Optimised Variational Counterdiabatic Driving

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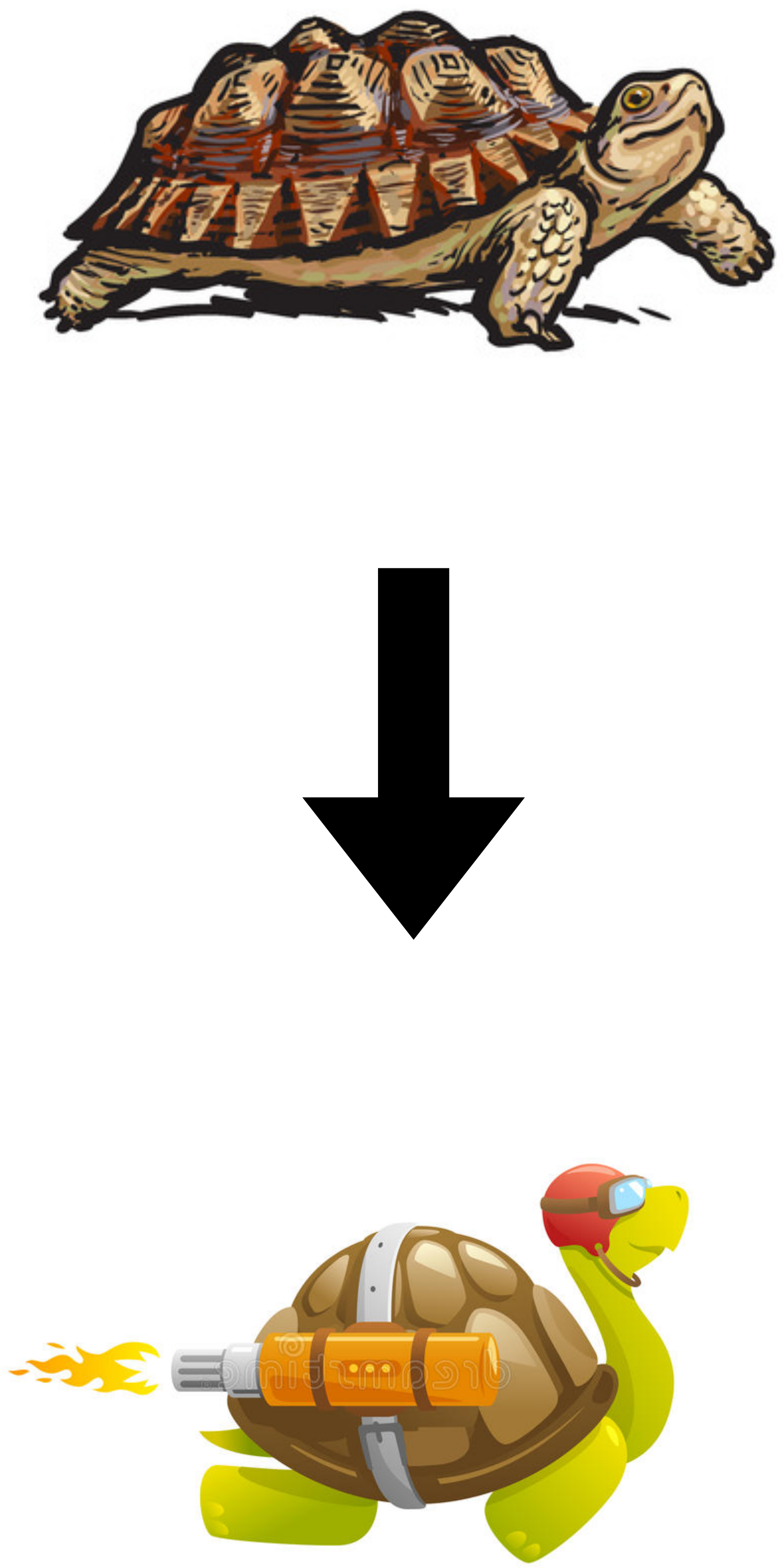
PASQuanS

Motivation

- Quantum computing applications using **adiabatic dynamics**:
    - State preparation
    - Optimisation problems
- **Problem**: coherence + timescales
    - Specifically limited by **adiabatic theorem**



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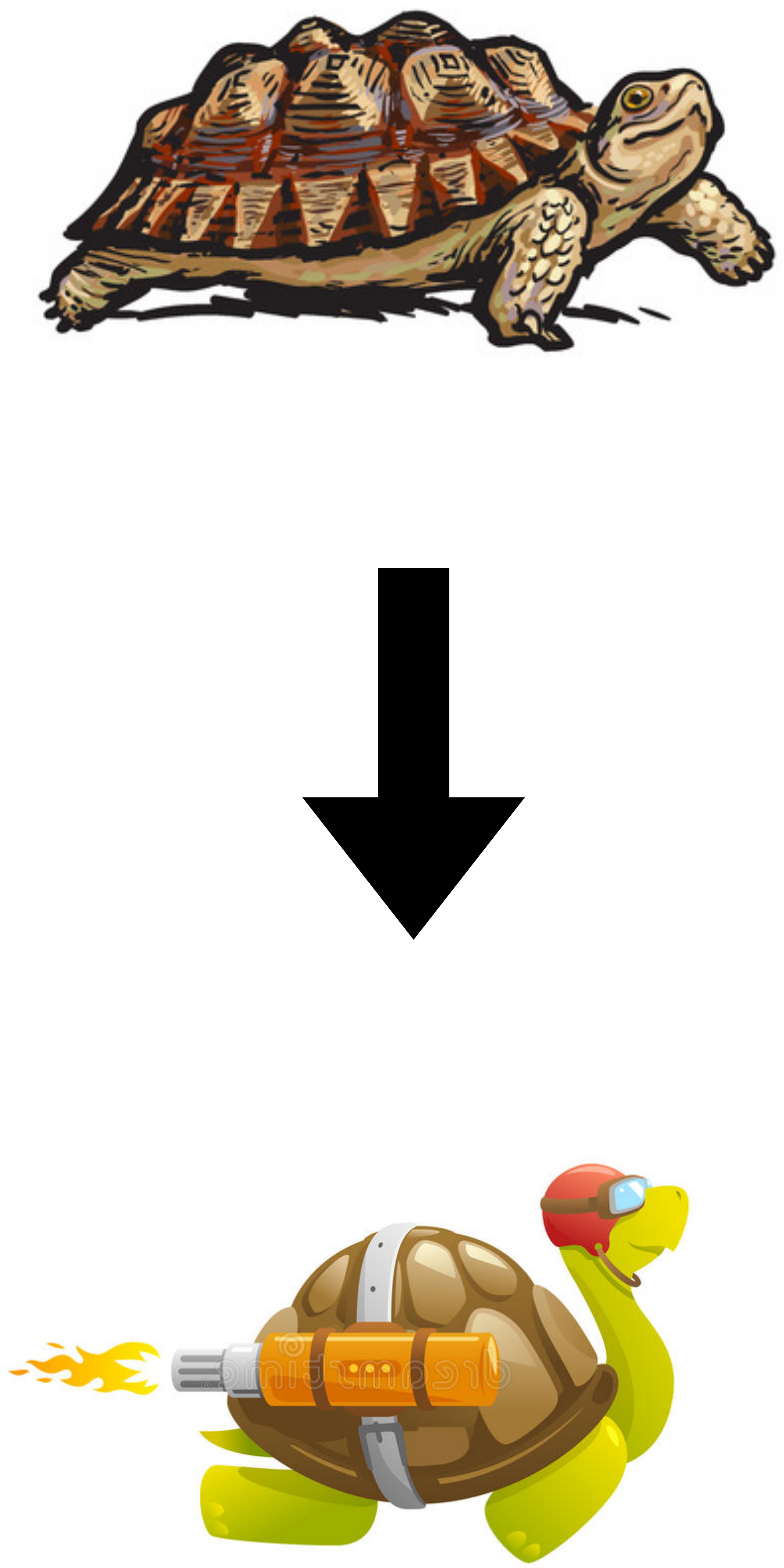
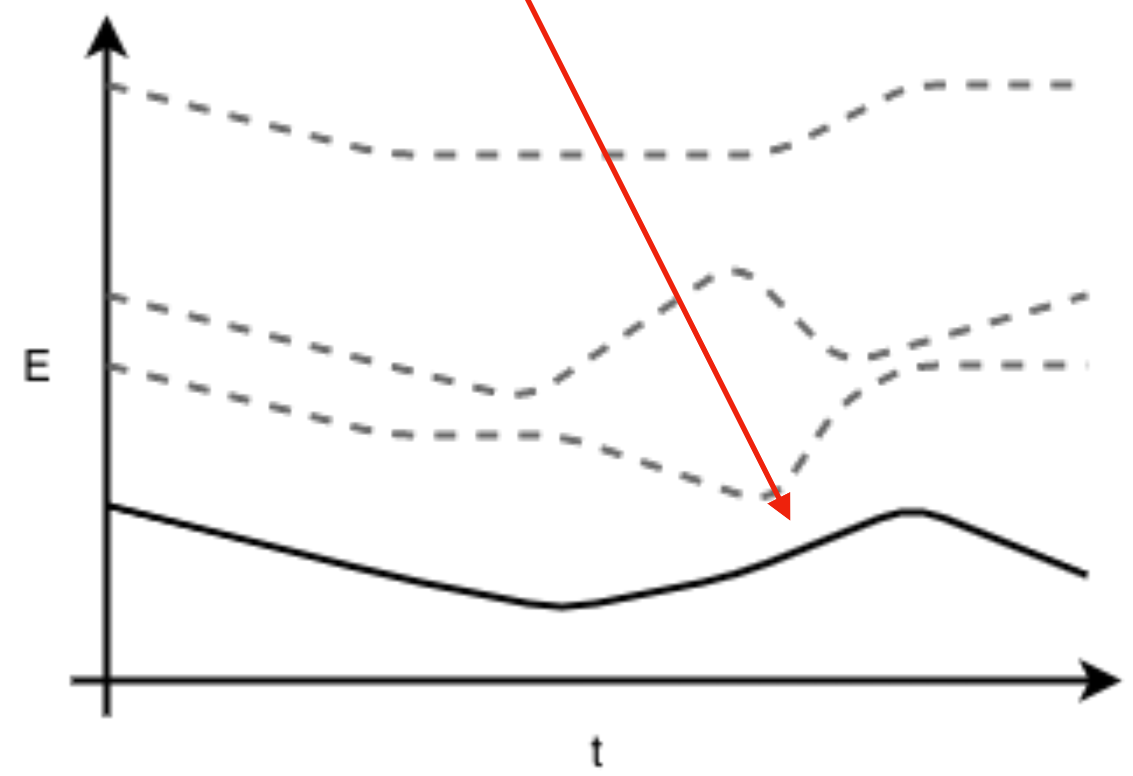


Goal: to speed up adiabatic processes using

(Variational) Counterdiabatic Driving + Optimal Control Methods

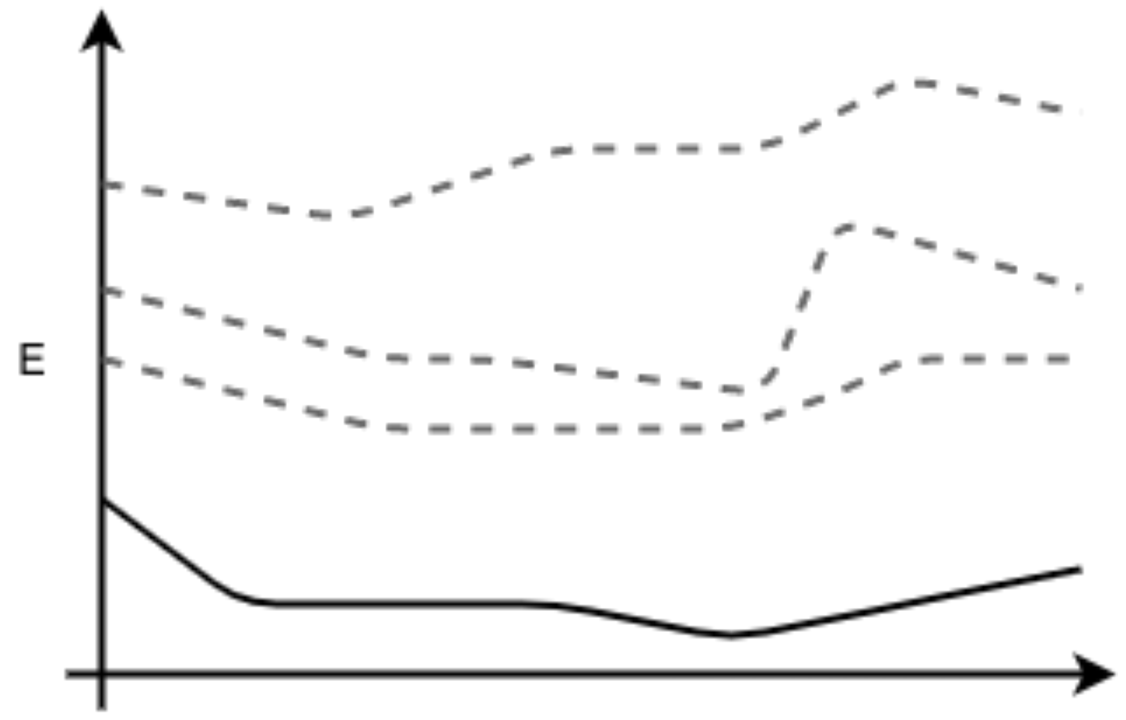
Counterdiabatic Driving

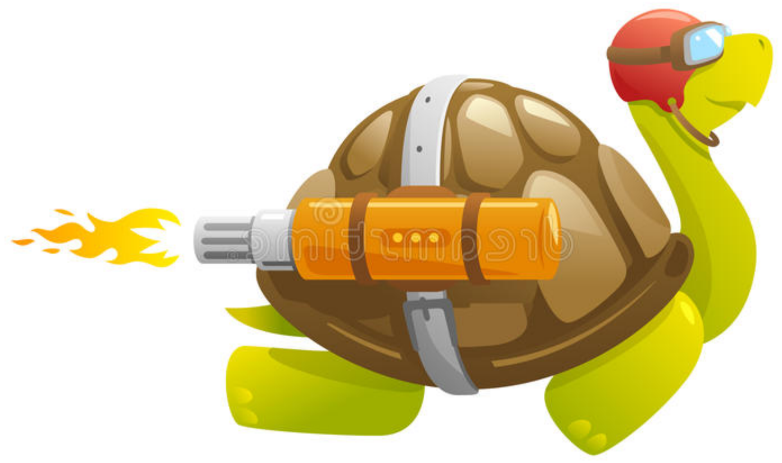
- Adiabatic theorem limits speed of quantum state evolution
  - if Hamiltonian varies too quickly, the system may transition to a higher eigenstate
- Counterdiabatic (CD) driving changes the energy landscape to avoid (or eliminate) small energy gaps



Add CD driving

(Not an easy thing to do!)





Counterdiabatic Driving

- For time-dependent moving frame: $|\tilde{\psi}\rangle = U^{\dagger}(\lambda)\,|\psi\rangle$
- Hamiltonian in moving frame:

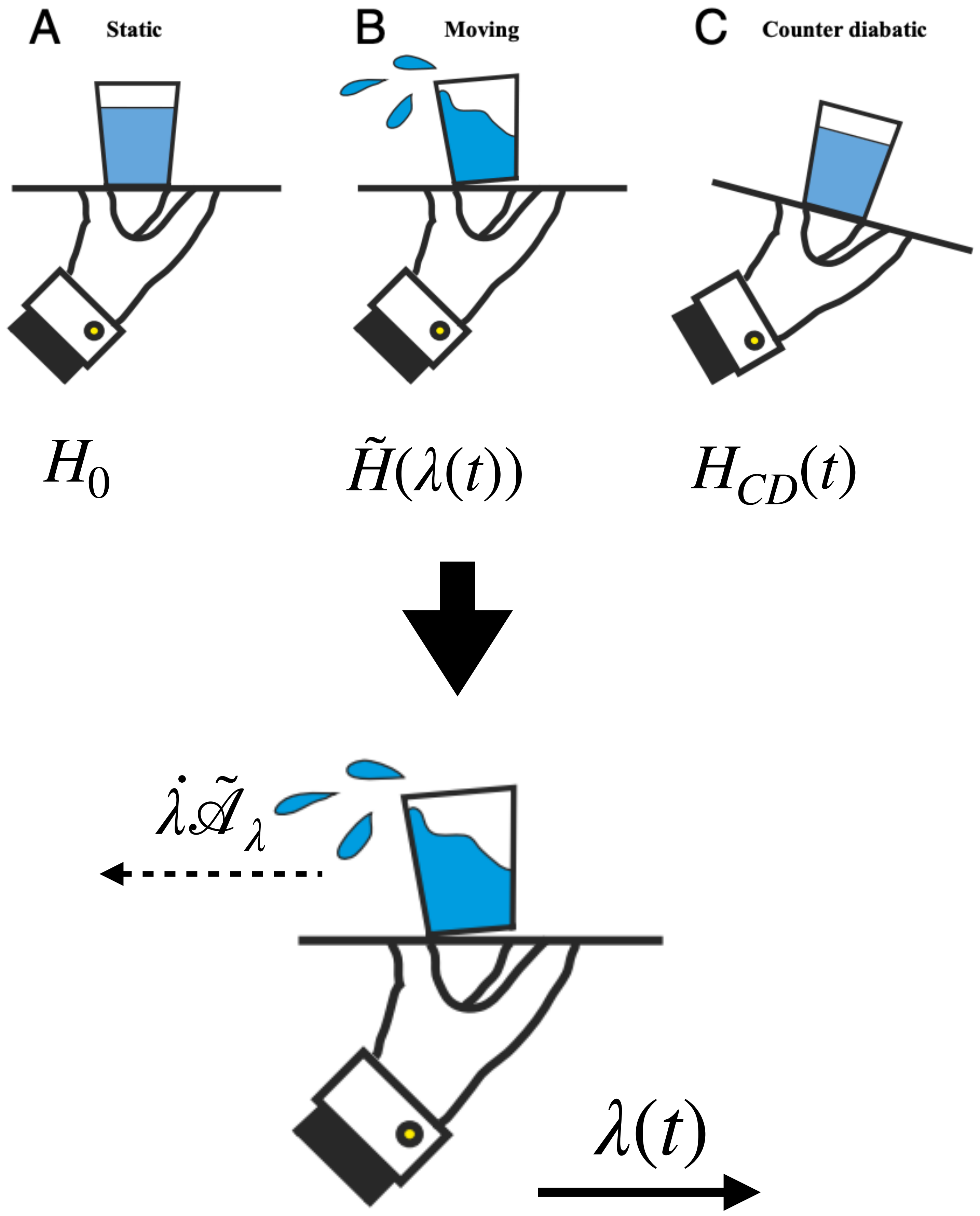
$$\tilde{H}_m(t) = \tilde{H}(\lambda(t)) - \dot{\lambda}\tilde{\mathscr{A}}_{\lambda}$$

$\tilde{H} = U^{\dagger}HU$
diagonal

adiabatic gauge potential responsible for transitions

- Want to counteract 'fictitious force' $\dot{\lambda}\tilde{\mathscr{A}}_{\lambda}$ — CD Hamiltonian:

$$H_{CD}(t) = H_0(\lambda(t)) + \dot{\lambda}\mathscr{A}_{\lambda}$$



Minimizing irreversible losses in quantum systems by local counterdiabatic driving, 2017 [PNAS]

Variational Counterdiabatic Driving (VCD)

- **Problem**: need exact knowledge of the system's eigenstates:

$$H(t) = \underbrace{\sum_n |n\rangle E_n \langle n|}_{H_0(t)} + \underbrace{i\hbar \sum_{m\neq n} \sum_n |m\rangle\langle m|\partial_t H_0|n\rangle\langle n|}_{H_{CD}(t)}$$

- **Solution**: devise *approximate* CD drive
  - Turn into **minimisation problem**
  - Want **local**, **physical** counter-terms, treat as a sort of perturbation theory

Ansatz: $\mathscr{A}_\lambda = \boldsymbol{\alpha}\hat{O}$

Optimise for $\boldsymbol{\alpha}$ **analytically**

$\hat{O} =$ some operator,
e.g. local $\hat{\sigma}^y$ terms for spins

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Optimal Control

What if approximate CD drive is not good enough?

- **Optimal control methods:**
    - Want to affect dynamics of a system in a controlled way.
    - Introduce physically motivated **function basis with optimisable parameters**
    - Build a drive that gives desired dynamics/ final state.

Chopped random-basis quantum optimization, 2011 [Phys. Rev. A]

Dressing the chopped-random-basis optimization: A bandwidth-limited access to the trap-free landscape, 2015 [Phys. Rev. A]

Data-driven gradient algorithm for high-precision quantum control, 2018 [Phys. Rev. A]

Optimal control of coupled spin dynamics: design of NMR pulse sequences by gradient ascent algorithms, 2005 [J. Magn. Reson.]

Optimised Variational Counterdiabatic Driving (OVCD)

- Add optimisable, highly controllable drive to problem Hamiltonian:

$$F(t) = \sum_{k}^{N_k} \beta^k \sin(k\omega t) \quad \longrightarrow \quad H_\beta(t) = H_0(\lambda(t)) + F(t)\sigma^i + \dot{\lambda}\mathcal{A}_\lambda$$

- Optimise β^k , e.g. via minimising for high fidelities with respect to final ground state:

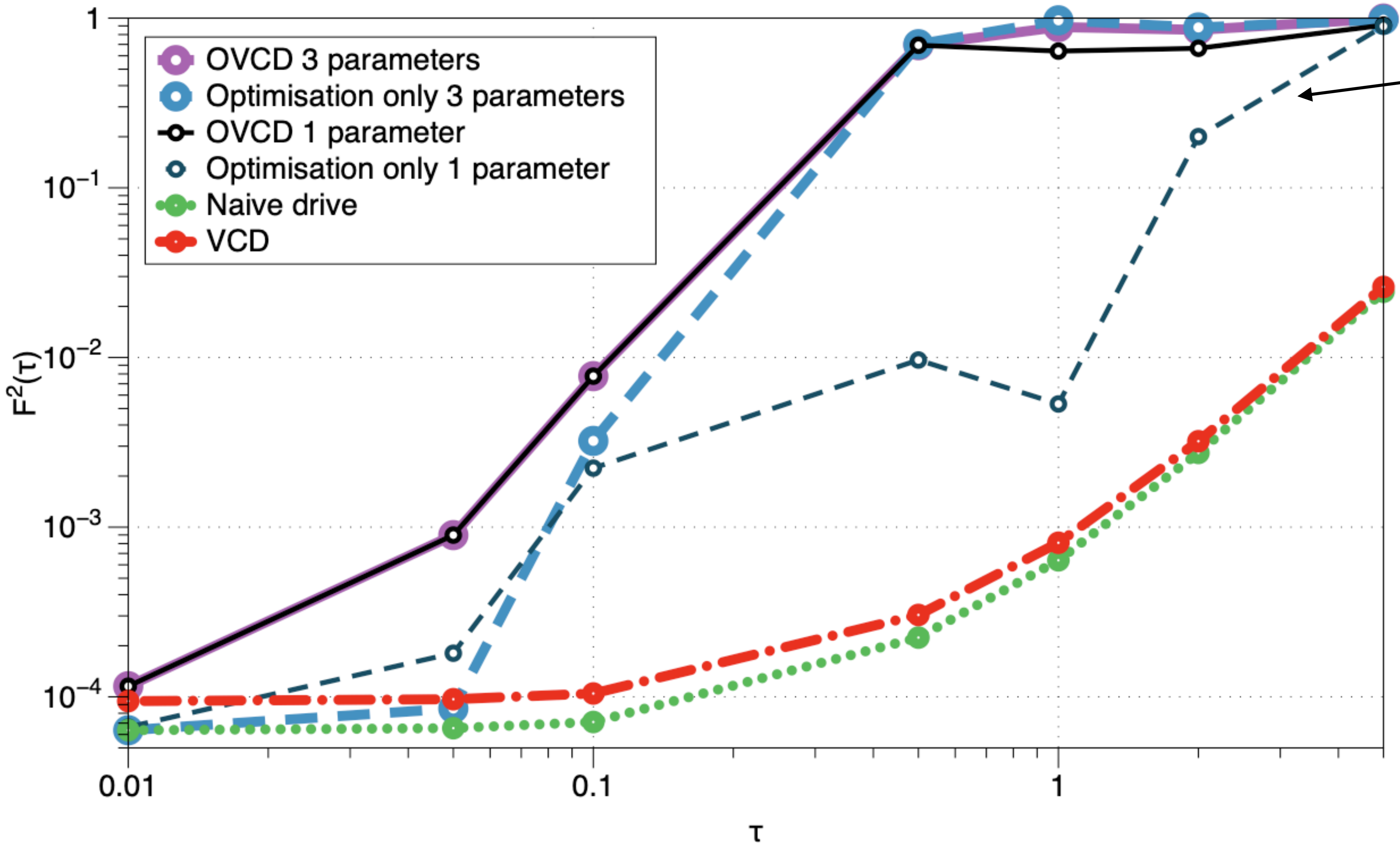
$$C(\boldsymbol{\beta}) = 1 - |\langle \psi_{CD} | \psi_{GS} \rangle|^2$$

- Allows for **far shorter evolution times** τ with **higher fidelities**.

Numerical Example: 1D Ising Model

$$H_0(t) = \sum_j \left[J\sigma_j^z\sigma_{j+1}^z + X(t)\sigma_j^x + Z(t)\sigma_j^z \right]$$

$\tau = \mathcal{O}(J^{-1})$



Compare with optimal control **only**, to see if we have an advantage.

$$X(t) = X_0 + X_f \sin^2\left(\frac{\pi}{2}\sin^2\left(\frac{\pi t}{2\tau}\right)\right)$$

$J = -1$

$X_0 = 0$

$X_f = 10$

$$Z(t) = Z_0 + \sum_k^{N_k} \beta^k \sin(k\omega t),$$

where $Z_0 = 0.02$

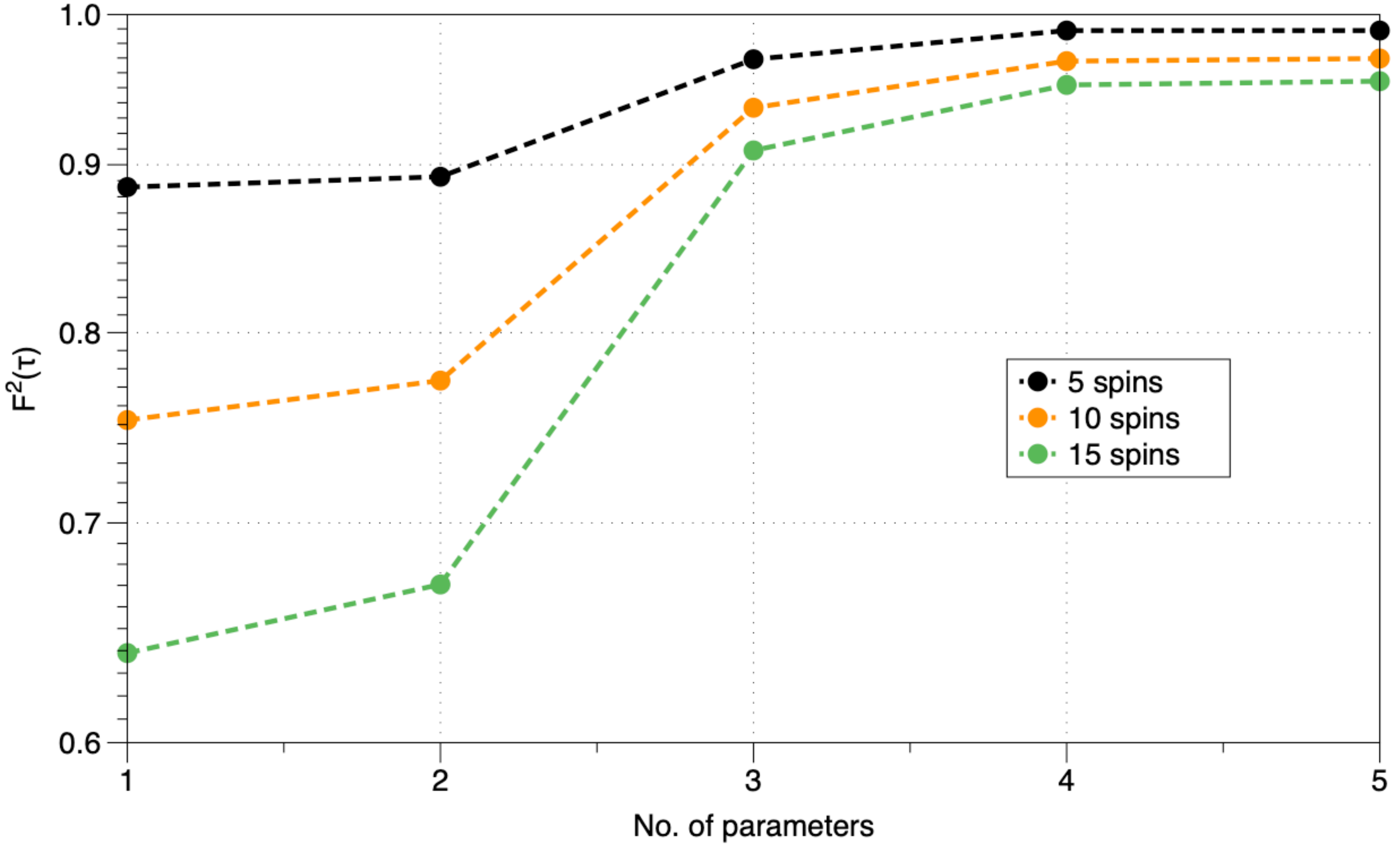
Numerical Example: 1D Ising Model

$$H_0(t) = \sum_j \left[J\sigma_j^z \sigma_{j+1}^z + X(t)\sigma_j^x + Z(t)\sigma_j^z \right]$$

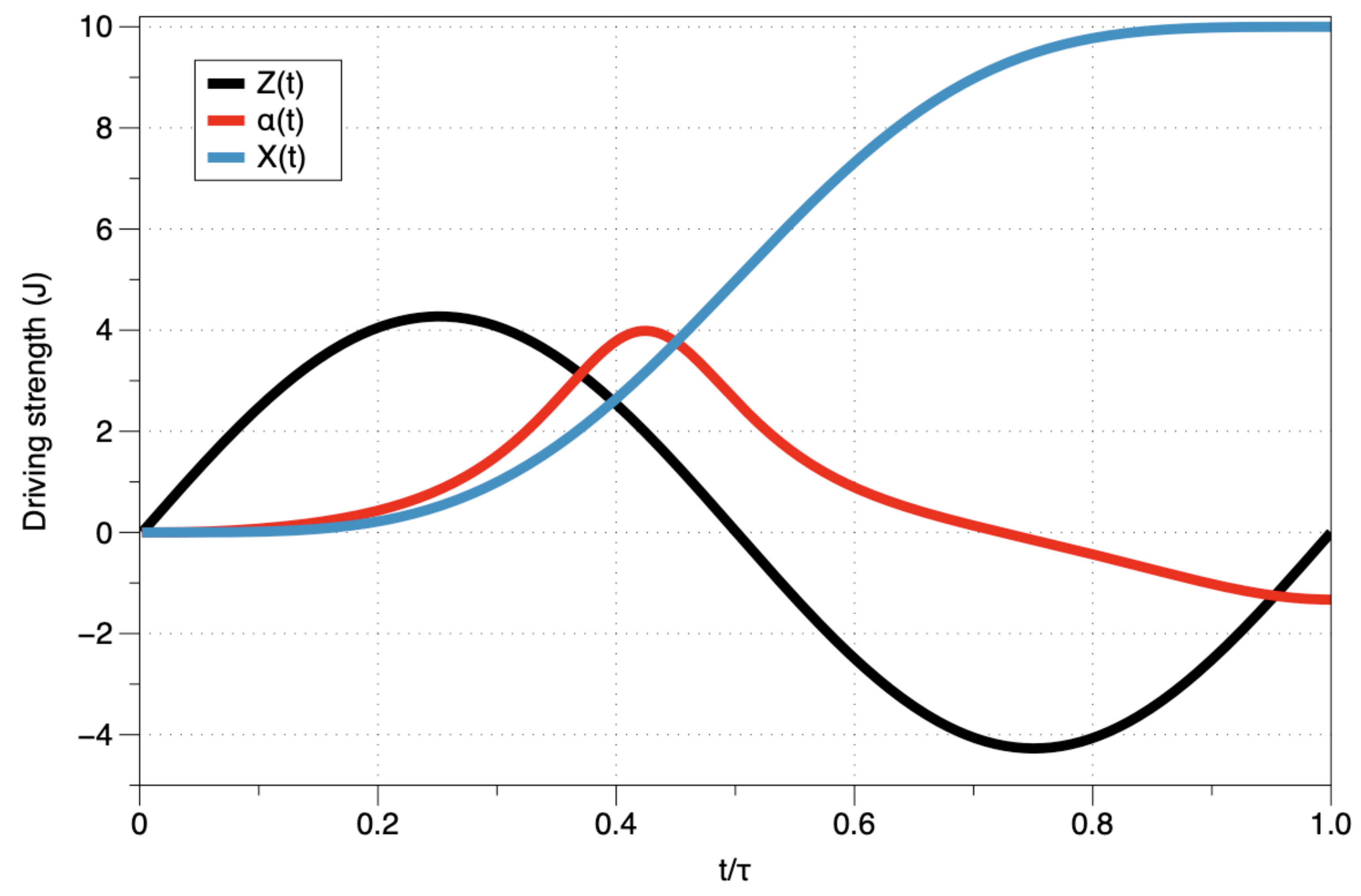
Maximum pulse amplitude: 10J

Short timescale: $\tau = 1J^{-1}$

Comparing System Size

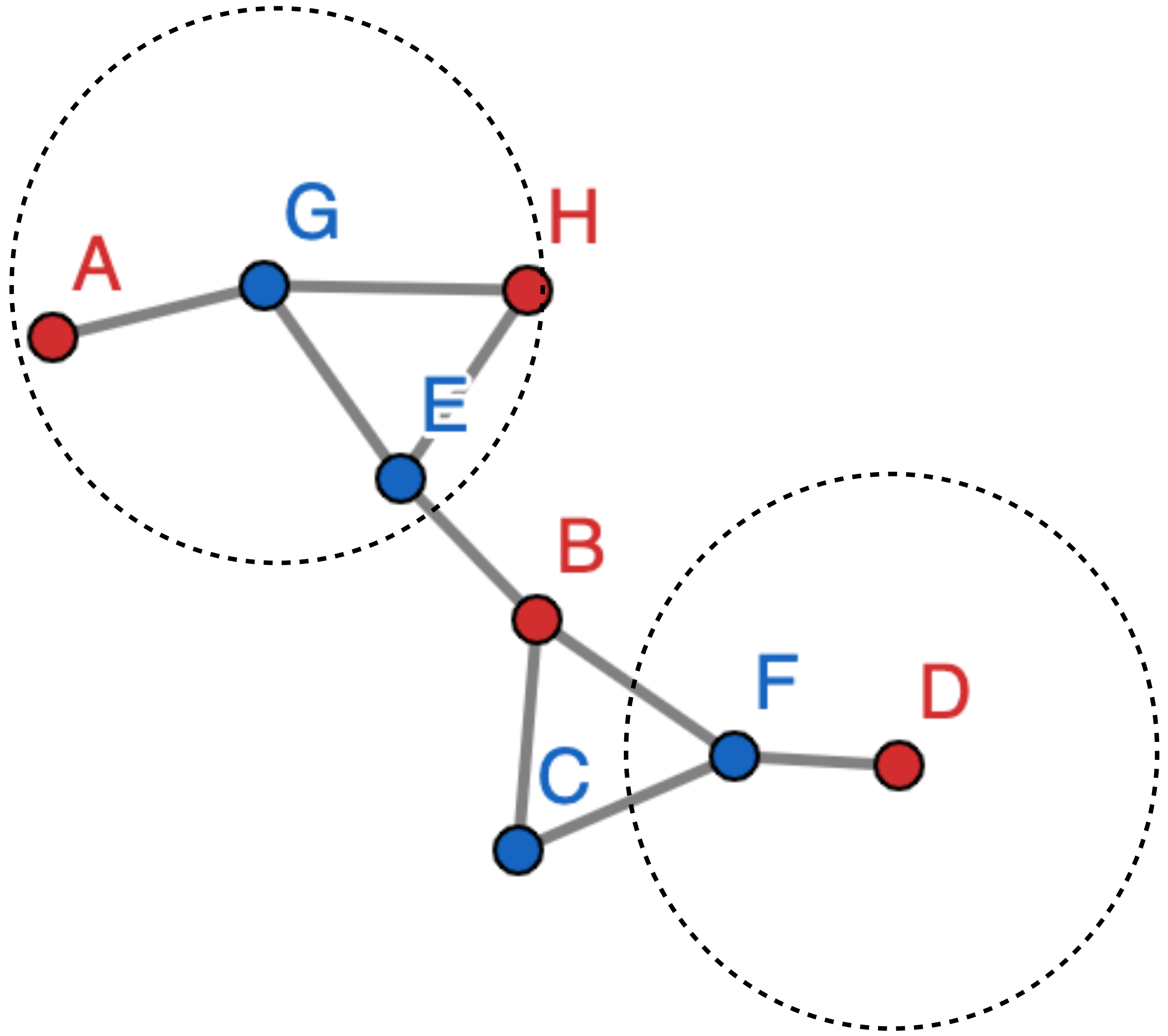


Example drives (1 parameter)



Other Examples and Future Outlook

- Applying OVCD to **graph problems** encoded in Hamiltonians:
  - e.g. **Maximal Independent Set (MIS)** - finding largest subset of vertices where no pair is connected by an edge
- Can perhaps get **better** approximation of analytic CD driving terms by *Floquet-Engineering* Hamiltonians
  - Problem: heating
- Look into **adiabatic state preparation**



Part of MIS

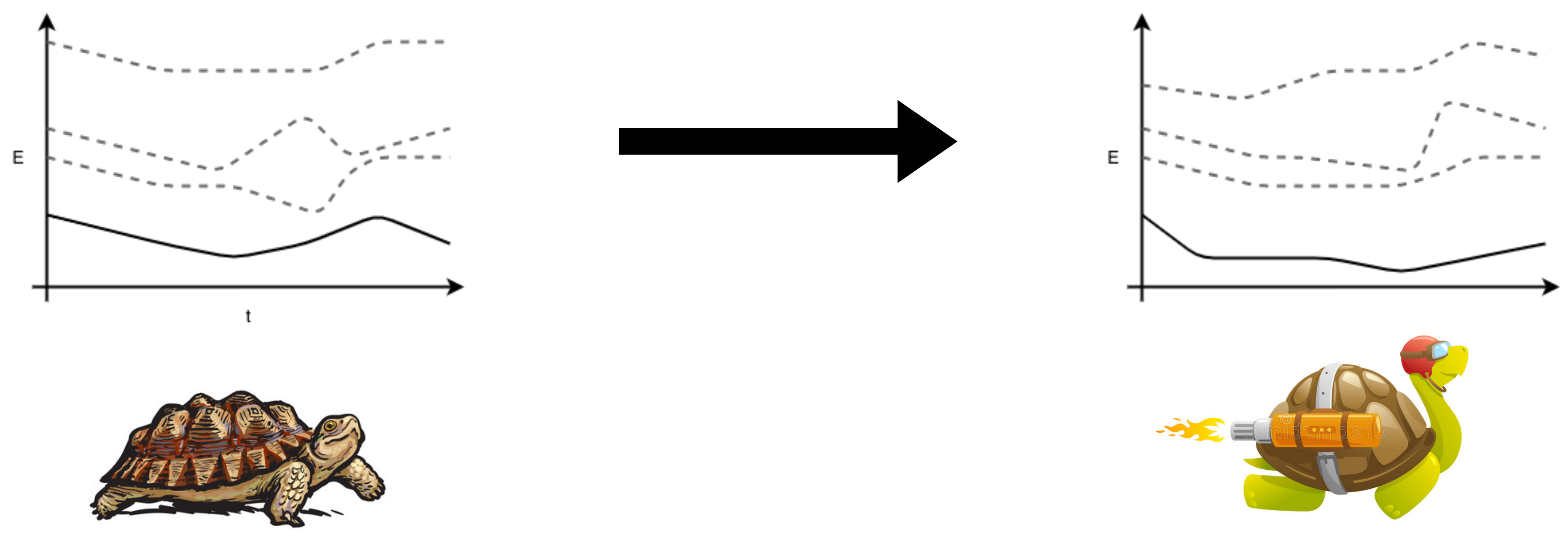
Not part of MIS

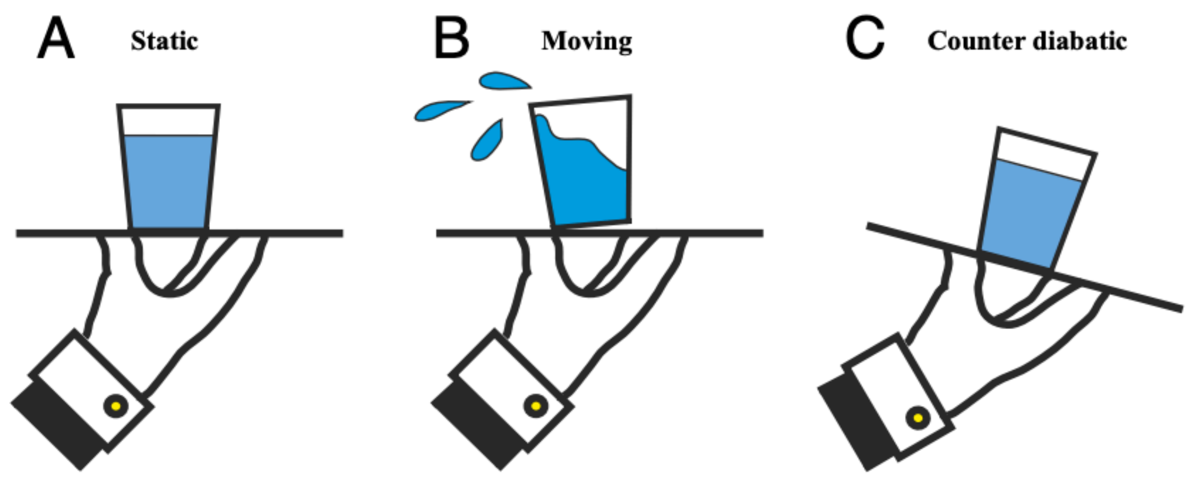
Quantum Optimization for Maximum Independent Set Using Rydberg Atom Arrays, 2018 [arXiv]

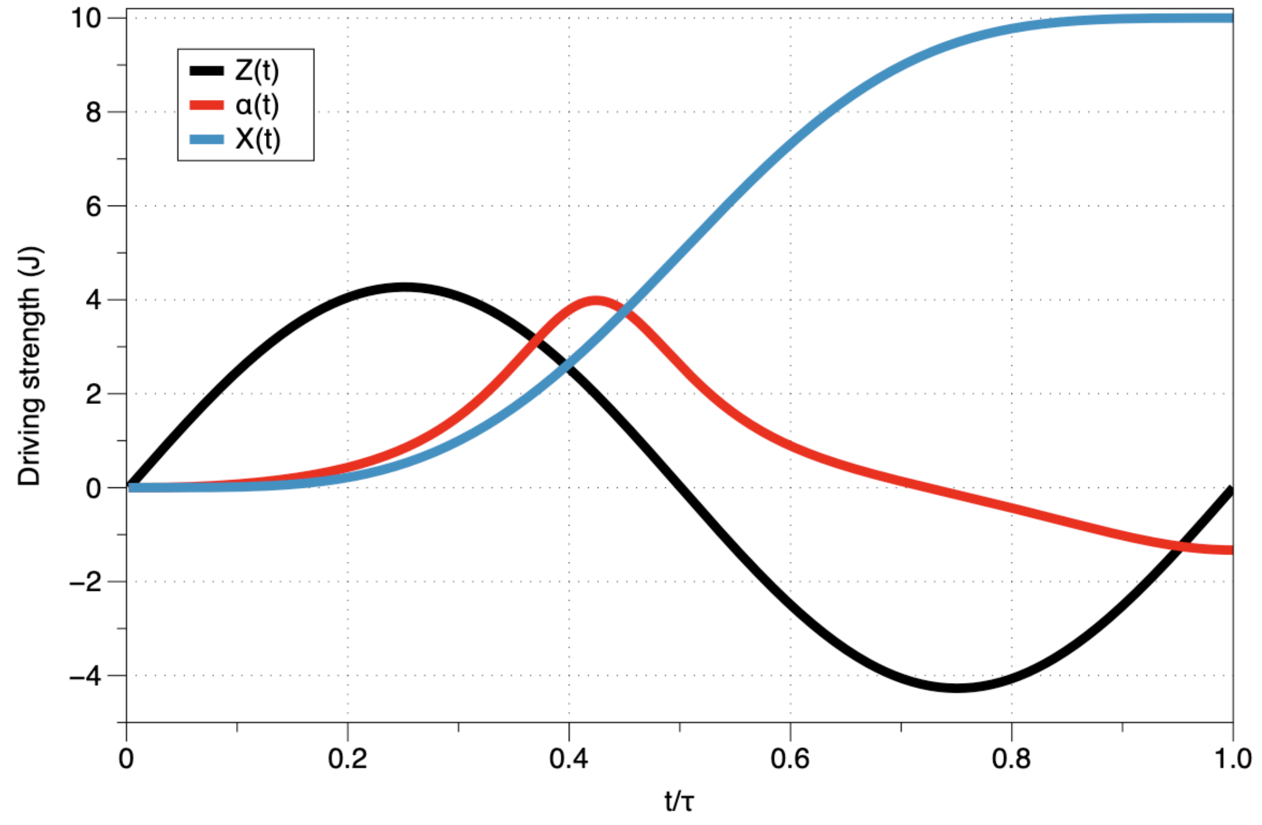
Floquet-engineering counterdiabatic protocols in quantum many-body systems, 2019 [Phys. Rev. Lett.]

Final Notes

- Adiabatic processes limited by adiabatic theorem
- Adding **CD driving** can help avoid/eliminate unwanted transitions, but…
    - … exact CD drive is difficult to derive/implement.
- Use **approximate (variational) CD drive**, treating it as a perturbation
- Improve approximate VCD using optimal control methods
    - Add highly controllable, parameterised drive







Thank you for listening :)