Structural Deep Network Embedding

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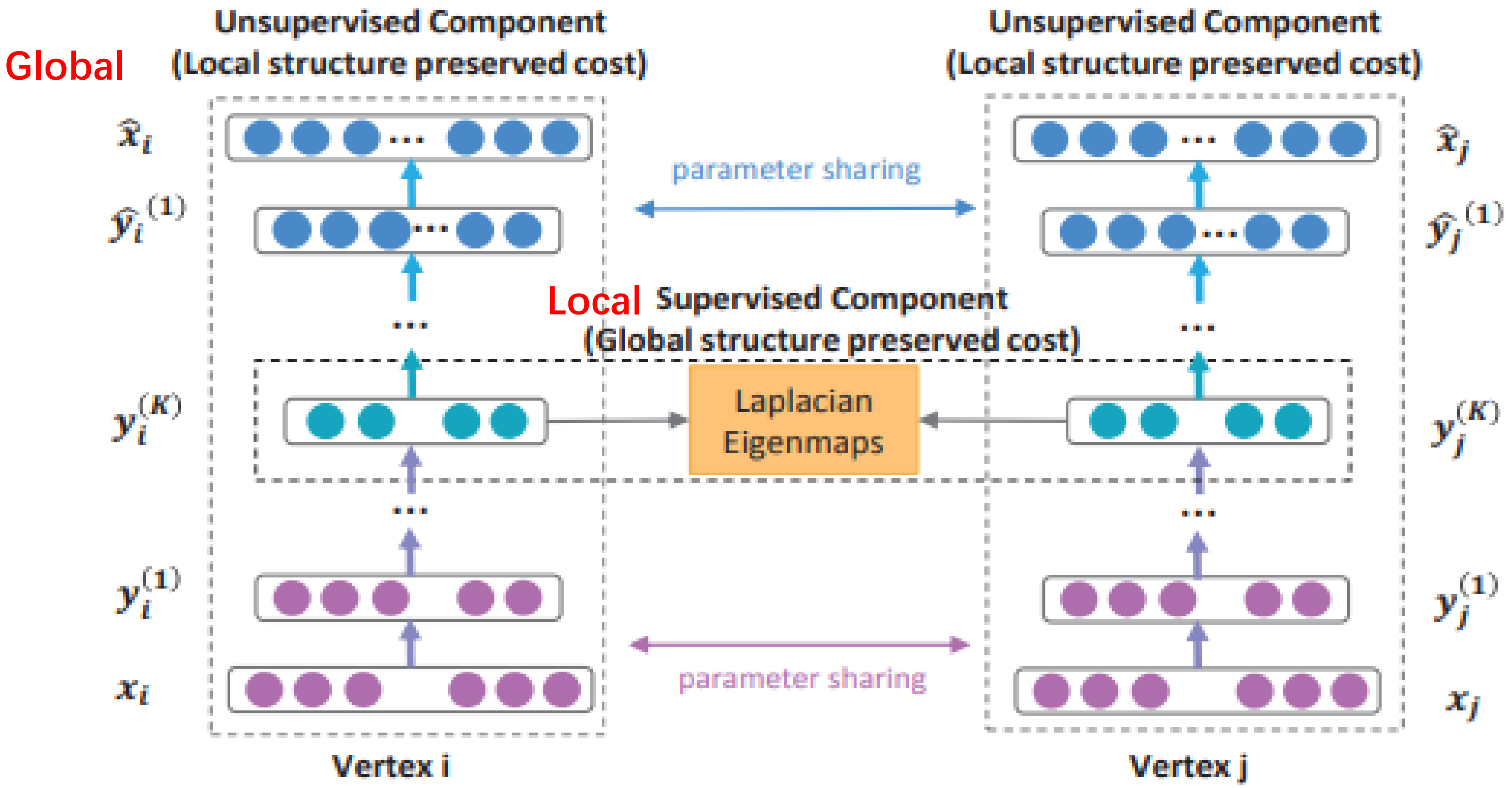
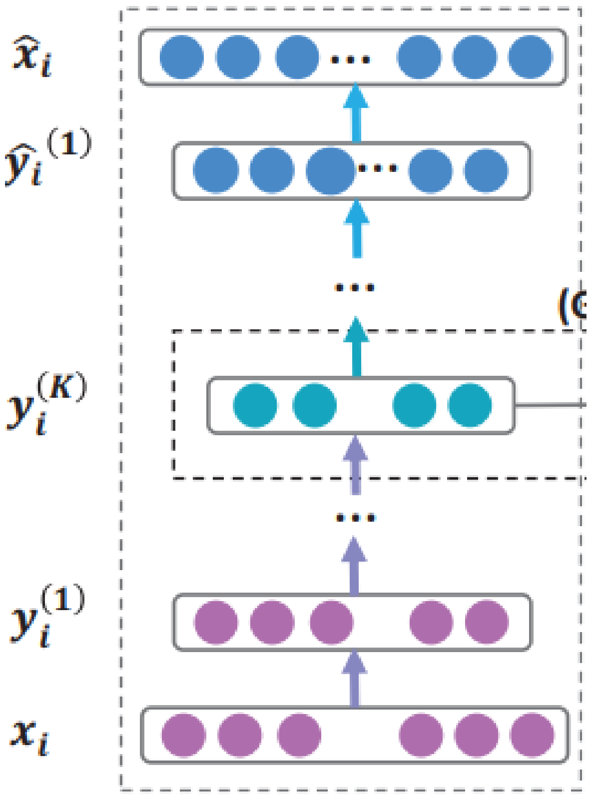
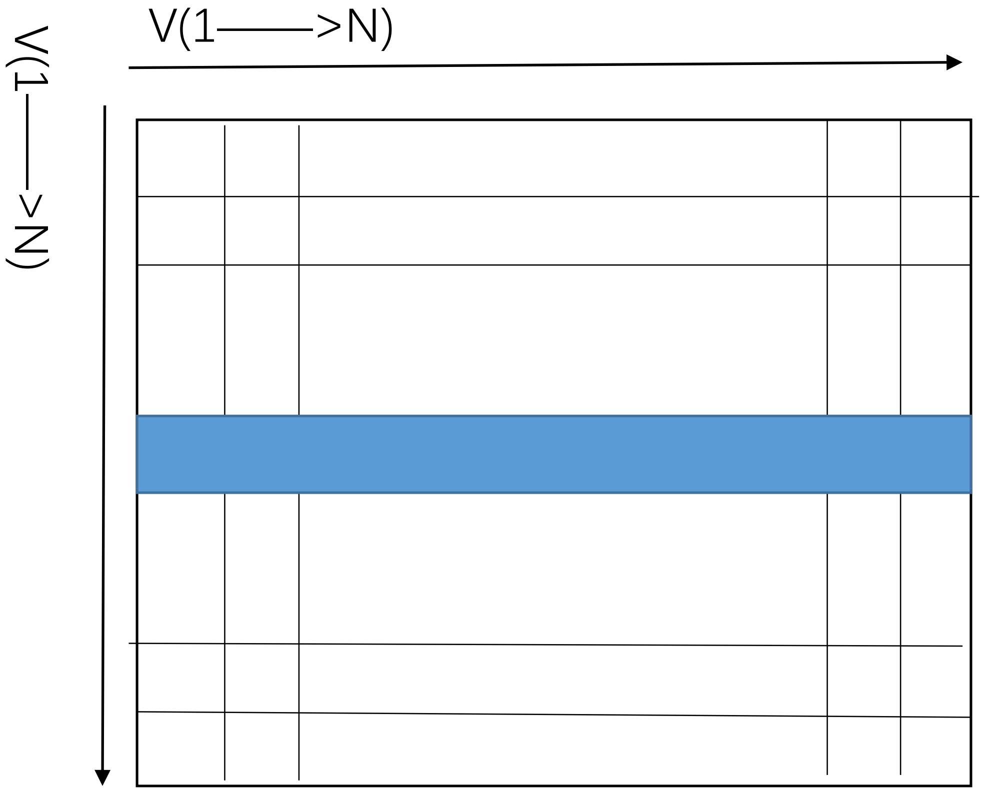
Application:

Reconstruction,Multi-label classification, Link prediction and Visualization.





Vertex i



Vi/Xi也就是Vi节点的邻接节点

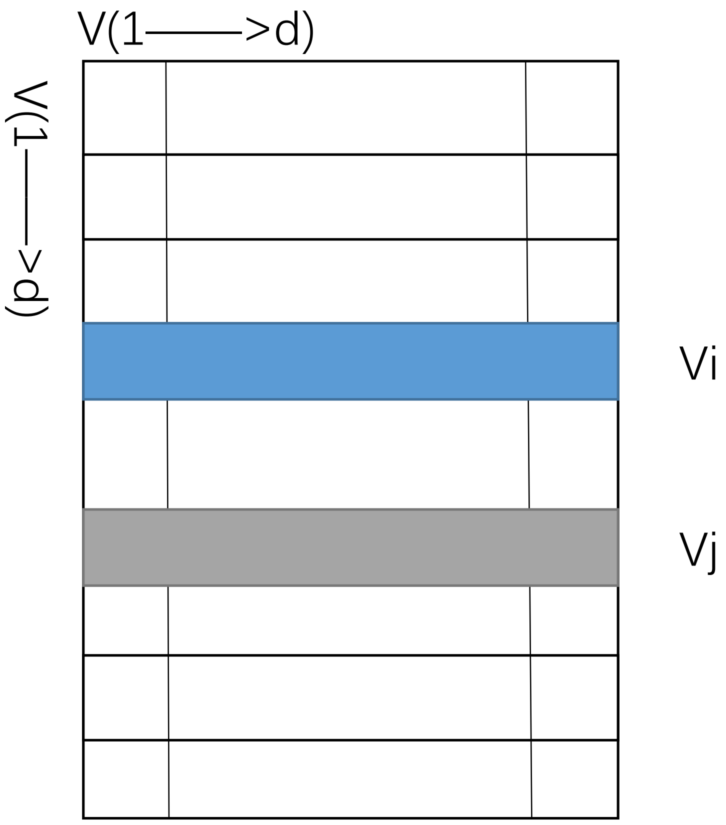
邻接矩阵为V*V　在图中V=N

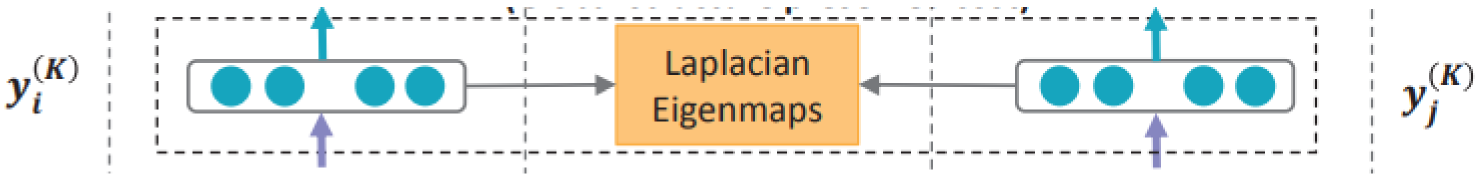
$$\mathcal{L}_{2nd} = \sum_{i=1}^{n} \|(\hat{\mathbf{x}}_i - \mathbf{x}_i) \odot \mathbf{b_i}\|_2^2$$
$$= \|(\hat{X} - X) \odot B\|_F^2$$

为什么说是全局结构?

the reconstruction criterion can smoothly capture the data manifolds and thus preserve the similarity between samples

the reconstruction process will make the vertexes which have similar neighborhood structures have similar latent representations.





$$\mathcal{L}_{1st} = \sum_{i,j=1}^{n} s_{i,j} \| \mathbf{y}_i^{(K)} - \mathbf{y}_j^{(K)} \|_2^2$$
$$= \sum_{i,j=1}^{n} s_{i,j} \| \mathbf{y}_i - \mathbf{y}_j \|_2^2$$

$$
\begin{aligned}
\mathcal{L}_{mix} &= \mathcal{L}_{2nd} + \alpha \mathcal{L}_{1st} + \nu \mathcal{L}_{reg} \\
&= \|(\hat{X} - X) \odot B\|_F^2 + \alpha \sum_{i,j=1}^{n} s_{i,j} \|\mathbf{y}_i - \mathbf{y}_j\|_2^2 + \nu \mathcal{L}_{reg}
\end{aligned} \tag{5}
$$

where $\mathcal{L}_{reg}$ is an $\mathcal{L}2$-norm regularizer term to prevent overfitting, which is defined as follows:

$$
\mathcal{L}_{reg} = \frac{1}{2} \sum_{k=1}^{K} (\|W^{(k)}\|_F^2 + \|\hat{W}^{(k)}\|_F^2)
$$

Community Preserving Network Embedding

Xiao Wang, Peng Cui, Jing Wang, Jian Pei, Wenwu Zhu, Shiqiang Yang

Application: node clustering and classification

Modularized Nonnegative Matrix Factorization (M-NMF)

incorporate the community structure into network embedding

M-NMF

microscopic structure:

first- and second-order proximities of nodes

mesoscopic structure:

the communities are detected by a modularity constraint term

M-NMF Model:

undirected network $G = (V, E)$ n nodes e edges,

adjacency matrix $\mathbf{A} = [A_{ij}] \in \mathbb{R}^{n \times n}$,

representations of nodes $\mathbf{U} \in \mathbb{R}^{n \times m}$, m $(m \leq n)$

microscopic structure:

$$\tilde{\mathbf{S}} = \mathbf{S}^{(1)} + \eta \mathbf{S}^{(2)}$$

$$\min \|\mathbf{S} - \mathbf{M}\mathbf{U}^T\|_F^2 \quad s.t. \quad \mathbf{M} \geq 0, \quad \mathbf{U} \geq 0. \qquad (3)$$

nonnegative basis matrix $\mathbf{M} \in \mathbb{R}^{n \times m}$

nonnegative representation matrix $\mathbf{U} \in \mathbb{R}^{n \times m}$,

mesoscopic structure:

K communities(k>2):

$$Q = tr(\mathbf{H}^T\mathbf{B}\mathbf{H}), \quad s.t. \quad tr(\mathbf{H}^T\mathbf{H}) = n,$$

The unified network embedding model:

$$\min_{\mathbf{M},\mathbf{U},\mathbf{H},\mathbf{C}} \|\mathbf{S}-\mathbf{M}\mathbf{U}^T\|_F^2 + \alpha\|\mathbf{H}-\mathbf{U}\mathbf{C}^T\|_F^2 - \beta tr(\mathbf{H}^T\mathbf{B}\mathbf{H}) \tag{4}$$
$$s.t., \mathbf{M} \geqslant 0, \mathbf{U} \geqslant 0, \mathbf{H} \geqslant 0, \mathbf{C} \geqslant 0, tr(\mathbf{H}^T\mathbf{H}) = n,$$

an auxiliary nonnegative matrix $\mathbf{C} \in \mathbb{R}^{k\times m}$, named community representation matrix, where the r-th row ($\mathbf{C}_r$) is the representation of community r.

Thanks

struc2vec: Learning Node Representations from Structural Identity

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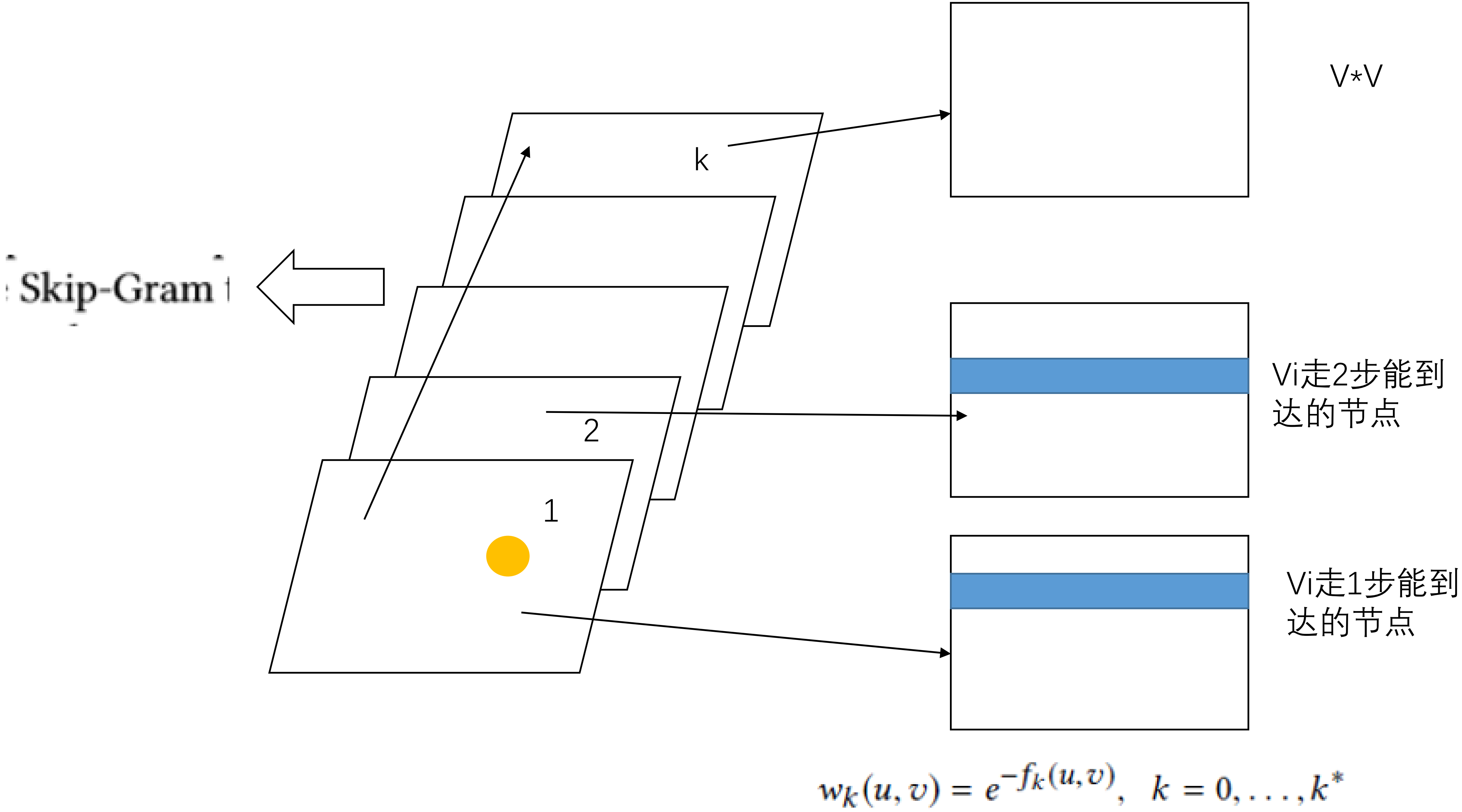
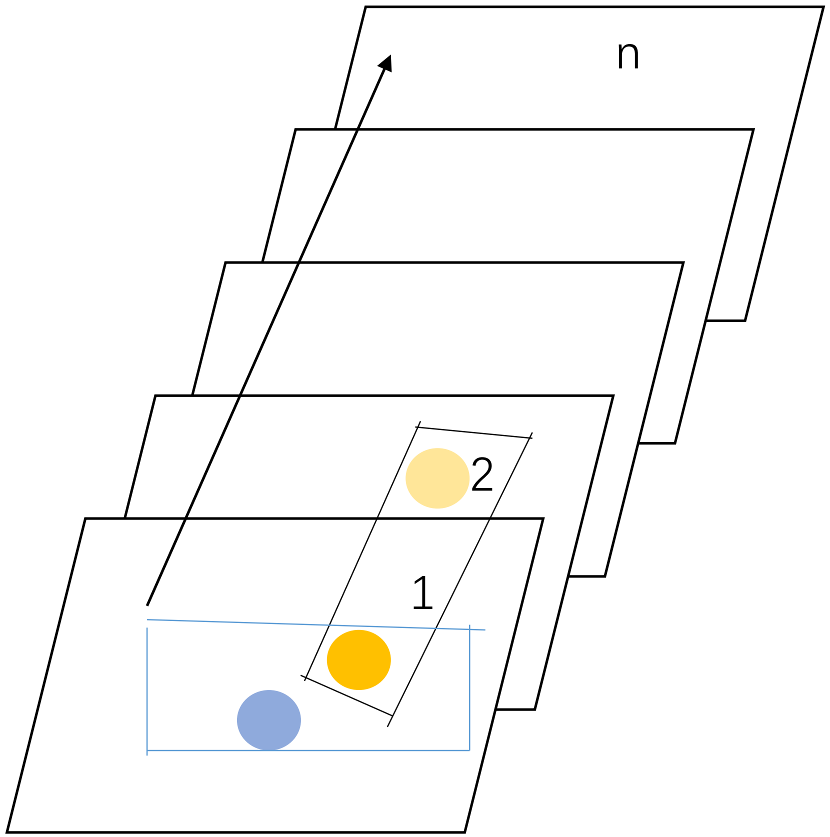


V*V
k
Skip-Gram
Vi走2步能到达的节点
2
1
Vi走1步能到达的节点
$w_k(u,v) = e^{-f_k(u,v)}, \quad k = 0, \ldots, k^*$



Probability q:

Given that it will stay in the current layer, the probability of stepping from node u to node v in layer k is given by:

$$p_k(u,v) = \frac{e^{-f_k(u,v)}}{Z_k(u)} \tag{6}$$

where $Z_k(u)$ is the normalization factor for vertex u in layer k, simply given by:

$$Z_k(u) = \sum_{\substack{v \in V \\ v \neq u}} e^{-f_k(u,v)} \tag{7}$$

Probability 1-q:

$$\begin{aligned} p_k(u_k, u_{k+1}) &= \frac{w(u_k, u_{k+1})}{w(u_k, u_{k+1}) + w(u_k, u_{k-1})} \\ p_k(u_k, u_{k-1}) &= 1 - p_k(u_k, u_{k+1}) \end{aligned} \tag{8}$$