

Maintenance analysis of transportation networks by the traffic transfer principle considering node idle capacity



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ARTICLE INFO

Keywords:

Maintenance
Transportation network
Failure path
Reliability

ABSTRACT

Traffic congestion is a universal challenge that affects urban transportation networks, which inevitably age and deteriorate. Maintenance is an essential method for alleviating road congestion. Most of the previous studies concentrate on node load and capacity analysis. The capacity of an idle node is also an important element that affects traffic congestion, such as road damage or traffic accident at the crossroads. To explore the effect of the capacity of an idle node on road congestion, this paper introduces a traffic transfer principle to improve road maintenance efficiency. The nodes of a traffic network can be ranked based on their failure severity. The failure paths of a traffic network can be identified through the internal connections between nodes. Using the transfer time as the weight of each edge and the service time as the weight of each node, this paper proposes a maintenance model to find the shortest repair path for minimizing the maintenance time. To evaluate the proposed model, four different types of road networks are adopted by comparing the maintenance time. The experimental results show the proposed model outperforms previous models.

1. Introduction

Urban transportation systems are becoming more and more complex with the rapid development of modern cities. Urbanization in many countries is becoming increasingly high, and the utilization of urban transportation networks becomes more essential for urban transportation systems. However, urban roads are relatively difficult to expand due to the limited and congested space. Traffic congestion may cause social problems, such as low efficiency of road network operation and traffic safety, which in turn hampers economic development. In many metropolis, such as Beijing and New Delhi, urban traffic congestion has become a challenging problem that needs tackling urgently. Even a single road congestion in a very large transportation network may cause huge damage to the society when it is not properly managed. Traffic congestion is also an instance of the butterfly effect. It is vital to develop a novel method to manage the resilience of traffic networks.

Congestion propagation in a transportation network can be due to the congestion of a certain intersection or street in the network [1]. Congestion may exacerbate if it is not lessened and it is frequently caused by cascading failures. A cascading failure is the failure in the network that causes other nodes to fail due to the coupling relationship

between the nodes [2]. If a cascading failure is not properly repaired, it can cause more serious damage and eventually lead to a large-scale congestion in the network.

In a transportation network, intersections are divided into different types based on their shapes, such as T-shaped, Y-shaped, round-shaped intersections, etc. The distribution diagrams of maintenance lanes for several common road types are shown in Fig. 1. The shaded parts in the figures in Fig. 1 are maintenance lanes. When a cascade failure occurs on the road, a maintenance vehicle can enter the maintenance lane to reach the place where the congestion needs to be cleared, and then the vehicle on the congested road is evacuated.

To solve the inaccessibility of maintenance vehicles when the roads are congested, this paper assumes that every road has a maintenance lane, which can be the lane designated for buses. Other types of vehicles can use the maintenance lane only when entering and exiting; otherwise, they must never be allowed to use it.

In order to alleviate traffic congestion and deal with the problem of vehicle transfer after congestion, this paper studies the propagation of road congestion and proposes a method to mitigate the congestion by considering the node idle capacity. When road congestion occurs, vehicles on the congested road will be transferred first to the neighboring

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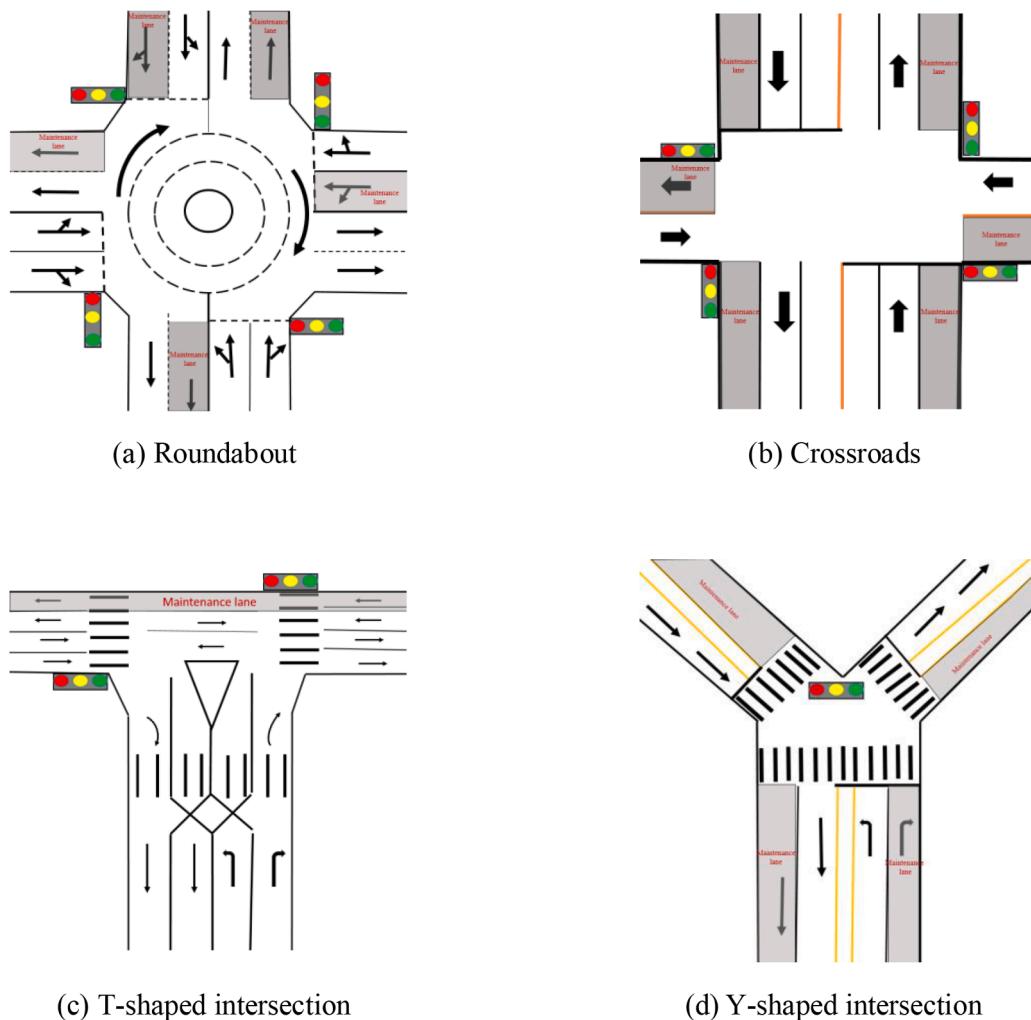


Fig. 1. Distribution diagram of maintenance lanes under various road types.

roads with larger idle capacity, alleviating the current road congestion. The contributions of this paper are summarized as follows.

used to verify the proposed method. Finally, the last section gives the conclusions and future work.

- 2 This paper investigates the capacity of idle nodes of a failed road network by constructing a failed network for failed nodes and repairing the failed network. The consideration of the capacity of idle nodes gives more options to the road network managers when failures are occurred in the road network.
 - 2 Several indicators for evaluating the node capacity of a transportation network and a traffic transfer principle are proposed, which helps calculate the failure scale caused by the initial failure node.
 - 2 We propose a maintenance model considering the failure paths and edges, which are calculated by using the internal connections between nodes and the priority index for maintenance. A relatively simple method is developed to solve the maintenance model. Different types of transportation networks are adopted and several groups of transportation networks are used as the input to verify the proposed maintenance model and experimental results show the efficiency of the proposed model.

The remainder of this paper is organized as follows. The related studies are introduced in Section 2. In Section 3, the traffic transfer principle based on node idle capacity is given. Section 4 discusses the failure process in the transportation network and establishes a road maintenance model. In Section 5, four different road networks are

1.1. Literature review

In this section, we divided the related studies into three categories, including cascading failures, maintenance, and the capacity of the failed road network, which are presented in the following subsections. In the end of this section, we identify research gaps.

1.2. Cascading failures of the failed road network

In terms of the traffic distribution after a cascading failure, the critical threshold of a network load distribution could be used to determine whether a cascading failure occurs, a cascading failure model considering load thresholds was established [3,5]. Shen et al. [4] considered that node failures can lead to the loss of flow to some extent. To obtain the optimal mutual flow redistribution rules that were beneficial to the robustness of the entire network, they proposed a cascading failure model of interdependent networks based on mutual traffic redistribution under fluctuant load. They studied the changes after cascading failures from different perspectives and analyzed the changing process of network cascading failures based on load dynamics node dependencies and node revenue [6,7,9]. Xie et al. [8] proposed a method for analyzing how the performance of systems influences the

Table 1
Notations used in this paper.

Set	Description
V	Set of nodes. $ V $ denotes the cardinality of node.
E	Set of edges. $ E $ denotes the cardinality of edge.
U	Set of upstream nodes. $ D $ denotes the cardinality of U .
V^*	The set of failed nodes in the failed network. $ V^* $ denotes the cardinality of V^* .
E^*	The set of failed edges in the failed network. $ E^* $ denotes the cardinality of E^* .
D_i	Set of downstream nodes of node i , $D_i = \{d_1, d_2, \dots, d_{ D }\}$
U_i	Set of upstream nodes of node i , $U_i = \{u_1, u_2, \dots, u_{ U }\}$
C_i	Node set connected to node i , $C_i = \{c_1, c_2, \dots, c_{ C }\}$
P_i	Set of paths traversed by node i . $ P $ denotes total number of paths traversed by node i
C_i	Set of nodes connected to node i . $ C $ denotes the cardinality of the node set connected to node i
Index	Description
$i, j, j' d, u$	Index of node
e	Index of edge
p	Index of path
Notation	Description
G	Initial transportation network
G^*	Failed network
α, β, γ	Constant value
p	Path taken by a node, $p = \{p_1, p_2, \dots, p_{ P }\}$
φ_i	The traffic imbalance coefficient of node i
ρ_{ic}	The traffic correction coefficient of each node connected to node i
h_i^p	Number of sequences of the node i on the path p
H_i^p	The total number of nodes traversed by node i on the path p
HVE_i	The hierarchy value of node i determined according to path p in the network
IT_i	Initial traffic volume of node i
NC_i	The capacity of node i
NIC_i	The idle capacity of node i
NC_j^i	The capacity of node j adjacent to node i
IT_j^i	The initial traffic of node j adjacent to node i
NIC_j^i	The idle capacity of node j adjacent to node i
IT_d^i	The initial traffic of the downstream node d of node i
IT_u^i	The initial traffic of upstream node u of node i
TTR_i	The traffic transfer rate of node i
R	Number of maintenance vehicles
T_i	Service time at node i
I	Node importance
Function	Description
$T(i, d)$	The transfer time between node i and upstream or downstream nodes.
$T(i, u)$	
$S(i, j)$	Transfer time saved from node i to node j
$T(O, i)$	Transfer time from maintenance center O to node i

protection against and mitigation of cascading failures, which considers system reliability and system durability in the mitigation of cascading failures.

1.3. Maintenance of the failed road network

Roadway pavement maintenance is essential for the safety of drivers and the reliability of highway infrastructure. In terms of road maintenance, many scholars have carried out research, such as, research on the maintenance of the damaged transportation network [10,11,20] and maintenance strategies considering the reliability of the transportation network [12,13,14]. In order to minimize the cost including inspection and maintenance in the total expected discounted cost within the network, research on regular maintenance of networks to improve maintenance efficiency [15,16,18], and on designing the optimal

maintenance strategy for the nodes and edges in the network [17,19] were conducted.

1.4. Reliability of the networks

In terms of network reliability research, work has been done on railway networks [21,24]. In order to improve network capacity and travel time reliability under normal and peak traffic conditions, the link capacity increase in dual-mode public transport networks can be determined [22]. The intrinsic association between nodes and edges in a network can provide key factors affecting network stability for improving network reliability [23,27]. Based on the road network capacity constrained by the road service level, Fang et al. [25] introduced a reliable bi-level programming research model, which can be used to evaluate and compare the performance of the road network under the service level requirements of different road segments. Cheng et al. [26] proposed a two-stage framework to estimate the overall reliability and failure modes of a disaster waste management system, taking into account the reliability of each route in the road network. The results obtained from the case study can be used for decision-making with information on the prioritization of routes in the system and the most likely failure modes.

1.5. Summary

With the above discussion, this paper aims to study a traditional cascade failure problem on transportation networks. It assumes that when large-scale traffic network congestion occurs, each road cannot operate as normal. Thus, it is necessary to slowly transfer the vehicles of each congested node to the adjacent node that is not congested. In the existing literature, few works of literature consider the free capacity of all adjacent nodes. This paper proposes a cascading failure model based on the idle capacity of adjacent nodes.

2. Traffic transfer principle based on node idle capacity

Before introducing the details of the traffic transfer principles, the set, index, notation and function used in this paper are listed in Table 1.

A transportation road network is usually composed of various types of intersections and roadways. The intersections and the roadways could be considered as nodes and edges, respectively. Then, we can construct an abstract graph G . A network can be represented as a directed network $G(V, E)$, where $V = \{v_1, \dots, v_{|V|}\}$ is the set of nodes and $E = \{e_1, \dots, e_{|E|}\}$ is the set of edges. An edge $e(i, j)$ is connected by nodes i and j . All edges are bidirectional and each edge has an inflow and outflow. In the transportation network, a plane intersection is referred to as a place where two or more roads intersect on the same plane. Fig. 2 gives an example.

2.1. Node idle capacity based on initial traffic

2.1.1. Initial traffic

As mentioned before, a transportation network can be represented by a graph composed of nodes and edges. During the traffic rush period, cars from different source nodes may simultaneously flow to the same sink nodes, such as office buildings or industrial zones. A finite sequence of edges that connects the source node and the sink node is referred to as a path. Usually, nodes with more cars passed by will have a higher possibility of incurring traffic jams. So, the set of upstream and downstream nodes of node i need to be included for calculating the initial traffic and is defined as follows.

$$HVE_i = \frac{\sum_{p=1}^{|P|} \frac{h_i^p}{H_i^p}}{|P|}, \quad i \in V \quad (1)$$

where HVE_i [30] represents the hierarchy value of node i . p

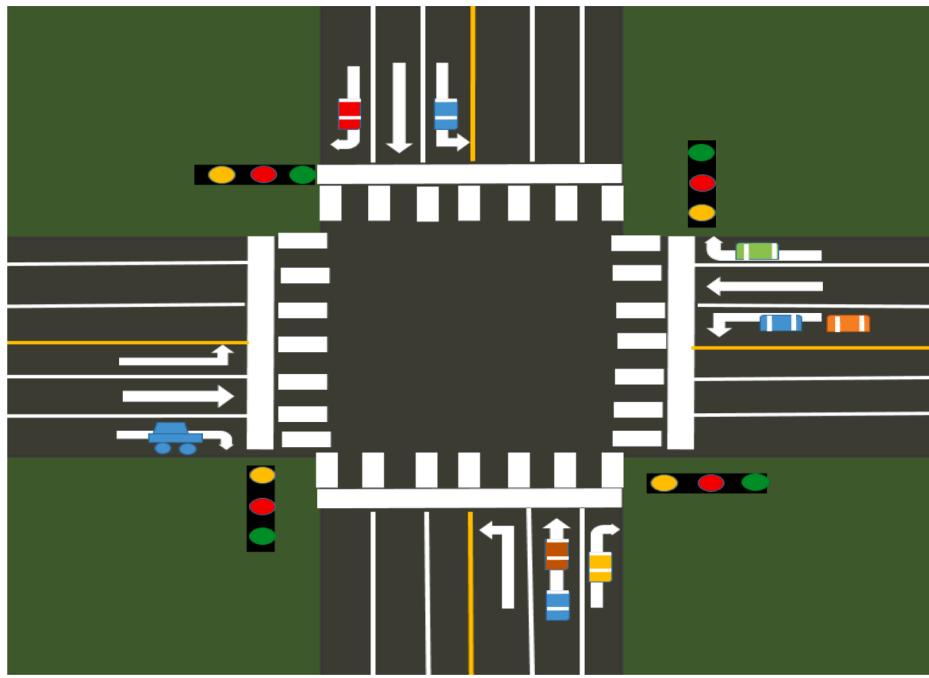


Fig. 2. An example of a crossroad.

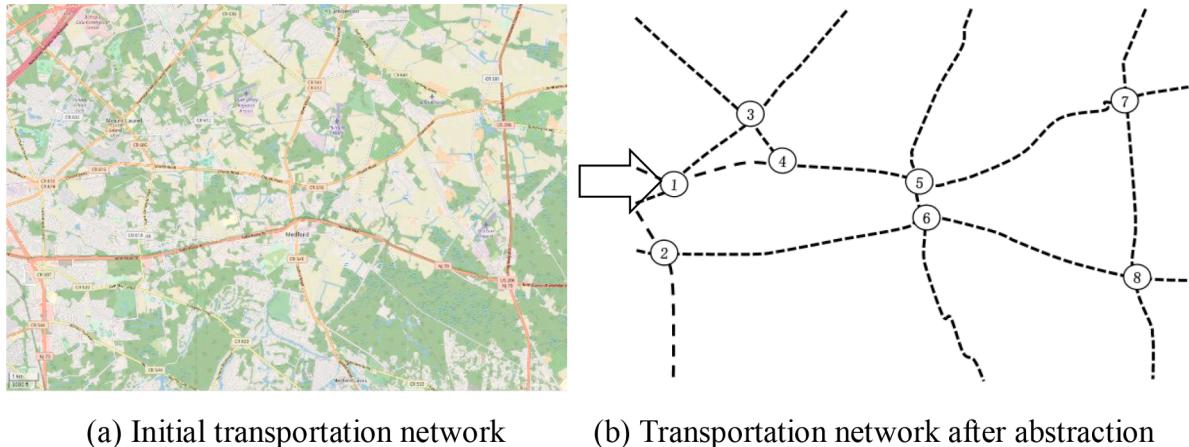


Fig. 3. An example of a transportation network.

Table 1a
The hierarchy value and initial traffic of each node.

Node	1	2	3	4	5	6	7	8
Hierarchy value	0.36	0.44	0.29	0.39	0.57	0.67	0.71	1
Initial traffic	36	44	29	39	57	67	71	100

represents the path taken by node i . h_i^p is the number of sequences of the node i on the path p . H_i^p is the total number of nodes traversed by node i on the path p . $|P|$ represents the total number of paths traversed by node i . Then the initial traffic value is defined as

$$IT_i = \gamma * HVE_i, i \in V \quad (2)$$

where γ denotes a zoom factor.

In this paper, the path refers to the route that can be traveled from a departure place to a destination. Usually, a node will not be traversed twice. Given a graph for any pair linking a source node and a sink node,

Table 2
Node importance.

Network	Node importance sequence
T_1	$I_{25} > I_{24} > I_{22} > I_{20} > I_{18} > I_{11} > I_{23} > I_{19} > I_{17} > I_{16} > I_7 > I_{21} > I_{14} > I_{12} > I_{10} > I_9 > I_8 > I_6 > I_5 > I_4 > I_2 > I_{15} > I_{13} > I_3 > I_1$
T_2	$I_{25} > I_{13} > I_{24} > I_{23} > I_{20} > I_{18} > I_{14} > I_{11} > I_{22} > I_{12} > I_8 > I_7 > I_4 > I_1 > I_{21} > I_9 > I_{17} > I_{16} > I_{15} > I_{10} > I_9 > I_5 > I_3 > I_6 > I_2$
T_3	$I_{25} > I_{24} > I_{23} > I_{15} > I_8 > I_1 > I_{22} > I_{21} > I_{16} > I_{12} > I_{11} > I_9 > I_{18} > I_{17} > I_{14} > I_7 > I_3 > I_2 > I_{20} > I_{19} > I_{10} > I_6 > I_5 > I_4$
T_4	$I_{24} > I_{25} > I_{12} > I_{23} > I_{18} > I_{13} > I_{22} > I_{19} > I_{16} > I_{15} > I_{11} > I_9 > I_5 > I_4 > I_{21} > I_{20} > I_{17} > I_{14} > I_8 > I_6 > I_3 > I_7 > I_2 > I_1 > I_{10}$

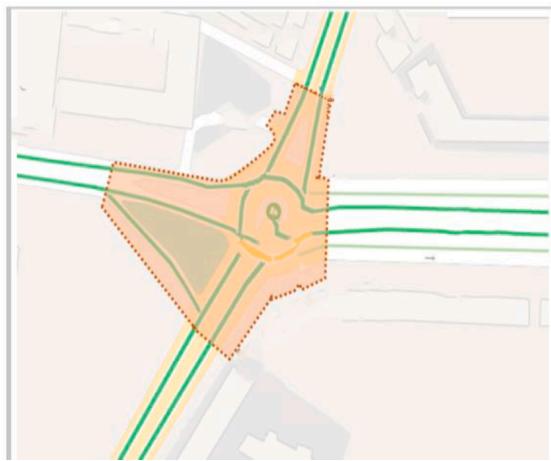


Fig. 4. Schematic diagram of node capacity range.

we can calculate its hierarchy value. Here, we provide an example to explain how to calculate the hierarchy value and the initial traffic is shown in Fig. 3. In Fig. 3, Figure (a) is an intercepted complex transportation network in reality, for the convenience of calculation, we abstract Fig. 3(a) into a simple topology, as shown in Fig. 3(b) and Table 1a

We use the above equations to calculate hierarchy values for the example shown in Fig. 3. The results show in Table 2 and the details of the calculation process is shown in Appendix A.

A single node is usually traversed by multiple paths and different paths are composed of different numbers of nodes. A node may located in different positions in terms of the sequences in each path for different paths. A higher hierarchy value of a node means that the node is closer to the downstream of the given network; otherwise, it is closer to the upstream. By analogy, the network can also be seen as a combination of

nodes at different levels. To calculate the location and traffic of a node in the network, it is necessary to average the levels of the same node in different paths, and the average value is equivalent to the level of the node in the network. To put the actual capacity and the hierarchy value at the same quantitative level, in this paper, for the convenience of calculation, we set the zoom factor $\gamma = 100$.

2.1.2. Node idle capacity

In this paper, node capacity denotes the maximal number of vehicles that can be parked within a certain area of the node satisfying the safe separation distance constraint. Fig. 4 shows an example of node capacity and the dotted polygon denotes the areas that vehicles could be occupied.

According to the “Technical Standard of Highway Engineering” of China [28], the capacity of expressways are: the annual average daily traffic of four lanes is 25,000–55,000 vehicles. The annual average daily traffic of six lanes is 45,000–80,000 vehicles. The annual average daily traffic of eight lanes is 60,000–100,000 vehicles. This paper involves the study of the traffic of each lane in various directions, and the traffic of each lane is unevenly distributed. In order to reduce the difference between different lanes in the same direction, according to the road capacity correction coefficient in the “Urban Road Design Code” [29], we define the traffic imbalance coefficient (φ_i). The node traffic imbalance coefficient reflects the degree of imbalance in a specific part of the transportation network. A smaller value of the φ_i means that the traffic difference between each lane connected to the node i is minor. In order to distinguish the traffic distribution of each lane in node i , we give a traffic distribution diagram of each lane which is shown in Fig. 5.

In Fig. 5, the red box in the middle represents node i , c_1, c_2, c_3 , and c_4 are the nodes connected to node i , which may be upstream nodes or downstream nodes. The edges connecting two nodes in the network are bidirectional, and the two directions include the inflow direction and the outflow direction, such as $e_{ic_1}, e_{ic_2}, e_{ic_3}$, and e_{ic_4} . Each direction consists of a straight lane, a left-turn lane, and a right-turn lane. The

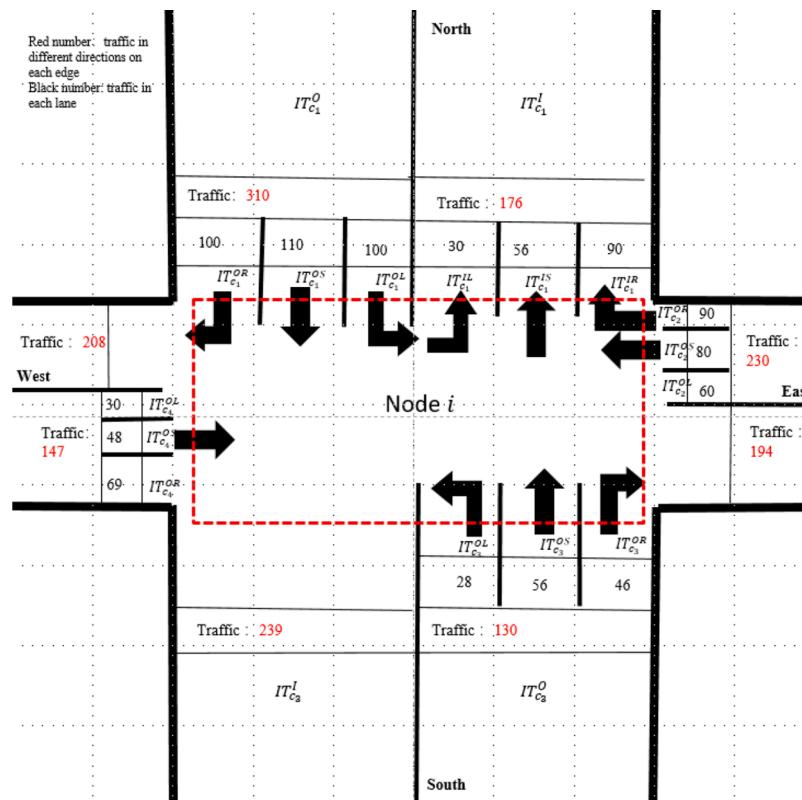


Fig. 5. Node traffic division diagram.

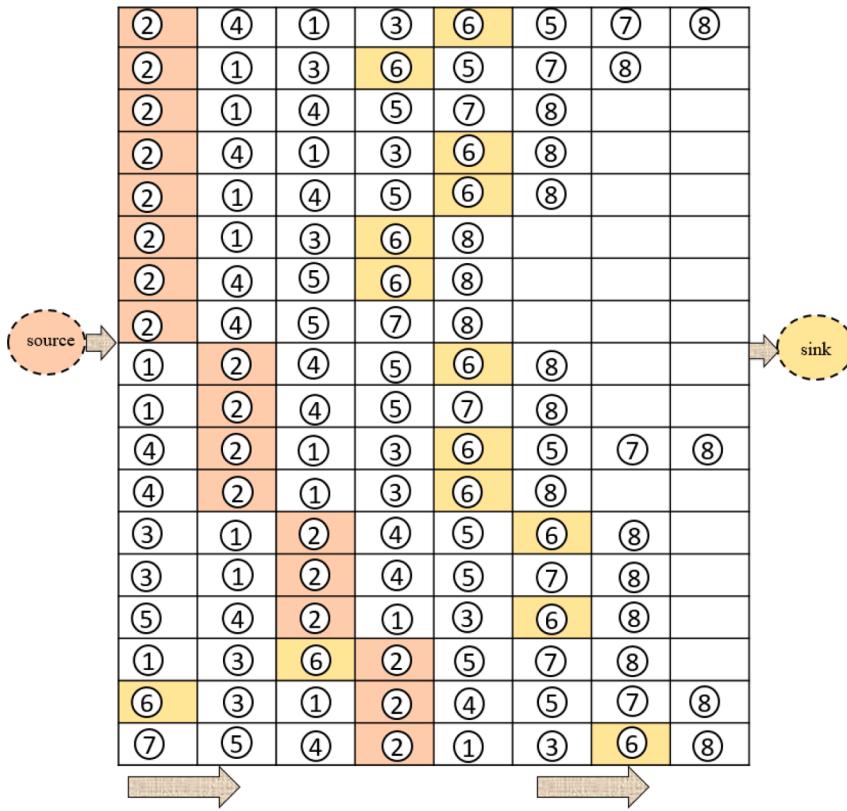


Fig. 6. Upstream and downstream node distinction.

edges connecting the nodes are composed of six lanes, and each lane has its own traffic. Let us use the north direction as an example, the traffic of the straight lane, the traffic of the left-turn lane, and the traffic of the right-turn lane in the inflow direction are respectively represented by $IT_{ic_1}^{IS} = IT_{ic_3}^{OS}$, $IT_{ic_1}^{IL} = IT_{ic_1}^{OL}$, and $IT_{ic_1}^{IR} = IT_{ic_2}^{IR}$. The traffic of the straight lane, the traffic of the left-turn lane, and the traffic of the right-turn lane in the outflow direction are respectively represented by $IT_{ic_1}^{OS}$, $IT_{ic_1}^{OL}$, and $IT_{ic_1}^{IR}$. The total traffic in the inflow direction is represented by $IT_{ic_1}^I$, and the total traffic in the outflow direction is represented by $IT_{ic_1}^O$. The negative value of the node's traffic means that the inflow is greater than the outflow.

The outflow of node i is the inflow of its adjacent nodes and C_i is a node set connected to node i . To avoid calculating traffic repeatedly, we simply calculate the traffic imbalance coefficient based on the traffic of each lane in the inflow direction and the initial traffic. The traffic imbalance coefficient is the ratio of the maximum traffic in the inflow lane of node i to the initial traffic of node i .

$$\left\{ \begin{array}{l} IT_{ic}^{IS} = \frac{IT_i}{\rho_{ic}} \\ IT_{ic}^{IL} = IT_{ic}^{IR} \\ \varphi_i = \frac{\max\{IT_{ic}^{IS}, IT_{ic}^{IR}, IT_{ic}^{IL}\}}{IT_i} \end{array} , c \in C_i \right. \quad (3)$$

In order to reduce the deviation of the traffic of different lanes, we define the traffic correction coefficient (ρ). ρ_{ic} represents the traffic correction coefficient of each node connected to node i . In order to facilitate the calculation, we define $IT_{ic}^{IL} = IT_{ic}^{IR}$. Since the traffic proportion on each lane is different and the distribution is uneven, this paper defines the traffic imbalance coefficient. The maximum value is more representative, so the ratio of the lane with the largest traffic in the lane to the total traffic in this direction is selected as the traffic imbalance coefficient in this direction.

From the above contents, we know that in the process of constructing a transportation network. Node capacity affects the smooth operation of roads. A node with a larger capacity means more vehicles could be passed simultaneously. Therefore, the capacity of a node has the following relationship with the initial traffic of the node:

$$NC_i = \frac{IT_i}{\varphi_i}. \quad (4)$$

The calculation formula of node idle capacity is $NC_i - IT_i = NIC_i$.

2.2. Traffic transfer principle

When a node in the transportation network is congested in the real world, the vehicles that originally intended to pass the failed node will re-plan its route to avoid the area where the failed node is located. In the case of insufficient planning route time, vehicles that have not obtained the node failure in advance can only choose to wait in the lines or transfer to other nodes adjacent to the failed node. The initial traffic and capacity of these adjacent nodes are different. When the vehicle chooses to transfer to other nodes adjacent to the failed node, the traffic that the failed node should have born will be transferred to the adjacent node. The traffic of the adjacent node will also change.

The traffic transfer rate is related to the initial traffic of the upstream node set and the downstream node set of a failed node. In order to better distinguish the flow of traffic after a node failure, we define upstream nodes and downstream nodes. In order to intuitively describe the difference between upstream nodes and downstream nodes, we give an example, as shown in Fig. 6. We introduce two virtual nodes: a source node and a sink node. A vehicle starts from a source node in the transportation network, traverses different nodes in the middle, and ends at a sink node. Under this condition, the hierarchy value of the source node is zero, and the hierarchy value of the sink node is one. Taking node 2 in Fig. 3 as an example, there are 18 paths, including node 2. The arrow in Fig. 6 refers to the direction of each path. We mark node 2 and node 6 in

different colors in the path. According to the hierarchy value of node 2 and node 6, and the position in Fig. 6, we find that node 2 is closer to the source node, and node 6 is closer to the sink node. Among multiple nodes, in order to distinguish between upstream nodes and downstream nodes, we regard the node closer to a source node as the upstream node, and the node closer to a sink node as the downstream node. Therefore, we believe that node 2 is the upstream node and node 6 is the downstream node. It can be seen from Table 2 that the nodes with the hierarchy value closer to 1 are easier to store higher traffic.

The total number of edges connecting all upstream nodes of node i is called the node's in-degree value (k_i^{in}). Moreover, the total number of edges connecting all downstream nodes of node i is called the node's out-degree value (k_i^{out}). In this paper, we adopted $k_i = \frac{k_i^{out}}{k_i^{in}}$, where k_i represents the node's ability for accepting external traffics and maintaining its own stability (Zhang et al. [30]).

$$k_i = \begin{cases} \frac{k_i^{out}}{k_i^{in}} & \text{if } k_i^{in} \neq 0 \\ \max\{k_1^{in}, \dots, k_{|V|}^{in}\} & \text{if } k_i^{in} = 0 \\ \max\{k_1^{out}, \dots, k_{|V|}^{out}\} & \text{if } k_i^{out} = 0 \end{cases} \quad (5)$$

k_i represents an index that measures the possibility of congestion at a node. When the out-of-degree value is greater, the degree value is also greater. Inspired by the path resistance function, the average value between two nodes is taken as the transfer time on the edge between the two nodes.

The traffic transfer rate is also affected by the transfer time between the failed node and its upstream or downstream nodes. In real life, the transfer time of a node in transportation network is affected by many factors, such as the difference in the number of lanes, waiting time for traffic lights, weather conditions, and other factors. The transfer time defined in this paper includes the transfer time between two nodes at the intersection. To make a reasonable plan for node traffic, the US Highway Administration proposed the Bureau of Public Roads Function [31], which shows the functional relationship between travel time, capacity, and traffic. Inspired by this function, we define transfer time as follows.

$$T(i,j) = \frac{k_i \left[1 + \alpha \left(\frac{IT_i}{NC_i} \right)^\beta \right] + k_j \left[1 + \alpha \left(\frac{IT_j}{NC_j} \right)^\beta \right]}{2} \quad (6)$$

$T(i,j)$ represents the transfer time between node i and node j , $j \in D_i$ or U_i . α and β are two input parameters.

According to formula (3), the above formula can be simplified as.

$$T(i,j) = \frac{k_i \left[1 + \alpha (\varphi_i)^\beta \right] + k_j \left[1 + \alpha (\varphi_j)^\beta \right]}{2}. \quad (7)$$

In a certain period, the more vehicles head in a certain direction, the more likely the nodes in that direction will be congested. In order to find the initial failure node of the next round of failure from the adjacent nodes, we define the traffic transfer rate as follow.

$$TRR_i = \frac{|\sum_{d \in D_i} IT_d^i - \sum_{u \in U_i} IT_u^i|}{\sum_{d \in D_i} T(i,d) + \sum_{u \in U_i} T(i,u)} \quad (8)$$

In formula (8), IT_d^i is the initial traffic of the downstream node d of node i , and IT_u^i is the initial traffic of upstream node u of node i . When a node is congested, vehicles are more inclined to travel to a node with a higher traffic transfer rate. Therefore, prioritizing the allocation of vehicles to places with a higher traffic transfer rate can eliminate congestion faster when allocating traffic in this paper.

According to formula (8), we can calculate the traffic transfer rate of all nodes. According to the traffic transfer rate of each node, we propose the following traffic transfer principles:

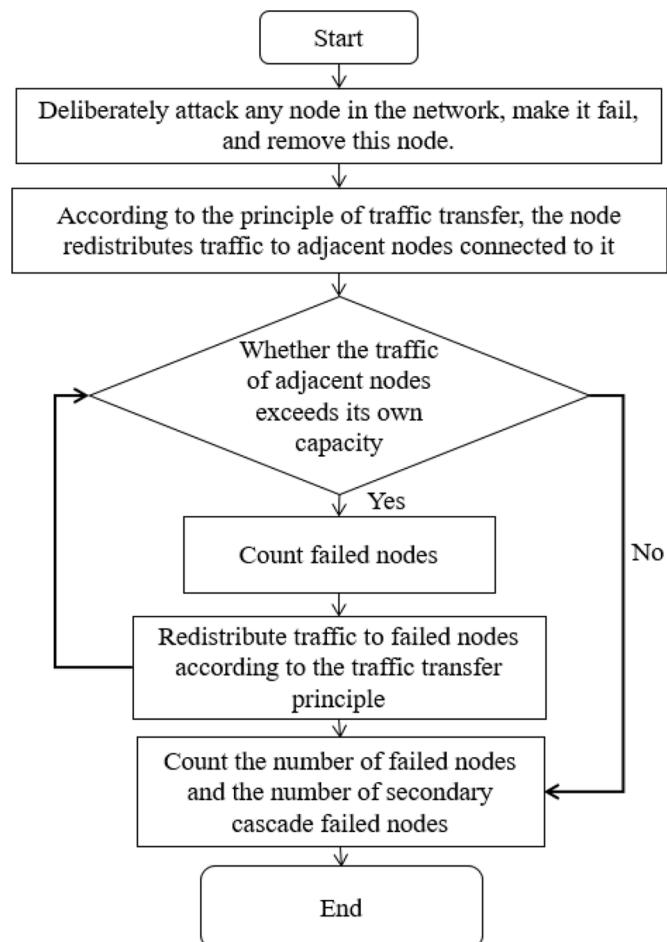


Fig. 7. Traffic transfer flowchart.

Step 1: In a given transportation network, the function of node i is impaired. And some vehicles fail to obtain road damage information in time, resulting in more and more traffic at this node, which reaches or exceeds the capacity of node i , then node i fails.

Step 2: The set of adjacent nodes of node i is represented by C_i , $C_i = U_i \cup D_i$. $\sum_{j \in C_i} NIC_j^i = \sum_{j \in C_i} (NC_j^i - IT_j^i)$ is the summation of the idle capacity of nodes adjacent to the failed node i , namely, the maximum range of traffic that can be transferred. If $\sum_{j \in C_i} NIC_j^i = \sum_{j \in C_i} (NC_j^i - IT_j^i)$ can accommodate NC_i , all nodes are operating normally except for node i .

Otherwise, node i and all adjacent nodes of node i are failed. Thus, we need to go to Step 3, in which NC_i is the traffic to be transferred.

Step 3: This step calculates the traffic to be transferred $\sum_{j \in C_i} (NC_j^i - IT_j^i) - NC_i$ by using the node j with the largest TRR in C_i , $j \in C_i$ and all the adjacent nodes C_j of node j .

Step 4: When looking for adjacent nodes of node j , it is necessary to remove the previously failed node. If $\left\{ \sum_{j \in C_j} (NC_j^i - IT_j^i) - \left[\sum_{j \in C_i} (NC_j^i - IT_j^i) - NC_i \right] \right\} < 0$, all adjacent nodes of node j are failed. Thus, go to Step 3, otherwise, go to next step.

Step 5: When the traffic to be transferred is 0, the failure is terminated. The number of failed nodes is counted.

In order to more intuitively see how the traffic transfers after the node fails, we give the flowchart shown in Fig. 7.

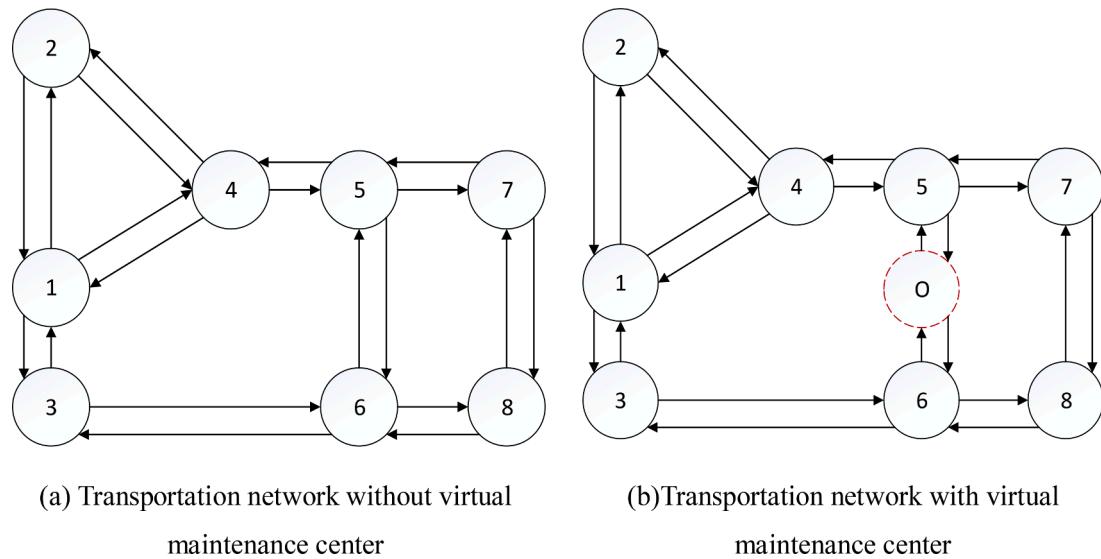


Fig. 8. Simplified diagram of urban transportation network.

3. Maintenance analysis of transportation network

3.1. Failure process in transportation network

According to the above traffic transfer principles, we can get the number of failed nodes and the geographic location of each failed node. We remove all the failed nodes in the original transportation network. Finally, we construct a failed network composed of these failed nodes. The steps are as follows.

Step 1: According to the complex network theory, we abstract a complex network into a transportation network, which is regarded as the initial transportation network, denoted by $G(V, E)$.

Step 2: The capacity of each node is determined by the initial traffic of each node. And we calculate the traffic transfer rate of each node, and process it in descending order.

Step 3: Suppose that a certain node in the network causes congestion, and its traffic is greater than or equal to its capacity. We can find the set of upstream and downstream nodes of the failed node. Then we need to filter out the node with a higher traffic transfer rate from the set. The detailed traffic transfer steps have been given in the above traffic transfer principles.

Step 4: When selecting a node with a higher traffic transfer rate in the upstream or downstream node sets, we remove the previously used nodes and continue step 3. Until there are no existing new failed nodes, then the cascading failure process is terminated.

Step 5: Sort out all failed nodes and set up a failed network composed of all failed nodes, denoted by G^* . The set of failed nodes is represented by V^* , and the set of failed connected edges is represented by E^* .

Fig. 3 shows an example of a real-life transportation road network, and Fig. 3 is abstracted into a transportation network as shown in Fig. 8(a). In Fig. 8(b), we give a virtual maintenance center O .

We can find the failure nodes based on the calculated initial traffic, capacity, and traffic transfer rate of each node. Let us assume that the traffic of node 3 exceeds its capacity, which leads to its failure. When node 3 fails, the nodes connected to it are node 1 and node 6. The idle capacity of these two nodes is equal to 21. The traffic to be transferred at node 3 is 53. Obviously, the idle capacity of adjacent nodes cannot accommodate the to-be-transferred traffic of node 3. So node 1 and node 6 are failing. Then we need to update the to-be-transferred traffic: $53 - 21 = 32$. Next, we compare TTR_1 and TTR_6 with finding that TTR_6

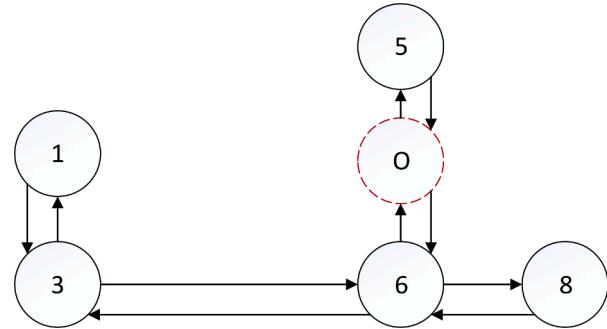


Fig. 9. Failure network.

$> TTR_1$. Therefore, we take node 6 as the initial failure node for the next round failure, find all adjacent nodes of node 6—node 3, node 5, and node 8. Since node 3 has failed, only node 5 and node 8 are considered. The idle capacity of node 5 and node 8 is 31, the traffic to be transferred is 32, update the to-be-transferred traffic: $32 - 32 - 31 = 1$. The idle capacity of node 5 and node 8 cannot fully bear the to-be-transferred traffic, so node 5 and node 8 fail. Then we continue to compare TTR_5 and TTR_8 . We find $TTR_8 > TTR_5$, and then take node 8 as the initial node to find the adjacent nodes of node 8—node 6, and node 7. Because node 6 has failed and only node 7 is considered, the idle capacity of node 7 is 14 and the to-be-transferred traffic is 1. So, node 7 can handle the to-be-transferred traffic and the failure is terminated. The failed nodes are $\{1, 3, 5, 6, 8\}$. The failure network diagram is identified, which is shown in Fig. 9.

3.2. Maintenance modeling for the failure network

In this study, the input of a maintenance network is the above failure network. Let directed graph $G^* = (V^*, E^*)$, which denotes a failure network, where V^* represents the set of failed nodes, and the number of failed nodes is $|V^*|$. O represents the maintenance center. All maintenance vehicles depart from the maintenance center and return to the maintenance center after finishing the maintenance task. From the failure network, the geographic location of each failed node is known. E^* represents the set of failed edges, $E^* = \{e(i,j) : i, j \in V^*, i \neq j\}$. The maintenance center operates multiple maintenance homogenous vehicles. Each failed node only accepts one service for one maintenance vehicle, which can be considered as a vehicle routing problem [32].

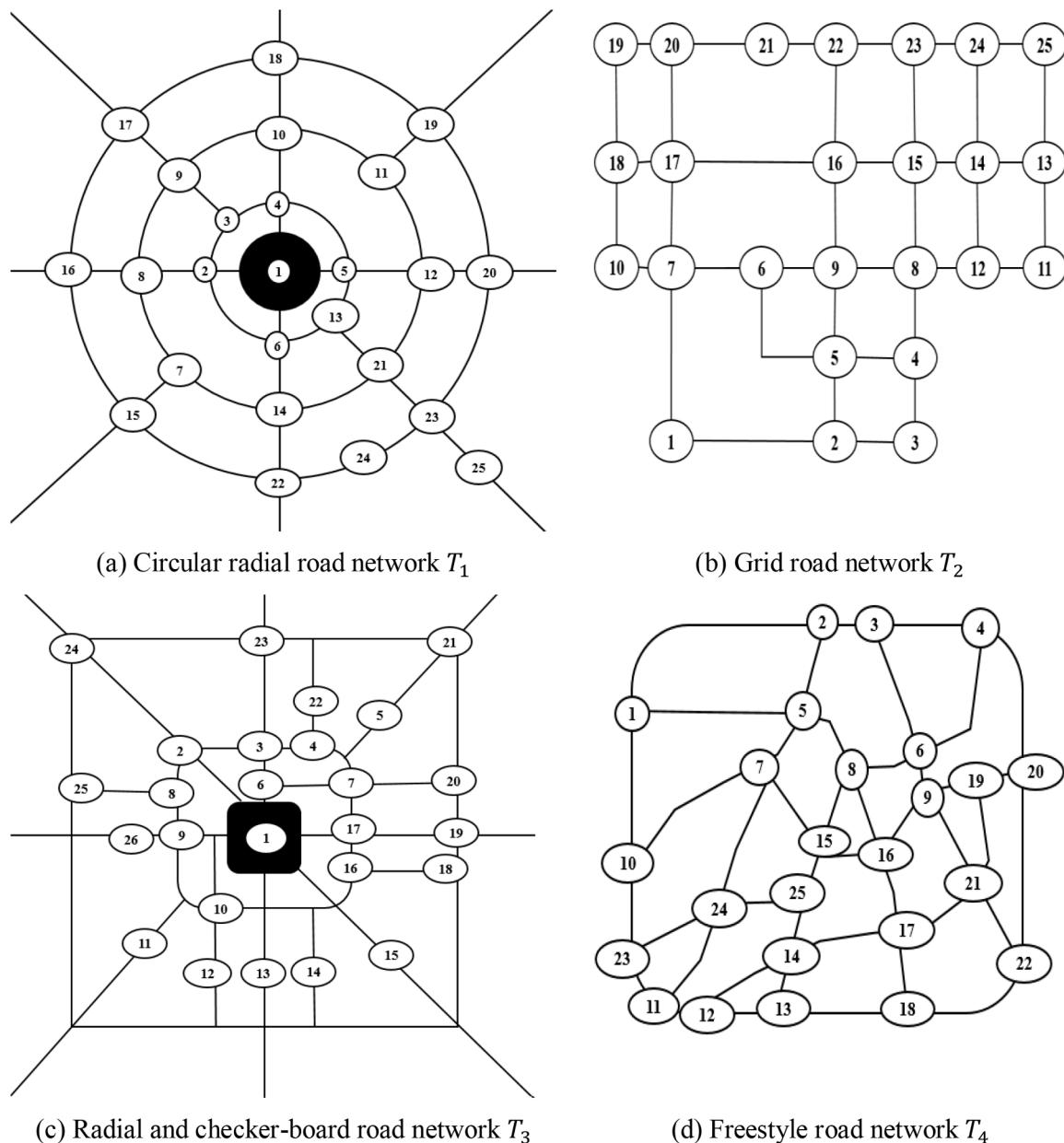


Fig. 10. Different road network types.

Based on the above description, the following assumptions are proposed.

- We represent a damaged road link by a node located in the middle of the corresponding edge. Therefore, repairing a road connection is equivalent to repairing a node.
- This paper only considers the vehicles scheduling problem of a single maintenance center, so it is assumed that there is only one maintenance center in the transportation network, expressed as O .
- Road maintenance will temporarily affect the normal operation of vehicles. All maintenance vehicles depart from the maintenance center and finally return to the maintenance center.
- All repaired vehicles are homogenous.
- When a maintenance vehicle leaves the node, it indicates that the node has been repaired.

This paper aims to find the shortest maintenance time, which includes the transfer time between two nodes and the service time of the failed nodes. As before mentioned, the transfer time between two nodes

is shown in formulas (6) and (7). Service time is the length of time that the maintenance vehicle provides maintenance service at the node, which is an input value. Based on the above assumptions, we build a maintenance model to investigate maintenance path planning to recover the road functions as soon as possible. In this paper, we use time as the weight of each node or edge in the failure network to find the maintenance path based on the shortest completion time.

Decision variables:

$$x_{ij}^r = \begin{cases} 1, & \text{If the maintenance vehicle } r \text{ goes from node } i \text{ to node } j \\ 0, & \text{otherwise} \end{cases}$$

Objective function:

$$\min \sum_{i=1}^{|V^*|} \sum_{j=1}^{|V^*|} \sum_{r=1}^R x_{ij}^r * T(i,j) + \sum_{i=1}^{|V^*|} \sum_{j=1}^{|V^*|} \sum_{r=1}^R x_{ij}^r * T(O,i) + \sum_{i=1}^{|V^*|} T_{i,j} \in D_i \text{ or } U_i. \quad (9)$$

Subject to

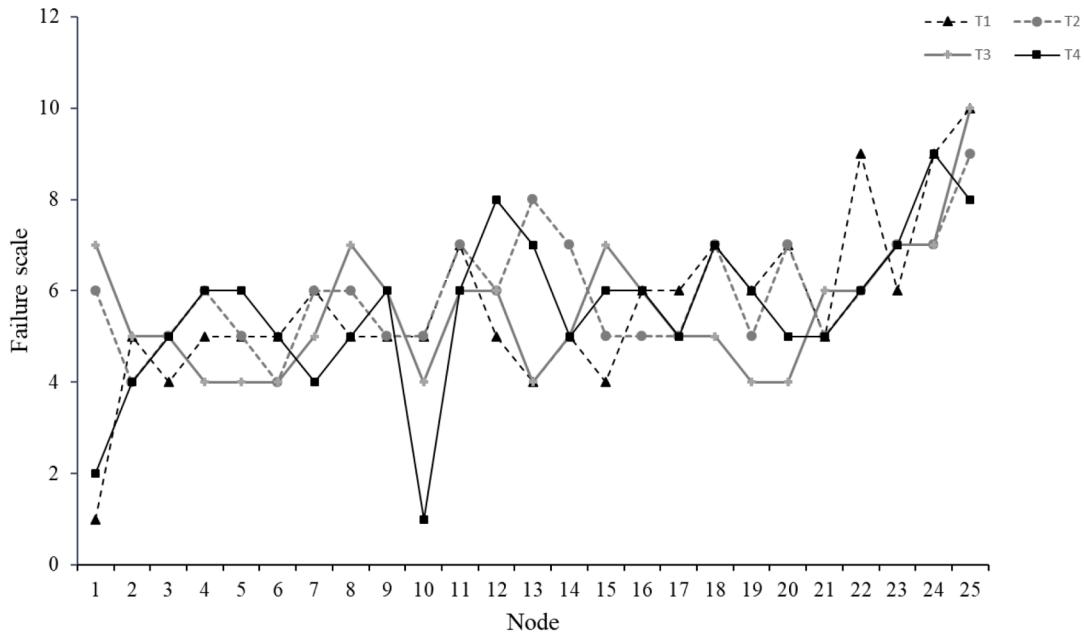


Fig. 11. Failure scale.

$$\sum_{j=1}^{|V^*|} \sum_{r=1}^R x_{0j}^r \leq R, \quad i \neq j. \quad (10)$$

$$\sum_{i=1}^{|V^*|} \sum_{r=1}^R x_{i0}^r \leq R, \quad i \neq j. \quad (11)$$

$$\sum_{i=1}^{|V^*|} \sum_{r=1}^R x_{ij}^r = 1, \quad j \in V^* \setminus \{0\}, \quad i \neq j. \quad (12)$$

$$\sum_{j=1}^{|V^*|} \sum_{r=1}^R x_{ij}^r = 1, \quad i \in V^* \setminus \{0\}, \quad i \neq j. \quad (13)$$

$$\sum_{i=0}^{|V^*|} x_{ij}^r = \sum_{i=0}^{|V^*|} x_{ji}^r, \quad j \in V^*, \quad r = 1, 2, 3 \dots R, \quad i \neq j. \quad (14)$$

$$T_i^r + T(i,j) + T_j^r - T_i^r \leq (1 - x_{ij}^r)M, \quad i \in V^*, \quad r = 1, 2, 3 \dots R, \quad i \neq j. \quad (15)$$

$$\sum_{i \in S} \sum_{j \in S} x_{ij}^r \leq |S| - 1,$$

$$2 \leq |\Omega| \leq |V^*| - 1, \quad \Omega \in V^*, \quad i, j \in V^*, \quad i \neq j, \quad r = 1, 2, 3 \dots R. \quad (16)$$

$$X_{ij}^r \in \{0, 1\}, \quad i, j \in V^*, \quad r = 1, 2, 3 \dots R, \quad i \neq j. \quad (17)$$

The objective function (9) represents the completion time. Constraint (10) ensures that at most R maintenance vehicle(s) depart from the maintenance center (node O). Constraint (11) ensures that at most R maintenance vehicle(s) return to the maintenance center (node O). Constraint (12) and (13) are the flow conservation constraints. Constraint (14) states that if vehicle r visits failed node j , it must also depart from failed node j . Constraint (15) assures that if vehicle r visits node j after node i , the service start time for node j cannot begin earlier than the service start time for node i , plus the maintenance time at node i and the transfer time from node i to node j . T_i^r is the time when vehicle r arrives at node i . T_j^r is the time when vehicle r arrives at node j . M is a very large value. Constraint (16) is the sub-tour elimination constraint. $|\Omega|$ is the set composed of all the subsets of the failed node set, eliminating the solution that satisfies other constraints but does not constitute

a complete path. Constraint (17) states that X_{ij}^r is a binary variable, 1 indicating that vehicle r travels node i to node j , and 0 indicating that no travel is incurred.

When solving the model, we take the transfer time as the weight of each edge, and the service time at a node as the weight of each node to find the shortest maintenance time and the shortest repair path. The above problem is solved by the Dijkstra algorithm. The Dijkstra algorithm is a widely used method for solving the shortest path, which can calculate the shortest path from one node to all other nodes. Specific steps are as follows.

Step 1: Divide all the nodes in the graph into two sets of S and U : “the visited nodes set” are put in S ; “the not-yet-visited nodes set” are put in U with the original condition of all distribution sites set.

Step 2: Change the starting point O (the maintenance center) as a permanent label and move from U to S . Set the starting point’s $P(O) = \text{NULL}$. The maintenance time of starting point $T(O) = 0$, the transfer time of starting point $w(O, \sim) = 0$, setting $i = O$; The maintenance time and transfer time of all other nodes j : $T(j) = \infty$, $w(i, j) = \infty$, $w(i, j)$ refers to the transfer time between node i and node j . $w(i, j)$ is the weight matrix. Thereinto, $U(i)$ is the upstream node of node i and $\Gamma(i)$ is the collection of all i .

Step 3: Update all the nodes that is labeled as temporary in $\Gamma(i)$ the total time of node O to node j is $t(O, j) = T(j) + w(O, j)$. If $t(O, i) < t(O, j)$, then $P(j) = i$.

Step 4: Choose the path with the smallest $t(O, j)$ from U .

Step 5: set node j as the permanent label, move it from set U to set S and let $i = j$.

Step 6: If $i = D$, then it is the shortest time from the starting point O to maintenance node D , $t(O, D)$ is the minimum total time; if $i \neq D$, return to Step 2 to continue the calculation.

The advantage of Dijkstra’s Algorithm is that it does not need to go through all nodes to find the shortest route. If the shortest route has found out the target distribution site, the distribution routes to the distribution site will necessarily spend more time than this route and the sub-path of this shortest route will necessarily become the shortest route.

Table 3

Path taken by each node.

(continued on next page)

4. Numerical examples

To verify the proposed model, four different types of transportation networks are adopted, which shows in Fig. 10. For each type of road network, we set the nodes equal to 25. The topological adjacency matrix of these four networks is used to calculate the capacity and initial traffic of nodes, traffic transfer rate. The parameter setting is that $\rho_{ic} = 1.2, \alpha = 3, \beta = 2$.

We can get the failure scale caused by each node according to the traffic transfer principles. The failure scale caused by four different networks is shown in Fig. 11.

We find that the fluctuation range of the failure scale caused by the four networks is small. The deviation is relatively small and hence the proposed model has universality. From the above figure, we can find that large-scale cascading failures generally occur in the middle and downstream location of the transportation network. Such as, the transportation network T_2 , we found that the failure scale caused by node 1 to node 12 as the initial failure node has always been in a trend of decreasing first and then increasing. The scale of the cascading failure caused is (1, 7), and the failure scale reaches a peak at node 13. The scale of failure caused by node 13 is 8. In the middle and downstream stages, that is, from node 14 to node 21, the scale of failure continues to be in a trend of decline first and then increase. In the downstream stage, it has been in an increasing trend, reaching a peak at node 25. The scale of failure caused by node 25 is 9. The initial failure nodes at different locations have different impacts on the network. Compared with the upstream node as the initial failure node, the downstream node as the initial failure node will bring greater changes, because the cascading failure dominated by downstream nodes has a greater impact on the network topology. From the perspective of the topology theory, the initial failure node located downstream of the network will cause a large range of fluctuations in the remaining members ability in transferring traffic. Finally, from the above analysis, we conclude that the specific degree of influence is: downstream failed node > middle-downstream node > middle-upstream node > upstream node.

According to the failure scale caused by each node as the initial failure node, we can sort the importance of the nodes. The nodes located in the middle and downstream nodes have more complicated in-degree and out-degree distribution and their traffic transfer rate is more susceptible to the influence of the surrounding area. The scale of failures caused by the middle and downstream nodes is relatively larger. The upstream node generally has a simpler topology, and the scale of failure caused is relatively small. The larger the failure scale, the more important the position in the network. Based on the scale of failure caused by each initial failure node, the node importance in various situations is ranked, which are summarized in Table 3.

To illustrate the versatility of the content in the paper, four common types of transportation networks are selected to verify the proposed method. The order of the nodes of the four types of transportation networks is randomly allocated, and the size of the failure has no relationship with the order of node allocation. The failure scale is comprehensively considered based on the node's traffic, capacity, and idle capacity of adjacent nodes, traffic transfer rate, and transfer time. Fig. 11 shows the failure scale obtained through comprehensive considerations using the above indicators for each transportation network type. The number of failed nodes caused by each node as the initial failure node. From Fig. 11, the results of the four types of transportation network have very little difference. Hence, we conclude that the proposed traffic transfer principle based on idle capacity is universal.

In the following, we analyze the maintenance path. First, we find the shortest path from maintenance center O to each node. Then, $S(i,j)$ is calculated. Finally, the maintenance path based on the shortest path is found. Let us use T_2 as an example. The numbers inside each circle in Fig. 12 represent the transfer time between any two nodes, and O is a virtual maintenance center. The maintenance center is located between node 16 and node 17, and can directly reach to node 16 and node 17.

Table 3 (continued)

1→2→6→5→7→8;1→3→4→5→7→8;1→4→5→7→8;
2→1→3→4→5→7→8;2→1→4→5→7→8;
2→6→5→7→8;3→1→2→6→5→7→8;3→1→4→5→
7→8;3→4→1→2→6→5→
7→8;3→4→5→7→8;4→1→2→6→5→7→8;4→3→1→2→6→5→7→8;4→5→7→
8;5→7→8;6→2→1→3→4→5→7→8;6→2→1→4→5→
7→8;6→5→7→8;7→8;7→5→4→1→2→6→8;7→5→
4→3→1→2→6→8;7→5→6→8;

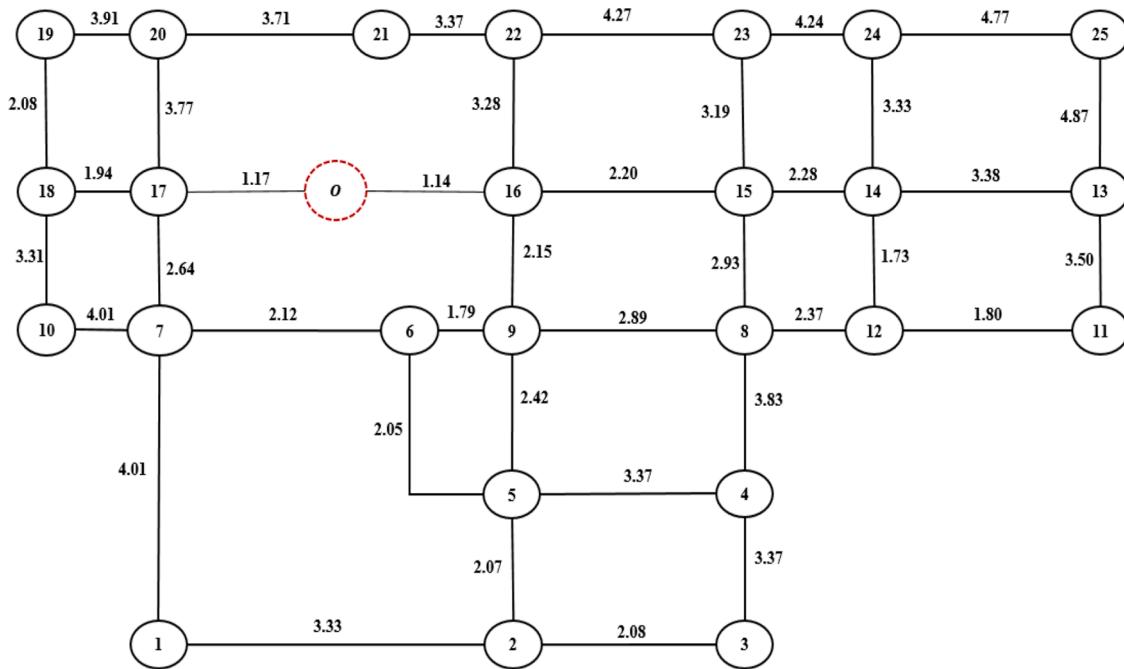


Fig. 12. Maintenance analysis of grid road network T_2 .

According to formula (6), the transfer time of the two directly connected nodes can be obtained. The transfer time of $i \rightarrow j$ and $j \rightarrow i$ are equal. Each node's service time is assigned according to the importance of each node, and the service time of the node with a larger scale of failure will be longer. The shortest transfer time and the service time of each node are shown in Table 4.

According to the transfer time and the service time of each node, which are shown in Table 4. We can get the shortest maintenance path. Such as, the node 20 in Fig. 12, when node 20 is the initial failure node, the nodes that need to be repaired are nodes 20, 17, 19, 21, 7, 16, and 18. And those to be repaired nodes are shown in Fig. 13. The numbers in red denote the service time of each node, and the numbers in black represent the transfer time among nodes. There are four maintenance paths: $O - 20 - 21 - O$, $O - 7 - 17 - O$, $O - 18 - 19 - O$, and $O - 16 - O$. The total maintenance time of the four paths is 23.88 h. If we use the proposed method repair the road network, we get the following three maintenance paths: $O - 16 - 21 - O$, $O - 17 - 7 - O$, $O - 18 - 19 - 20 - O$. The total maintenance time of the three paths is 18.78 h. By comparing the maintenance paths and maintenance time before and after using the proposed method, we find that the proposed method can effectively shorten the total time, which proves the effectiveness of the proposed method.

In addition, let the node 20 in the T1 network be the initial failure node, and the failure scale be caused by node 20 is node 20, node 12, node 19, node 23, node 11, node 18, and node 10. There are two original maintenance routes (route 1: 10–11–12; route 2: 18–19–20–23). We found that, it takes 51 h to obtain the two maintenance routes. After adopting Dijkstra's algorithm, we get two maintenance routes (route 1: 10–18–19–20–23; route 2: 11–12), the computational time is 40 h. By adopting the Dijkstra's algorithm, the computational time is reduced by 21%.

We take another group of experiments. Let node 20 in the T3 network be the initial failure node. The failure scale is caused by node 20 is node 20, node 7, node 19, node 21. Since the repair of some nodes can only be reached through other non-failed nodes, the maintenance route will

include non-failed nodes. In the T3 network, there is one maintenance route: 19–20–21–22–23–24–7. The total time of this original maintenance route is 52 h. After adopting Dijkstra's algorithm, the computational time is 39 h. Based on the shortest time we get two maintenance routes (route 1: 10–18–19–20–23; route 2: 11–12). By adopting the Dijkstra's algorithm, the computational time is reduced by 25%.

Take node 20 in the T4 network as the initial failure node for research, and the failure scale is caused by node 20 is node 20, node 4, node 19, node 21, and node 22. There are two original maintenance routes (route 1: 4–20–19; route 2: 4–3–21–22). We found that, it takes 46 h to obtain the two maintenance routes. After adopting Dijkstra's algorithm, we get one maintenance route: 4–20–19–21–22, the computational time is 37 h. By adopting the Dijkstra's algorithm, the computational time is reduced by 19%.

5. Conclusions and future work

Transportation network maintenance is an important issue for ensuring the operation of transportation networks. This paper proposed several traffic transfer principles to construct a novel model for minimizing the maintenance time of transportation networks, considering node idle capacity. To verify the proposed model, four different types of road networks are adopted in the experiment. From the result analysis, we found that the location close to the middle and downstream nodes has a greater impact on the entire network in transportation networks. Large-scale cascading failures usually occur in the middle and downstream of the transportation network. The nodes located downstream of the transportation network can accommodate more traffic. Compared with the upstream node as the initial failure node, the downstream node as the initial failure node will bring more significant changes, which can explain the level dominated by the downstream node to a certain extent. Connection failures have a greater impact on the network topology. Besides that, we also found that the larger the size of the node that caused the failure, the more important its position in the network. In this paper, the failure scale is used to measure the importance of each node

Table 4
The shortest transfer time and service time of each node.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0	0	7.82	7.78	9.86	9.08	5.71	5.08	3.81	6.18	3.29	6.42	9.15	7.35	9	5.62	3.34	1.14	1.17	3.11	5.19	4.94	7.79	4.42	6.53	8.95	13.72
1	0	0	3.33	5.41	8.77	5.4	6.13	4.01	10.71	7.82	8.02	14.88	13.08	16.82	11.16	8.96	6.65	8.59	10.67	10.42	14.13	12.24	14.35	16.77	21.54	
2		0	2.08	5.44	2.07	4.12	6.24	7.38	4.49	10.25	11.55	9.75	14.5	11.12	8.84	6.64	8.88	10.82	12.9	12.65	13.29	9.92	12.03	14.45	19.22	
3		0	0	3.37	4.15	6.2	8.32	7.2	6.57	12.33	11.37	9.57	14.68	11.3	10.13	8.72	10.96	12.9	14.98	14.73	15.37	12	13.32	14.63	19.4	
4		0	0	3.37	5.42	7.54	5.79	11.55	8	1.77	6.2	11.31	7.93	6.76	7.94	10.18	12.12	14.2	13.95	14.59	11.22	9.95	11.26	16.03		
5		0	0	2.05	4.17	5.31	2.42	8.18	9.48	7.68	12.43	9.05	6.77	4.57	6.81	8.75	10.83	10.58	11.22	7.85	9.96	12.38	17.15			
6		0	0	2.12	4.68	1.79	6.13	8.85	7.05	11.8	8.42	6.14	3.94	4.76	6.7	8.78	8.53	10.59	7.22	9.33	11.75	16.52				
7		0	0	6.8	3.91	4.01	10.97	9.17	12.81	9.43	7.15	4.95	2.64	4.58	6.66	6.41	10.12	8.23	10.34	12.76	17.53					
8		0	0	2.89	10.81	4.17	2.37	7.48	4.1	2.93	5.04	7.35	9.29	11.37	11.12	11.69	8.32	6.12	7.43	12.2						
9		0	0	0	7.92	7.06	5.26	10.01	6.63	4.35	2.15	4.46	6.4	8.48	8.23	8.8	5.43	7.54	9.96	14.73						
10		0	0	0	14.98	13.18	15.42	12.04	9.76	7.56	5.25	3.31	5.39	9.02	12.73	10.84	12.95	15.37	20.14							
11		0	0	1.8	3.5	3.53	5.81	8.01	10.32	12.26	14.34	14.09	14.66	11.29	9	6.86	8.37									
12		0	5.11	1.73	4.01	6.21	8.52	10.46	12.54	12.29	12.86	9.49	7.2	5.06	9.83											
13		0	0	3.38	5.66	7.86	10.17	12.11	14.19	13.94	14.51	11.14	8.85	6.71	4.87											
14		0	0	2.28	4.48	6.79	8.73	10.81	10.56	11.13	7.76	5.47	3.33	8.1												
15		0	0	0	2.2	4.51	6.45	8.53	8.28	8.85	5.48	3.19	5.61	10.38												
16		0	0	0	2.31	4.25	6.33	6.08	6.65	3.28	5.39	7.81	12.58													
17		0	0	1.94	0	4.02	3.77	7.48	5.59	7.7	10.12	14.89														
18		0	0	0	2.08	5.71	9.42	7.53	9.64	12.06	16.83															
19		0	0	0	3.91	7.62	9.61	11.72	14.14	18.91																
20		0	0	0	3.71	7.08	11.35	13.89	18.66																	
21		0	0	0	3.37	7.64	11.88	16.65																		
22		0	0	0	4.27	8.51	13.28																			
23		0	0	0	4.24	9.01																				
24		0	0	0	4.77																					
25		0	0	0	0																					

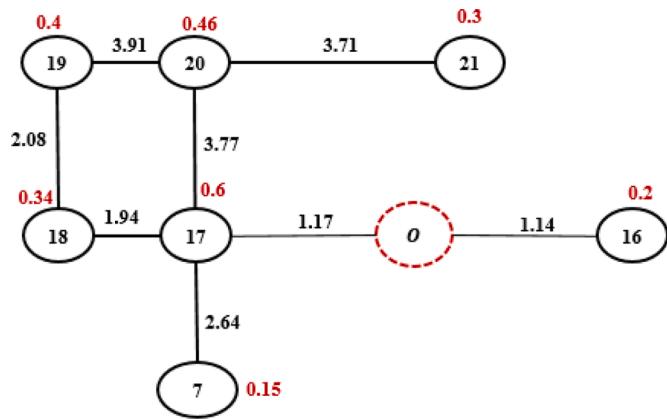


Fig. 13. Maintenance of failed transportation network.

in the network. First, we considered the scale of failure caused by each node as the initial failure node. The range of failure scale is between (1, 10). The larger the scale of failure caused by node congestion, the greater the traffic flow through the node and the more important this node may. Therefore, we should strengthen the nodes' maintenance in the core location of the transportation network. The maintenance time based on the failed network is shorter than that based on the normal network.

From the numerical experiments, we can find that the computational time is very high due to the complicity of the studied problem itself. However, the result analysis still illustrates the significance of the proposed policies. Due to the VRP (vehicle routing problem) problem, which is a sub-problem of this study, is NP-hard, it makes the computational time very long. The core of this study is for studying road maintenance policies, and future studies could be extended to improve the efficiency of solving VRP.

The future work can consider the following aspects. (1) The protection strategies for dredging urban road congestion in different degrees based on the background of intelligent transportation could be developed. (2) The degree of urban congestion into slight congestion and severe congestion could be divided. Aiming at slight congestion, our future works could establish a single maintenance center, multiple dredging personnel, and multiple dredging tasks to minimize dredging time. In response to severe congestion, multiple failure nodes, multiple dredging personnel, multiple maintenance centers, and multiple dredging tasks could be considered. (3) Another possible method is that maintenance centers in different regions can cooperate across regions to complete dredging tasks.

CRediT authorship contribution statement

Hongyan Dui: Writing – original draft, Conceptualization, Methodology, Validation. **Shuanshuan Chen:** Visualization, Investigation, Resources. **Yanjie Zhou:** Conceptualization, Project administration, Formal analysis, Writing – review & editing. **Shaomin Wu:** Supervision, Validation.

Declaration of Competing Interest

The authors have no conflicts of interest to declare.

Acknowledgments

The authors gratefully acknowledge the financial support for this research from the National Natural Science Foundation of China (Nos. 72071182, U1904211), the Ministry of Education's Humanities and Social Sciences planning fund (No. 20YJA630012), the Key Science and Technology Program of Henan Province (No. 222102520019), the

Program for Science & Technology Innovation Talents in Universities of Henan Province (No. 22HASTIT022), and the Program for young

backbone teachers in Universities of Henan Province (No. 2021GGJS007).

Appendix A: An example of the calculation process for the initial traffic value

In Fig. 3, node 8 is regarded as a sink node and other nodes are regarded as source nodes. We can find all the paths for each pair of source node and sink node for the transportation network. The paths are shown in Table 1.

In this paper, “all the paths” refers to: In real life, each vehicle has a departure place and a destination. In the topology of this paper, we regard the departure place as the source node and the destination as the sink node. For example, node 1 - node7 in Fig. 1 are all source nodes, and node 8 is regarded as a sink node. Taking node 1 as an example, the path from node 1 to node 8 is:

1 → 2 → 6 → 8; 1 → 2 → 6 → 5 → 7 → 8; 1 → 3 → 4 → 5 → 6 → 8; 1 → 3 → 4 → 5 → 7 → 8;
8; 1 → 4 → 5 → 6 → 8; 1 → 4 → 5 → 7 → 8; The path from node 2 - node 7 to node 8 through node 1 is: 2 → 1 → 3 → 4 → 5 → 6 → 8;
2 → 1 → 3 → 4 → 5 → 7 → 8; 2 → 1 → 4 → 5 → 6 → 8;
2 → 1 → 4 → 5 → 7 → 8; 3 → 1 → 2 → 6 → 8; 3 → 1 → 2 → 6 → 5 → 7 → 8; 3 → 1 → 4 → 5 → 6 → 8; 3 → 1 → 4 → 5 → 7 → 8; 3 → 4 → 1 → 2 → 6 → 8; 3 → 4 → 1 → 2 → 6 →
5 → 7 → 8; 4 → 1 → 2 → 6 → 8; 4 → 1 → 2 → 6 → 5 → 7 → 8; 4 → 3 → 1 → 2 → 6 → 8; 4 → 3 → 1 → 2 → 6 → 5 → 7 → 8; 5 → 4 → 1 → 2 →
6 → 8; 5 → 4 → 3 → 1 → 2 → 6 → 8; 6 → 2 → 1 → 3 → 4 → 5 → 7 → 8; 6 → 2 → 1 → 4 → 5 → 7 → 8; 7 → 5 → 4 → 1 → 2 → 6 → 8; 7 → 5 → 4 → 3 → 1 → 2 → 6 → 8;

In this paper, all paths from the source node 1-7 to the sink node 8 that include node 1 are referred to as all the paths of node 1.

It can be seen from the above data that there are 26 paths through node 1. Taking any one of the lines 2 → 1 → 3 → 4 → 5 → 6 → 8 as an example, the path consists of five nodes in total and the sequence number of node 1 is two. Therefore, on this path, $\frac{h_1^1}{H_1^1} = 2/7$. According to this method, $\frac{h_i^1}{H_i^1}$, which measures the values of the remaining 25 paths, can be obtained. Finally, according to the hierarchy value formula, we obtain that the hierarchy value of node 1 is 0.36. According to the hierarchy value of the node, the upstream and downstream node sets of a node can be distinguished. According to the number of paths, the location of each node can be known. A node with the hierarchy value closer to 1 indicates that the position is closer to the downstream node. A node with the hierarchy value closer to 0 means that it is closer to the upstream node. Taking the node in Fig. 3 as an example, node 1 is adjacent to node 2, node 3, and node 4. According to Table 2, the hierarchy value of each node can be known. For node 1, the hierarchy value of node 2 and node 4 are both greater than the hierarchy value of node 1, which indicates that nodes 2 and 4 are downstream nodes of node 1. Moreover, the hierarchy value of node 3 is less than the hierarchy value of node 1, indicating that node 3 is the upstream node of node 1. In addition, the hierarchy value formula includes the number of all paths passing through a node and the number of sequences, and the total number of nodes passing through a path. These indicators are comprehensive. In order to quantify the traffic of the network, the hierarchy value of the node is introduced.

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