Structural Neighborhood Based Classification of Nodes in a Network

基于结构邻域的网络节点分类

KDD

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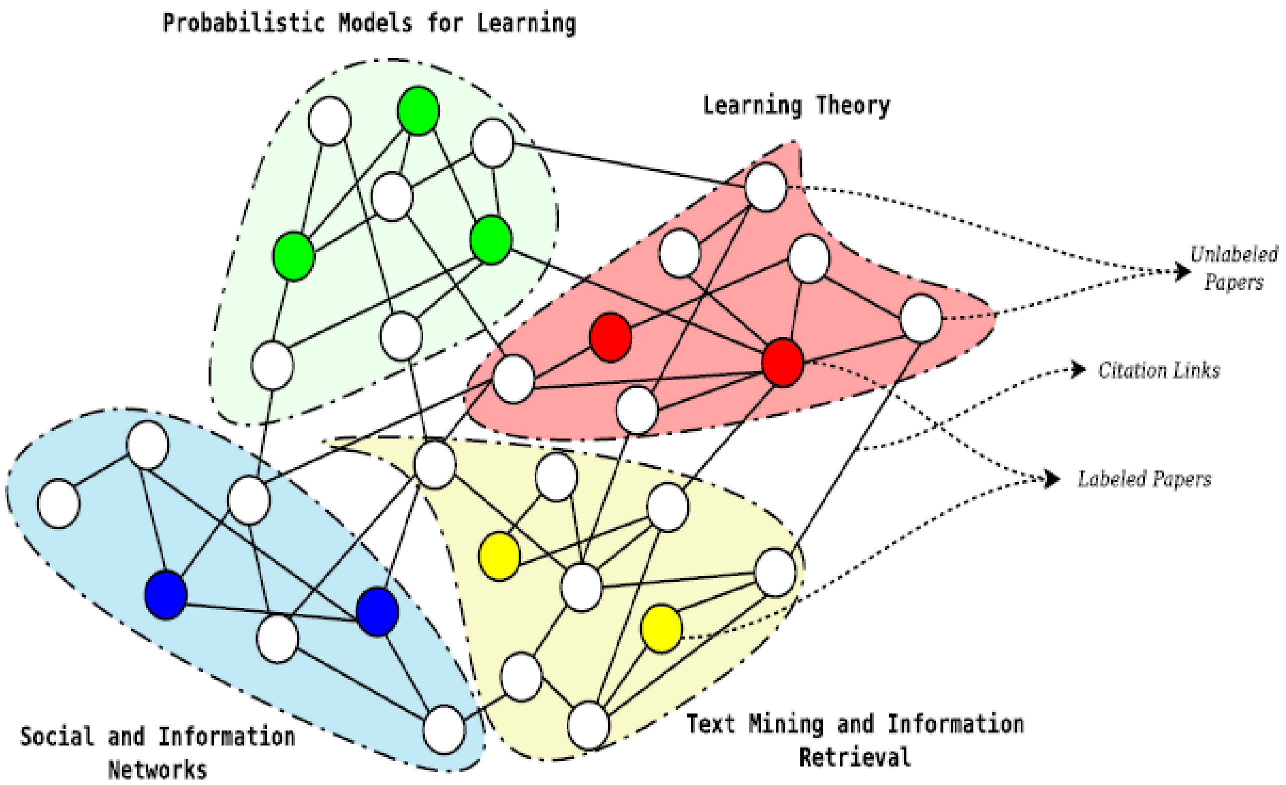
Purpose



- propose a novel structural neighborhood-based classifier learning using a random walk under sparse network

(1)

- homophily [1]: neighboring nodes in a network are supposed to be similar to each other.

- To exploit homophily in networks, we take a comprehensive view of classification, where a node is classified based on how other nodes in its extended neighborhood are labeled.

[1] A. Anagnostopoulos, R. Kumar, and M. Mahdian.Influence and correlation in social networks. In KDD,pages 7–15. ACM, 2008.

(2)

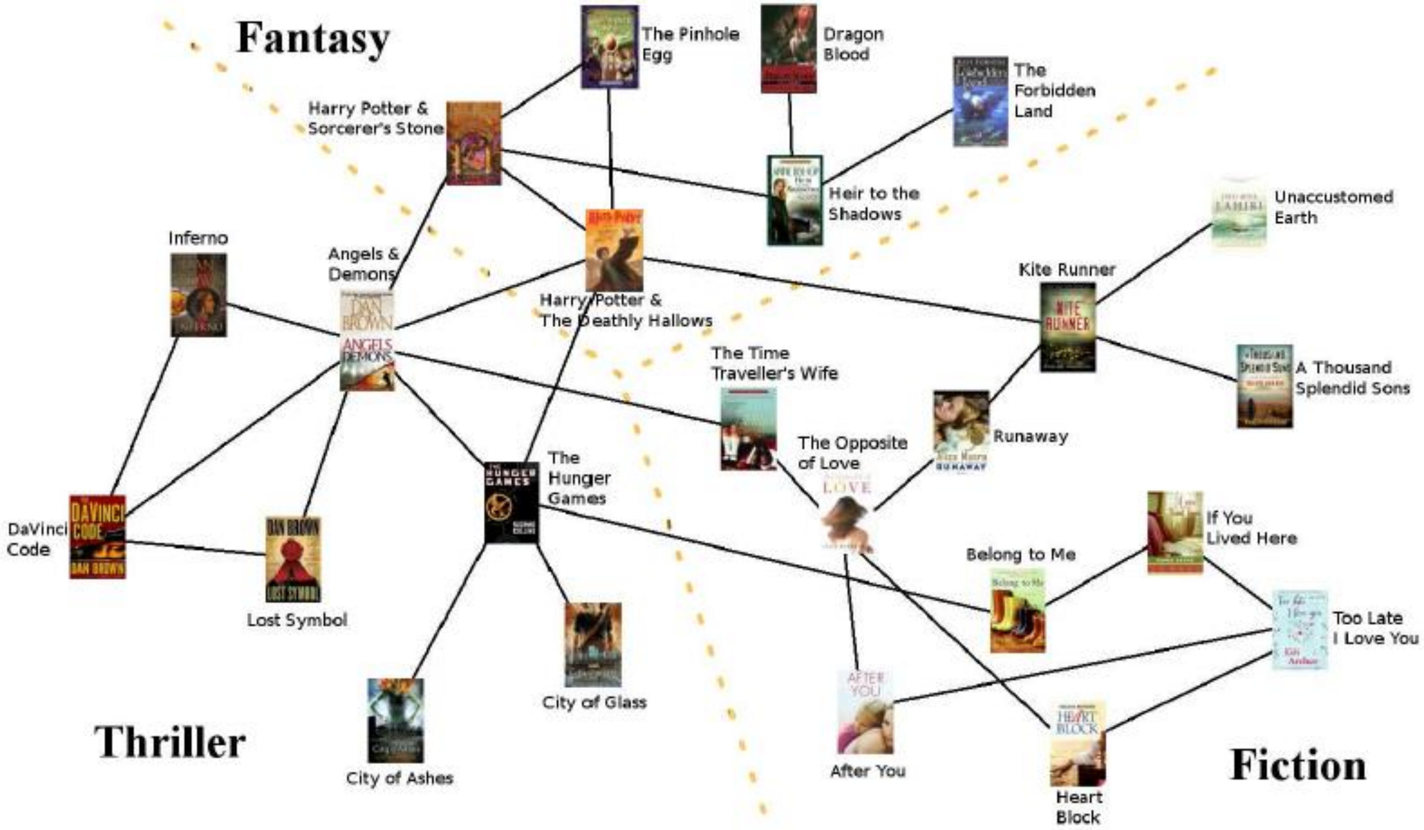
- A simple approach: use the number of neighbors from the respective classes and label the node based on majority voting.

- However,if the network is sparsely.It did not well.the label is sparse too.

What affect weight？

- Propose a method: weight vote instead of count

S. A. Macskassy and F. Provost. A simple relationalclassifier. In MRDM at KDD, 2003.



- It is observed that high degree nodes are generally the source of linkage noise in networks.

- They think should a link to a low or medium degree node should be considered more reliable

- So！！！Degree could affect the weight

Some Prepared

- In this paper, we will be working with a binary classification problem

- Let $C+$ and $C-$ represent the sets of positive and negative examples respectively in the binary classification problem under consideration.

DEFINITION 1. *A network is modeled as a graph* $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{W})$, *where* $\mathcal{V}$ *is the set of* $|\mathcal{V}| = n$ *interacting units (nodes or actors), and* $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ *is the set of edges indicating relationship or interactions among the nodes.* $\mathcal{W} \in \mathbb{R}^{n \times n}$, *where* $\mathcal{W}_{ij}$ *indicates affinity or strength of the relationship between nodes* $v_i, v_j \in \mathcal{V}$.

For a sparsely labeled network, the classification problem is formally stated as follows.

DEFINITION 2. *Given a graph* $\mathcal{G}(\mathcal{V}, \mathcal{E})$ *and a set of labeled nodes* $\mathcal{V}_l$ $(\subsetneq \mathcal{V})$ *with corresponding (ordered) set of labels* $\mathcal{Y}_l \in \mathcal{C}^{|\mathcal{V}_l|}$, *where* $\mathcal{C} = \{\mathcal{C}_1, \mathcal{C}_2, \ldots, \mathcal{C}_k\}$, *the set of* k *labels* $\mathcal{C}_1$ *to* $\mathcal{C}_k$. *The objective is to learn a model for inferring labels of the unlabeled nodes* $\mathcal{V}_u = \mathcal{V} \setminus \mathcal{V}_l$.

We start by defining the adjacency based representation of a graph. Given an undirected and binary-weighted graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{v_1, v_2, \ldots, v_n\}$ and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, the corresponding adjacency matrix A is defined as follows,

$$A_{ij} = \begin{cases} 1, & \text{if } (v_i, v_j) \in \mathcal{E} \\ 0, & \text{otherwise} \end{cases} \quad .$$

In the case of a weighted graph $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{W})$, links carry a non-negative weight specified by $\mathcal{W}$. Adjacency Matrix A for these graphs is modified as $A_{ij} = \mathcal{W}_{ij}$. Based on the link attributes of a node, we define node v_i in vector space notation as $a_i = [A_{ji}]_{n \times 1}$ where $j \in \{1, 2, \cdots, n\}$.

Definition 3. ***First-Level Neighborhood*** *of a node* v_i *is the set* $\mathcal{N}_i^1$ *s.t.* $v_j \in \mathcal{N}_i^1$ *if and only if there exists an edge in the graph connecting* v_i *and* v_j*, i.e.,* $(v_i, v_j) \in \mathcal{E}$.

the follow problem

约束最优化问题

$$\min_{w,b} \quad \frac{\lambda}{2}||p||^2 \quad + \quad \frac{1}{|\mathcal{V}_l|}\sum_{i\in\mathcal{V}_l} \varepsilon_i \tag{2}$$

$$\begin{aligned} \text{such that} \quad & y_i(w^\intercal m_i + b) \geq 1 - \varepsilon_i \;, \\ & \varepsilon_i \geq 0 \;, \text{ and} \\ & m_i = A\frac{a_i}{d_i} \end{aligned}$$

(1) Structural Neighborhood: For a node v_i having class label $y_i \in \{-1, 1\}$, the aggregated scores from the nodes having label y_i in the first-level neighborhood ($\mathcal{N}_i^1$) should be more than the aggregated scores from rest of the nodes in the first-level neighborhood. Thus, for optimal w and b, we have, for all i,

$$r_i = y_i \left(w \bullet x_i + b \right)$$

$$y_i \sum_{j \in \mathcal{N}^1_{i,y_i}} A(v_i, v_j)(w^\mathsf{T} \cdot a_j + b) \;\geq\; -y_i \sum_{j \in \mathcal{N}^1_{i,-y_i}} A(v_i, v_j)(w^\mathsf{T} \cdot a_j + b),$$

where $A(v_i, v_j) = A_{ij}$ indicates the weight of edge joining nodes v_i and v_j. This can be equivalently rewritten as,

$$y_i \left(\sum_{j \in \mathcal{N}^1_{i,y_i}} A_{ij}(w^\mathsf{T} \cdot a_j + b) \;+\; \sum_{j \in \mathcal{N}^1_{i,-y_i}} A_{ij}(w^\mathsf{T} \cdot a_j + b) \right) \geq\; 0,$$

$$\implies y_i \left(\sum_{j \in \mathcal{V}} A_{ij}(w^\mathsf{T} \cdot a_j + b) \right) \;\geq\; 0.$$

Rearranging the terms, we get,

$$y_i \;\left(\; w^\mathsf{T} \cdot A \cdot a_i + d_i \; b \;\right) \;\geq\; 0,$$

where $d_i \;=\; \sum_{j \in \mathcal{V}} A_{ij}$.

$$\implies y_i \left(\frac{1}{d_i} w^\mathsf{T} \cdot A \cdot a_i + b \right) \;\geq\; 0$$

Let $M = [m_1, m_2, \cdots, m_n] = A^2 D^{-1}$, where $D \in \mathbb{R}n \times n$ defined as $D_{ij} = \begin{cases} d_i & \text{if } \; i = j \\ 0 & \text{otherwise} \end{cases}.$

Then, we have the following in a linearly separable case:

$$y_i(w^\mathsf{T} m_i + b) \geq 0.$$

The above can be interpreted as mapping the adjacency information a_i to a new space as $m_i = A\dfrac{a_i}{d_i}$, and then learning a decision boundary. For node v_i, we define empirical loss ε_i (≥ 0) such that

$$y_i(w^\mathsf{T} m_i + b) \geq 1 - \varepsilon_i.$$

Mean empirical loss, that is to be minimized, is given by

$$f(w, b) = \frac{1}{|\mathcal{V}_l|} \sum_{i \in \mathcal{V}_l} \varepsilon_i. \tag{1}$$

转化为了约束
最优化问题

Degree Dependent Regularization

- W depends on degree
- Add Degree Dependent Regularization p
- $p = (p_1, p_2, \ldots, p_n)$ 其中 penalty for node v_i is $p_i = g(w_i, d_i)$.

- Linear Weighted Degree **(LWD):**

$$g(w_i, d_i) := |w_i| d_i$$

- Linear Weighted Root Degree **(LWRD):**

$$g(w_i, d_i) := |w_i| \sqrt{d_i}$$

- Linear Weighted Root Log Degree **(LWRLD):**

$$g(w_i, d_i) := |w_i| \sqrt{\log_2 d_i}$$

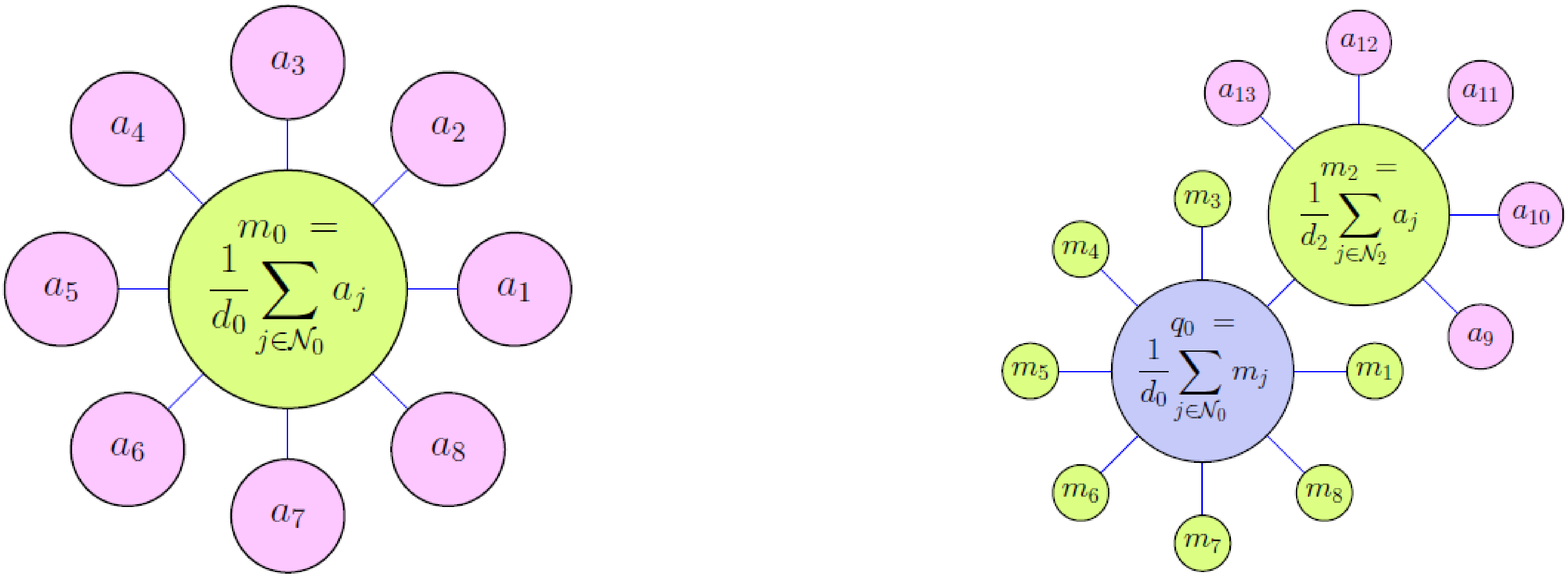
The objective function

Add a parameter λ

$$\min_{w,b} \quad \frac{\lambda}{2}||p||^2 \quad + \quad \frac{1}{|\mathcal{V}_l|}\sum_{i\in\mathcal{V}_l} \varepsilon_i \qquad (2)$$

$$\text{such that} \quad y_i(w^\mathsf{T} m_i + b) \geq 1 - \varepsilon_i \ ,$$
$$\varepsilon_i \geq 0 \ , \text{ and}$$
$$m_i = A\frac{a_i}{d_i}$$

Definition 4. ***r^{th}-Level Neighborhood*** *of a node v_i is defined as a multiset $\mathcal{N}_i^r$ s.t. $v_k \in \mathcal{N}_i^r$ if and only if there is an edge in graph $\mathcal{G}$ connecting nodes v_k and v_j where node $v_j \in \mathcal{N}_i^{r-1}$, and multiplicity of v_k in $\mathcal{N}_i^r$ is given by the cardinality of set $\{v_j | (v_k, v_j) \in \mathcal{E}$ and $v_j \in \mathcal{N}_i^{r-1}\}$.*



(a) First-Level Neighborhood

(b) Second-Level Neighborhood

$$y_i \left(\sum_{j \in \mathcal{N}^1_{i,y_i}} A_{ij}(w^\mathsf{T} \cdot m_j + b) \; + \sum_{j \in \mathcal{N}^1_{i,-y_i}} A_{ij}(w^\mathsf{T} \cdot m_j + b) \right) \geq \; 0, \quad \forall i = 1, \ldots, n$$

$$y_i \quad \left(\frac{1}{d_i} w^\mathsf{T} \cdot A \cdot m_i + b \right) \quad \geq \quad 0, \quad \forall i = 1, \ldots, n$$

where, $d_i \;\; = \;\; \sum_{j \in \mathcal{V}} A_{ij}$.

Let $Q = [q_1, q_2, \cdots, q_n] = MAD^{-1} = A(AD^{-1})^2$, where M and D are defined in section 3. Then, for the linearly separable case, we have,

$$y_i(w^\mathsf{T} q_i + b) \geq 0, \quad \forall i = 1, \ldots, n$$

where $q_i = A^2 D^{-1} \dfrac{a_i}{d_i}$ based on second-level neighborhood of a node. We can argue using inductive logic that while using r-level neighbors, the same will be mapped to $A(AD^{-1})^{r-1} \dfrac{a_i}{d_i}$. For classification using r-level neighbors, the adjacency information contained in the matrix A as a whole gets mapped to $A(AD^{-1})^r$.

How to Random Walk

- Random Walk：

with increasing values of r, the transition probabilities between any pair of nodes start converging towards the stationary probability distribution.

- Lazy Random Walk:

（1） length is not fixed.

（2） Use dampening factor at each hop, which controls the termination of the random walk

- If dampening factor is (… the random walk terminates at each hop …), and … the next hop with probab… enta… obtained using the above pr… given b…

if γ is chosen to be close to zero, it is equivalent to learning with the given adjacency representation alone.

On the other extreme, if gamma is large (≈ 1), the same as random walk

$$q_i = (1-\gamma)A\left(e_i + \gamma\frac{a_i}{d_i} + \gamma^2(AD^{-1})\frac{a_i}{d_i} + \gamma^3(AD^{-1})^2\frac{a_i}{d_i} + \dots\right)$$

e_i being a n-dimensional unit vector with i^{th} entry as 1 and remaining as zeros.

Structured RandomWalk

- higher degree implies a higher termination probability and a lesser dampening factor

$$\gamma_i = \frac{1}{\log_2 d_i}.$$

We define matrix Γ as,

$$\Gamma = diag(\gamma_1, \gamma_2, \ldots, \gamma_{|\mathcal{V}|}).$$

The representation then becomes

$$q_i = A\Big(e_i + \Gamma\tfrac{a_i}{d_i} + (AD^{-1})\Gamma^2\tfrac{a_i}{d_i} + (AD^{-1})^2\Gamma^3\tfrac{a_i}{d_i} + \ldots\Big)(I - \Gamma).$$

Let Q be the matrix obtained by stacking column vectors q_i corresponding to all the nodes. Then, we have

$$Q = A(I - AD^{-1}\Gamma)^{-1}(I - \Gamma).$$

The core of Algorithm

Taking structured random walk into account the objective in (2) is modified as:

$$\min_{w,b} \quad \frac{\lambda}{2}||p||^2 \quad + \quad \frac{1}{|\mathcal{V}_l|}\sum_{i\in\mathcal{V}_l} \quad \varepsilon_i \tag{4}$$

$$\text{s.t.} \quad y_i(w^{\mathsf{T}}q_i + b) \geq 1 - \varepsilon_i \ ,$$
$$\varepsilon_i \geq 0 \ , \text{ and}$$
$$q_i = A\Big(e_i + \Gamma\tfrac{a_i}{d_i} + (AD^{-1})\Gamma^2\tfrac{a_i}{d_i} + (AD^{-1})^2\Gamma^3\tfrac{a_i}{d_i} + \ldots\Big)(I-\Gamma)$$

Subgradient of the above is given by,

$$\nabla_t = \lambda p\frac{\partial p}{\partial w} - \frac{1}{|\mathcal{V}_l|}\sum_{i\in\mathcal{V}_l} \mathbb{1}[y_i w_t^{\mathsf{T}} q_i < 1] y_i q_i,$$

where $\mathbb{1}$ denotes indicator function. Using gradient descent, the iterative update rule for w is given by

$$w_{t+1} = w_t - \eta_t \nabla_t$$

where η_t is the learning rate for the t^{th} iteration. We use stochastic gradient descent mini-batch update algorithm with a variable learning rate η_t given by $\eta_t = \dfrac{1}{2+\lambda t}$.

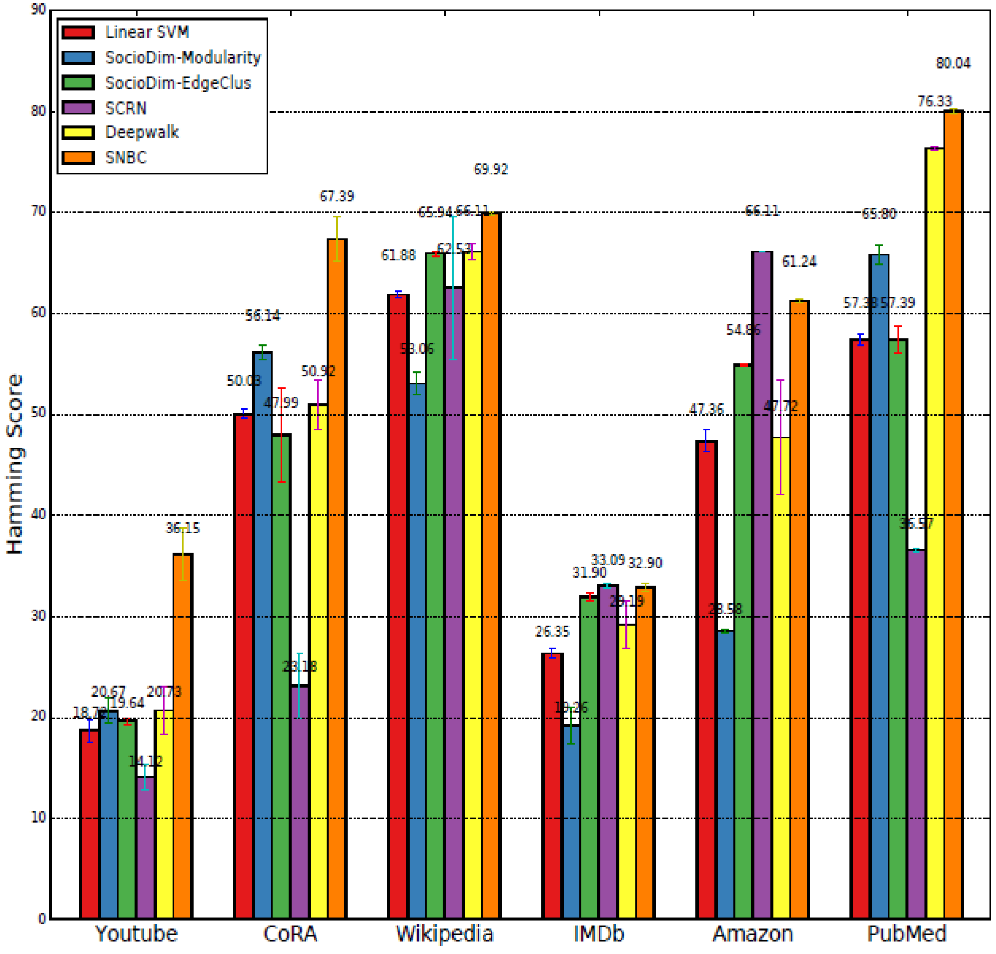
Experienment

If for the i_{th} node, T_i is the set of true labels, and P_i is set of predicted labels

$$\text{Hamming Score} = \sum_i \frac{|T_i \cap P_i|}{|T_i \cup P_i|}$$

$$\text{Micro-}F_1 \text{ Score} = \frac{2\sum_i |T_i \cap P_i|}{\sum_i |T_i| + \sum_i |P_i|}$$

$$\text{Macro-}F_1 \text{ Score} = \frac{1}{k}\sum_{j=1}^{k} \frac{2\sum_{i\in\mathcal{C}_j} |T_i \cap P_i|}{\sum_{i\in\mathcal{C}_j} |T_i| + \sum_{i\in\mathcal{C}_j} |P_i|}$$


Hamming Score
Linear SVM
SocioDim-Modularity
SocioDim-EdgeClus
SCRN
Deepwalk
SNBC
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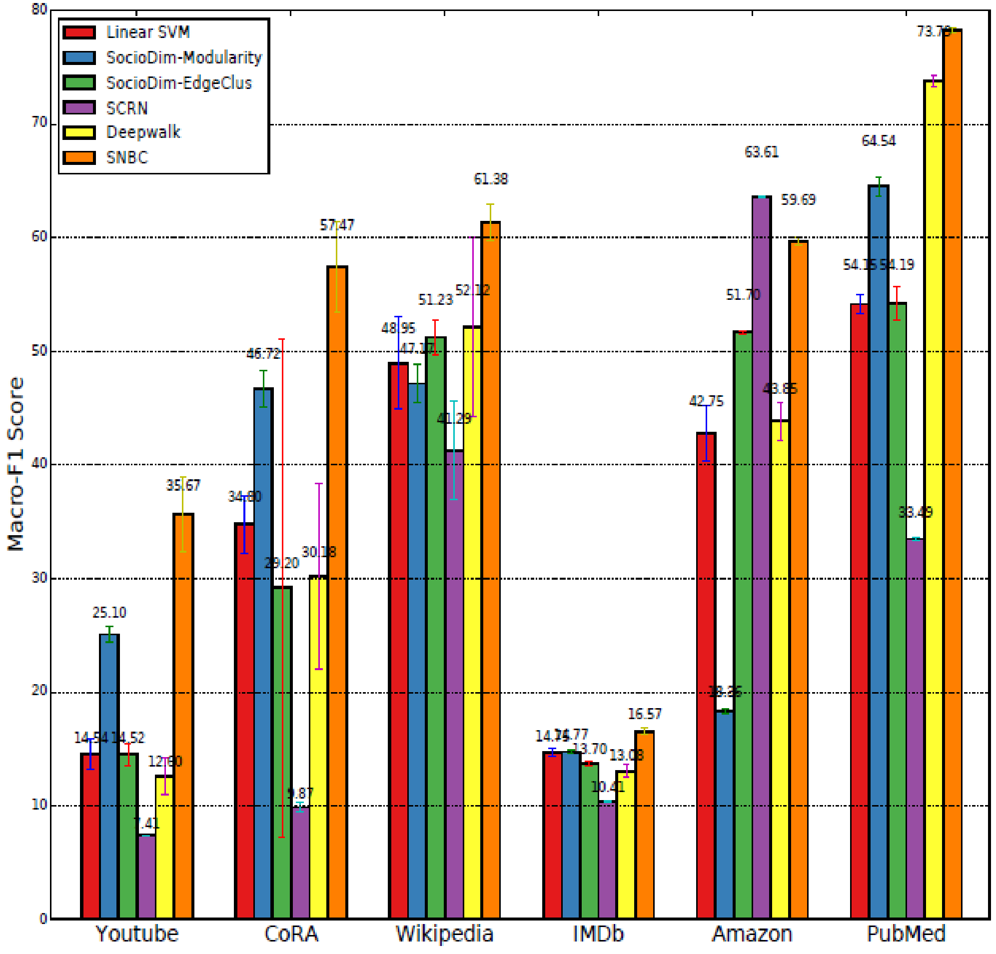
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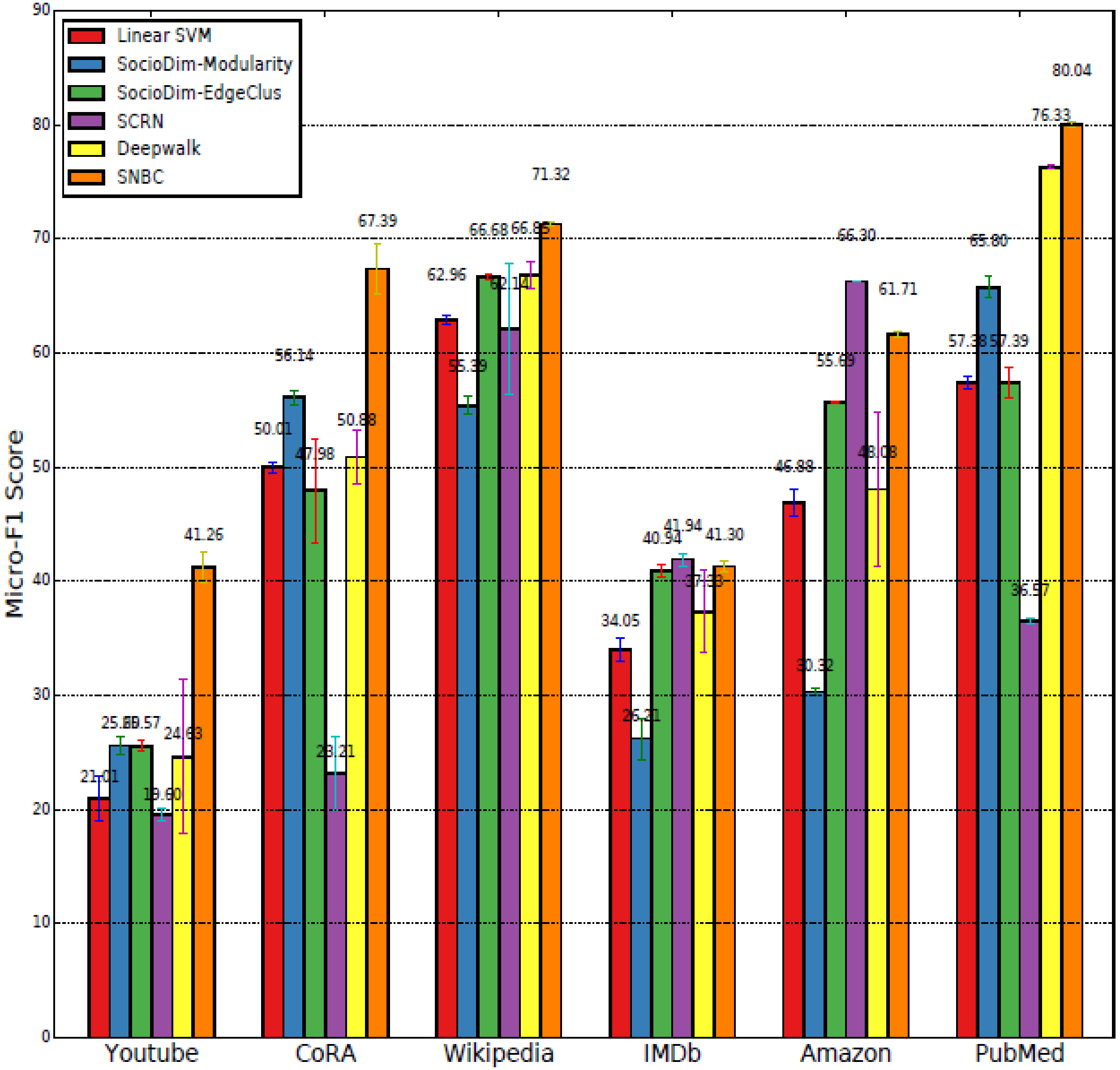
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Thank you！