Programmability of Covariant Quantum Channels

Martina Gschwendtner, Andreas Bluhm, and Andreas Winter

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Presenter: Parsa Rangriz
Sharif University of Technology

Authors

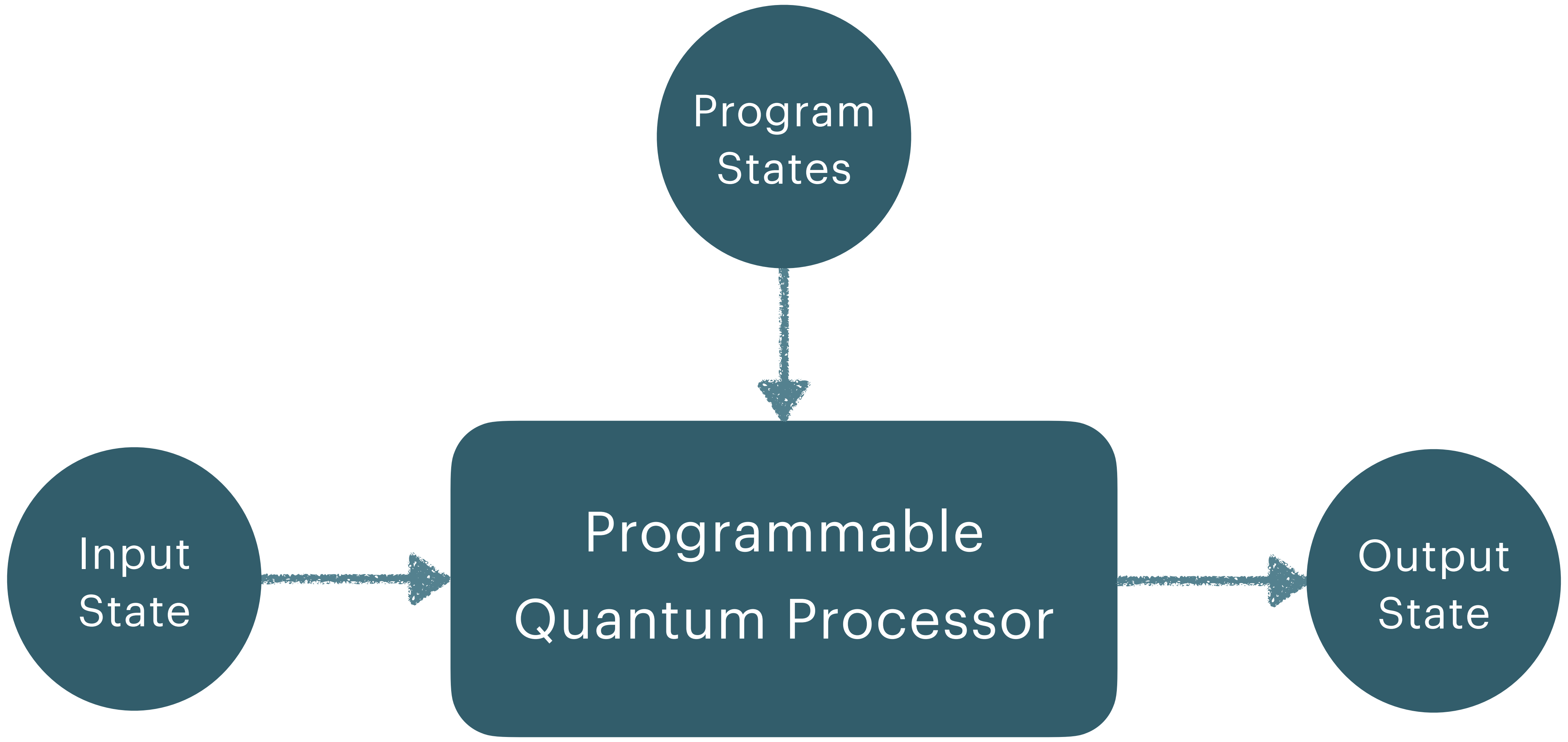
Martina Gschwendtner
Technical University of Munich

Andreas Bluhm
University of Copenhagen

Andreas Winter
Universitat Autònoma de Barcelona

Programmable Quantum Processor

- Programmable quantum processors are devices which can apply desired quantum operations, specified by the user via program states, to arbitrary input states.



Nielsen and Chuang's No-Programming Theorem

- It is not possible to implement infinitely many unitary channels exactly with finite-dimensional program register, i.e. exact universal programmable quantum processors are impossible.

$$|\psi\rangle \otimes |\mathscr{P}\rangle \to U|\psi\rangle \otimes |\mathscr{P}'\rangle$$

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Symmetry

- Symmetries are of fundamental importance in physics, since they give rise to conserved quantities via Noether's theorem.
- In open systems, these symmetries arise as covariant quantum channels and are studied using tools from quantum information theory.

Emmy Noether

Symmetry - II

- The No-Programming Theorem states that it is not possible to build a device which can implement all unitary channels, or in fact any infinite set of unitaries, exactly and with a finite-dimensional program register.

- In this work, they consider a setting where the No-Pogramming Theorem is not applicable because we do not want our processor to implement all unitary channels but a family with a certain symmetry consisting of possibly noisy quantum operations.

Preliminaries

- Definition 1: (Unitary representation). Let G be a compact group. A unitary representation of G is a continuous homomorphism from G to the unitary operators $\mathscr{U} := \{U | U \subset \mathscr{U}(H)\}$ on some complex, d-dimensional Hilbert space H.

- Definition 2: (Irreducible representation). A unitary representation U of a group G on a finite-dimensional vector space H is called irreducible representation (irrep) if and only if the only invariant subspaces of $\mathscr{U}$ are $\{0\}$ and the whole space.

- A fundamental result in representation theory states that we can decompose any unitary representation of a compact group into a direct sum of irreducible representations.

Preliminaries - II

- Definition 3: (UV-covariant quantum channel). Let G be a compact group and let U and V be representations on Hilbert spaces H_1 and H_2 . Let $T : B(H_1) \to B(H_2)$ be a quantum channel. We call T, UV-covariant if

- $$T(U_g A U_g^*) = V_g T(A) V_g^*, \quad \forall A \in B(H_1), \forall g \in G.$$

- The set of all UV-covariant channels is represented by

$$\mathcal{T}_{UV} := \{T : B(H_1) \to B(H_2), UV\text{-covariant quantum channel}\}$$

- We define the set of all Choi-Jamiolkowski states corresponding to quantum channels $T \in \mathcal{T}_{UV}$ as

$$\mathcal{J}_{UV} := \{c_T \in B(H_1 \otimes H_2) : c_T := (I \otimes T)(|\Omega\rangle\langle\Omega|), \forall T \in \mathcal{T}\}$$

An Introduction to Exact and Approximate Programmability

- There is a lemma which states that $T \in \mathcal{T}_{UV}$ is equivalent to $[c_T, \bar{U}_g \otimes V_g] = 0$ for all $g \in G$. Due to this correspondence, we consider representations of the form $\bar{U} \otimes V$ with $U_g \in \mathcal{U}_1, g \in G$ and the commutant

$$K := \{X \in B(H_1 \otimes H_2) \,|\, [X, \bar{U}_g \otimes V_g] = 0, \forall g \in G\}$$

- Let K be as define above and let U be an irrep of a compact group G on H_1. Let V be a representation of G on H_2. Then $K \cap D(H_1 \otimes H_2) = \mathcal{J}_{UV}$. Moreover, if V is an irrep, any $T \in \mathcal{T}_{UV}$ is unital
- The channels implemented by a processor that is covariant with respect to the special unitary group $SU(H_1)$ are unital using a similar argument. Then the authors considered how to construct covariant programmable quantum processors in the case where K is abelian.

ϵ-Programmable Quantum Processor (ϵ-PQP)

- Let H_1 and H_2 be separable Hilbert spaces. Then we call $P \in$ CPTP$(H_1 \otimes H_P, H_2)$, with finite-dimensional H_P, an ϵ-programmable quantum processor for a set $C \subset$ CPTP(H_1, H_2) of channels (ϵ-PQP$_C$), if for every quantum channel $T \in C$ there exists a state $\pi_T \in D(H_P)$ such that

$$\frac{1}{2}\|P(\,\cdot\otimes\pi_T) - T(\,\cdot\,)\| \leq \epsilon.$$

- For $\epsilon = 0$ we say that P exactly implements the class C, and address it as a PQP$_C$.

What we will see?

- Bounds for Approximate Programmability.
- Exact and Approximate Programmability