arXiv:2203.01948

Counterdiabatic Optimised Local Driving
(COLD)

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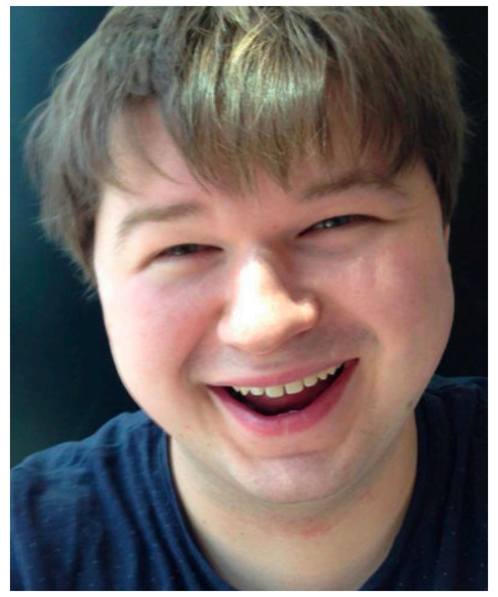


Adiabatic processes

- **Slowly** varying $H(\lambda)$ according to some $\lambda(t)$

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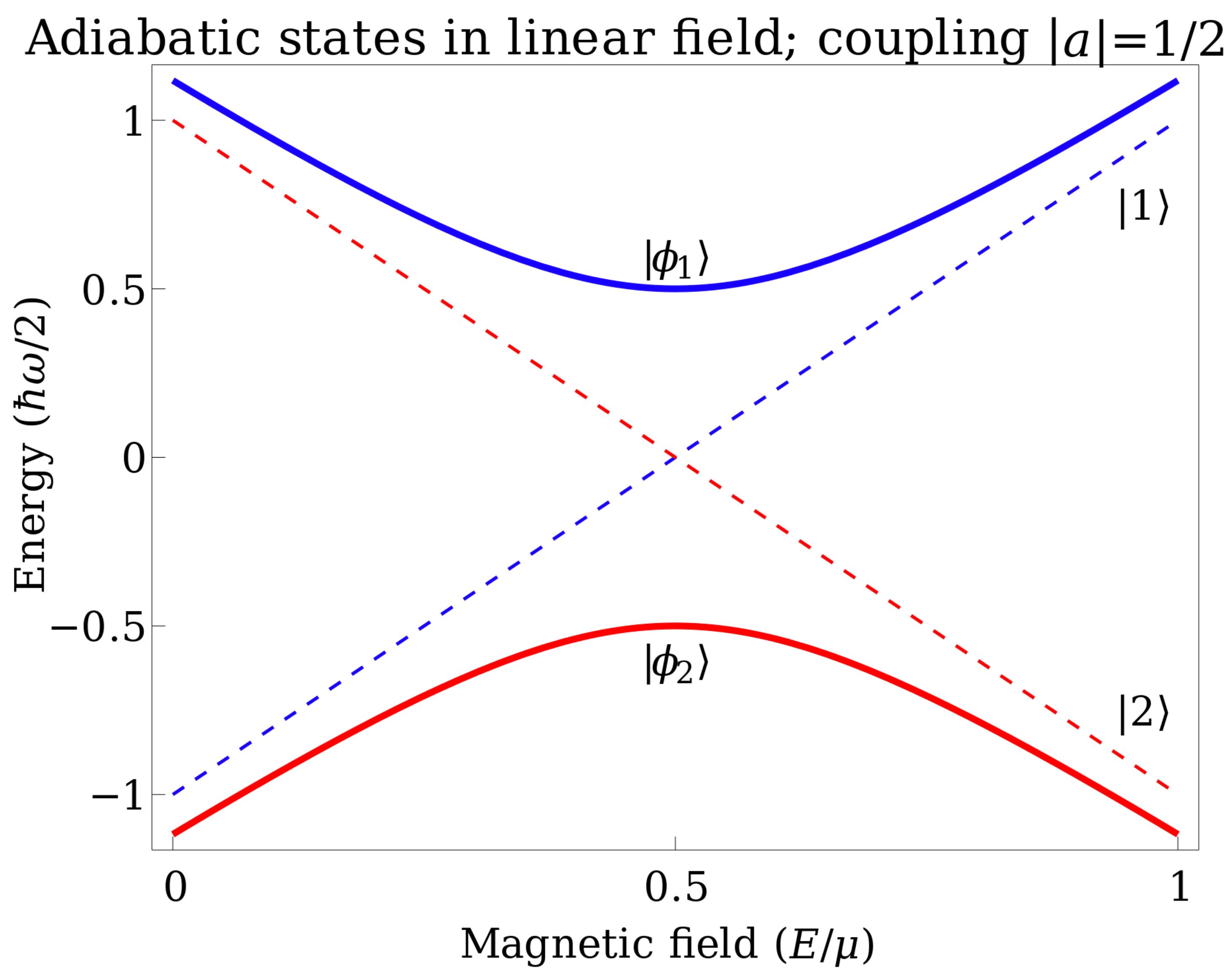
  - System in eigenstate of $H(\lambda_0)$ $\xrightarrow{\text{(Adiabatic evolution)}}$ *corresponding* eigenstate of $H(\lambda_f)$ (**adiabatic theorem**)



https://en.wikipedia.org/wiki/Adiabatic_theorem

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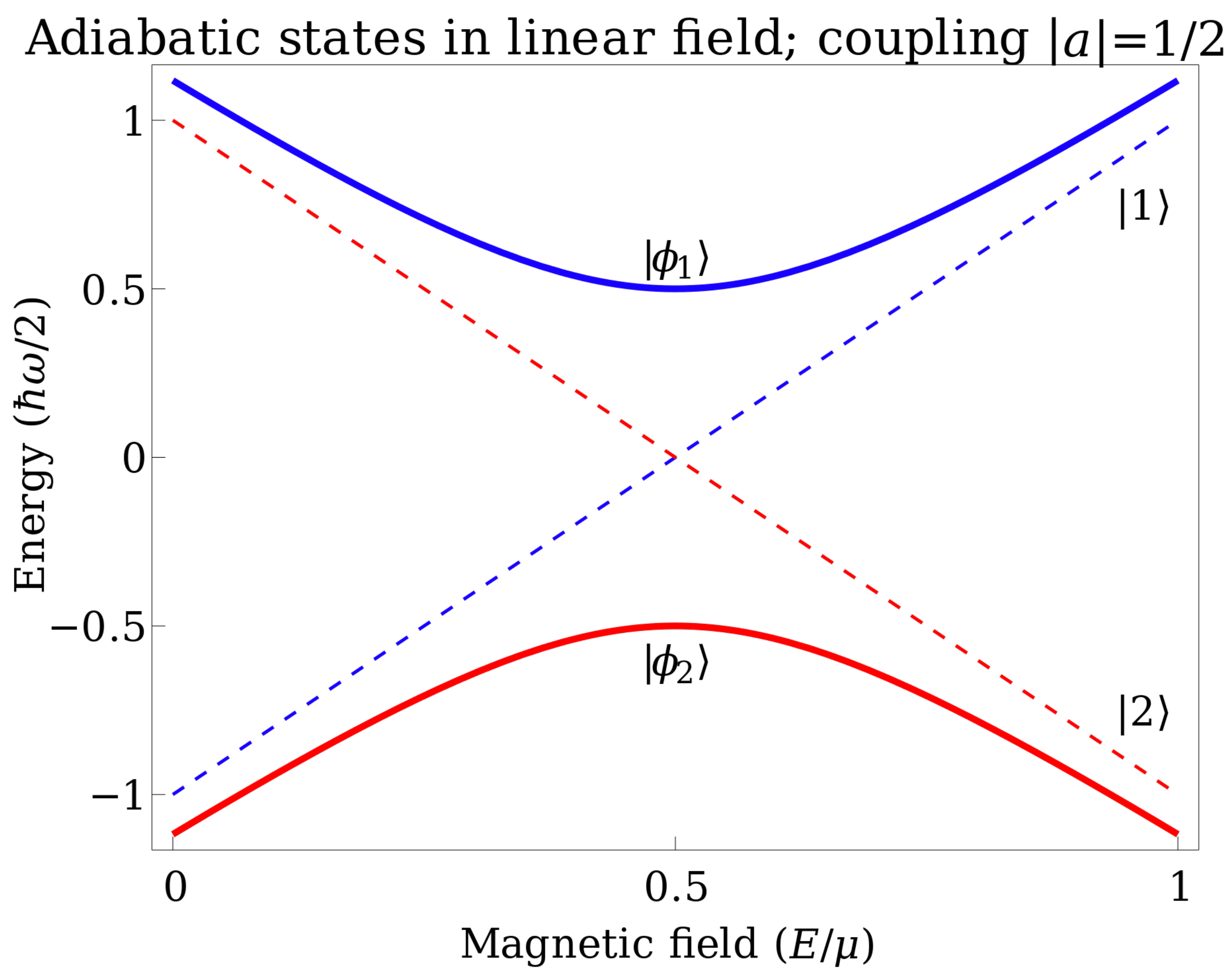

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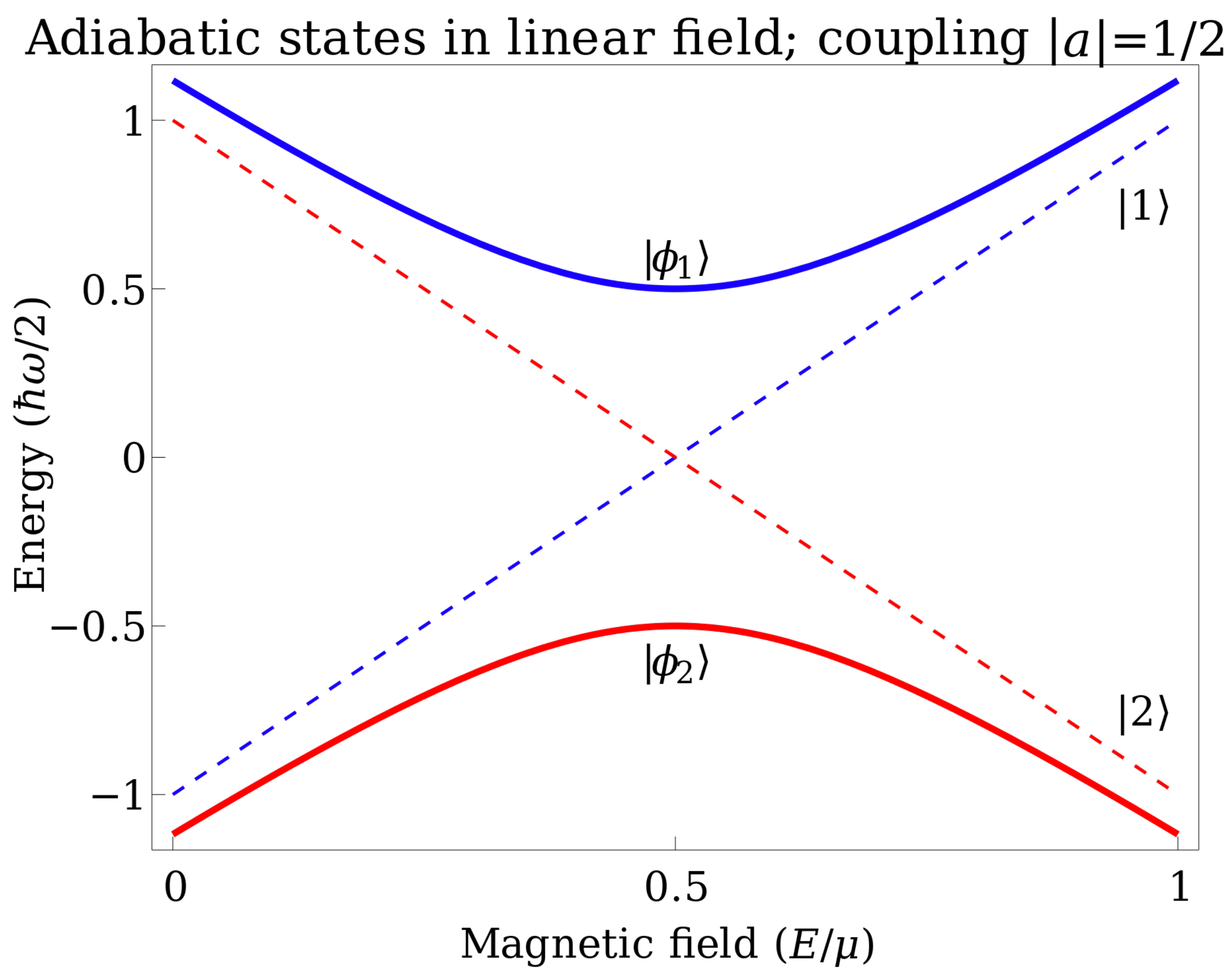

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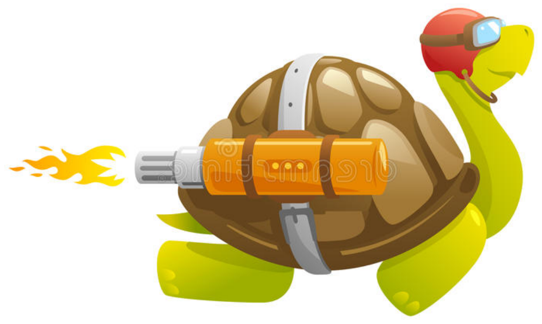
!

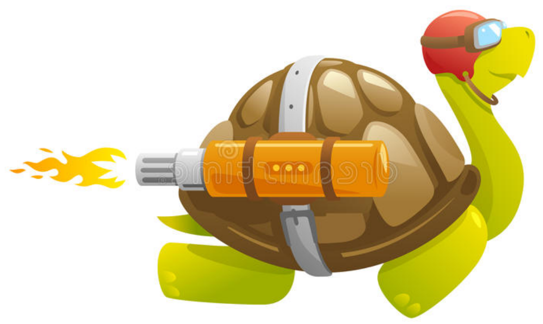
Trying to perform an adiabatic evolution **fast** leads to **losses**: transitions out of the required eigenstate

Speeding things up



- What do we want?

Speeding things up



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    - Adiabatic dynamics!

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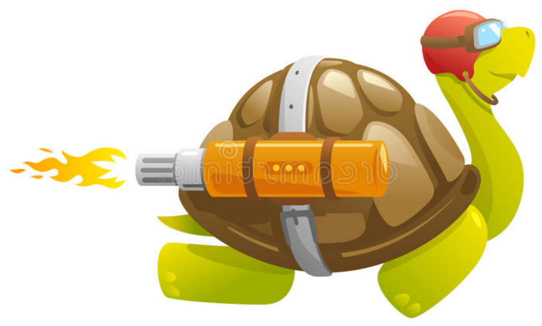

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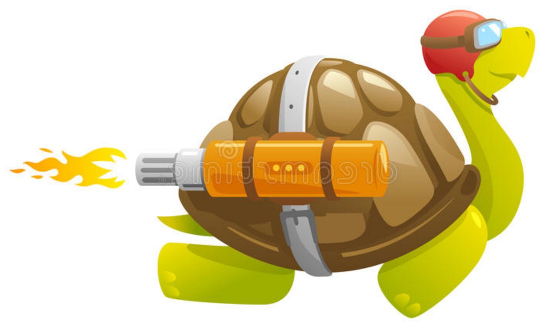

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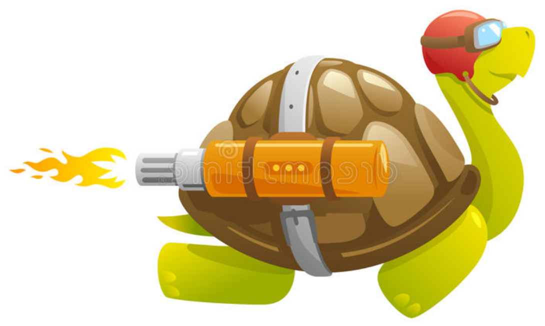

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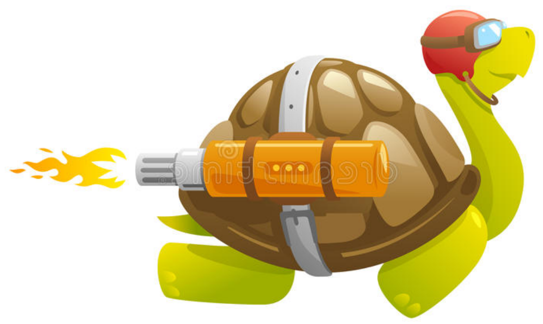

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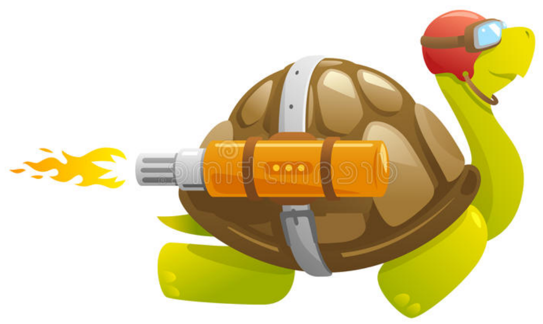
Shortcuts to adiabaticity

Rev. Mod. Phys. 91, 045001 (2019)

Optimal control methods

Eur. Phys. J. D 69, 1 (2015).

Speeding things up



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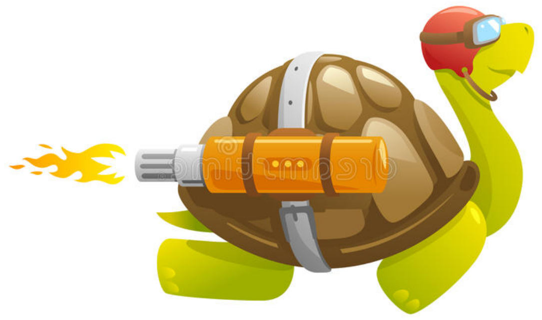
Shortcuts to adiabaticity

Optimal control methods

Local counterdiabatic driving

PNAS 114, E3909 (2017)

Speeding things up



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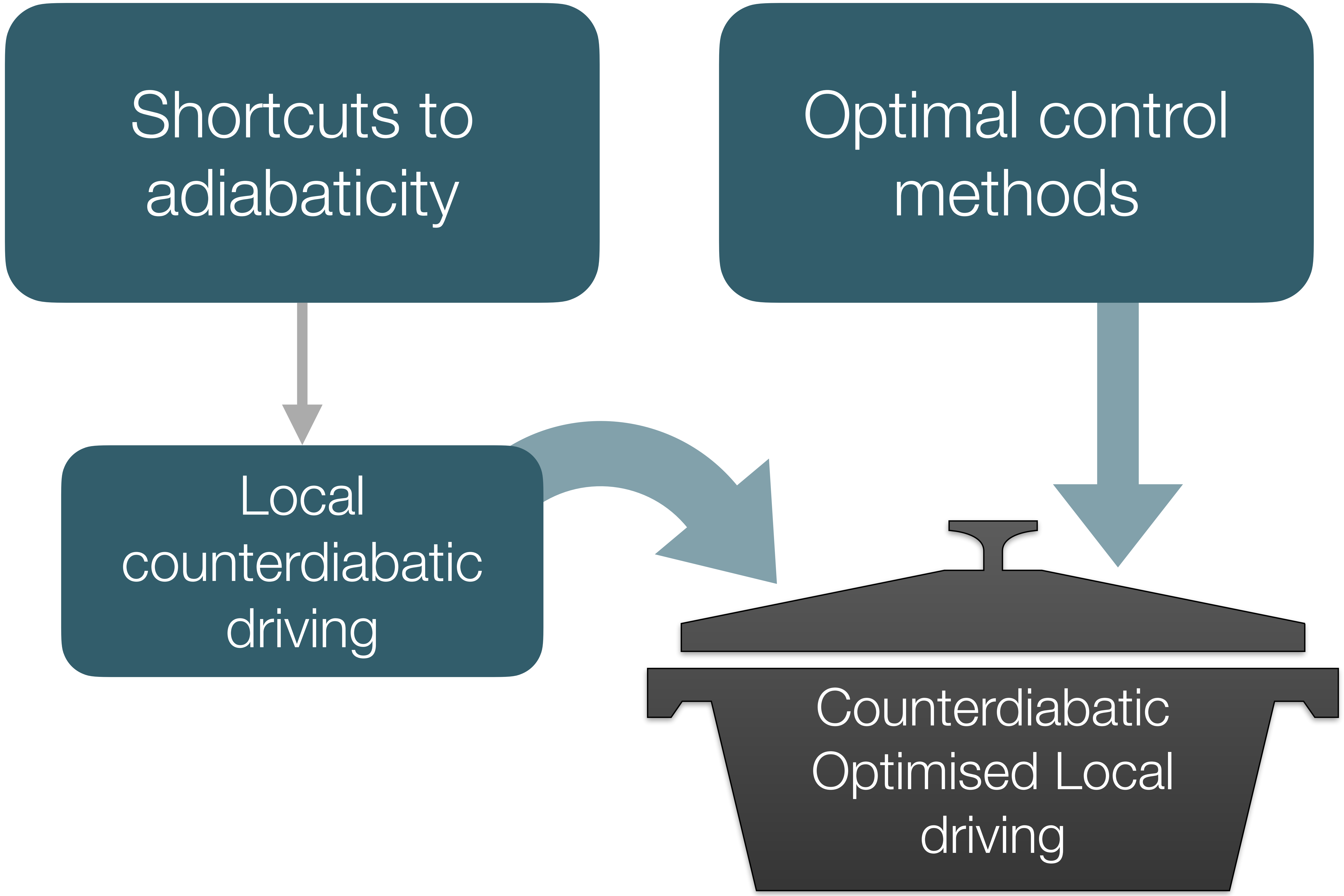

arXiv:2203.01948 (2022)

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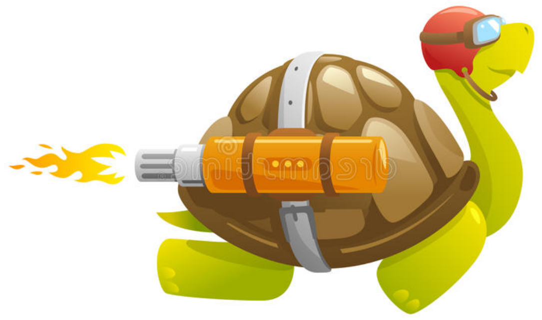

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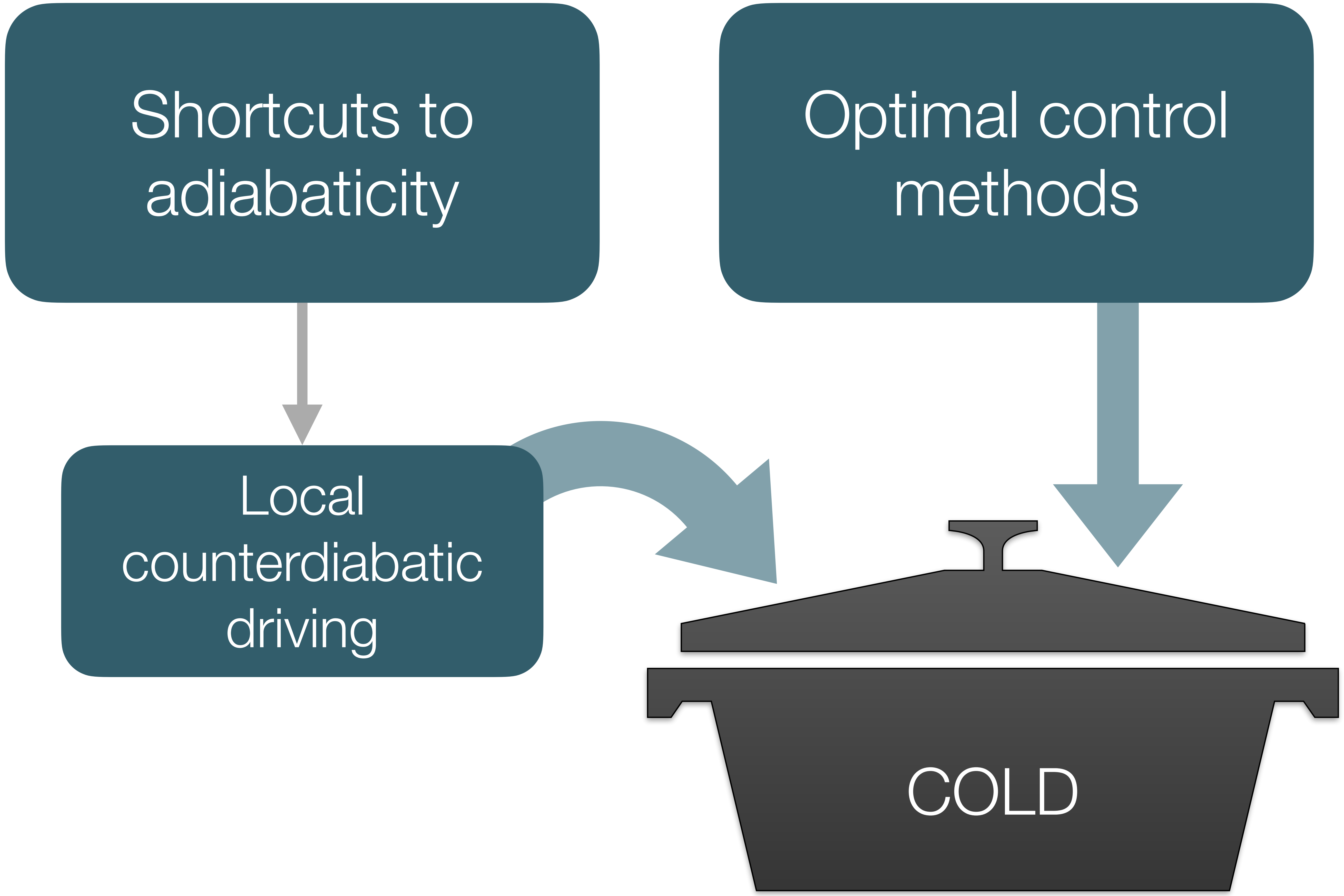

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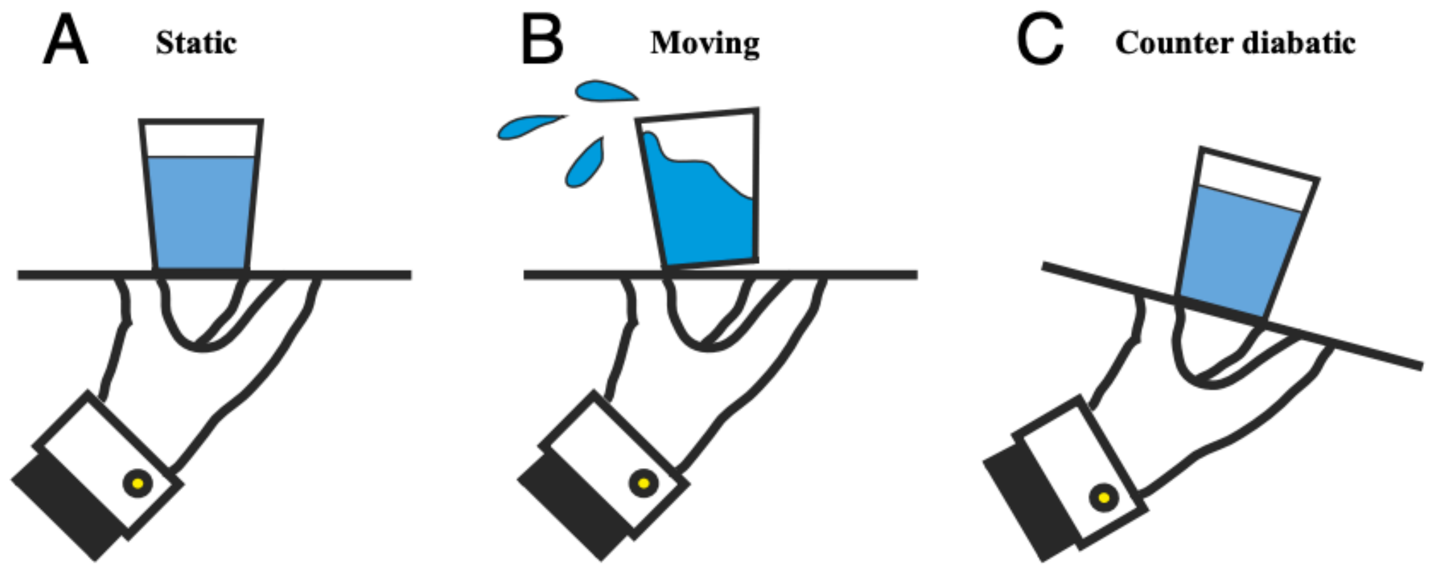
Counterdiabatic driving (CD)

- For time-dependent moving frame: $|\tilde{\psi}\rangle = U(\lambda)\,|\psi\rangle$



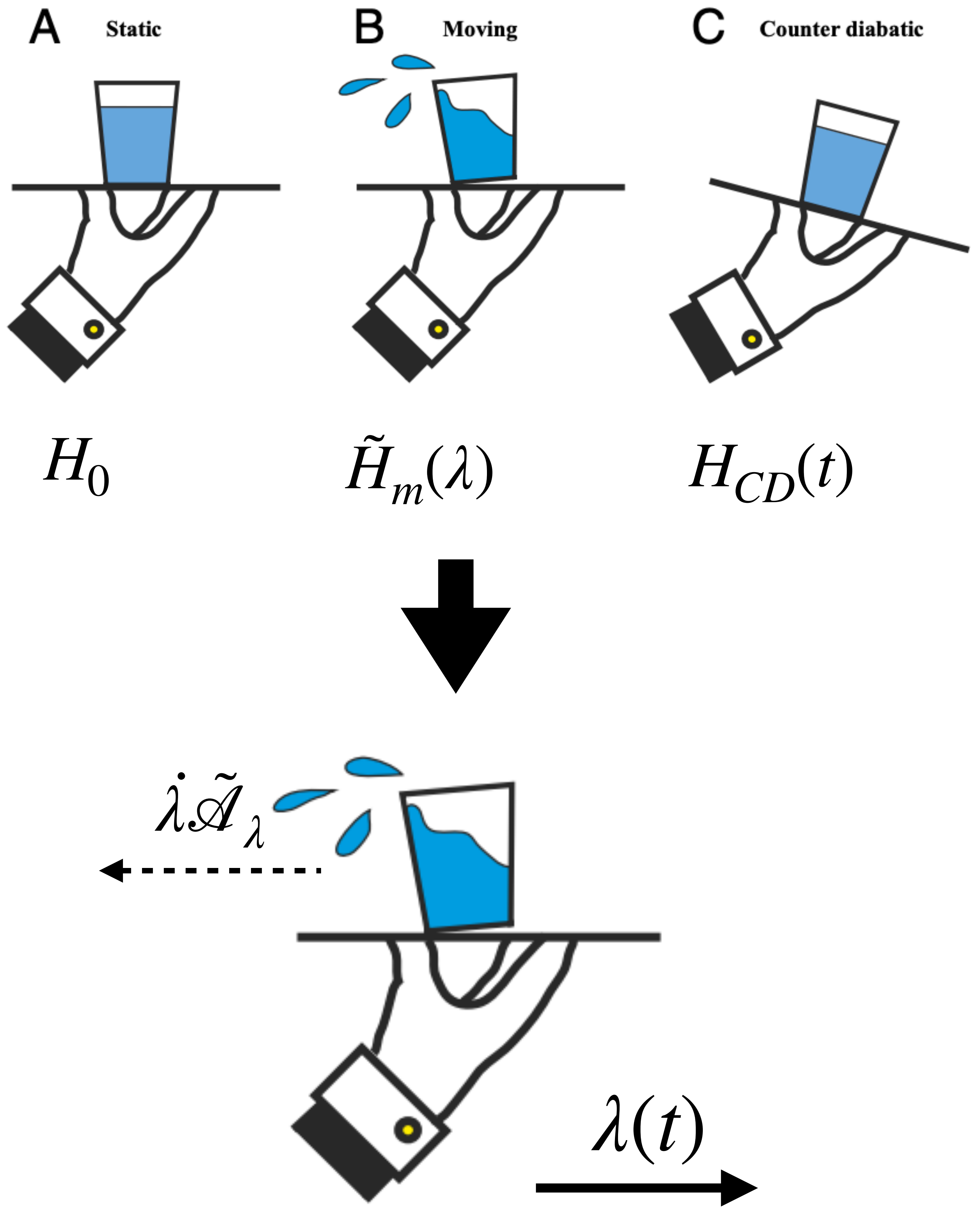
H_0 $\tilde{H}_m(\lambda)$ $H_{CD}(t)$

Counterdiabatic driving (CD)

- For time-dependent moving frame: $|\tilde{\psi}\rangle = U(\lambda)\,|\psi\rangle$

- In moving frame:

$$\tilde{H}_m(t) = \tilde{H}(\lambda) - \dot{\lambda}\mathcal{A}_\lambda(t)$$



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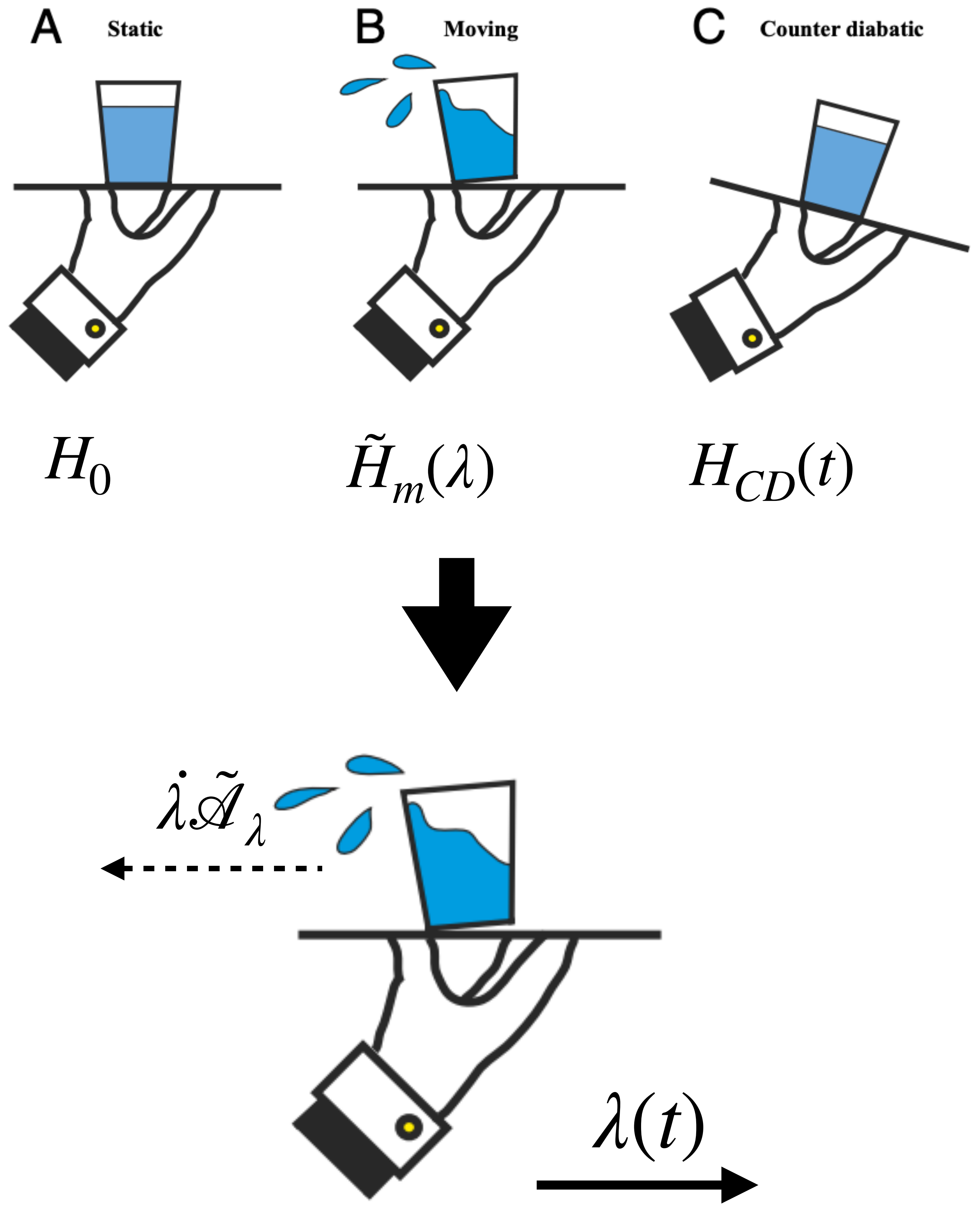
diagonal

adiabatic gauge potential



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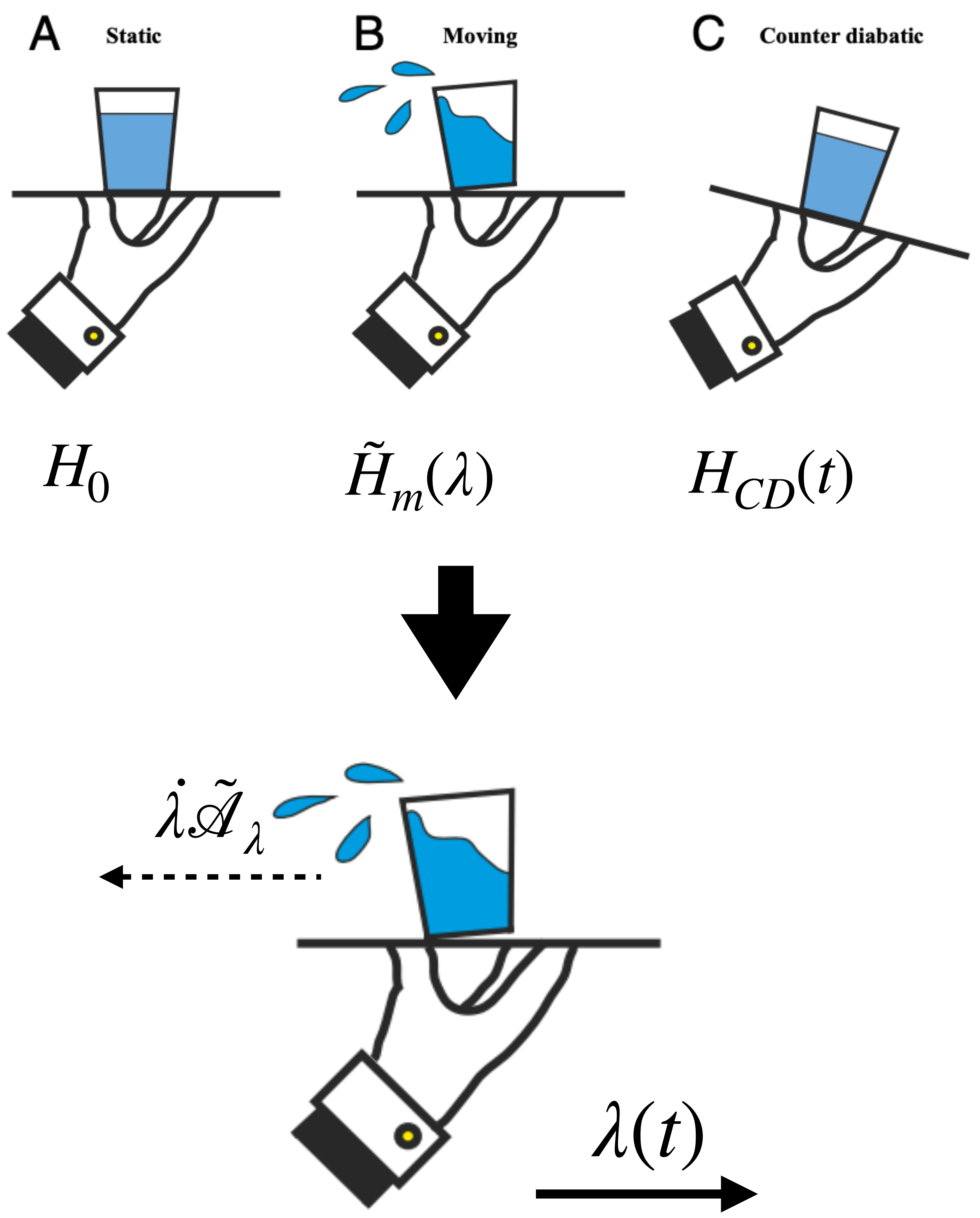
diagonal *adiabatic gauge potential*

- Want to counteract ‘fictitious force’ $\dot{\lambda}\tilde{\mathscr{A}}_\lambda$:

$$H_{CD}(t) = \tilde{H}_m(t) + \dot{\lambda}\mathscr{A}_\lambda = \tilde{H}(\lambda)$$



Local counterdiabatic driving (LCD)

- **Problem**: $\mathscr{A}_\lambda$ may require exponential fine-tuning/full knowledge of instantaneous eigenstates:

$$\mathscr{A}_\lambda \sim |\partial_\lambda \psi\rangle\langle\psi|$$

PNAS 114, E3909 (2017)

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$$\mathscr{A}_\lambda \sim \sigma^y$$

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$$\mathscr{A}_\lambda \sim \sigma_i^y + \sigma_i^y \sigma_{i+1}^z + \sigma_i^z \sigma_{i+1}^y + \sigma_i^y \sigma_{i+1}^x + \sigma_i^x \sigma_{i+1}^y$$

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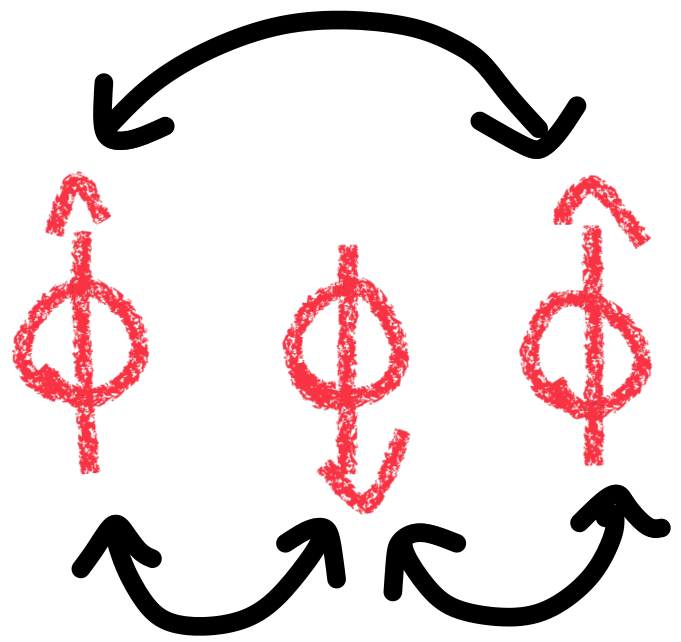

$$\mathscr{A}_\lambda \sim \sigma_i^y + \sigma_i^y \sigma_{i+1}^z + \ldots + \sigma_i^y \sigma_{i+1}^z \sigma_{i+2}^z + \sigma_i^y \sigma_{i+1}^x \sigma_{i+2}^z + \ldots$$

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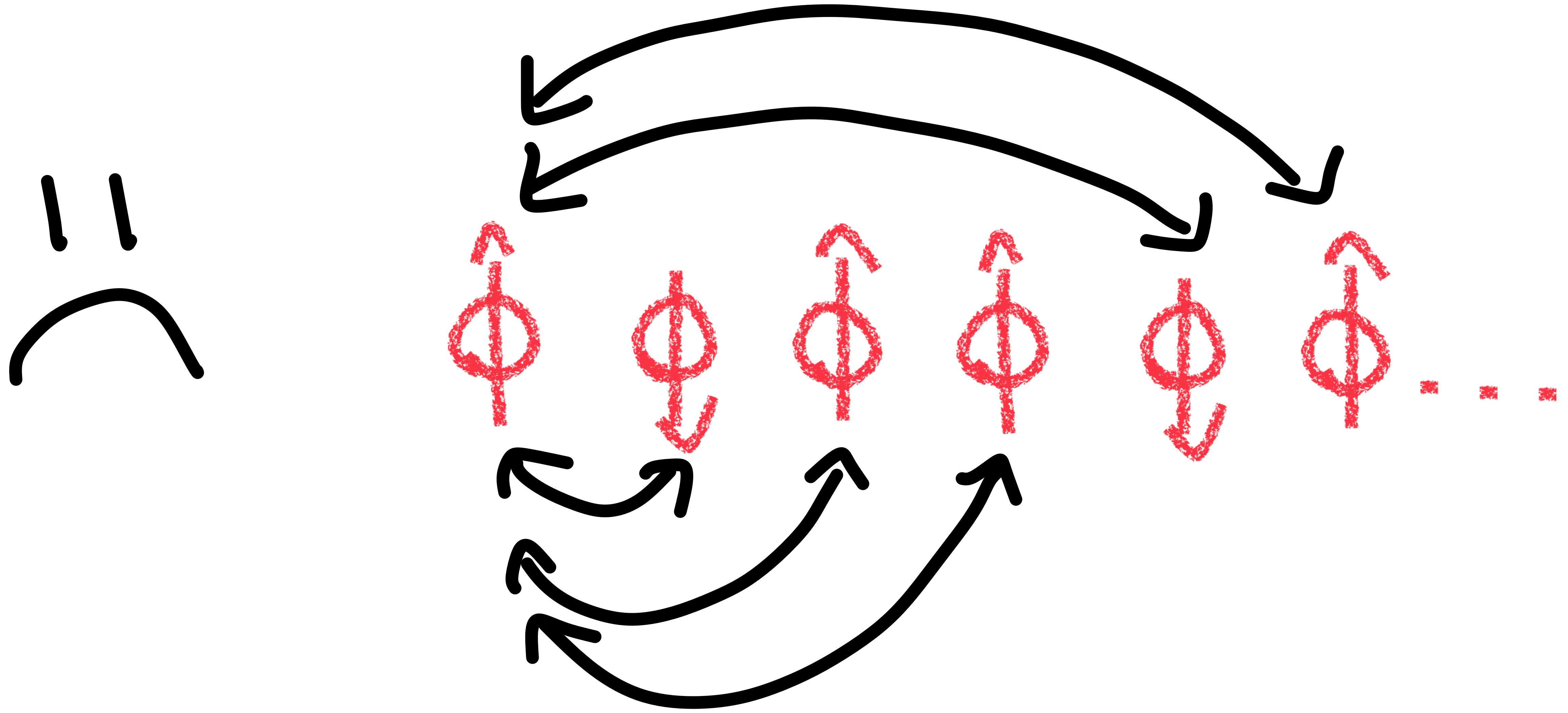

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  - Choose Ansatz operator basis $\{\mathscr{O}_{LCD}\}$ for $\mathscr{A}_\lambda$ and perform **minimisation***

PNAS 114, E3909 (2017)

Local counterdiabatic driving (LCD)

- **Solution**: devise *approximate* CD

- Choose Ansatz operator basis $\{\mathcal{O}_{LCD}\}$ for $\mathcal{A}_\lambda$ and perform **minimisation***
  - e.g. spin system 1st order: $\mathcal{A}_\lambda^{(1)} = \sum_j \alpha\, \sigma_j^y$
  - 2nd order: $\mathcal{A}_\lambda^{(2)} = \sum_{\langle i,j \rangle} \left[\gamma(\sigma_i^x \sigma_j^y + \sigma_i^y \sigma_j^x) + \zeta(\sigma_i^z \sigma_j^y + \sigma_i^y \sigma_j^z) \right]$

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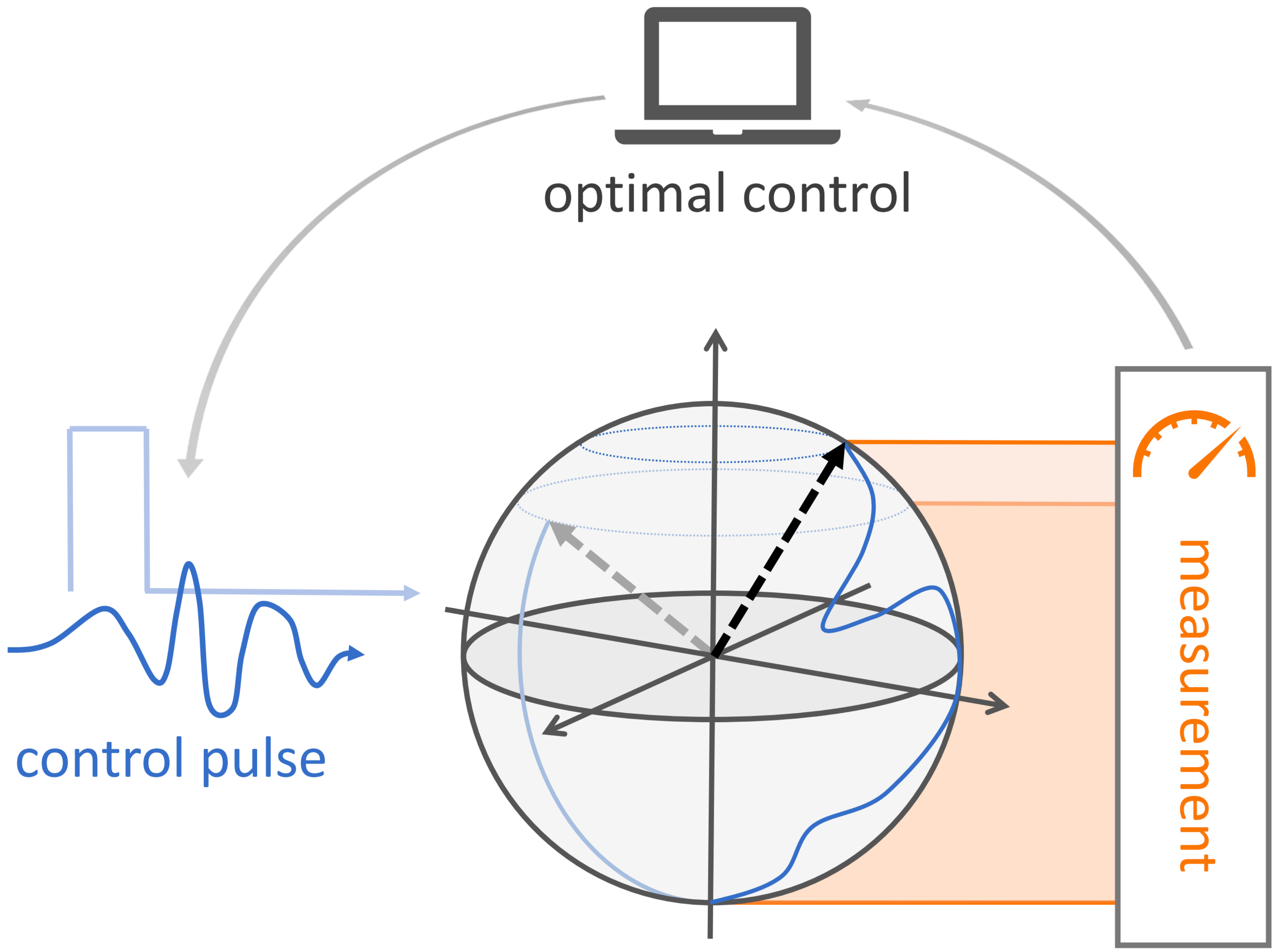
Optimal control

- Add control field to system:

$$H_\beta(t, \beta) = \tilde{H}(\lambda) + \beta \mathcal{O}_{\text{opt}}$$



https://qusco-itn.eu/2020/06/19/intro_nvc/

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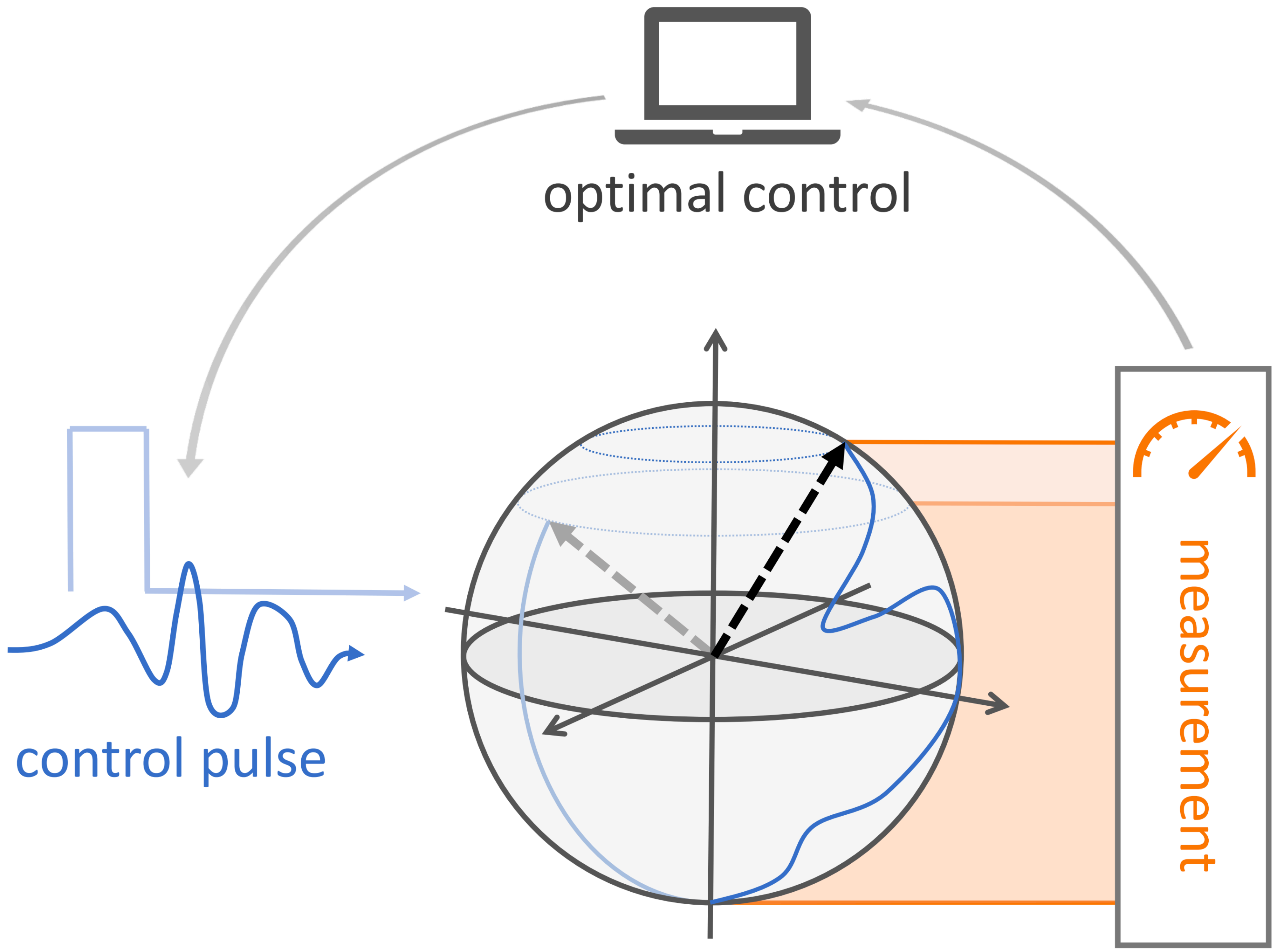
- Optimise parameters β according to chosen metric, e.g. final state fidelity:

$$\mathcal{C}(|\psi_f\rangle) = 1 - \left| \langle \psi_T | \psi_f \rangle \right|^2$$



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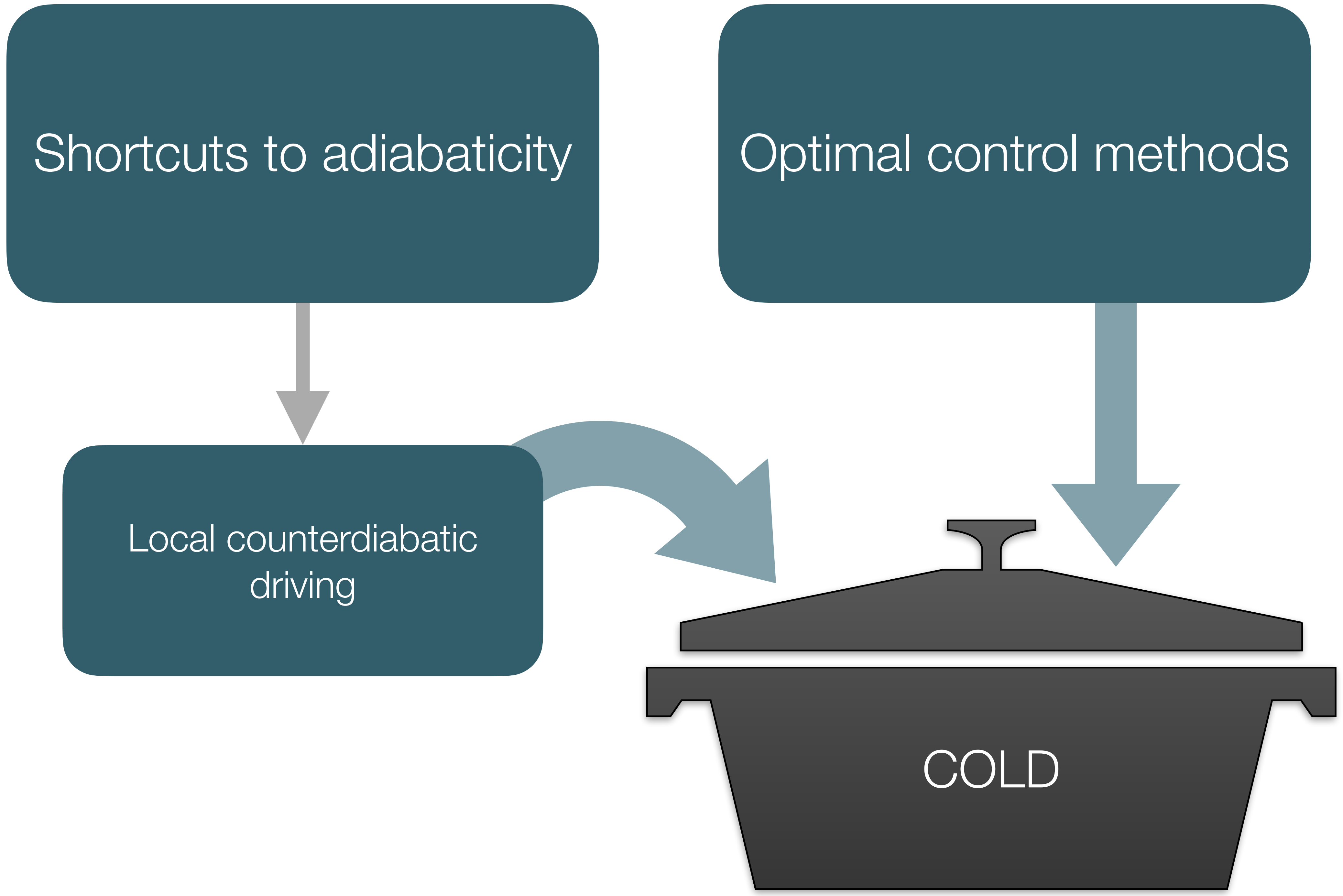

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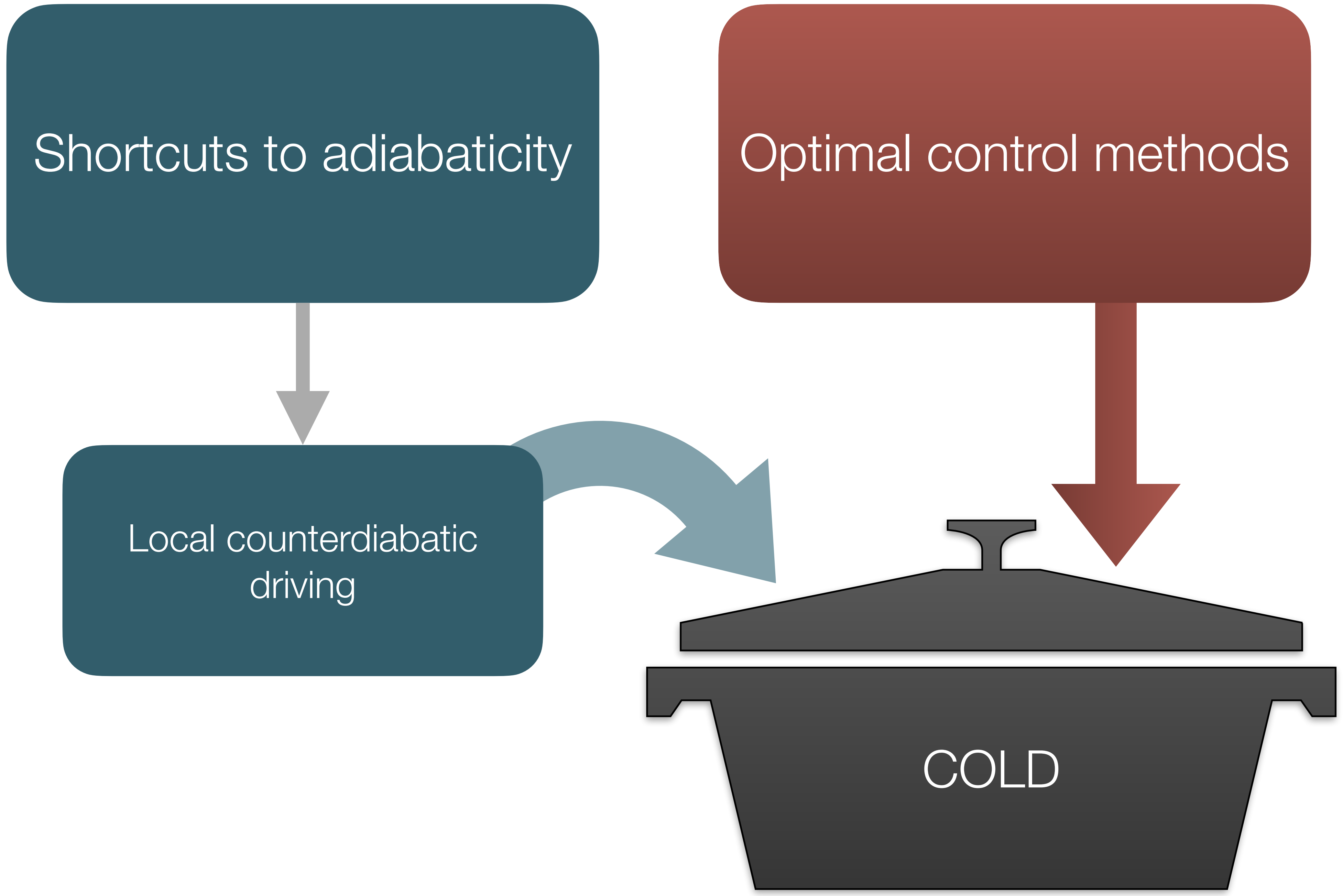

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$$H_{\beta}(\lambda, \beta) = H(\lambda) + \beta \mathcal{O}_{\text{opt}}$$

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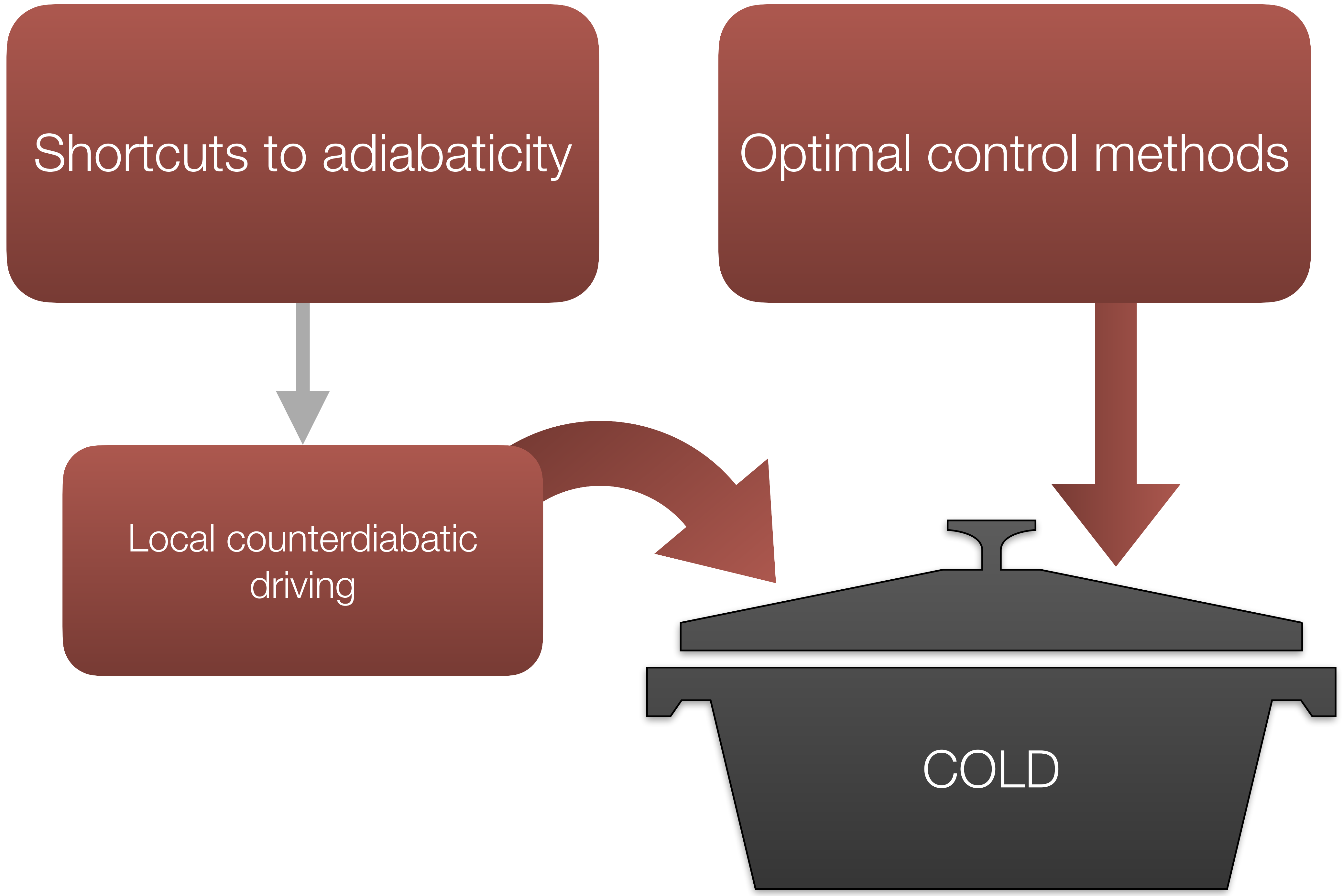
$$H_{\text{COLD}}(\lambda, \beta) = H_{\beta}(\lambda, \beta) + \sum_j \alpha(\lambda, \beta) \mathcal{O}_{\text{LCD}}$$



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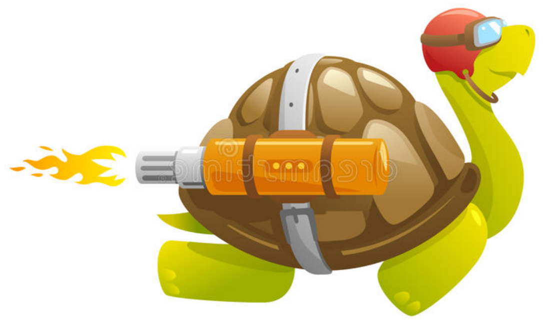
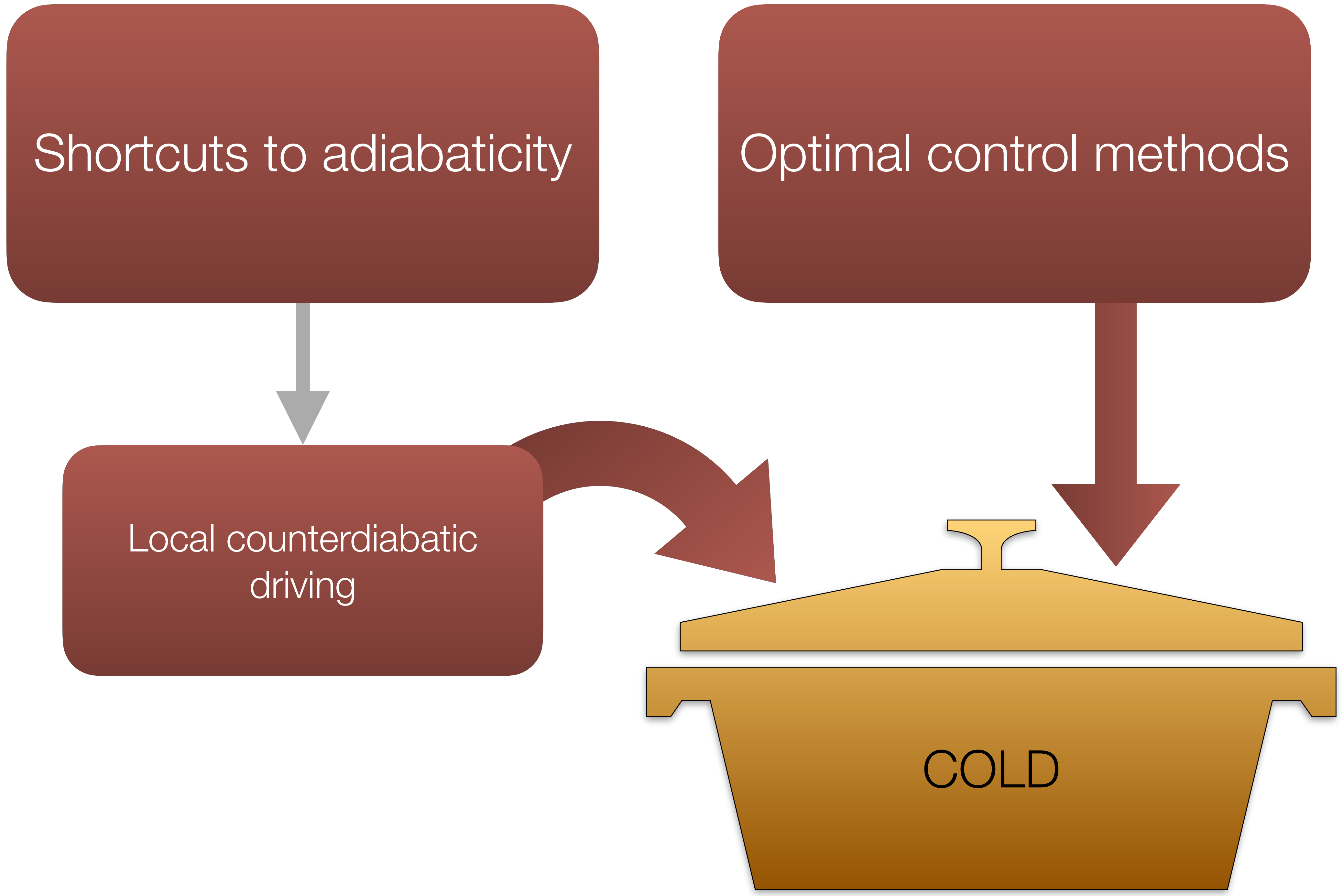
4. Optimise parameters β





Results: 2-spin annealing

(best of 500 optimisations!)

Bare Hamiltonian:

$$H(\lambda) - 2J\sigma_1^z\sigma_2^z - h(\sigma_1^z + \sigma_2^z) + 2h\lambda(t)(\sigma_1^x + \sigma_2^x)$$

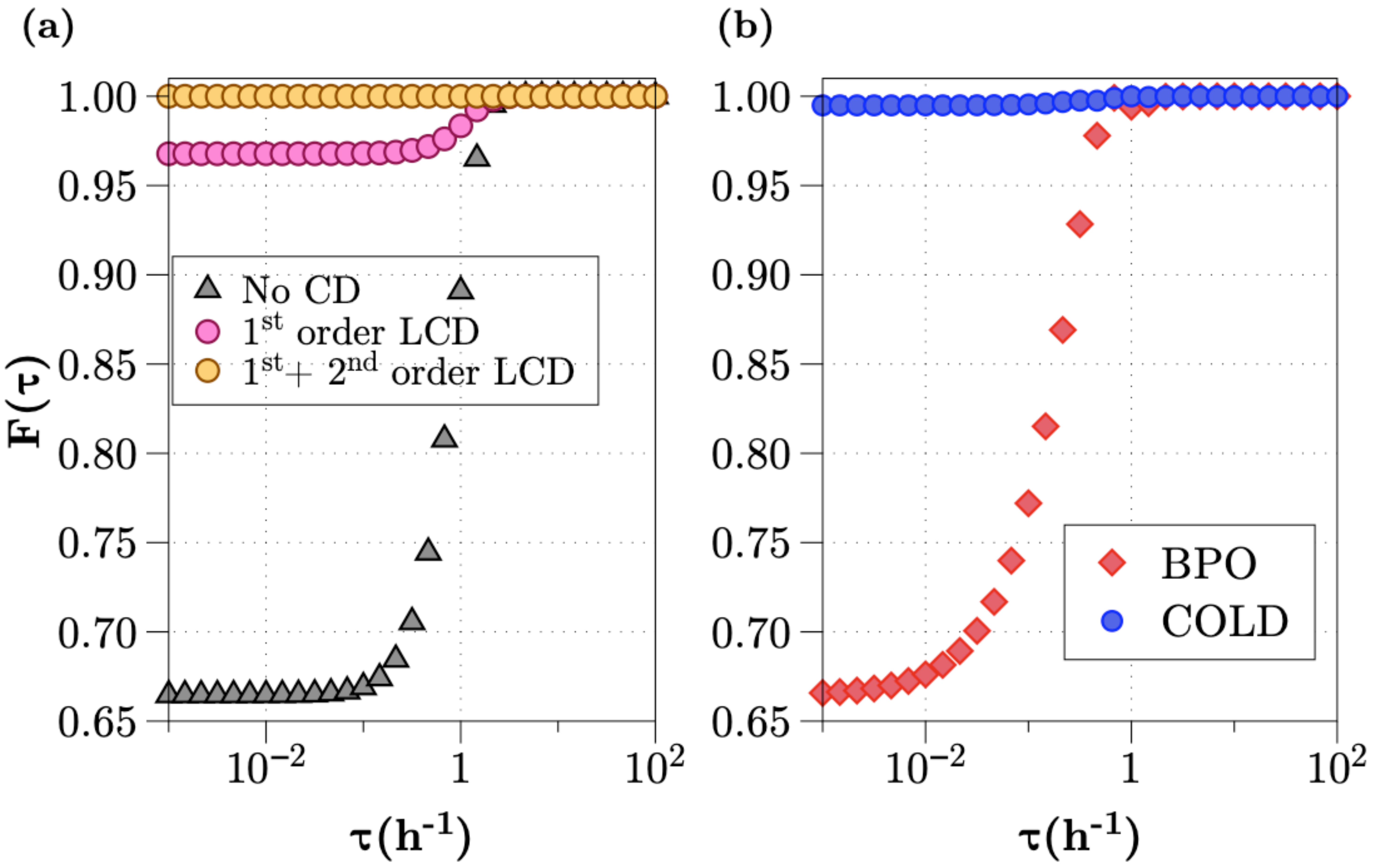
Optimal control Hamiltonian:

$$H_\beta(t) = H_0(t) + \sum_{k=1}^{N_k} \beta^k \sin(\pi k t/\tau) \sum_i \sigma_i^z$$

1st Order LCD:

$$\mathscr{A}(\lambda) = \alpha(\sigma_1^y + \sigma_2^y)$$

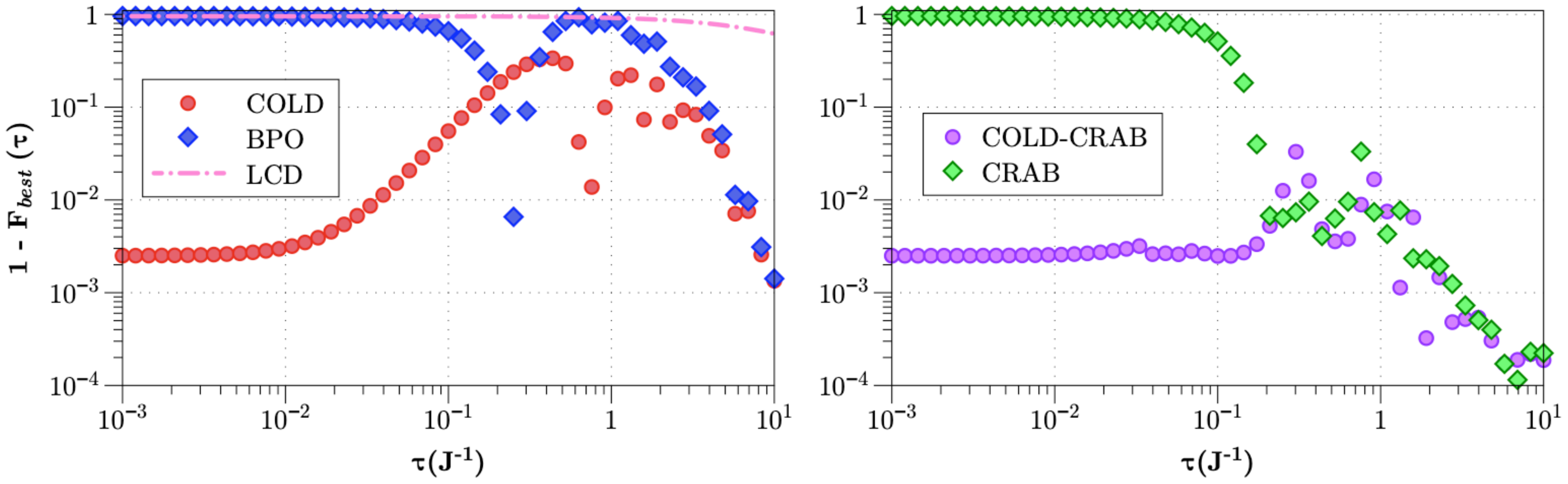
$$\alpha = -\frac{h^2}{4(h\lambda)^2 + h^2 + 4J^2}$$



Results: Ising spin chain

$$H(\lambda) = -J\sum_{j}^{N-1}\sigma_j^z\sigma_{j+1}^z + Z_0\sum_j^N\sigma_j^z + \lambda X_f\sum_j^N\sigma_j^x,$$

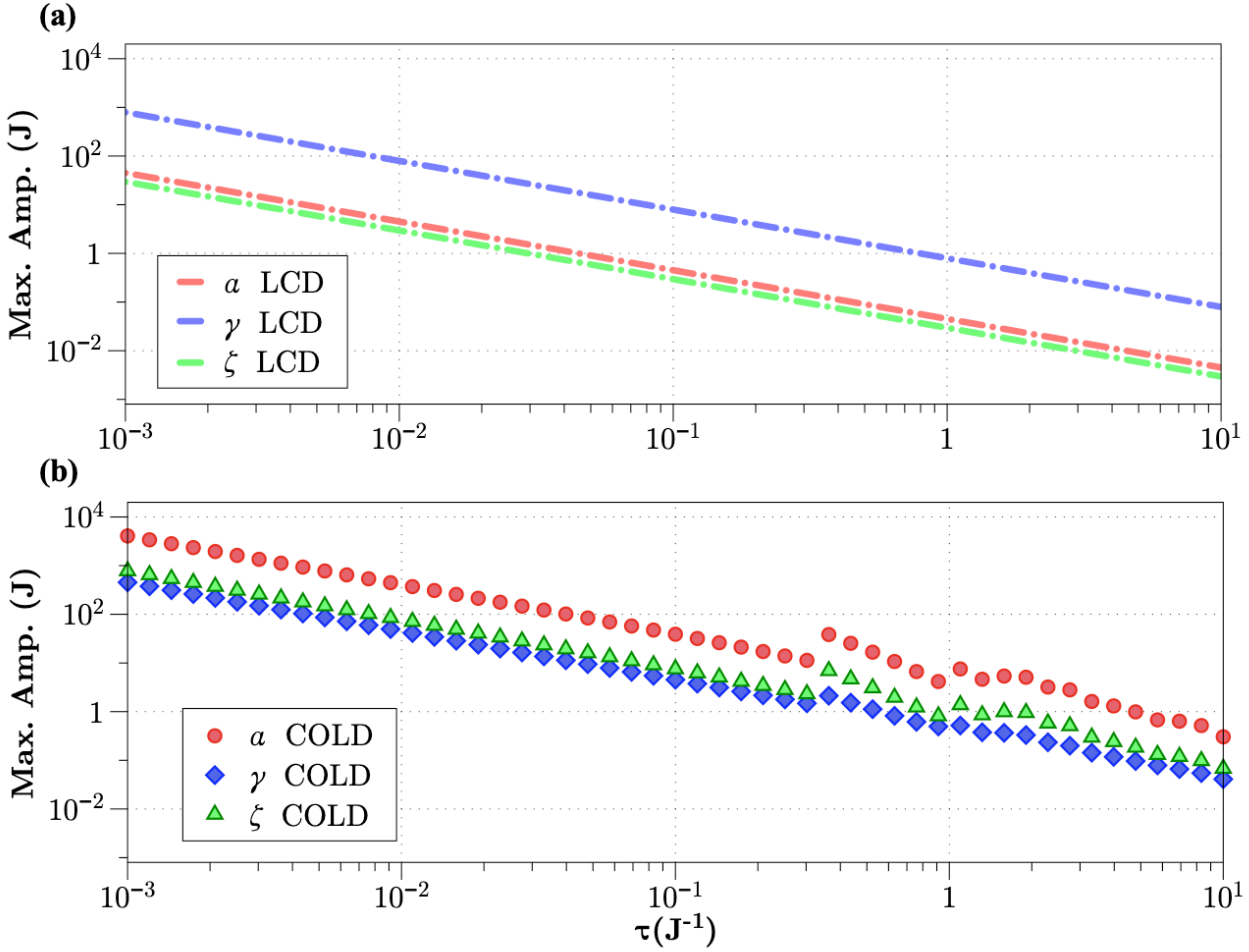
(best of 500 optimisations!)



CRAB method: **Phys. Rev. A 84, 022326 (2011)**

Results: Maximum CD amplitudes

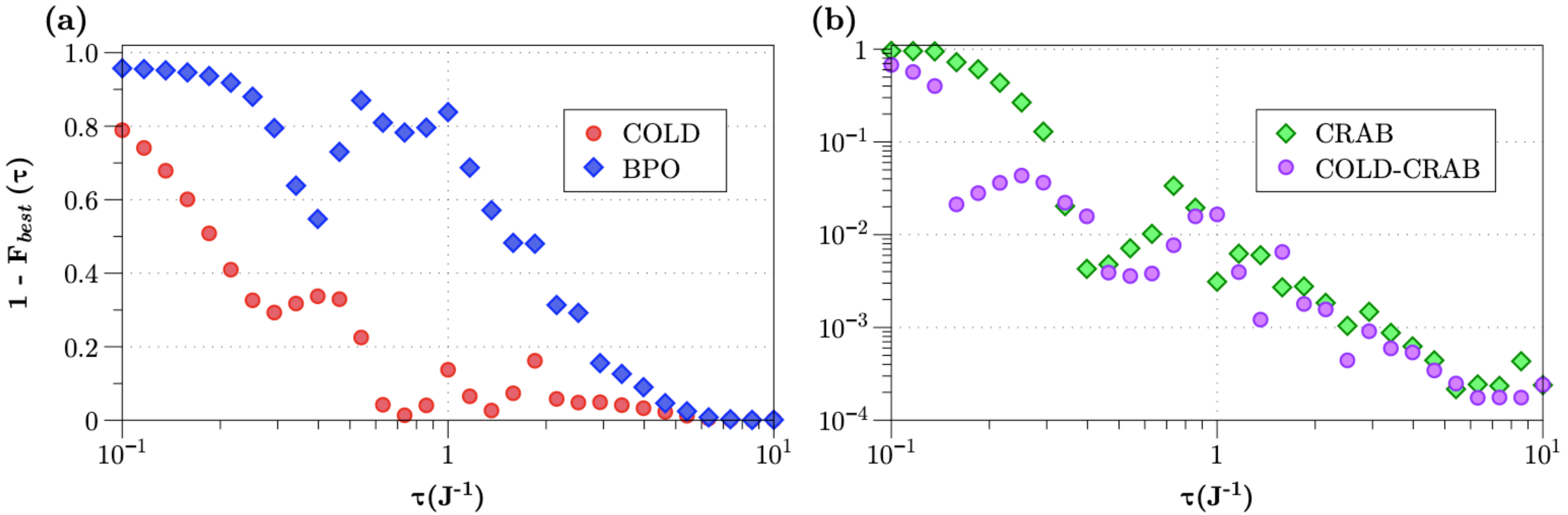
- Shorter the driving time -> higher **power** requirements!
- Optimising for lower order LCD reduces amplitude of higher orders
- **Idea**: use higher order LCD amplitude integral as cost function for optimiser!



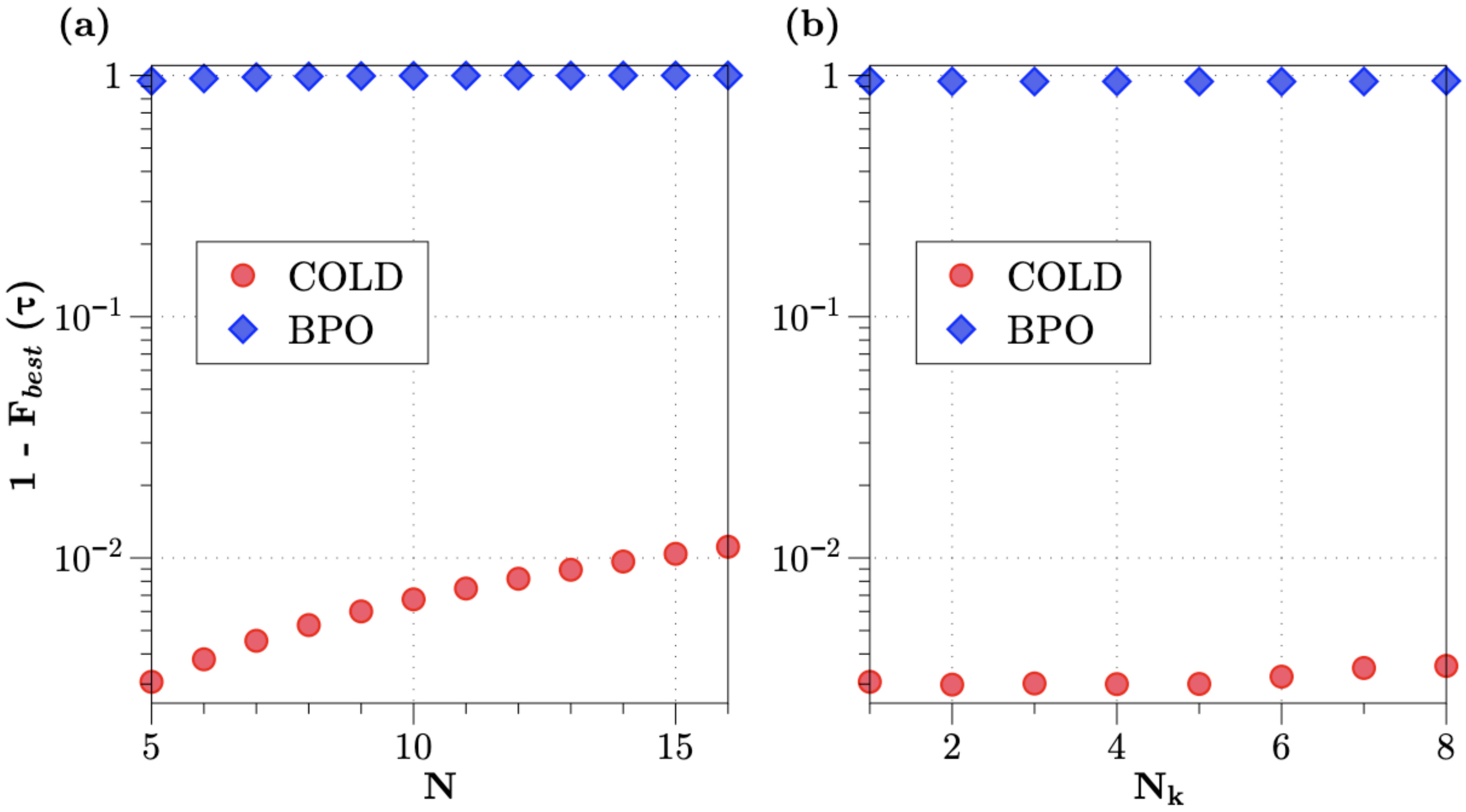
Results: Ising spin chain (constrained)

Limit maximum amplitude of CD to $|X_f|$

(best of 500 optimisations!)



Results: Ising spin chain scaling



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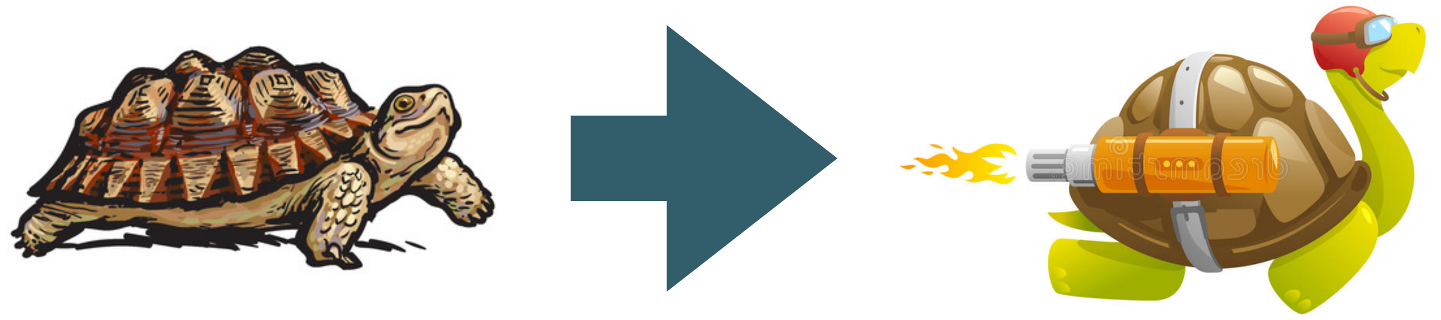
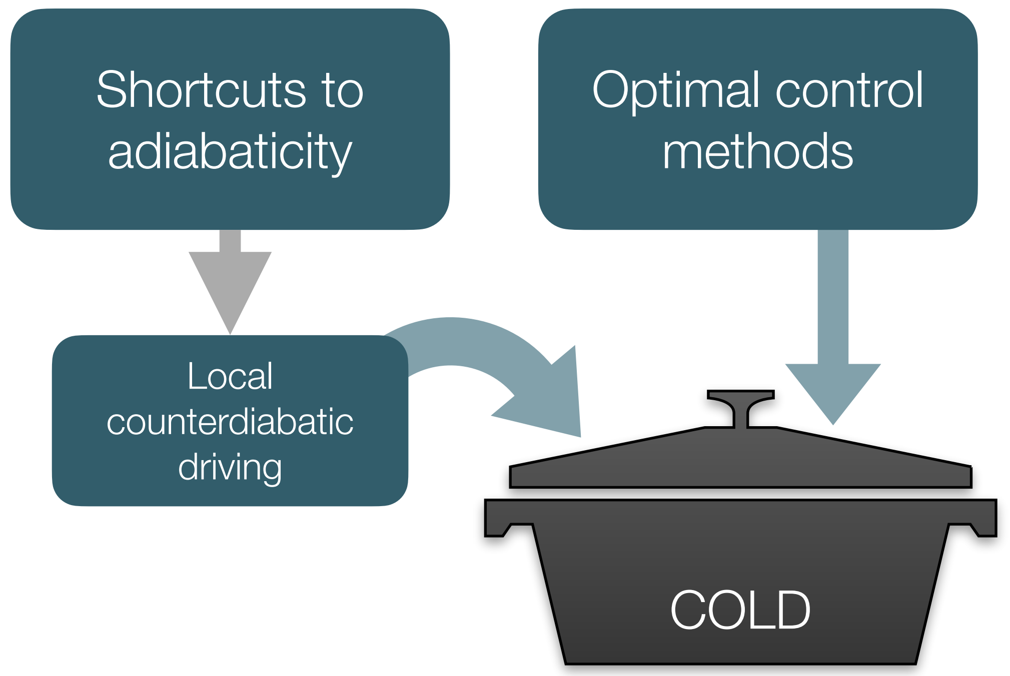
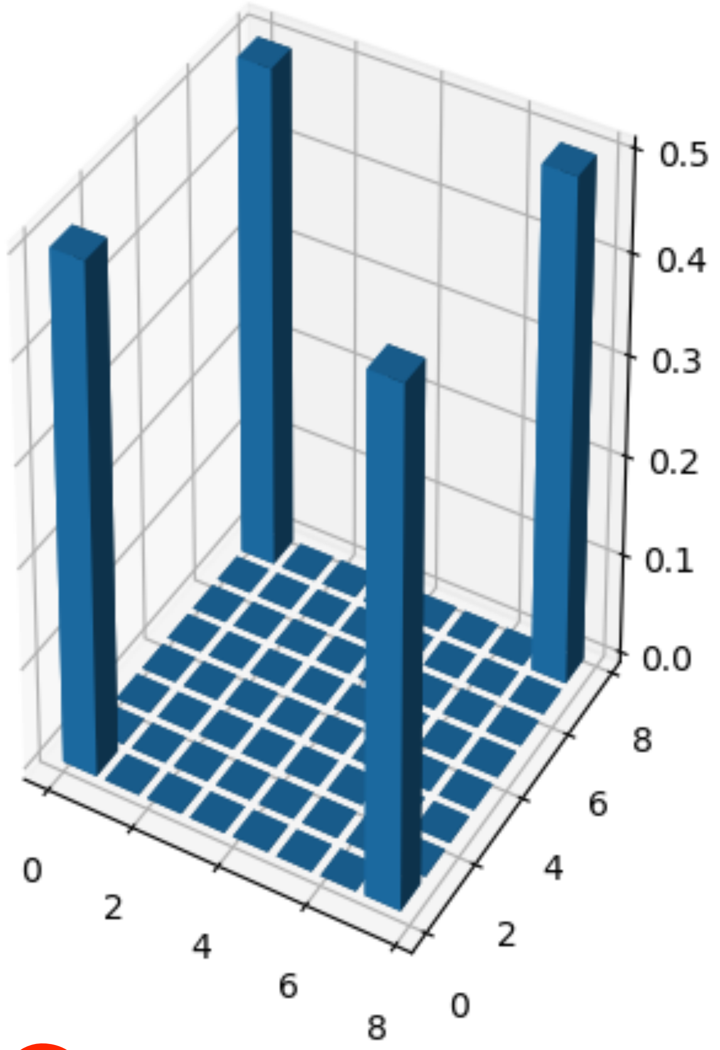
$$\tau = 10^{-2} J^{-1}$$

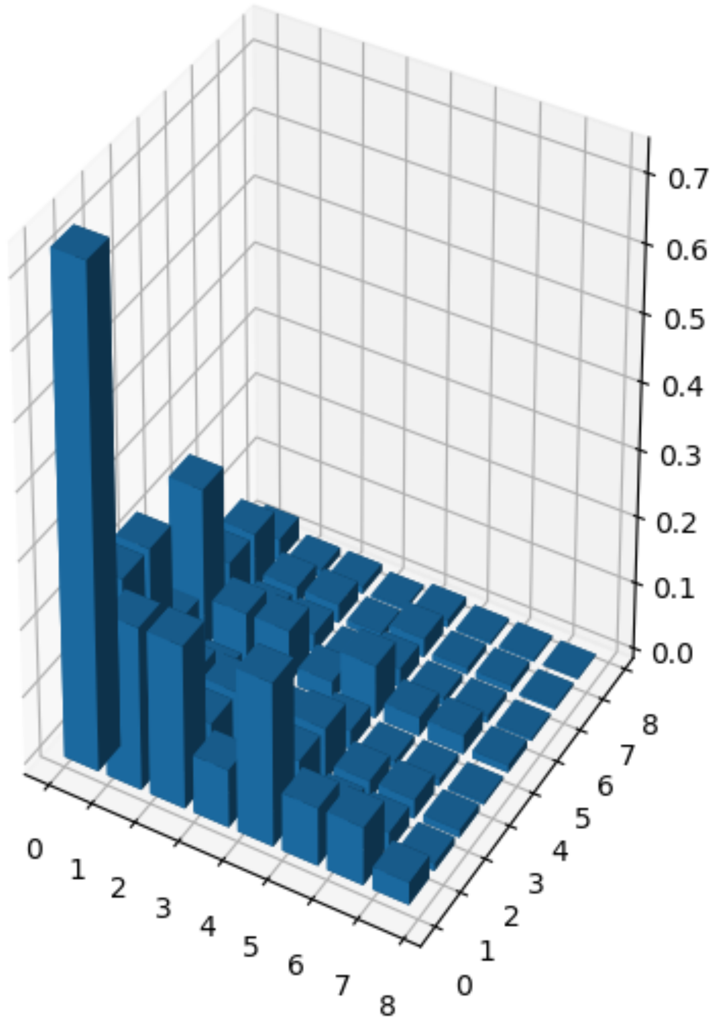
Summary



- **COLD**: new method for speeding up adiabatic processes
- Improves upon LCD and optimal control methods
- New avenues to explore:
    - combinations with different optimal control methods
    - minimisation of higher order LCD
    - experimental implementations 🧪







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@Hyperboleva

Coming soon!
Frustrated spins