PROJECT 3

PROBLEM 6

a) MAX. NUMBER OF FREE PARAMETERS TO DEFINE HMM

MATRICES: I, T, E

I NUMBER OF DIFFERENT LATENT STATES: K
NUMBER OF DIFFERENT EMISSION STATES: M

I - INITIAL STATE PROBABILITIES: K × 1 MATRIX
⇒ K - 1 FREE PARAMETERS

T - TRANSITION PROBABILITIES: K × K MATRIX
⇒ K · (K-1) FREE PARAMETERS (∀ ROW HAS TO SUM UP TO 1).

E - EMISSION PROBABILITES: K × M MATRIX
⇒ K · (M-1) FREE PARAMETERS

OVERALL: $(K-1) + K(K-1) + K(M-1) = (K-1)(K+1) + K(M-1)$

FREE PARAMETERS

b) $T = \begin{bmatrix} 0.3 & 0.7 \\ 0.2 & 0.8 \end{bmatrix}$

EIGENVALUE EQUATION $\pi^t T = \pi^t$

$T^t \pi = \pi$

$T^t \pi - I\pi = 0$

$$\begin{vmatrix} 0.3 - \lambda & 0.2 \\ 0.7 & 0.8 - \lambda \end{vmatrix} = 0$$

$(0.3 - \lambda)(0.8 - \lambda) - 0.14 = 0$
$0.24 - 0.3\lambda - 0.8\lambda + \lambda^2 - 0.14 = 0$
$\lambda^2 - 1.1\lambda + 0.1 = 0$

$\lambda_1 = 1 \quad \lambda_2 = 0.1$

EIGENVECTORS

$\lambda_1 = 1$

$$\left(\begin{array}{cc|c} -0.7 & 0.2 & 0 \\ 0.7 & -0.2 & 0 \end{array}\right)$$

$-0.7 x_1 = -0.2 x_2$
$x_1 = \frac{0.2}{0.7} x_2 = \frac{2}{10} \cdot \frac{10}{7} x_2 = \frac{2}{7} x_2 \qquad v_1 = \begin{pmatrix} \frac{2}{7} \\ 1 \end{pmatrix}$

$\lambda_2 = 0.1$

$$\left(\begin{array}{cc|c} 0.2 & 0.2 & 0 \\ 0.7 & 0.7 & 0 \end{array}\right)$$

$0.2 x_1 = -0.2 x_2$
$x_1 = -1 x_2 \qquad v_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

STATIONARY DISTRIBUTION COMPUTED FROM v_1 (VECTOR WITH $\lambda_1 = 1$)

$$\pi = \begin{pmatrix} \frac{\frac{2}{7}}{1 + \frac{2}{7}} \\ \frac{1}{1 + \frac{2}{7}} \end{pmatrix} = \begin{pmatrix} \frac{\frac{2}{7}}{\frac{9}{7}} \\ \frac{1}{\frac{9}{7}} \end{pmatrix} = \begin{pmatrix} \frac{2}{9} \\ \frac{7}{9} \end{pmatrix}$$