Project 6

Contents

Problem 16

Gibbs sampler (b)

```
# Encoding conditional probabilities
p <- list(C=0.5, S.C1=0.1, S.C0=0.5, R.C1=0.8, R.C0=0.2,
          W.S1_R1=0.99, W.S1_R0=0.9, W.S0_R1=0.9, W.S0_R0=0.01)
p$C.R1_S1_W1 <- p$R.C1*p$S.C1*p$C/(p$R.C1*p$S.C1*p$C+p$R.C0*p$S.C0*(1-p$C))
p$C.R0_S1_W1 <- (1-p$R.C1)*p$S.C1*p$C/((1-p$R.C1)*p$S.C1*p$C+(1-p$R.C0)*p$S.C0*(1-p$C))
p$R.C1_S1_W1 <- p$W.S1_R1*p$R.C1/(p$W.S1_R1*p$R.C1+p$W.S1_R0*(1-p$R.C1))
p$R.C0_S1_W1 <- p$W.S1_R1*p$R.C0/(p$W.S1_R1*p$R.C0+p$W.S1_R0*(1-p$R.C0))
```

```
Gibbs_sampler <- function(N=100) {
    samples <- matrix(NA, ncol=2, nrow=N, dimnames=list(NULL, c("R", "C")))
    # Initialisation (beginning state)
    samples[1,] <- c(TRUE, TRUE)

    for(n in 2:N) {
        # Random sampling to choose which variable (R or C) is updated
        indicator <- sample(c("R", "C"), size=1, prob=c(0.5, 0.5))
        if(indicator == "R") {
            # Sampling from P(R_{n+1}|C_{n},S=T,W=T)
            if(samples[(n-1), "C"]) sampling_prob <- p$R.C1_S1_W1 else sampling_prob <- p$R.C0_S1_W1

            sampled_R <- sample(c(TRUE, FALSE), size=1, prob=c(sampling_prob, (1-sampling_prob)))
            samples[n,] <- c(sampled_R, samples[(n-1), "C"])
        } else {
            # Sampling from P(C_{n+1}|R_{n},S=T,W=T)
            if(samples[(n-1), "R"]) sampling_prob <- p$C.R1_S1_W1 else sampling_prob <- p$C.R0_S1_W1

            sampled_C <- sample(c(TRUE, FALSE), size=1, prob=c(sampling_prob, (1-sampling_prob)))
            samples[n,] <- c(samples[(n-1), "R"], sampled_C)
        }
```

```r
    }

    return(samples)
}
```

```r
set.seed(42)
N <- 100
samples <- Gibbs_sampler(N)
```

```r
result <- table(samples[, "R"], samples[, "C"])/N
rownames(result) <- c("R0", "R1")
colnames(result) <- c("C0", "C1")
result
```

```
##
##        C0   C1
##   R0 0.59 0.05
##   R1 0.18 0.18
```

Marginal probability (c)

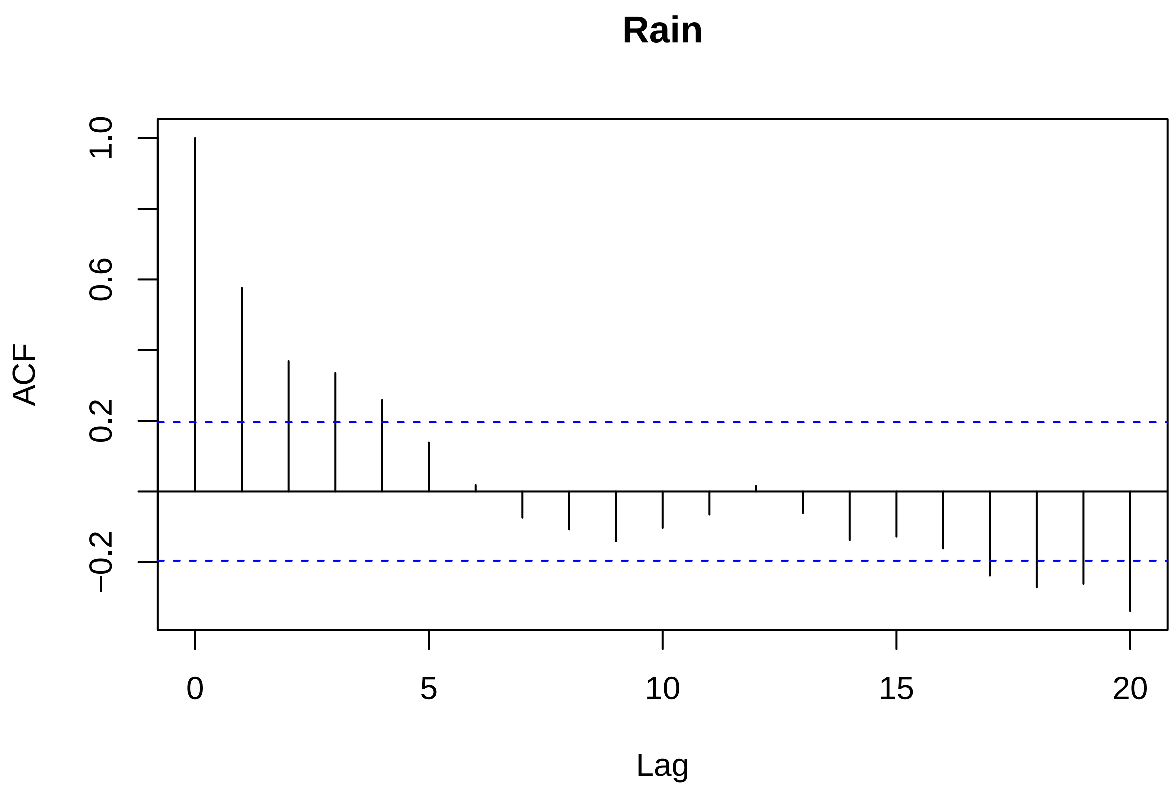
Computing marginal probability P(R=T|S=T,W=T).

```r
marginal.R.S1_W1 <- sum(result[2,])
marginal.R.S1_W1
```

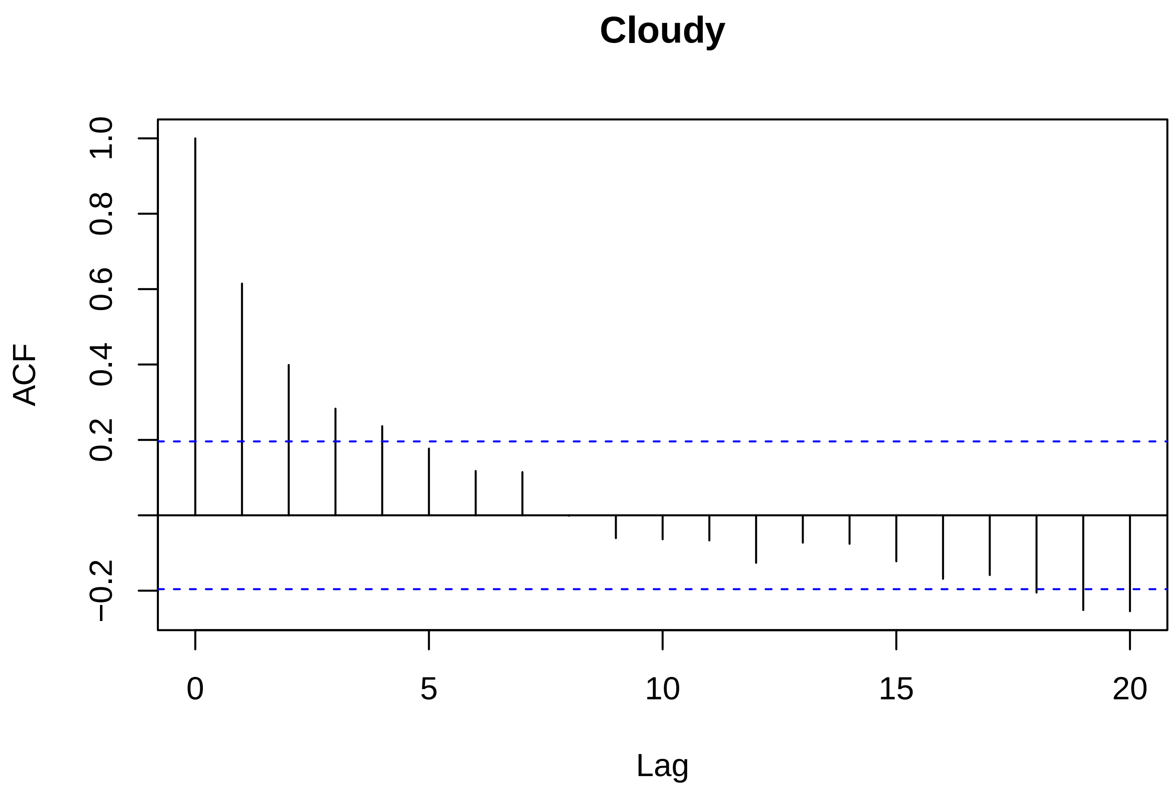
```
## [1] 0.36
```

Auto-correlation and effective sample size (ESS) (d)

```r
ro_R <- acf(as.numeric(samples[, "R"]), main="Rain")
```




```r
ro_C <- acf(as.numeric(samples[, "C"]), main="Cloudy")
```




```
ESS_R <- ro_R$n.used/(1+2*sum(ro_R$acf))
ESS_C <- ro_C$n.used/(1+2*sum(ro_C$acf))
print(paste("Effective sample size for chain 'rain':", ESS_R))
```

```
## [1] "Effective sample size for chain 'rain': 44.4718962322423"
```

```
print(paste("Effective sample size for chain 'cloudy':", ESS_C))
```

```
## [1] "Effective sample size for chain 'cloudy': 27.5547672392332"
```

Gibbs sampling of 50000 samples (e)

```
set.seed(42)
N <- 50000
samples1 <- Gibbs_sampler(N)

result1 <- table(samples1[, "R"], samples1[, "C"])/N
rownames(result1) <- c("R0", "R1")
colnames(result1) <- c("C0", "C1")

samples2 <- Gibbs_sampler(N)

result2 <- table(samples2[, "R"], samples2[, "C"])/N
rownames(result2) <- c("R0", "R1")
colnames(result2) <- c("C0", "C1")
```

```
result1
```

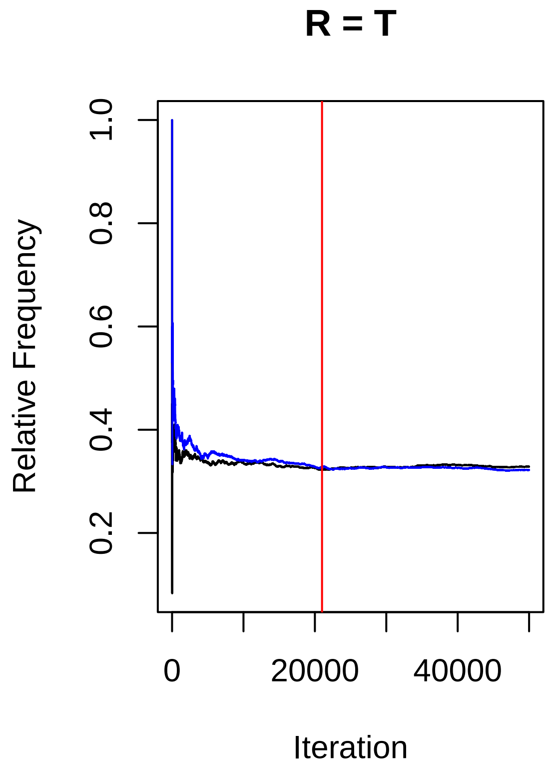
```
##
##           C0      C1
##   R0 0.63588 0.03570
##   R1 0.17904 0.14938
```

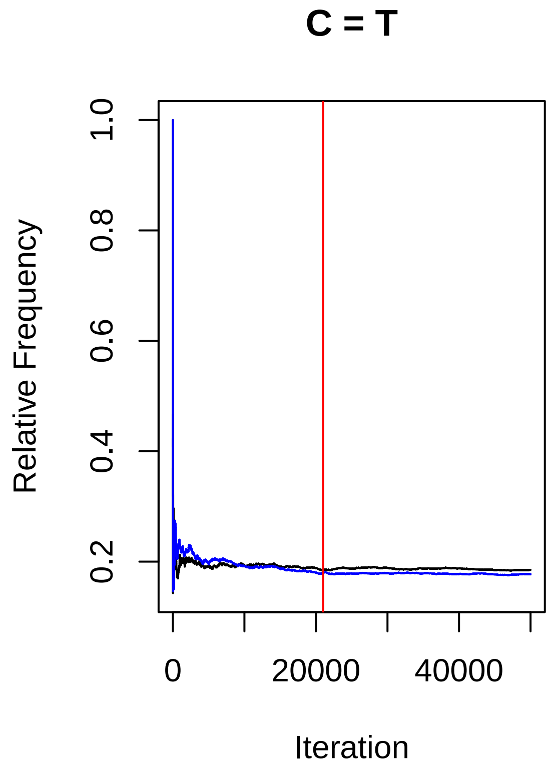
```
result2
```

```
##
##           C0      C1
##   R0 0.64450 0.03334
##   R1 0.17826 0.14390
```

Relative frequencies to detect burn-in phase (f)

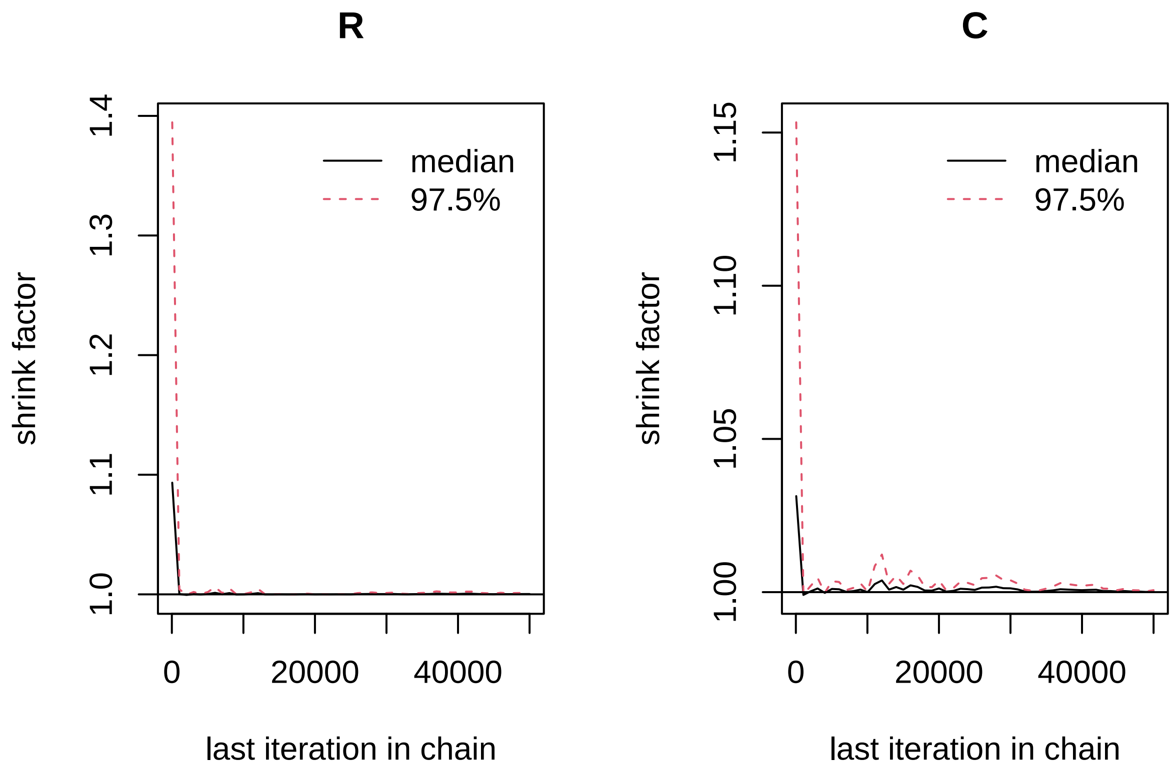
```
# Plotting relative frequencies of R = T and C = T
burn_in_phase <- 21000
par(mfrow=c(1,2))
plot(1:N, cumsum(samples1[, "R"])/seq_len(N), type="l", xlab="Iteration", ylab="Relative Frequency", mai
lines(cumsum(samples2[, "R"])/seq_len(N), type='l', col='blue')
abline(v=burn_in_phase, col="red")
plot(1:N, cumsum(samples1[, "C"])/seq_len(N), type="l", xlab="Iteration", ylab="Relative Frequency", mai
lines(cumsum(samples2[, "C"])/seq_len(N), type='l', col='blue')
abline(v=burn_in_phase, col="red")
```





Potential scale reduction factor (g)

```r
library(coda)
mcmcs <- mcmc.list(mcmc(data=samples1, start=2, end=N),
                   mcmc(data=samples2, start=2, end=N))
gelman.plot(mcmcs)
```




```r
gelman.diag(mcmcs)
```

```
## Potential scale reduction factors:
##
##   Point est. Upper C.I.
## R          1          1
## C          1          1
##
## Multivariate psrf
##
## 1
```

After around 10000 iterations, the fluctuations around the potential scale reduction factor are negligible, therefore a suggested burn-in time is 10000.

Re-estimation of marginal probability (h)

Computing marginal probability P(R=T|S=T,W=T) excluding the samples from the burn-in phase.

```r
burn_in_phase <- 10000
result1 <- table(samples1[(burn_in_phase+1):N, "R"], samples1[(burn_in_phase+1):N, "C"])/(N-burn_in_pha
```

```
rownames(result1) <- c("R0", "R1")
colnames(result1) <- c("C0", "C1")
result1
```

```
##
##            C0       C1
##   R0 0.637625 0.035600
##   R1 0.179275 0.147500
```

```
result2 <- table(samples2[(burn_in_phase+1):N, "R"], samples2[(burn_in_phase+1):N, "C"])/(N-burn_in_pha
rownames(result2) <- c("R0", "R1")
colnames(result2) <- c("C0", "C1")
result2
```

```
##
##            C0       C1
##   R0 0.649850 0.032700
##   R1 0.176575 0.140875
```

```
marginal1.R.S1_W1 <- sum(result1[2,])
marginal2.R.S1_W1 <- sum(result2[2,])
print("Marginal probability using the first run of 50000 iterations and excluded burn-in phase:")
```

```
## [1] "Marginal probability using the first run of 50000 iterations and excluded burn-in phase:"
```

```
print(marginal1.R.S1_W1)
```

```
## [1] 0.326775
```

```
print("Marginal probability using the second run of 50000 iterations and excluded burn-in phase:")
```

```
## [1] "Marginal probability using the second run of 50000 iterations and excluded burn-in phase:"
```

```
print(marginal2.R.S1_W1)
```

```
## [1] 0.31745
```

Analytical solution of marginal probability (i)

```
S1_W1 <- p$W.S1_R0*p$S.C0*(1-p$R.C0)*(1-p$C) +
         p$W.S1_R0*p$S.C1*(1-p$R.C1)*p$C +
         p$W.S1_R1*p$S.C0*p$R.C0*(1-p$C) +
         p$W.S1_R1*p$S.C1*p$R.C1*p$C

R1_S1_W1 <- p$W.S1_R1*p$S.C0*p$R.C0*(1-p$C) +
            p$W.S1_R1*p$S.C1*p$R.C1*p$C

analytical_marginal.R.S1_W1 <- R1_S1_W1 / S1_W1

analytical_marginal.R.S1_W1
```

```
## [1] 0.3203883
```

The marginal probabilities of the (h) part are closer to the analytical solution, meanwhile probability of (c) part deviates more. It is intuitive, since the sample size in (c) was small (100).

```
library(rmarkdown)
render("Gibbs.Rmd", pdf_document(TRUE), "Indilewitsch_Toidze_Houhamdi_Pudziuvelyte_Project6.pdf")
```