CO7212 Game Theory in Computer Science

Vickrey Auctions, Complexity of Computing Nash Equilibria, and Applications of Game Theory in CS

Vickrey Auctions

- Bidders submit bids for an item.
- Every bidder i has a private valuation v(i) for the item.
- Payoff to bidder i is v(i)-p if he gets the item and pays p, and 0 if he does not get the item.
- Vickrey Auction: Award item to highest bidder, set the price equal to the second-highest bid.

Truthfulness of Vickrey Auctions

- Vickrey auctions are truthful: Bidding the true value of v(i) is a (weakly) dominant strategy.
  - If bidder i wins the item and the second-highest bid is x (so bidder i pays x and has payoff v(i)-x):
    - Bidding less than x, he would lose the item and get payoff 0.
    - Bidding any other value greater than x, he will still win the item and pay x, so his payoff is unchanged
  - If bidder i does not win the item, and the highest bid is y > v(i):
    - Bidding any other value less than y, bidder i's payoff is still 0.
    - Bidding a value greater than y, bidder i wins the item for price y and has negative payoff

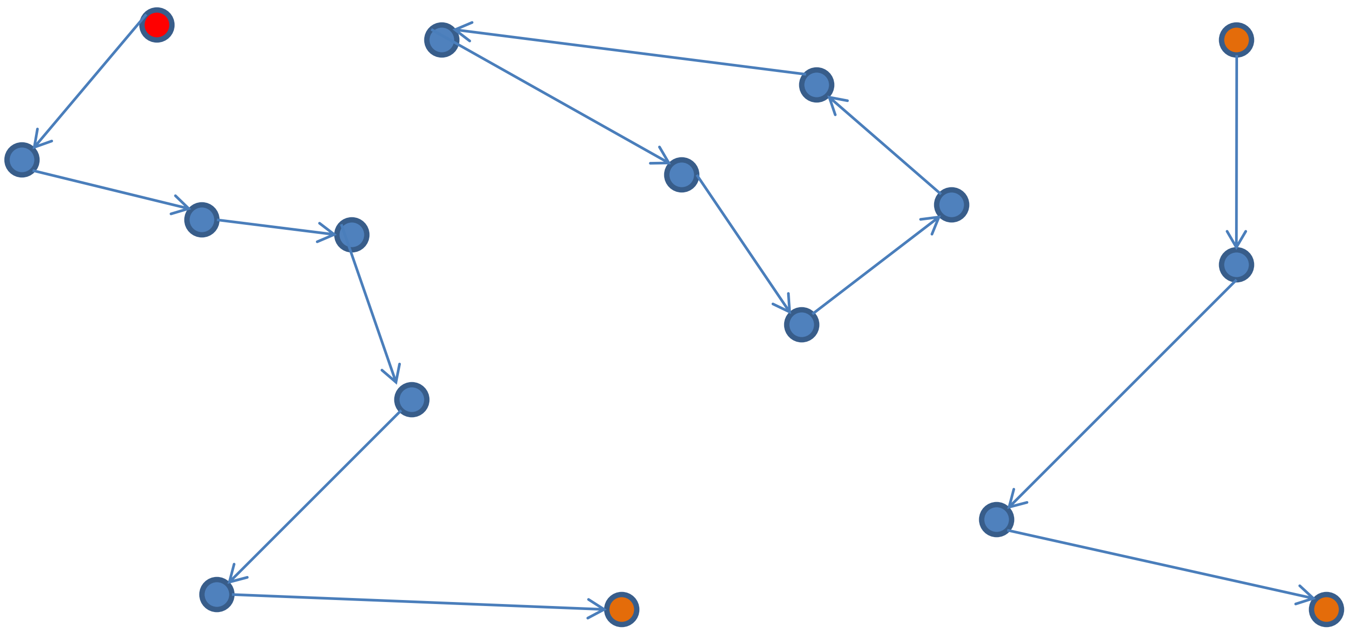
The Complexity of Finding Equilibria

- How difficult is it to find a Nash equilibrium of a given game?
- Is there an efficient algorithm that can be used to calculate Nash equilibria?
- **Problem NASH**: Given a bimatrix game G, compute any Nash equilibrium of G.

Lemke-Howson Algorithm

- Best known algorithm for finding Nash equilibria of bimatrix games.
- Works by moving from vertex to vertex of a certain polytope.
- Every vertex has at most one incoming edge and at most one outgoing edge.
- Any vertex, except the start vertex, with only one incident edge, gives a Nash equilibrium.
- Can require exponentially many steps to find a Nash equilibrium.

Lemke-Howson Illustration



NP-Completeness

- **NP** = Class of decision problems that can be solved in polynomial time by a non-deterministic Turing machine
- **NP-hard:** A problem P is NP-hard if a polynomial-time algorithm that solves P implies that all problems in NP can be solved in polynomial time.
- **NP-complete:** A problem that is NP-hard and in NP.

Can NASH be NP-hard?

- Every game has a Nash equilibrium, so the decision problem “Does a given game G have a Nash equilibrium?” cannot be NP-hard. (The answer is always YES.)
- Therefore, NP-hardness is not the right concept for showing that it is hard to compute Nash equilibria.
- But related problems can be proved NP-hard.

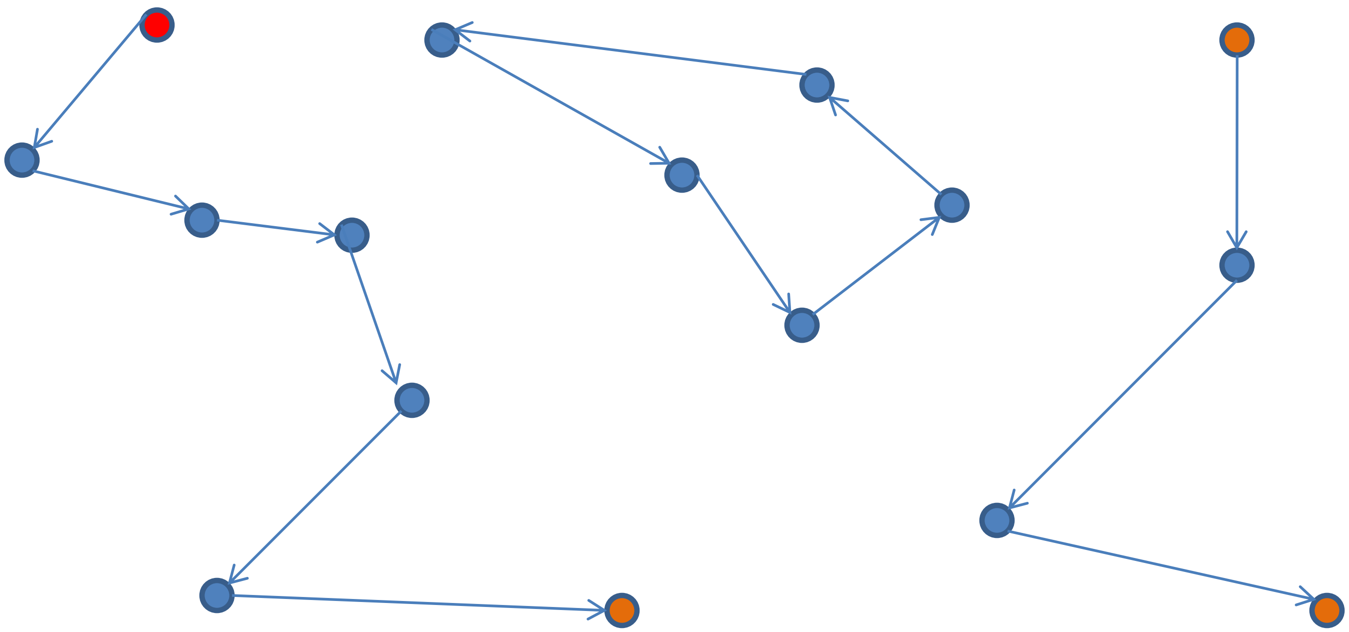
NP-Complete Problems

- Given a two-player game in strategic form, does it have:
  - At least two Nash equilibria?
  - A NE in which player I has payoff at least a certain amount?
  - A NE in which the two players have total payoff at least a certain amount?
  - A NE in which strategy s has positive probability?
  - A NE in which strategy s has probability 0?
  - Etc.

The Class PPAD

- Problems that can be represented as follows:
  - Directed graph on an exponentially large set of vertices.
  - Each vertex has indegree and outdegree at most one. Easy to determine neighbours of a vertex.
  - One known source (indegree 0), called the "standard source"
  - Any sink of the graph (outdegree 0), or any source other than the standard source is a solution.

PPAD Illustration



PPAD-Hardness of NASH

- The problem NASH is PPAD-complete. (Daskalakis et al, 2006; Goldberg and Papadimitriou, 2006; Chen and Deng, 2006)
- If NASH could be solved in polynomial time, all problems in PPAD could be solved in polynomial time.
- It is considered very unlikely that PPAD-complete problems (including NASH) admit polynomial-time algorithms.

Zero-Sum Games

- In zero-sum games, Nash equilibria are simply the combinations of any max-min strategy of player I and any min-max strategy of player II.
- Max-min and min-max strategies can be computed efficiently by **linear programming**.
- Therefore, problem NASH is efficiently solvable for zero-sum games.

Applications of Game Theory in Computer Science

- John von Neumann was instrumental in initiating game theory (1944) and the use of digital computers and their software (algorithms) (1946).
- Especially after the advent of the Internet, topics at the intersection of game theory and computer science are becoming increasingly important.

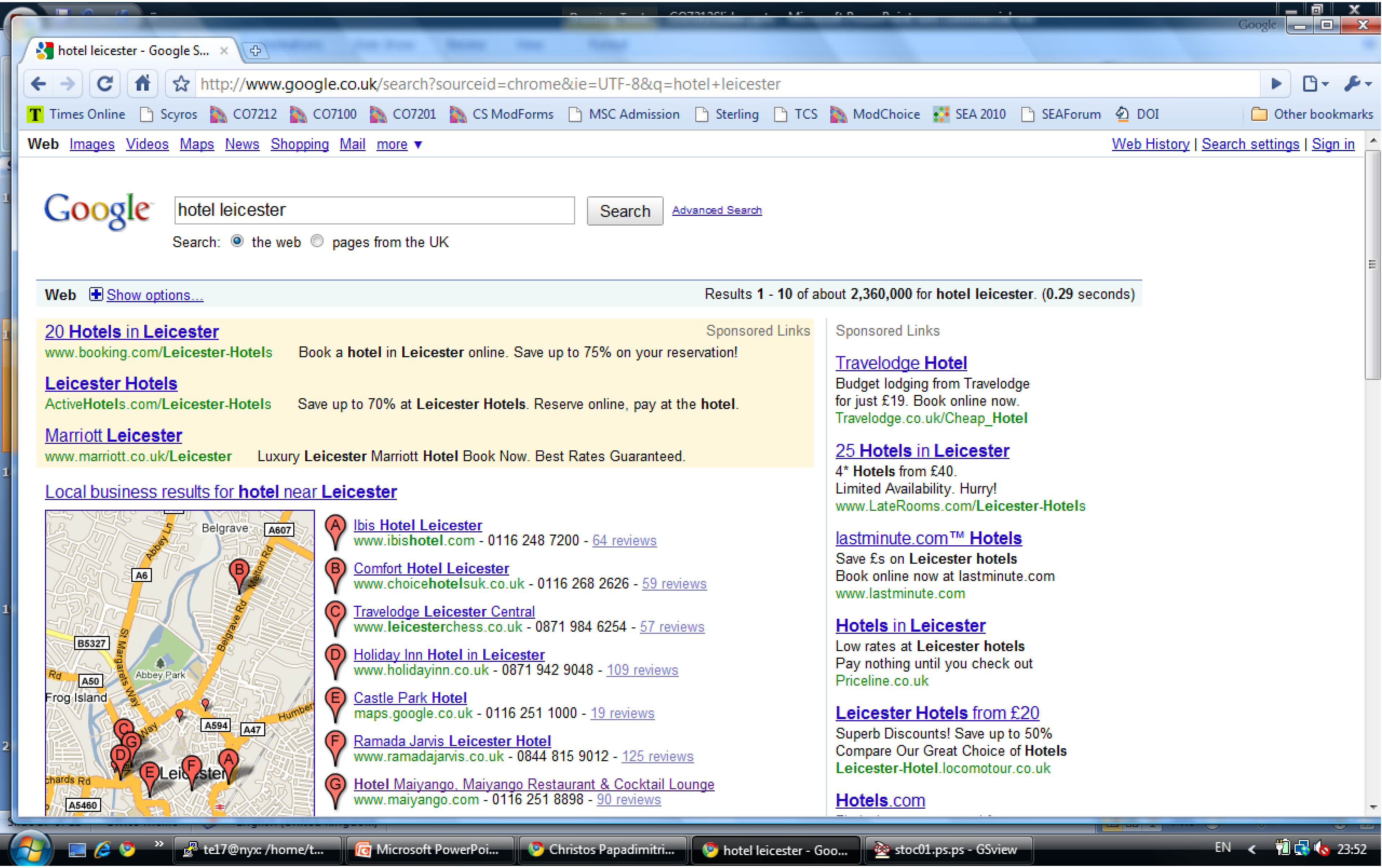
The Internet

- Large, distributed system without central control.
- Large socio-economic complexity: Built, operated and used by a multitude of diverse economic interests, in varying relationships of collaboration and competition with each other.
- Interaction of many agents (Internet Service Providers (ISPs), Routers, Users), and new types of markets (e.g. e-bay).

Games in the Internet

- TCP congestion control: The resulting bandwidth sharing is the equilibrium of which game?
- Interactions between ISPs: How are links between ISPs formed in the Internet?

Google AdWords



Peer-to-peer Networks

- Users want to download files, but may be reluctant to contribute their bandwidth and files to the network.
- Mechanism design is needed to motivate selfish users to collaborate.

Wireless Ad-Hoc Networks

- Networks without pre-installed infrastructure
- Every node acts as a router that forwards messages to other nodes
- Selfish nodes have no interest in forwarding traffic for other nodes
- Game theoretic concepts can be used to analyse mechanisms that provide incentives for nodes to cooperate.

Traffic Planning and Routing

- Satellite navigation systems will be increasingly networked and can then be coordinated centrally.
- This has the potential of improving traffic flow as different drivers can be given individual route recommendations.
- Route recommendations representing a Nash equilibrium are stable (drivers have no incentive to use other routes).

Games in Theoretical Computer Science

- Some types of games (Ehrenfeucht games, mean payoff games) are used in logic and model checking, for example in checking whether two models are equivalent
- A variant of von Neumann's minimax theorem is used to prove lower bounds on the competitive ratio (worst-case deviation fromt he optimum) of randomised online algorithms.