Counterdiabatic, Better, Faster, Stronger:

Overcoming Losses in Quantum Processes

PhD Thesis

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This thesis is the result of the author's original research. It has been composed by the author and has not been previously submitted for examination which has led to the award of a degree.

Abstract

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Acronyms and abbreviations

STA	Shortcuts to Adiabaticity
CD	Counterdiabatic driving
LCD	Local Counterdiabatic Driving
COLD	Counterdiabatic Optimised Local Driving
BPO	Bare Powell Optimisation
CRAB	Chopped Randomised Basis
ARP	Adiabatic Rapid Passage

Lay Summary

"With magic, you can turn a frog into a prince. With science, you can turn a frog into a Ph.D and you still have the frog you started with."

Terry Pratchett

At the moment, this is full of quotes from Star Trek I'd like to use:

- "A library serves no purpose unless someone is using it." Mr. Atoz, "All Our Yesterdays"
- "Computers make excellent and efficient servants, but I have no wish to serve under them." Mr. Spock, "The Ultimate Computer"
- "Insufficient facts always invite danger." Mr. Spock, "Space Seed"
- "Change is the essential process of all existence." Mr. Spock, "Let That Be Your Last Battlefield"
- "Instruments register only through things they're designed to register. Space still contains infinite unknowns." Mr. Spock, "The Naked Time"

And now for some Terry Pratchett:

- "Sometimes scientists change their minds. New developments cause a rethink. If this bothers you, consider how much damage is being done to the world by people for whom new developments do not cause a rethink."

Preface/Acknowledgements

I would like to acknowledge

Chapter 1

Introduction

1.1 Thesis overview

1.2 Publications and manuscripts

The majority of this work is based on the following publications and manuscripts:

(1) **Counterdiabatic Optimised Local Driving**, *Ieva Čepaitė, Anatoli Polkovnikov, Andrew J. Daley, Callum W. Duncan. PRX Quantum 4, 010309, 2023.* Eprint arxiv:2203.01948.

(2) **Many-body spin rotation by adiabatic passage in spin-1/2 XXZ chains of ultracold atoms**, *Ivana Dimitrova, Stuart Flannigan, Yoo Kyung Lee, Hanzhen Lin, Jesse Amato-Grill, Niklas Jepsen, Ieva Čepaitė, Andrew J. Daley, Wolfgang Ketterle.* Eprint arxiv:2301.00218.

(3) **Numerical approaches for non-adiabatic terms in quantum annealing**, *Ewen D. C. Lawrence, Sebastian Schmid, Ieva Čepaitė, Peter Kirton, Callum W. Duncan* Eprint arxiv:0000.00000.

1.3 Talks and presentations

- *"Solving Partial Differential Equations (PDEs) with Quantum Computers"*, Atomic Weapons Establishment, (March 2020)

- *"A Continuous Variable Born Machine"*, Pittsburgh Quantum Institute Virtual Poster Session, Online (April 2020)
- *"A Continuous Variable Born Machine"*, Quantum Techniques in Machine Learning, Online (November 2020)
- *"Variational Counterdiabatic Driving"*, University of Strathclyde and University of Waterloo Joint Virtual Research Colloquium on Quantum Technologies, Online (November 2020)
- *"A Continuous Variable Born Machine"*, Bristol QIT Online Seminar Series, Online (March 2021)
- *"Optimised counderdiabatic driving with additional terms"*, APS March Meeting, Online (March 2021)
- *"Counterdiabatic Optimised Local Driving"*, DAMOP, Orlando (May 2022)
- *"Counterdiabatic Optimised Local Driving"*, QCS Hub Project Forum, Oxford (January 2023)
- *"Counterdiabatic Optimised Local Driving"*, APS March Meeting, Las Vegas (March 2023)
- *"Counterdiabatic Optimised Local Driving"*, INQA Seminar, Online (March 2023)

Chapter 2

Background: Quantum Adiabaticity

> I saw this movie about a bus that had to SPEED around a city, keeping its SPEED over fifty, and if its SPEED dropped, it would explode! I think it was called 'The Bus That Couldn't Slow Down'.
>
> Homer Simpson

2.1 Quantum Adiabaticity

The concept of quantum adiabaticity is the central starting point of the work presented in this thesis. While in classical thermodynamics, an adiabatic process is essentially one where no heat or mass is transferred between a system and its environment, the quantum adiabatic theorem concerns itself more with the speed at which changes in a system Hamiltonian occur.

2.1.1 The adiabatic theorem

Imagine a quantum system that begins in the non-degenerate ground state of a time-dependent Hamiltonian. According to the the quantum adiabatic theorem, it will *remain* in the instantaneous ground state provided the Hamiltonian changes sufficiently slowly. To take an intuitive example, we can consider a spin in a magnetic field that is rotated from the x direction to the z direction during some total time τ. The Hamiltonian might be written as:

$$H(t) = -\cos\left(\frac{\pi t}{2\tau}\right)\sigma^x - \sin\left(\frac{\pi t}{2\tau}\right)\sigma^z. \tag{2.1}$$

If the spin starts in the ground state of $H(0)$ (pointing in the x direction, $|\psi(0)\rangle = |+\rangle$), then as the magnetic field is rotated, the spin starts precessing about the new direction of the field. This moves the spin toward the z axis but also produces a component out of the xz plane. As the total time for the rotation gets longer (i.e. the rotation gets slower compared to the precession), the state maintains a tighter and tighter orbit around the field direction. In the limit of $\tau\infty$, the state of the spin tracks the magnetic field perfectly, always in the ground state of $H(t)$ for all t.

While this may feel like an intuitive story, the physics governing it is far more interesting than what can be seen at first glance. Let us first take a general case: for a dimensionless parameter $\lambda \in [0, 1]$, let $H(\lambda)$ be a Hermitian operator that varies smoothly as a function of λ. This λ may be the magnetic field orientation or any other parameter that can be varied in a quantum system. In fact, let's take $\lambda = \frac{t}{\tau}$, so that when $\tau \gg 1$, $H(\lambda)$ varies very slowly as a function of the time. An initial quantum state $|\psi(0)\rangle$ evolves according to the Schrödinger equation:

$$i\frac{\mathrm{d}\,|\psi(\lambda)\rangle}{\mathrm{d}\lambda} = \tau H(\lambda)\,|\psi(\lambda)\rangle \tag{2.2}$$

2.1.2 The adiabatic gauge potential

2.2 Approximations of the AGP

2.2.1 Shortcuts to adiabaticity

Here I'm going to talk about Shortcuts to Adiabaticity (
STA)

2.2.2 Transitionless Driving

The form of the dynamical Hamiltonian enforcing this is [**?**]:

$$H_{\mathrm{CD}}(t) = H_0(t) + i\hbar \sum_n (|\partial_t n\rangle \langle n| - \langle n|\partial_t n\rangle |n\rangle \langle n|), \tag{2.3}$$

2.2.3 Counterdiabatic driving

2.2.4 Variational counterdiabatic driving

Following the methods of Ref. [**?**], the problem of finding the optimal adiabatic gauge potential can be cast as the minimisation of the Hilbert-Schmidt norm of the operators

$$G_\lambda = \partial_\lambda H_\beta + i[\mathcal{A}_\lambda, H_\beta], \tag{2.4}$$

which is equivalent to minimisation of the action

$$\mathcal{S}(\mathcal{A}_\lambda) = \mathrm{Tr}\left[G_\lambda(\mathcal{A}_\lambda)^2\right], \tag{2.5}$$

with respect to $\mathcal{A}_\lambda$.

2.2.5 Nested commutator expansion

2.2.6 Krylov methods

Chapter 3

Background: Optimal Control

Here I'll talk about optimal control.

In the context we consider, we employ quantum optimal control to optimise the function $f(\psi, \boldsymbol{\beta})$ in the Schrödinger equation

$$\dot{\psi} = f(\psi, \boldsymbol{\beta}), \tag{3.1}$$

A commonly used cost function in state preparation is related to the fidelity of the final, post-evolution state $|\psi_f\rangle$ with respect to the target state:

$$\mathcal{C}(|\psi_f\rangle) = 1 - |\langle\psi_T|\psi_f\rangle|^2 . \tag{3.2}$$

The full Hamiltonian of the control system is then:

$$H_\beta(t, \boldsymbol{\beta}) = H_0(t) + \boldsymbol{\beta}(t)\mathcal{O}_{\text{opt}}. \tag{3.3}$$

3.1 CRAB

3.2 GRAPE

Chapter 4

Counterdiabatic optimised local driving

Chapter 5

Applications

5.1 Two-spin annealing

5.2 Ising chain

5.3 Transport in a synthetic lattice

5.4 Preparing GHZ states

5.5 Random Ising graphs

5.5.1 Maximal Independent Set

5.6 Spin-1/2 XXZ chains of ultracold atoms

Chapter 6

Higher order AGP as a cost function

Chapter 7

Conclusion

Appendix A

Stuff That Didn't Fit Anywhere Else