output, and a tax on output is essentially equivalent to a tax on emissions.

Third, the tax/subsidy and the quota approaches are equally effective in achieving an optimal outcome.

However, the government must have a great deal of information about the benefits and costs of the externality to set the optimal levels of either the quota or the tax.

When the government does not possess this information the two approaches typically are not equivalent.

Fostering bargaining over externalities: enforceable property rights

Another approach to the externality problem aims at a less intrusive form of intervention:

Ensure that conditions are met **for the parties to reach an optimal agreement on the level of the externality**.

Suppose that we establish enforceable property rights with regard to the externality-generating activity.

For example, that we assign the right to an "externality-free" environment to consumer 2.

Then consumer 1 is unable to engage in the externality-producing activity without consumer 2's permission.

For simplicity, imagine that the bargaining between the parties takes a form in which consumer 2 makes consumer 1 a take-it-or-leave-it offer, demanding a payment of T in return for permission to generate externality level h.

Consumer 1 will agree to this demand if and only if she will be at least as well off as she would be by rejecting it.

That is, if and only if $\phi_1(h) - T \geq \phi_1(0)$.

Hence, consumer 2 will choose her offer (h, T) to solve

$$\text{Max}_{h \geq 0, T} \quad \phi_2(h) + T$$
$$\text{s.t. } \phi_1(h) - T \geq \phi_1(0)$$

The constraint is binding in any solution to this problem. In particular:

$$T = \phi_1(h) - \phi_1(0)$$

Therefore, consumer 2's optimal offer involves the level of h that solves

$$\text{Max}_{h \geq 0} \ \phi_2(h) + \phi_1(h) - \phi_1(0).$$

The solution is precisely h°, the socially optimal level.

Note that the precise allocation of these rights between the two consumers is inessential to the achievement of optimality.

Suppose, for example, that consumer 1 instead has the right to generate as much of the externality as she wants.

In the absence of any agreement, consumer 1 will generate externality level h^*.

Now consumer 2 will need to offer a $T < 0$ (i.e., to pay consumer 1) to have $h < h^*$.

In particular, consumer 1 will agree to externality level h if and only if:

$$\phi_1(h) - T \geq \phi_1(h^*).$$

As a consequence, consumer 2 will offer to set h at the level that solves:

$$\text{Max}_h \left(\phi_2(h) + \phi_1(h) - \phi_1(h^*)\right)$$

Once again, the optimal externality level h° results.

The allocation of rights affects only the final wealth of the two consumers by altering the payment made by consumer 1 to consumer 2.

In the first case, consumer 1 pays $\phi_1(h'') - \phi_1(0) > 0$ to be allowed to set $h^\circ > 0$

In the second, she "pays" $\phi_1(h'') - \phi_1(h^*) < 0$ in return for setting $h^\circ < h^*$.

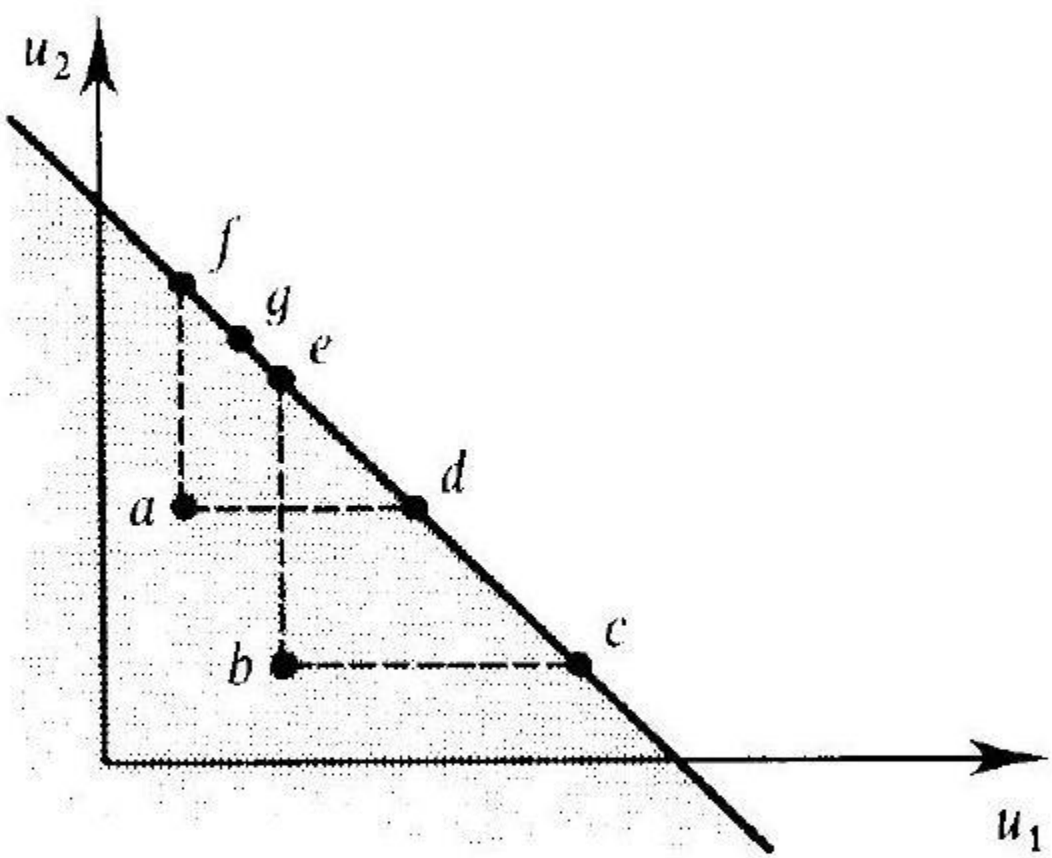


Figure 11.B.3: The final distribution of utilities under different property rights institutions and different bargaining procedures.

This is an instance of what is known as the **Coase theorem** [for Coase (1960)]:

If trade of the externality can occur, then bargaining will lead to an efficient outcome no matter how property rights are allocated.

The existence of both well-defined and enforceable property rights is essential for this type of bargaining to occur.

If property rights are not well defined, it will be unclear whether consumer 1 must gain consumer 2's permission to generate the externality.

If property rights cannot be enforced (perhaps the level of h is not easily measured), then consumer 1 has no need to purchase the right to engage in the externality-generating activity from consumer 2.

For this reason, **proponents of this type of approach focus on the absence of these legal institutions as a central impediment to optimality**.

This solution to the externality problem has a significant advantage over the tax and quota schemes in terms of the level of knowledge required of the government.

The consumers must know each other's preferences, but the government need not.

> For bargaining over the externality to lead to efficiency, it is important that the consumers know this information.
>
> When the agents are to some extent ignorant of each others' preferences, bargaining need not lead to an efficient outcome.

Two further points:

First, in the case in which the two agents are firms, one form that an efficient bargain might take is the sale of one of the firms to the other.

The resulting merged firm would then fully internalize the externality in the process of maximizing its profits.

This conclusion presumes that the owner of a firm has full control over all its functions. In more complicated (but realistic) settings in which this is not true, say because owners must hire managers whose actions cannot be perfectly controlled, the results of a merger and of an agreement over the level of the externality need not be the same.

See Holmstrom and Tirole (1989) for a discussion of these issues in the theory of the firm.

Second, note that all three approaches require that the externality-generating activity be measureable.

This is not a trivial requirement; in many cases, such measurement may be either technologically infeasible or very costly (consider the cost of measuring air pollution or noise).

A proper computation of costs and benefits should take these costs into account. **If measurement is very costly, then it may be optimal to simply allow the externality to persist.**

Externalities and Missing Markets

The observation that bargaining can generate an optimal outcome suggests a **connection between externalities and missing markets**.

A market system can be viewed as a particular type of trading procedure.

Suppose that property rights are well defined and enforceable and that a competitive market for the right to engage in the externality-generating activity exists.

For simplicity, assume that consumer 2 has the right to an externality-free environment.

Let p_h denote the price of the right to engage in one unit of the activity.

In choosing how many of these rights to purchase, say h_1, consumer 1 will solve

$$\text{Max}_{h_1 \geq 0} \ \phi_1(h_1) - p_h h_1$$

which has the first-order condition

$$\phi_1'(h_1) = p_h$$

In deciding how many rights to sell, h_2, consumer 2 will solve

$$\text{Max}_{h_2 \geq 0} \ \phi_2(h_2) + p_h h_2$$

which has the first-order condition

$$\phi_2'(h_2) = -p_h$$

In a competitive equilibrium, the market for these rights must clear: $h_1 = h_2$.

Hence, the level of rights traded in this competitive rights market, say h^{**}, satisfies

$$\phi_1'(h^{**}) = -\phi_2'(h^{**})$$

We see that h^{**} equals the optimal level h°.

The equilibrium price of the externality is $p_h^* = \phi_1'(h^\circ) = -\phi_2'(h^\circ)$.

Consumer 1 and 2's equilibrium utilities are then $\phi_1(h^\circ) - p_h^* h^\circ$ and $\phi_2(h^\circ) + p_h^* h^\circ$, respectively.

The market therefore works as a particular bargaining procedure for splitting the gains from trade.

If a competitive market exists for the externality, then optimality results.

Externalities can be seen as being inherently tied to the absence of certain competitive markets.

Indeed, our definition of an externality explicitly required that an action chosen by one agent must directly affect the well-being or production capabilities of another.

Once a market exists for an externality, however, each consumer decides for herself how much of the externality to consume at the going prices.

The idea of a competitive market for the externality in the present example is rather unrealistic.

In a market with only one seller and one buyer, price taking would be unlikely.

However, most important externalities are produced and felt by many agents.

Thus, we might hope that in these multilateral settings, price taking would be a more reasonable assumption and, as a result, that a competitive market for the externality would lead to an efficient outcome.

We will consider multilateral externalities: the correctness of this conclusion depends on whether the externality is "private" or "public" in nature.

Before coming to this, however, we first study the nature of public goods.

Public Goods

Definition: A public good is a commodity for which use of a unit of the good by one agent does not preclude its use by other agents.

Public goods are nondepletable:

Consumption by one individual does not affect the supply available for other individuals.

Knowledge provides a good illustration.

The use of a piece of knowledge for one purpose does not preclude its use for others.

Commodities studied up to this point have been assumed to be of a private, or depletable, nature;

A distinction can also be made according to whether exclusion of an individual from the benefits of a public good is possible.

Every private good is automatically excludable, but public goods may or may not be.

The patent system, for example, is a mechanism for excluding individuals (although imperfectly) from the use of knowledge developed by others.

On the other hand, it might be technologically impossible, or at the least very costly, to exclude some consumers from the benefits of national defense or of a project to improve air quality.

Our discussion here will focus primarily on the case in which exclusion is not possible.

A public "good" need not necessarily be desirable; that is, we may have public bads (e.g., foul air). In this case, we should read the phrase "does not preclude" to mean "does not decrease."

Conditions for Pareto Optimality

Consider a setting with I consumers and one public good, in addition to L traded goods of the usual, private, kind.

Partial equilibrium perspective: the quantity of the public good has no effect on the prices of the L traded goods and that each consumer's utility function is quasilinear with respect to the same numeraire, traded commodity.

We can therefore define, for each consumer i, a derived utility function over the level of the public good.

Let x denote the quantity of the public good.

Denote consumer i 's utility from the public good by $\phi_i(x)$.

Assume that this function is twice differentiable, with $\phi_i''(x) < 0$ at all $x \geq 0$.

Precisely because we are dealing with a public good, the argument x does not have an i subscript.

The cost of supplying q units of the public good is $c(q)$.

Assume that $c(\cdot)$ is twice differentiable, with $c''(q) > 0$ at all $q \geq 0$.

Take $\phi_i'(\cdot) > 0$ for all i and $c'(\cdot) > 0$: wlog, public good is desirable and costly.

In this quasilinear model, any Pareto optimal allocation must maximize aggregate surplus.

Therefore must involve a level of the public good that solves

$$\text{Max}_{q \geq 0} \sum_{i=1}^{I} \phi_i(q) - c(q)$$

The necessary and sufficient first-order condition for the optimal quantity q° is then

$$\sum_{i=1}^{I} \phi_i'(q^{\circ}) = c'(q^{\circ})$$

This condition is the classic optimality condition for a public good first derived by Samuelson (1954; 1955).

> **At the optimal level of the public good the sum of consumers' marginal benefits from the public good is set equal to its marginal cost.**

For a private good, where each consumer's marginal benefit from the good is equated to its marginal cost.

Inefficiency of Private Provision of Public Goods

Consider a public good provided by means of private purchases by consumers.

We imagine that a market exists for the public good and that each consumer i chooses how much of the public good to buy, denoted by $x_i \geq 0$, taking as given its market price p.

The total amount of the public good purchased by consumers is then $x = \sum_i x_i$.

Formally, we treat the supply side as consisting of a single profit-maximizing firm with cost function $c(\cdot)$ that chooses its production level taking the market price as given.

> (We can also think of the supply behavior of this firm as representing the industry supply of J price-taking firms whose aggregate cost function is $c(\cdot)$.)

At a competitive equilibrium involving price p^*, each consumer i 's purchase of the public good x_i^* must maximize her utility:

$$\operatorname{Max}_{x_i \geq 0} \ \phi_i\left(x_i + \sum_{k \neq i} x_k^*\right) - p^* x_i$$

In determining her optimal purchases, consumer i takes as given the amount of the private good being purchased by each other consumer.

> There is a bit of game theory here: this is how we find a Nash equilibrium.

Consumer i 's purchases x_i^* must therefore satisfy the necessary and sufficient first-order condition

$$\phi_i'\left(x_i^* + \sum_{k \neq i} x_k^*\right) = p^*, \text{ with equality if } x_i^* > 0$$

Letting $x^* = \sum_i x_i^*$ denote the equilibrium level of the public good, for each consumer i we must therefore have

$$\phi_i'(x^*) \leq p^*, \text{ with equality if } x_i^* > 0$$

The firm's supply q^*, on the other hand, must solve:

$$\operatorname{Max}_{q \geq 0} \ (p^* q - c(q))$$

and therefore must satisfy the standard necessary and sufficient first-order condition

$$p^* \leq c'(q^*), \text{ with equality if } q^* > 0$$

At a competitive equilibrium, $q^* = x^*$

Thus, letting $\delta_i = 1$ if $x_i^* > 0$ and $\delta_i = 0$ if $x_i^* = 0$, we have:

$$\sum_i \dot{\delta}_i[\phi_i'(q^*) - c'(q^*)] = 0$$

Recalling that $\phi_i'(\cdot) > 0$ and $c'(\cdot) > 0$, this implies that whenever $I > 1$ and $q^* > 0$ (so that $\delta_i = 1$ for some i) we have

$$\sum_{i=1}^{I} \phi_i'(q^*) > c'(q^*)$$

Whenever $q^\circ > 0$ and $I > 1$, the level of the public good provided is too low; that is, $q^* < q^0$.

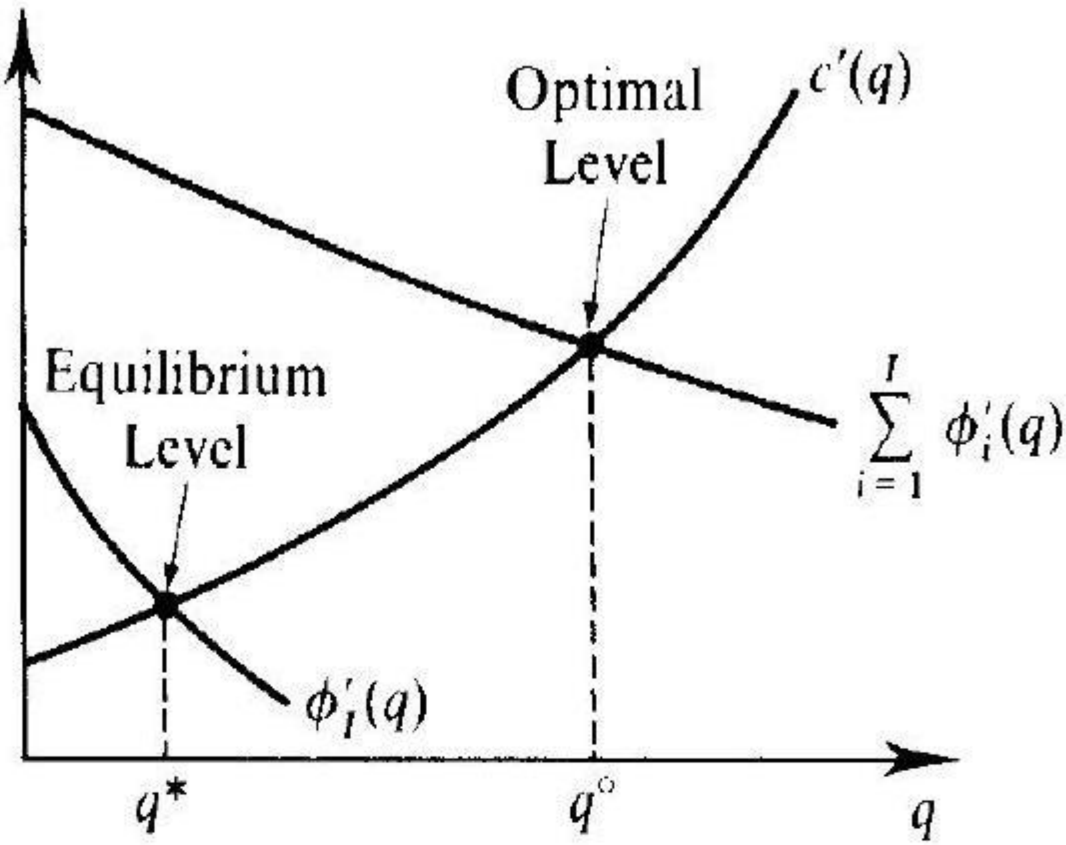


Figure 11.C.1: Private provision leads to an insufficient level of a desirable public good.

The cause of this inefficiency can be understood in terms of our discussion of externalities.

Each consumer's purchase of the public good provides a direct benefit not only to the consumer herself but also to every other consumer.

Hence, private provision creates a situation in which externalities are present.

The failure of each consumer to consider the benefits for others of her public good provision is often referred to as the free-rider problem:

> **Each consumer has an incentive to enjoy the benefits of the public good provided by others while providing it insufficiently herself.**

In the present model, the free-rider problem takes a very stark form.

To see this most simply, suppose that we can order the consumers according to their marginal benefits, in the sense that $\phi_1'(x) < \cdots < \phi_I'(x)$ at all $x \geq 0$.

Then optimality can hold with equality only for a single consumer and, moreover, this must be the consumer labeled I.

Therefore, only the consumer who derives the largest (marginal) benefit from the public good will provide it; all others will set their purchases equal to zero in the equilibrium.

The equilibrium level of the public good is then the level q^* that satisfies $\phi_I'(q^*) = c'(q^*)$.

Figure 11.C.1 depicts both this equilibrium and the Pareto optimal level. Note that the curve representing $\sum_i \phi_i'(q)$ geometrically corresponds to a vertical summation of the individual curves representing $\phi_i(q)$ for $i = 1, \ldots, I$.

(Whereas in the case of a private good, the market demand curve is identified by adding the individual demand curves horizontally).

The inefficiency of private provision is often remedied by governmental intervention in the provision of public goods.

Just as with externalities, this can happen not only through quantity-based intervention (such as direct governmental provision) but also through "price-based" intervention in the form of taxes or subsidies.

For example, suppose that there are two consumers with benefit functions $\phi_1(x_1 + x_2)$ and $\phi_2(x_1 + x_2)$, where x_i is the amount of the public good purchased by consumer i, and that $q^\circ > 0$.

A subsidy to each consumer i per unit purchased of $s_i = \phi'_{-i}(q^\circ)$ faces each consumer with the marginal external effect of her actions and so generates an optimal level of public good provision by consumer i.

Formally, if $(\tilde{x}_1, \tilde{x}_2)$ are the competitive equilibrium levels of the public good purchased by the two consumers given these subsidies, and if $\tilde{p}$ is the equilbrium price, then consumer i 's purchases of the public good, $\tilde{x}_i$, must solve:

$$\text{Max}_{x_i \geq 0} \ \phi_i\left(x_i + \tilde{x}_j\right) + s_i x_i - \tilde{p} x_i$$

and so $\tilde{x}_i$ must satisfy the necessary and sufficient first-order condition

$$\phi_i'(\tilde{x}_1 + \tilde{x}_2) + s_i \leq \tilde{p}, \text{ with equality of } \tilde{x}_i > 0.$$

Substituting for s_i, and using the fact that price equals marginal cost and the market-clearing condition that $\tilde{x}_1 + \tilde{x}_2 = \tilde{q}$, we conclude that $\tilde{q}$ is the total amount of the public good in the competitive equilibrium given these subsidies if and only if

$$\phi_i'(\tilde{q}) + \phi'_{-i}(q^\circ) \leq c'(\tilde{q})$$

with equality for some i if $\tilde{q} > 0$.

Use $\sum_{i=1}^{I} \phi_i'(q^\circ) \leq c'(q^\circ)$ to see that $\tilde{q} = q^\circ$.

Note that both optimal direct public provision and this subsidy scheme require that the government know the benefits derived by consumers from the public good.

I.e., their willingness to pay in terms of private goods.

Lindahl Equilibria

Although private provision of the sort studied above results in an inefficient level of the public good, there is in principle a market institution that can achieve optimality.

Suppose that, for each consumer i, we have a market for the public good "as experienced by consumer i."

That is, we think of each consumer's consumption of the public good as a distinct commodity with its own market.

We denote the price of this personalized good by p_i.

Note that p_i may differ across consumers.

Suppose also that, given the equilibrium price p_i^{**}, each consumer i sees herself as deciding on the total amount of the public good she will consume, x_i, so as to solve

$$\text{Max}_{x_i>0}\ \phi_i(x_i) - p_i^{**}x_i$$

Her equilibrium consumption level x_i^{**} must therefore satisfy the necessary and sufficient first-order condition

$$\phi_i'(x_i^{**}) \le p_i^{**},\ \text{ with equality if } x_i^{**} > 0$$

The firm is now viewed as producing a bundle of I goods with a fixed-proportions technology (i.e., the level of production of each personalized good is necessarily the same).

Thus, the firm solves

$$\text{Max}_{q\geq 0}\left(\sum_{i=1}^{1} p_i^{**}q\right) - c(q)$$

The firm's equilibrium level of output q^{**} therefore satisfies the necessary and sufficient first-order condition

$$\sum_{i=1}^{I} p_i^{**} \le c'(q^{**}), \text{with equality if } q^{**} > 0$$

We have then:

$$\sum_{i=1}^{I} \phi_i'(q^{**}) \le c'(q^{**}), \text{with equality if } q^{**} > 0$$

The equilibrium level of the public good consumed by each consumer is exactly the efficient level: $q^{**} = q^{\circ}$.

This type of equilibrium in personalized markets for the public good is known as a Lindahl equilibrium, after Lindahl (1919).

To understand why we obtain efficiency, note that once we have defined personalized markets for the public good, each consumer, taking the price in her personalized market as given, fully determines her own level of consumption of the public good.

Externalities are eliminated.

Yet, despite the attractive properties of Lindahl equilibria, their realism is questionable.

Note, first, that the ability to exclude a consumer from use of the public good is essential if this equilibrium concept is to make sense.

Otherwise a consumer would have no reason to believe that in the absence of making any purchases of the public good she would get to consume none of it.

Moreover, even if exclusion is possible, these are markets with only a single agent on the demand side.

As a result, price-taking behavior of the sort presumed is unlikely to occur.

The idea that inefficiencies can in principle be corrected by introducing the right kind of markets is a very general one.

In particular cases, however, this "solution" may or may not be a realistic possibility.

Multilateral Externalities

In most cases, externalities are felt and generated by numerous parties. This is particularly true of those externalities, such as industrial pollution, smog caused by automobile use, or congestion, that are widely considered to be "important" policy problems. In this section, we extend our analysis of externalities to these multilateral settings.

An important distinction can be made in the case of multilateral externalities according to whether the externality is depletable (or private, or rivalrous) or nondepletable (or public, or nonrivalrous). Depletable externalities have the feature that experience of the externality by one agent reduces the amount that will be felt by other agents. For example, if the externality takes the form of the dumping of garbage on people's property, if an additional unit of garbage is dumped on one

11. Thus, the possibility of exclusion can be important for efficient supply of the public good, even though the use of an exclusion technology is itself inefficient (a Pareto optimal allocation cannot involve any exclusion). piece of property, that much less is left to be dumped on others. [12] Depletable externalities therefore share the characteristics of our usual (private) sort of commodity. In contrast, air pollution is a nondepletable externality; the amount of air pollution experienced by one agent is not affected by the fact that others are also experiencing it. Nondepletable externalities therefore have the characteristics of public goods (or bads).

In this section we argue that a decentralized market solution can be expected to work well for multilateral depletable externalities as long as well-defined and enforceable property rights can be created. In contrast, market-based solutions are unlikely to work in the nondepletable case, in parallel to our conclusions regarding public goods in Section 11.C.

We shall assume throughout this section that the agents who generate externalities are distinct from those who experience them. This simplification is inessential but eases the exposition and facilitates comparison with the previous sections (Exercise 11.D.2 asks you to consider the general case). For ease of reference, we assume here that the generators of the externality are firms and that those experiencing the externality are consumers. We also focus on the special, but central, case in which the externality generated by the firms is homogeneous (i.e., consumers are indifferent to the source of the externality). (Exercise 11.D.4 asks you to consider the case in which the source matters.)

We again adopt a partial equilibrium approach and assume that agents take as given the price vector p of L traded goods. There are J firms that generate the externality in the process of production. As discussed in Section 11.B, given price vector p, we can determine firm j 's derived profit function over the level of the externality it generates, $h_j \geq 0$, which we denote by $\pi_j(h_j)$. There are also I consumers, who have quasilinear utility functions with respect to a numeraire, traded commodity. Given price vector p, we denote by $\phi_i(\tilde{h}_i)$

consumer i 's derived utility function over the amount of the externality $\tilde{h}_i$ she experiences. We assume that $\pi_j(\cdot)$ and $\phi_i(\cdot)$ are twice differentiable with $\pi_j''(\cdot) < 0$ and $\phi_i''(\cdot) < 0$. To fix ideas, we shall focus on the case where $\phi_i'(\cdot) < 0$ for all i, so that we are dealing with a negative externality.

Depletable Externalities

We begin by examining the case of depletable externalities. As in Section 11.B, it is easy to see that the level of the (negative) externality is excessive at an unfettered competitive equilibrium. Indeed, at any competitive equilibrium, each firm j will wish to set the extcrnality-generating activity at the level h_j^* satisfying the condition

$$\pi_j\left(h_j^*\right) \leq 0, \text{ with equality if } h_j^* > 0. \text{ [13]}$$

In contrast, any Pareto optimal allocation involves the levels $\left(\tilde{h}_1^\circ, \ldots, \tilde{h}_I^\circ, h_1^\circ, \ldots, h_J^\circ\right)$

12. ∧ distinction can also be made as to whether a depletable externality is allocable. For example, acid rain is depletable in the sense that the total amount of chemicals put into the air will fall somewhere, but it is not readily allocable because where it falls is determined by weather patterns. Throughout this section, we take depletable externalities to be allocable. The analytical implications of nonallocable depletable externalities parallel those of nondepletable ones.

13. The firms are indifferent about which consumer is affected by their externality. Therefore, apart from the fact that $\sum_i \tilde{h}_i = \sum_j h_j^*$, the particular values of the individual $\tilde{h}_i$'s are indeterminate. that solve [14]

$$\max_{\substack{(\hat{h}_1 \ldots, \hat{h}_J) \geq 0 \\ (\tilde{h}_1, \ldots, \tilde{h}_I) > 0}} \sum_{i=1}^{I} \phi_i(\tilde{h}_i) + \sum_{j=1}^{J} \pi_j(h_j)$$

$$\text{s.t.} \sum_{j=1}^{J} h_j = \sum_{i=1}^{I} \tilde{h}_i.$$

The constraint in (11.D.2) reflects the depletability of the externality: If $\tilde{h}_i$ is increased by one unit, there is one unit less of the externality that needs to be experienced by others. Letting μ be the multiplier on this constraint, the necessary and sufficient first-order conditions to problem (11.D.2) are

$$\phi_i'(\tilde{h}_i') \leq \mu, \text{ with equality if } \tilde{h}_i^\circ > 0, i = 1, \ldots, I,$$

and

$$\mu \leq -\pi_j'(h_j''), \text{ with equality if } h_j^o > 0, j = 1, \ldots, J$$

Conditions (11.D.3) and (11.D.4), along with the constraint in problem (11.D.2), characterize the optimal levels of externality generation and consumption. Note that they exactly parallel the efficiency conditions for a private good derived in Chapter 10, conditions (10.D.3) to (10.D.5), where we interpret $-\pi_j'(\cdot)$ as firm j 's marginal cost of producing more of the externality. If well-defined and enforceable property rights can be specified over the externality, and if I and J are large numbers so that price taking is a reasonable hypothesis, then by analogy with the analysis of competitive markets for private goods in Chapter 10, a market for the externality can be expected to lead to the optimal levels of externality production and consumption in the depletable case.

Nondepletable Externalities

We now move to the case in which the externality is nondepletable. To be specific, assume that the level of the externality experienced by each consumer is $\sum_j h_j$, the total amount of the externality produced by the firms.

In an unfettered competitive equilibrium, each firm j 's externality generation h_j^* again satisfies condition (11.D.1). In contrast, any Pareto optimal allocation involves externality generation levels $(h_1^\circ, \ldots, h_J')$ that solve

$$\mathrm{Max}_{(h_1, \ldots, h_,) \geq 0} \sum_{i=1}^{I} \phi_i\left(\sum_j h_j\right) + \sum_{j=1}^{J} \pi_j(h_j)$$

This problem has necessary and sufficient first-order conditions for each firm j 's optimal level of externality generation, h_j°, of

$$\sum_{i=1}^{I} \phi_i'\left(\sum_j h_j^\circ\right) \leq -\pi_j'\left(h_j^\circ\right), \text{ with equality if } h_j^\circ > 0$$

14. The objective function in (11.D.2) amounts to the usual difference between benefits and costs arising in the aggregate surplus measure. Note, to this effect, that $-\pi_j(\cdot)$ can be viewed as firm j 's cost function for producing the externality. Condition (11.D.6) is exactly analogous to the optimality condition for a public good, condition (11.C.1), where $-\pi_j'(\cdot)$ is firm j 's marginal cost of externality production. [15]

By analogy with our discussion of private provision of public goods in Section 11.C, the introduction of a standard sort of market for the externality will not lead here, as it did in the bilateral case of Section 11.B, to an optimal outcome. The free-rider problcm reappears, and the equilibrium level of the (negative) externality will exceed its optimal level. Instead, in the case of a multilateral nondepletable externality, a market-based solution would require personalized markets for the externality, as in the Lindahl equilibrium concept. However, all the problems with Lindahl equilibrium discussed in Section 11.C will similarly afflict these markets. As a result, purely market-based solutions, personalized or not, are unlikely to work in the case of a depletable externality. [16]

In contrast, given adequate information (a strong assumption!), the government can achieve optimality using quotas or taxes. With quotas, the government simply sets an upper bound on each firm j 's level of externality generation equal to its optimal level h_j. On the other hand, as in Section 11.B, optimality-restoring taxes face each firm with the marginal social cost of their externality. Here the optimal tax is identical for each firm and is equal to $t_h = -\sum_i \phi_i'\left(\sum_j h_j^\circ\right)$ per unit of the externality generated. Given this tax, each firm j solves

$$\operatorname{Max}_{h_j \geq 0}\ \pi_j\left(h_j\right) - t_h h_j$$

which has the necessary and sufficient first-order condition

$$\pi_j'(h_j) \leq t_h, \text{ with equality if } h_j > 0.$$

Given $t_h = -\sum_i \phi_i'(\sum_j h_j'')$, firm j 's optimal choice is $h_j = h_j^\circ$.

A partial market-based approach that can achieve optimality with a nondepletable multilateral externality involves specification of a quota on the total level of the externality and distribution of that number of tradeable externality permits (each permit grants a firm the right to generate one unit of the externality). Suppose that $h^\circ = \sum_j h_j^\circ$ permits are given to the firms, with firm j receiving $\bar{h}_j$ of them. Let p_h^* denote the equilibrium price of these permits. Then each firm j 's demand for permits, h_j, solves $\text{Max}_{h_j \geq 0} \left(\pi_j(h_j) + p_h^*(\bar{h}_j - h_j)\right)$ and so satisfies the necessary and sufficient first-order condition $\pi_j'(h_j) \leq p_h^*$, with equality if $h_j > 0$. In addition, market clearing in the permits market requires that $\sum_j h_j = h^\circ$. The competitive equilibrium in the market for permits then has price $p_h^* = -\sum_i \phi_i'(h^\circ)$ and each firm j using h_j'' permits and so yields an optimal allocation. The advantage of this scheme relative to a strict quota method arises when the government has limited information about the $\pi_j(\cdot)$ functions and cannot tcll which particular firms can efficiently bear the burden of externality reduction, although it has enough information, perhaps of a statistical sort, to allow the computation of the optimal aggregate level of the externality, h°.

15. Recall that the single firm's cost function $c(\cdot)$ in Section 11. C could be viewed as the aggregate cost function of J separate profit-maximizing firms. Were we to explicitly model these J firms in Section 11.C, the optimality conditions for public good production would take exactly the form in (11.D.6) with $c_j'(h_j'')$ replacing $-\pi_j'(h_j'')$.

16. The public nature of the externality leads to similar free-rider problems in any bargaining solution. (See Exercise 11.D.6 for an illustration.)

Private Information and Second-Best Solutions

In practice, the degree to which an agent is affected by an externality or benefits from a public good will often be known only to her. The presence of privately held (or asymmetrically held) information can confound both centralized (e.g., quotas and taxes) and decentralized (c.g., bargaining) attempts to achieve optimality. In this section, we provide an introduction to these issues, focusing for the sake of specificity on the case of a bilateral externality such as that studied in Section 11.B. Following the convention adopted in Section 11.D, we shall assume here that the externalitygenerating agent is a firm and the affected agent is a consumer. (For a more general treatment of some of the topics covered in this section, see Chapter 23.)

Suppose, then, that we can write the consumer's derived utility function from externality level h (see Section 11.B for more on this construction) as $\phi(h,\eta)$, where $\eta \in \mathbb{R}$ is a parameter, to be called the consumer's type, that affects the consumer's costs from the externality. Similarly, we let $\pi(h,\theta)$ denote the firm's derived profit given its type $\theta \in \mathbb{R}$. The actual values of θ and η are privately ohserved: Only the consumer knows her type η, and only the firm observes its type 0. The ex ante likelihoods (probability distributions) of various values of θ and η are, however, publicly known. For convenience, we assume that θ and η are independently distributed. As previously, we assume that $\pi(h,\theta)$ and $\phi(h,\eta)$ are strictly concave in h for any given values of θ and η.

Decentralized Bargaining

Consider the decentralized approach to the externality problem first. In general, bargaining in the presence of bilateral asymmetric information will not lead to an efficient level of the externality. To see this, consider again the case in which the consumer has the right to an externality-free environment, and the simple bargaining process in which the consumer makes a take-it-or-leave-it offer to the firm. For simplicity, we assume that there are only two possible levels of the externality, 0 and $\bar{h} > 0$, and we focus on the case of a negative externality in which externality level $\bar{h}$, relative to the level 0 , is detrimental for the consumer and beneficial for the firm (the analysis is readily applied to the case of a positive externality).

It is convenient to define $h(\theta) = \pi(\bar{h},\theta) - \pi(0,\theta) > 0$ as the measure of the firm's benefit from the extcrnality-generating activity when its type is θ. Similarly, we let $c(\eta) = \phi(0,\eta) - \phi(\bar{h},\eta) > 0$ give the consumer's cost from externality level $\bar{h}$. In this simplified setting, the only aspects of the consumer's and firm's types that matter are the values of h

and c that these types generate. Hence, we can focus directly on the various possible values of b and c that the two agents might have. Denote by $G(b)$ and $F(c)$ the distribution functions of these two variables induced by the underlying probability distributions of θ and η (note that, given the independence of θ and η, b and c are independent). For simplicity, we assume that these distributions have associated density functions $g(b)$ and $f(c)$, with $g(b) > 0$ and $f(c) > 0$ for all $b > 0$ and $c > 0$.

Since the consumer has the right to an externality-free environment, in the absence of any agreement with the firm she will always insist that the firm set $h = 0$ (recall that $c > 0$). However, in any arrangement that guarantees Pareto optimal outcomes for all values of h and c, the firm should be allowed to set $h = \bar{h}$ whenever $b > c$. Now consider the amount that the consumer will demand from the firm when her cost is c in exchange for permission to engage in the externality-generating activity. Since the firm knows that the consumer will insist on $h = 0$ if there is no agreement, the firm will agree to pay the amount T if and only if $b \geq T$. Hence, the consumer knows that if she demands a payment of T, the probability that the firm will accept her offer equals the probability that $b \geq T$; that is, it is equal to $1 - G(T)$. Given her cost $c > 0$ (and assuming risk neutrality), the consumer optimally chooses the value of T she demands to solve

$$\text{Max}_T \ (1 - G(T))(T - c)$$

The objective function of problem (11.E.1) is the probability that the firm accepts the demand, multiplied by the nel gain to the consumer when this happens $(T - c)$. Under our assumptions, the objective function in (11.E.1) is strictly positive for all $T > c$ and equal to zero when $T = c$. Therefore, the solution, say T_c^*, is such that $T_c^* > c$. But this implies that this bargaining process must result in a strictly positive probability of an inefficient outcome, since whenever the firm's benefit b satisfies $c < b < T_c^*$, the firm will reject the consumer's offer, resulting in an externality level of zero, even though optimality requires that $h = \bar{h}$[17,18]

Quotas and Taxes

Just as decentralized bargaining will involve inefficiencies in the presence of privately held information, so too will the use of quotas and taxes. Moreover, as originally noted by

Weitzman (1974), the presence of asymmetrically held information causes these two policy instruments to no longer be perfect substitutes for one another, as they were in the model of Section 11.B. [19]

To begin, note that given θ and η, the aggregate surplus resulting from externality level h (we return to a continuum of possible externality levels here) is $\phi(h, \eta) + \pi(h, \theta)$. Thus, the externality level that maximizes aggregate surplus depends in general on the realized values of (θ, η). We denote this optimal value by the function $h^\circ(\theta, \eta)$. Figure 11.E.1 depicts this optimum value for two different pairs of parameters, (θ', η') and (θ'', η'').

Suppose, first, that a quota level of $\hat{h}$ is fixed. The firm will then choose the level of the externality to solve

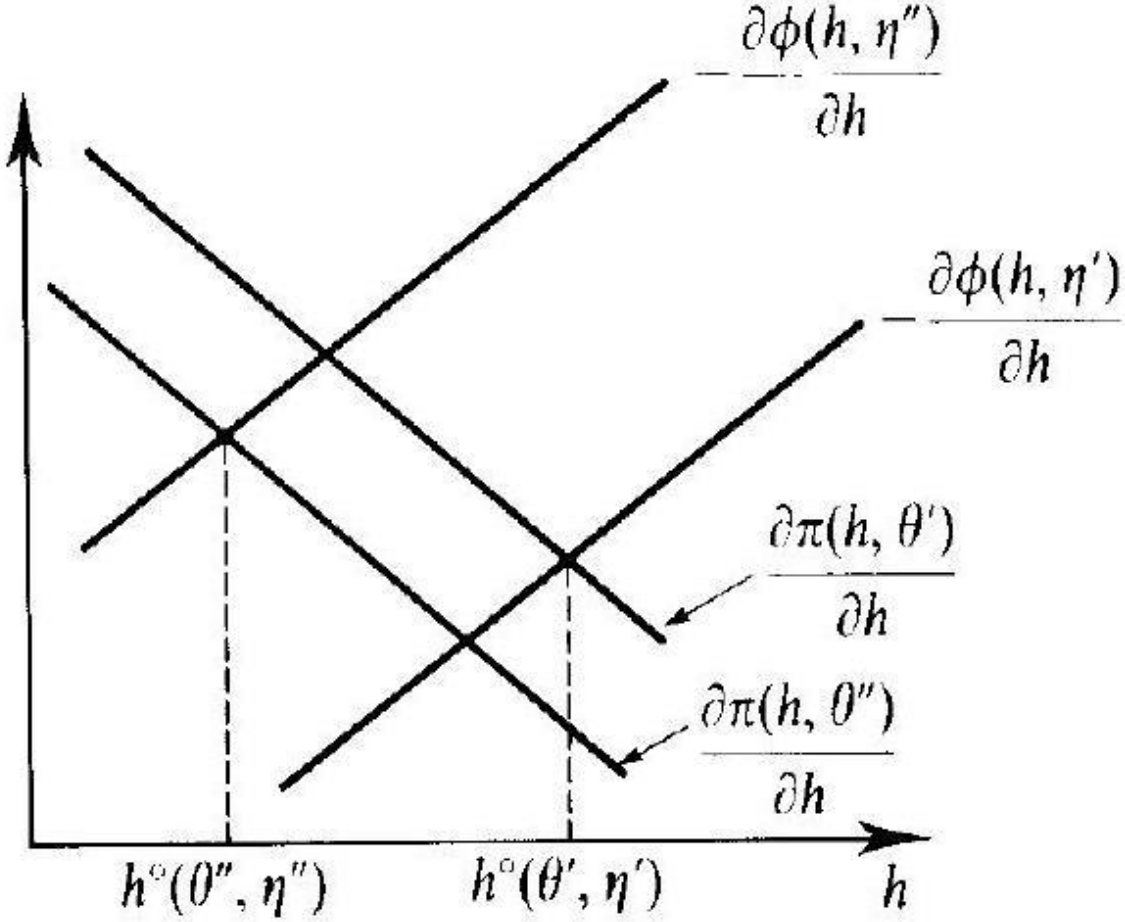
$$\begin{aligned} \text{Max}_{h \geq 0} \quad & \pi(h, \theta) \\ & \text{s.t. } h \leq \hat{h}. \end{aligned}$$

Denote its optimal choice by $h^q(\hat{h}, \theta)$. The typical effect of the quota will be to make

17. Note the similarity between problem (11.E.1) and the monopolist's problem studied in Section 12.B. Here the consumer's inability to discriminate among firms of different types leads her optimal offer to be one that yields an inefficient outcome.

18. We could, of course, also consider the outcomes from other, perhaps more elaborate, bargaining procedures. In Chapter 23, however, we shall study a result due to Myerson and Satterthwaite (1983) that implies that no bargaining procedure can lead to an efficient outcome for all values of b and c in this selting.

19. The discussion that follows also has implications for the relative advantages of quantityversus price-based control mechanisms in organizations.



17. Figure 11.E.1

The surplusmaximizing aggregate externality level for two different pairs of parameters, (θ',η') and $(0'',\eta'')$.

the actual level of the externality much less sensitive to the values of θ and η than is required by optimality. The firm's externality level will be completely insensitive to η. Morcover, if the level of the quota $\hat{h}$ is such that $\partial\pi(\hat{h},\theta)/\partial h > 0$ for all θ, we will have $h^q(\hat{h},\theta) = \hat{h}$ for every θ. The loss in aggregate surplus arising under the quota for types (θ,η) is given by

$$\phi\left(h^q(\hat{h},\theta),\eta\right) + \pi\left(h^4(\hat{h},\theta),\theta\right) - \phi(h^{\circ}(\theta,\eta),\eta) - \pi(h^{\circ}(\theta,\eta),\theta)$$

$$= \int_{h(\theta,\eta)}^{h^q(\hat{h},\theta)} \left(\frac{\partial \pi(h,\theta)}{\partial h} + \frac{\partial \phi(h,\eta)}{\partial h} \right) dh$$

This loss is represented by the shaded region in Figure 11.E.2 for a case in which the quota is set equal to $\hat{h} = h(\bar{\theta},\bar{\eta})$, the externality level that maximizes social surplus when θ and η each take their mean values, $\bar{\theta}$ and $\bar{\eta}$ [the dashed lines in the figure are the graphs of $\partial \pi(h,\bar{\theta})/\,\partial h$ and $-\,\partial \phi(h,\bar{\eta})/\,\partial h$ and the solid lines are the graphs of $\partial \pi(h,\theta)/\,\partial h$ and $-\,\partial \phi(h,\eta)/\,\partial h$; note that in the case depicted, the firm wishes to produce the externality up to the allowed quota $\hat{h}$].

Consider next the use of a tax on the firm of t units of the numeraire per unit of the externality. For any given value of θ, the firm will then choose the level of externality to solve

$$\text{Max}_{h \geq 0} \; \pi(h,\theta) - th$$

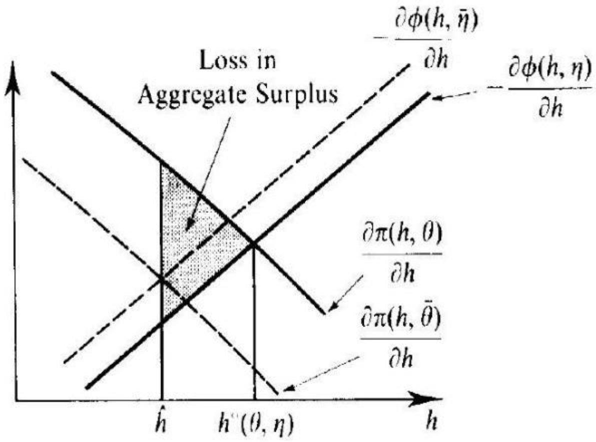
Denote its optimal choice by $h^t(t,\theta)$. The loss in aggregate surplus from the tax relative to the optimal outcome for types (θ,η) is therefore given by

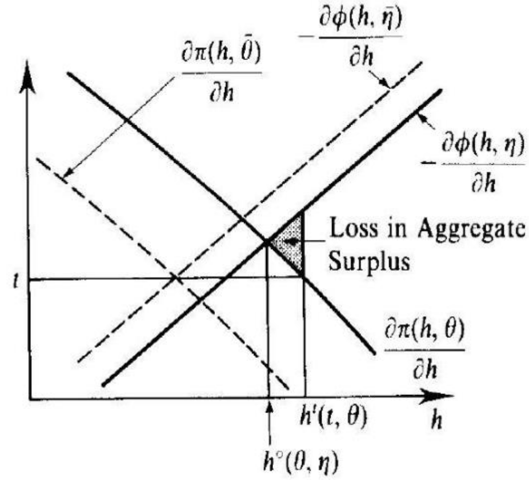
$$\phi(h'(t,\theta),\eta) + \pi(h'(t,\theta),\theta) - \phi(h^{\circ}(\theta,\eta),\eta) - \pi(h^{\circ}(\theta,\eta),\theta)$$

$$= \int_{h(\theta,\eta)}^{h^t(t,\theta)} \left(\frac{\partial \pi(h,\theta)}{\partial h} + \frac{\partial \phi(h,\eta)}{\partial h} \right) dh$$

Its value is depicted by the shaded region in Figure 11.E.3 for the same situation as in Figure 11.E.2, but now assuming that a tax is set at $t = -\,\partial \phi(h^{\circ}(\bar{\theta},\bar{\eta}),\bar{\eta})/\,\partial h$, the value that results in the maximization of aggregate surplus when $(\theta,\eta) = (\bar{\theta},\bar{\eta})$. Note that under a tax, as under a quota, the level of the externality is responsive to changes in the marginal benefits of the firm but not to changes in the marginal costs of the consumer.

Which of these instruments, quota or tax, performs better? The answer is that it depends. Imagine, for example, that η is a constant, say equal to $\bar{\eta}$. Then, for θ such that the benefits of the externality's use to the firm are high, a quota will typically miss the optimal externality level by not allowing the externality to increase above the quota level. On the other hand, because a fixed tax rate t does not reflect any





18. Figure 11.E.2 (left)

The loss in aggregate surplus under a quota for types (θ, η).

Flgure 11.E.3 (right)

The loss in aggregate surplus under a tax for types (θ, η). increasing marginal costs of the externality to the consumer at higher externality levels, for such values of θ the tax may result in excess production of the externality. Intuitively, when the optimal externality level varies little with θ, we expect a quota to be better. Figure 11.E.4(a), for example, depicts a case in which the marginal cost to the consumer of the externality is zero up to some point h^* and infinite thereafter. In this case, by setting a quota of $\hat{h} = h^*$, we can maximize aggregate surplus for any value of (θ, η), but no tax can accomplish this. A tax would have to be very high to guarantee that with probability one the externality level fixed by the firm is not larger than h^*. But if so, the resulting externality level would be too low most of the time.

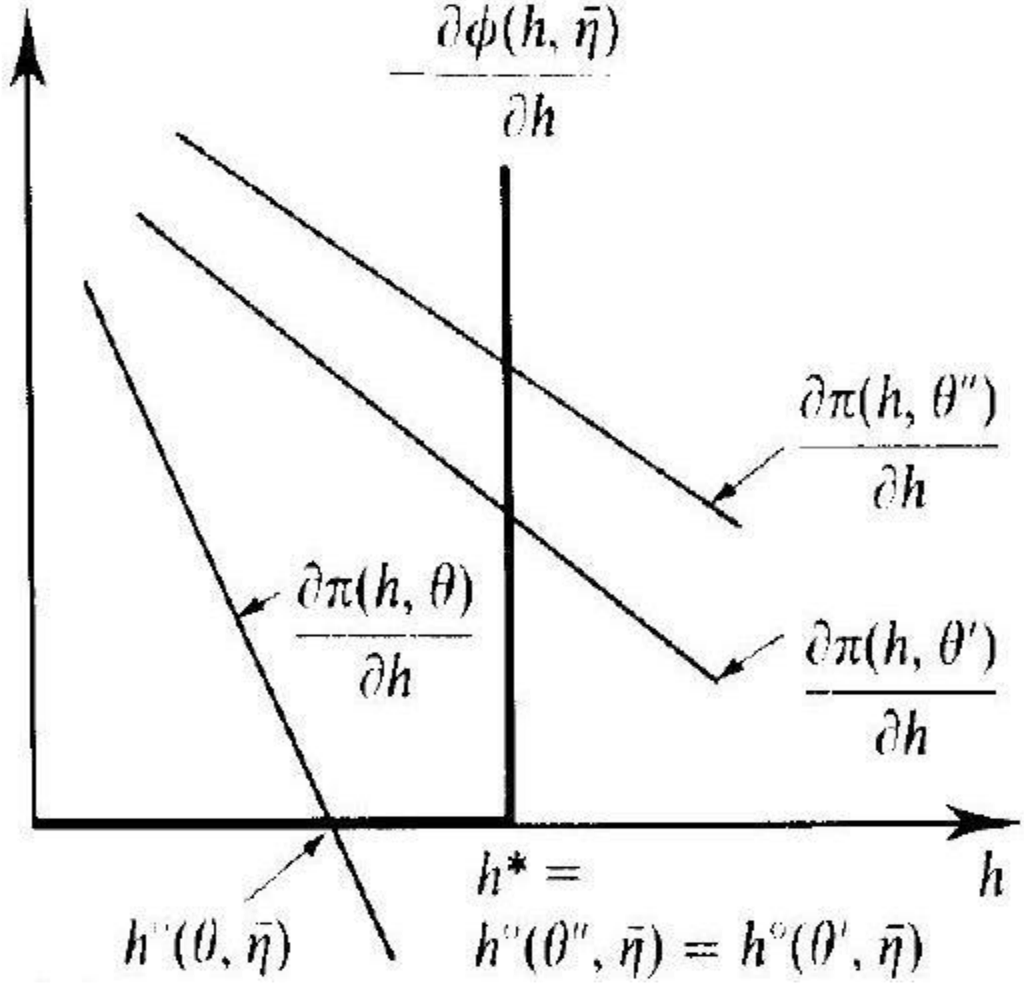
In contrast, in Figure 11.E.4(b) we depict a case in which the marginal cost to the consumer of the externality is independent of the level of h. In this case, a tax equal to this marginal cost $(t = t^*)$ achieves the surplus-maximizing externality level for all $(0, \eta)$, but no quota can do so.

If we take the expected value of aggregate surplus as our welfare measure, we therefore see from these two examples that either policy instrument may be preferable, depending on the circumstances. [20] (Exercise 11.E.1 asks you to provide a full analysis for a linear-quadratic example.) Note also that the bargaining procedure we have discussed will not result in optimality in either case depicted in Figure 11.E.4. [21] Thus, we have here two cases in which either a quota or a tax performs better than a particular decentralized outcome. [22]

20. In Chapter 13, we discuss in greater detail some of the issues that arise in making welfare comparisons in settings with privately held information. There we shall justify the maximization of expected aggregate surplus in this partial equilibrium setting as a requirement of a notion of ex ante Pareto optimality for the two agents. See also the discussion in Section 23.F.

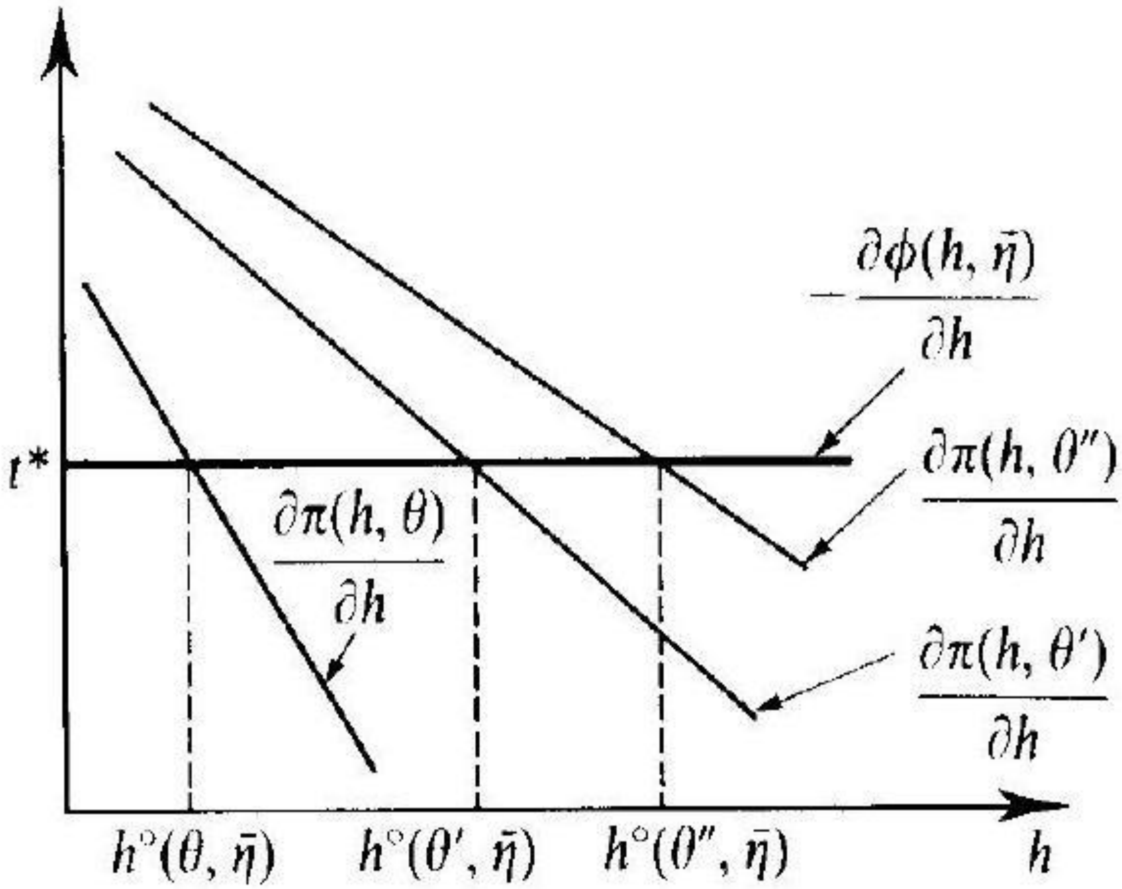
21. Strictly speaking, our previous discussion of bargaining assumed only two possible levels of the externality, while here we have a continuum of levels. This difference is not important. The inefficiency of the bargaining procedure previously studied would hold in this continuous environment as well.

22. We should emphasize that in these two examples other bargaining procedures will perform better than the procedure involving a take-it-or-leave-it offer by the consumer. For example, if a take-or-leave offer is made by the firm, then full optimality results in both of these cases because the type of the consumer is known with certainty. The conclusion of our discussion is therefore a qualitative one: With asymmetric information, it is difficult to make very general assertions about the relative performance of centralized versus decentralized approaches.



$-\frac{\partial \phi(h, \bar{\eta})}{\partial h}$
$\frac{\partial \pi(h, \theta'')}{\partial h}$
$\frac{\partial \pi(h, \theta)}{\partial h}$
$\frac{\partial \pi(h, \theta')}{\partial h}$
$h^* =$
h
$h^\circ(\theta, \bar{\eta})$
$h^\circ(\theta'', \bar{\eta}) = h^\circ(\theta', \bar{\eta})$

(a)



(b)

19. Figure 11.E.4

Two cases in which a quota or tax maximizes aggregate surplus for every realization of θ. (a) Quota $\hat{h} = h^*$ maximizes aggregate surplus for all o . (b) Tax $t = t^*$ maximizes aggregate surplus for all o .

In Exercisc 11.E.2, you are asked to extend the analysis just given to a case with two firms $(j = 1,2)$ generating an externality, where the two firms are identical except possibly for their realized levels of θ_j. The exercise illustrates the importance of the degree of correlation between the θj 's for the relative performance of quotas versus taxes. In comparing a uniform quota policy versus a uniform tax policy ("uniform" here means that the two firms face the same quota or tax rate), the less correlated the shocks across the firms, the better the tax looks. The reason is not difficult to discern. With imperfect correlation, a uniform tax has a benefit that is not achieved with a uniform quota: It allows for the individual levels of externalities generated to be responsive to the realized values of the θ_j 's. Indeed, with a uniform tax, the production of the total amount of externality generated is always efficiently distributed across the two firms.

The presence of multiple generators of an externality also raises the possibility that a market for tradeable emissions permits could be created, as discussed at the end of Section 11.D. This simple addition to the quota policy can potentially eliminate the inefficient distribution of externality generation across different generators that is often a feature of a quota policy. In particular, suppose that instead of simply giving each firm a quota level, we now give them tradeable externality permits entitling them to generate the same number of units of the externality as in the quota. Suppose also that each firm would always fully use its quota if no trade was possible. Then trade must result in at least as large a value of aggregate surplus as the simple quota scheme for any realization of the firms' and consumer's types, because we still get the same total level of emissions and we can never get a trade between firms that lowers aggregate profits. [23] Of course, the same bargaining problems that we studied above can prevent a fully efficient distribution of externality generation from arising; but if the firms know each others' values of θ_j or are numerous enough to act competitively in the market for these rights, then we can expect a distribution of the total externality generation that is efficient across generators. In fact, in the case where the statistical distribution of costs among the firms is known but the particular realizations for individual firms are not known, this type of policy can achieve a fully optimal outcome.

23. Note, however, that the assumption that the externalities generated by the different firms are perfect substitutes to the consumer is crucial to this conclusion. If this is not true, then the reallocation of externality generation can reduce aggregate surplus by lowering the well-being of the agents affected by the externality.

More General Policy Mechanisms

The tax and quota schemes considered above are, as we have seen, completely unresponsive to changes in the marginal costs of the externality to the affected agent (the consumer in this case). It is natural to wonder whether any other sorts of schemes can do better, perhaps by making the level of the externality responsive to the consumer's costs. The problem in doing so is that these benefits and costs are unobservable, and the parties involved may not have incentives to reveal them truthfully if asked. For example, suppose that the government simply asks the consumer and the firm to report their benefits and costs from the externality and then enforces whatever appears to be the optimal outcome based on these reports. In this case, the consumer will have an incentive to exaggerate her costs when asked in order to prevent the firm from being allowed to generate the externality. The question, then, is how to design mechanisms that control these incentives for misreporting and, as a consequence, cnable the government to achieve an efficient outcome. This problem is studied in a very general form in Chapter 23; here we confine ourselves to a brief examination of one well-known scheme.

Return to the case in which there are only two possible levels of the externality, 0 and $\bar{h}$. Can we design a scheme that achieves the optimal level of externality generation for every realization of b (the firm's benefit from the externality) and c (the consumer's cost)? We now verify that the answer is "yes."

Imagine the government setting up the following revelation mechanism: The firm and the consumer are each asked to report their values of b and c, respectively. Let $\hat{b}$ and $\hat{c}$ denote these announcements. For each possible pair of announcements $(\hat{b}, \hat{c})$, the government sets an allowed level of the externality as well as a tax or subsidy payment for each of the two agents. Suppose, in particular, that the government declares that it will set the allowed externality level h to maximize aggregate surplus given the announcements. That is, $h = \bar{h}$ if and only if $\hat{b} > \hat{c}$. In addition, if externality generation is allowed (i.e., if $h = \bar{h}$), the government will tax the firm an amount equal to $\hat{c}$ and will subsidize the consumer with a payment equal to $\hat{b}$. That is, if the firm wants to generate the externality (which it indicates by reporting a large value of h), it is asked to pay the externality's cost as declared by the consumer; and if the consumer allows the externality (by reporting a low value of c) she receives a payment equal to the externality's benefit as declared by the firm.

In fact, under this scheme both the firm and the consumer will tell the truth, so that an optimal level of externality generation will, indeed, result for every possible (b, c) pair. To see this, consider the consumer's optimal announcement when her cost level is ∴. If the firm announces some $\hat{b} > c$, then the consumer prefers to have the externality-generating activity allowed (she does $\hat{b} - c$ better than if it is prevented). Hence, her optimal announcement satisfies $\hat{c} < \hat{b}$; moreover, because any such announcement will give her the same payoff, she might as well announce the truth, that is, $\hat{c} = c < \hat{b}$. On the other hand, if the firm announces $\hat{b} \leq c$, the consumer prefers to have the externality level set to zero. Hence, she would like announce $\hat{c} \geq \hat{b}$; and again, because any of these announcements will give her the same payoff, she may as well announce the truth, that is, $\hat{c} = c \geq \hat{b}$. Thus, whatever the firm's announcement, truth-telling is an optimal strategy for the consumer. (Formally, telling the truth is a weakly dominant strategy for the consumer in the sense studied in Section 8. B. In fact, it is the consumer's only weakly dominant strategy; see Exercise 11.E.3.) A parallel analysis yields the same conclusion for the firm.

Exercise 11.E.4: Show that in the tax-subsidy part of the mechanism above we could add, without affecting the mechanism's truth-telling or optimality properties, an additional payment to each agent that depends in an arbitrary way on the other agent's announcement.

The scheme we have described here is an example of the Groves Clarke mechanism [due to Groves (1973) and Clarke (1971); see also Section 23.C] and was originally proposed as a mechanism for deciding whether to carry out public good projects. Some examples for the public goods context are contained in the exercises at the end of the chapter.

The Groves Clarke mechanism has two very attractive features: it implements the optimal level of the externality for every (b, c) pair, and it induces truth-telling in a very strong (i.e., dominant strategy) sense. But the mechanism has some unattractive features as well. In particular, it does not result in a balanced budget for the government: The government has a deficit equal to $(b - c)$ whenever $b > c$. We could use the flexibility offered by Exercise 11.E.4 to eliminate this deficit for all possible (b, c), but then we would necessarily create a budget surplus and therefore a Pareto inefficient outcome for some values of (b, c) (not all units of the numeraire will be left in the hands of the firm or the consumer).

In fact, this problem is unavoidable with this type of mechanism: If we want to preserve the properties that, for every (b, c), truth-telling is a dominant strategy and the optimal level of externality is implemented, then we generally cannot achieve budget balance for every (b, c). In Chapter 23 we discuss this issue in greater detail and also consider other mechanisms that can, under certain circumstances, get around the problem. (See also Exercise 11.E.5 for an analysis in which budget balance is required only on average.)

APPENDIX A: NONCONVEXITIES AND THE THEORY OF EXTERNALITIES

Throughout this chapter, we have maintained the assumption that preferences and production sets are convex, leading the derived utility and profit functions we have considered to be concave. With these assumptions, all the decision problems we have studied have been well behaved; they had unique solutions (or, more generally, convex-valued solutions) that varied continously with the underlying parameters of the problems (e.g., the prices of the L traded commodities or the price of the externality if a market existed for it). Yet, this is not a completely innocent assumption. In this appendix, we present some simple examples designed to illustrate that externalities may themselves generate nonconvexities, and we comment on some of the implications of this fact.

We consider here a bilateral externality situation involving two firms. We suppose that firm 1 may engage in an externality-generating activity that affects firm 2's production. The level of externality generated by firm 1 is denoted by h, and firm j 's profits conditional on the production of externality level h are $\pi_j(h)$ for $j = 1,2$. It is perfectly natural to assume that $\pi_1(\cdot)$ is concave: The level h could, for example,

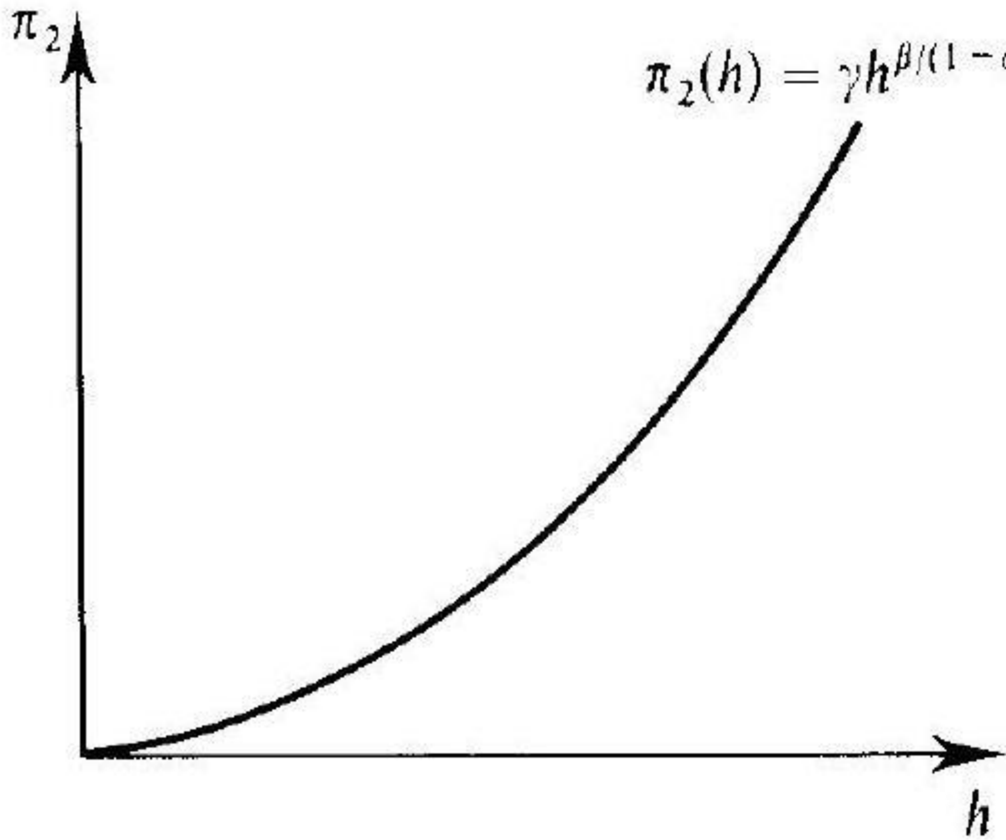


Figure 11.AA.1

The derived profit function of firm 2 (the externality recipient) in Example 11.AA.1 when $\alpha + \beta > 1$.

be equal to firm 1's output. [24] As Examples 11.AA.1 and 11.AA.2 illustrate, however, this may not be true of firm 2's profit function.

Example 11.AA.1: Positive Externalities as a Source of Increasing Returns. Suppose that firm 2 produces an output whose price is 1 , using an input whose price, for simplicity, we also take to equal 1 . Firm 2's production function is $q = h^{\beta} z^{\alpha}$, where $\alpha, \beta \in [0,1]$. Thus, the externality is a positive one. [25] Note that, for fixed h, the problem of firm 2 is concave and perfectly well behaved. Given a level of h, the maximized profits of firm 2 can be calculated to be $\pi_2(h) = \gamma h^{\beta/(1-\alpha)}$, where $\gamma > 0$ is a constant. In Figure 11.AA.1, we represent $\pi_2(h)$ for $\beta > 1 - \alpha$. We see there that firm 2's derived prolit function is not concave in h; in fact, it is convex. This reflects the fact that if we think of the externality h

as an input to firm 2's production process, then firm 2's overall production function exhibits increasing returns to scale because $\alpha + \beta > 1$.

Example 11.AA.2: Negative Externalities as a Source of Nonconvexities. In Example 11.AA.1, the nonconvexity in firm 2's production set, and the resulting failure of concavity in its derived profit function, were caused by a positive externality. In this example the failure of concavity of firm 2's derived profit function is the result of a negative externality.

Suppose, in particular, that $\pi_2'(h) \leq 0$ for all h, with strict inequality for some h, and that firm 2 has the option of shutting down when experiencing externality level h and receiving profits of zero. [26] In this case, the function $\pi_2(\cdot)$ can never be concave

24. Note also that we may well have $\pi_1(h) < 0$ for some levels $h \geq 0$ because $\pi_1(h)$ is firm 1's maximized profit conditional on producing externality level h (and so shutting down is not possible if $h > 0$).

25. More generally, we could think that there is an industry composed of many firms and that the externality is produced and felt by all firms in the industry (e.g., h could be an index, correlated with output, of accumulated know-how in the industry). Externalities were first studied by Marshall (1920) in this context. See also Chipman (1970) and Romer (1986).

26. In the more typical case of a multilateral externality, the ability of affected parties to shut down in this manner often depends on whether the externality is depletable. In the case of a nondepletable externality, such as air pollution, affected firms can always shut down and receive zero profits. In contrast, in the case of a depletable externality (such as garbage), where $\pi_j(h)$ reflects firm j 's profits when it individually absorbs h units of the externality, the absorption of the externality may itself require the use of some inputs (c.g., land to absorb garbage). Indeed, were this not the case for a depletable externality, the externality could always be absorbed in a manner that creates no social costs by allocating all of the externality to a firm that shuts down.

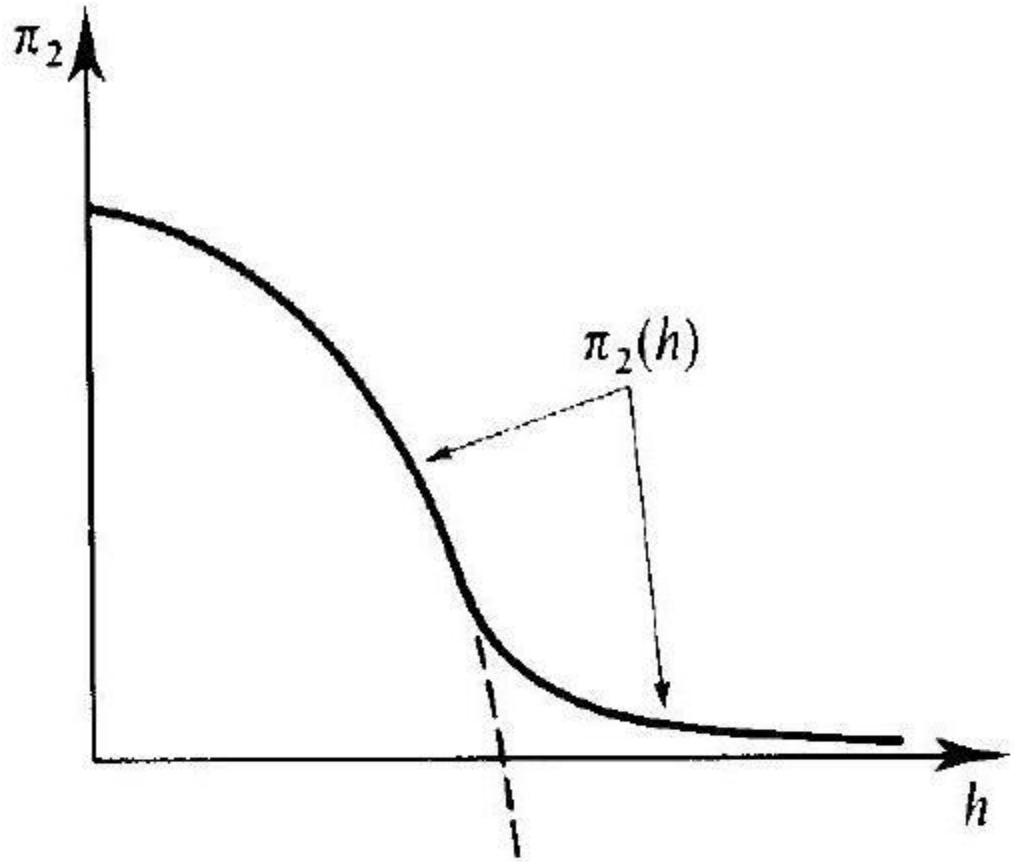


Figure 11.AA.2

If the recipient of a negative externality can shut down and earn zero profits for any level of the externality, then its derived profit function $\pi_2(h)$ cannot be concave over $h \in [0, \infty]$. over all $h \in [0, \infty)$, a point originally noted by Starrett (1972). The reason can be seen in Figure 11.AA.2: If $\pi_2(\cdot)$ were a strictly decreasing concave function, then it would have to become negative at some level of h (see the dashed curve), but $\pi_2(\cdot)$ must be nonnegative if firm 2 can always choose to shut down.

The failure of $\pi_2(\cdot)$ to be concave can create problems for both centralized and decentralized solutions to the externality problem. For example, if property rights over the externality are defined and a market for the externality is introduced in either Example 11.AA.1 or Example 11.AA.2, a competitive equilibrium may fail to exist (even assuming that the two agents act as price takers). Firm 2's objective function will not be concave, and so its optimal demand may fail to be well defined and continuous (recall our discussion in Section 10.C of the equilibrium existence problems caused by nonconvexities in firms' cost functions).

In contrast, laxes and quotas can, in principle, still implement the optimal outcome despite the failure of firm 2's profit function to be concave because their use depends only on the profit function of the externality generator (here, firm 1) being well behaved. In practice, however, nonconvexities in firm 2's profit function may create problems for these centralized solutions as well. Example 11.AA.3 illustrates this point.

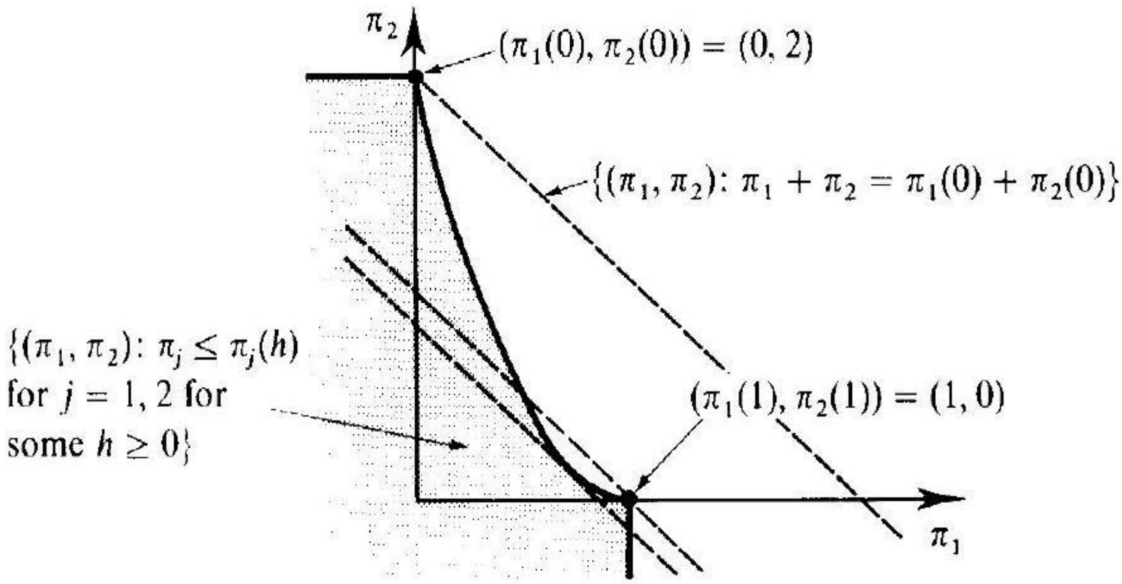
Example 11.AA.3: Externalities as a Source of Multiple Local Social Optima. It is, in principle, true that if the decision problem of the generator of an externality is concave, then the optimum can be sustained by means of quotas or taxes. But if $\pi_2(\cdot)$ is not concave, then the aggregate surplus function $\pi_1(h) + \pi_2(h)$ may not be concave and, as a result, the first-order conditions for aggregate surplus maximization may suffice only for determining local optima. In fact, as emphasized by Baumol and Oates (1988), the nonconvexities created by externalities may easily generate situations with multiple local social optima, so that identifying a global optimum may be a formidable task.

Suppose, for example, that the profit functions of the two firms are

$$\pi_1(h) = \begin{cases} h & \text{for } h \le 1 \\ 1 & \text{for } h > 1 \end{cases}$$

and

$$\pi_2(h) = \begin{cases} 2(1-h)^2 & \text{for } h \le 1 \\ 0 & \text{for } h > 1 \end{cases}$$



The function $\pi_2(\cdot)$ is not concave, something that the two previous examples have shown us can easily happen with externalities. The profit levels for the two firms that are attainable for different levels of h are depicted in Figure 11.AA.3 by the shaded set $\{(\pi_1, \pi_2)\colon \pi_j \le \pi_j(h)$ for $j = 1,2$ for some $h > 0\}$ (note that this definition allows for free disposal of profits). The social optimum has $h = 0$ (joint profits are then equal to 2), in which case firm 2 is able to operate in an environment free from the externality. This can be implemented by setting a tax rate on firm 1 of $t > 1$ per unit of the externality. But note that the outcome $h = 1$ (implemented by setting a tax rate on firm 1 of $t = 0$) is a local social optimum: As we decrease h, it is not until $h < \frac{1}{2}$ that we get an aggregate surplus level higher than that at $h = 1$. Hence, this latter outcome satisfies both the first-order and second-order conditions for the maximization of aggregate surplus (e.g., at this point, the marginal benefits of the externality exactly equal its marginal costs), and it will be easy for a social planner to be misled into thinking that she is at a welfare maximum.