
```

%Assignment 2 Part 1 A
%ELEC 4700
%Umeh Ifeanyi
%101045786

%initializing the dimensions of our matrices, ensuring ny is 3/2
    times nx
nx = 50;
ny = (3/2)*50;

%we are going to need two matrices for this part.
%Not just a G matrix,
%but a matrix we can use for the operations of the G matrix. the
    solution will
%be in the form  $Ax = b$ , and to get  $x$  this will be  $b \backslash A$ , in this case
    it
%will be  $G \backslash Op$ .

G = sparse(nx*ny,nx*ny);
Op = zeros(nx*ny,1);

%filling in the G matrix's bulk nodes and BC's using a loop, similar
%to what we did in PA-5 using the Finite Difference method
for x = 1:nx
    for y = 1:ny

        n = y + (x-1)*ny;

        if x == 1

            G(n,:) = 0;
            G(n,n) = 1;
            Op(n) = 1;

        elseif x == nx

            G(n,:) = 0;
            G(n,n) = 1;
            Op(n) = 0;

        elseif y == 1

            G(n, :) = 0;
            G(n, n) = -3;
            G(n, n+1) = 1;
            G(n, n+ny) = 1;
            G(n, n-ny) = 1;

        elseif y == ny

            G(n, n) = -3;

```

```

        G(n, n-1) = 1;
        G(n, n+ny) = 1;
        G(n, n-ny) = 1;

    else

        G(n, n) = -4;
        G(n, n+1) = 1;
        G(n, n-1) = 1;
        G(n, n+ny) = 1;
        G(n, n-ny) = 1;

    end
end
end

Voltage =      G\Op;

%now, need to create matrix to be surfed (x,y,voltage)

sol = zeros(nx,ny,1);

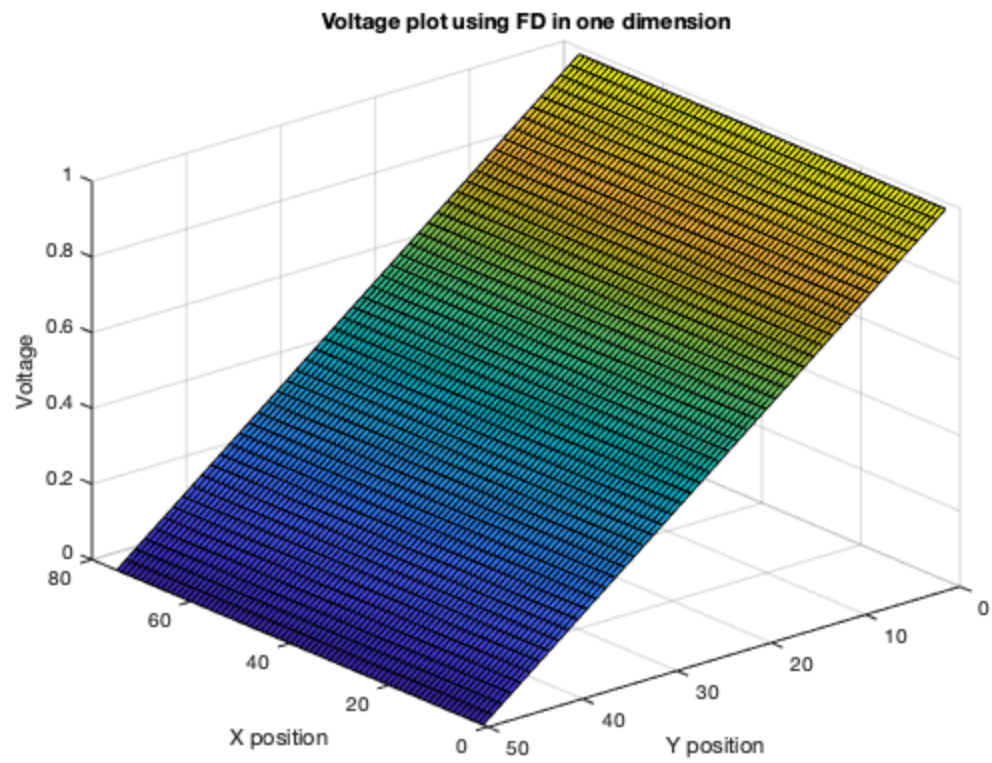
for x = 1:nx
    for y = 1:ny

        n = y + (x-1)*ny;
        sol(x,y) = Voltage(n);

    end
end

figure(1)
surf(sol)
title("Voltage plot using FD in one dimension")
xlabel("X position")
ylabel("Y position")
zlabel("Voltage")
view(-130,30)
%the end

```



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%Assignment 2 Part 1 B
%ELEC 4700
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%initializing the dimensions of our matrices, ensuring ny is 3/2
    times nx
nx = 50;
ny = (3/2)*50;

%In Part B), we are preparing and comparing two solutions. Again, we
    are
%using finite difference method, but this time in 2-D. We are then
    finding
%a solution using the analytical method, which works by iterating to
%complete the summation of an infinite series. It won't, however, be
%infinite in this case. I will provide more discussion at the end of
%this code to further explain, and to answer the questions asked in
    the
%assignment outline

G = sparse(nx*ny,nx*ny);
Op = sparse(nx*ny,1);

%filling in the G matrix's bulk nodes and BC's using a loop, similar
%to what we did in PA-5 using the Finite Difference method

for x = 1:nx
    for y = 1:ny
        n = y + (x-1)*ny;

        if x == 1
            G(n, :) = 0;
            G(n, n) = 1;
            Op(n) = 1;
        elseif x == nx
            G(n, :) = 0;
            G(n, n) = 1;
            Op(n) = 1;
        elseif y == 1
            G(n, :) = 0;
            G(n, n) = 1;
        elseif y == ny
            G(n, :) = 0;
            G(n, n) = 1;
        else
            G(n, n) = -4;
            G(n, n+1) = 1;
            G(n, n-1) = 1;
            G(n, n+ny) = 1;
            G(n, n-ny) = 1;
        end
    end
end

```

```

        end
    end

    Voltage = G\Op;

    %now, need to create matrix to be surfed (x,y,voltage)

    sol = zeros(nx,ny,1);

    for x = 1:nx
        for y = 1:ny
            n = y + (x-1)*ny;
            sol(x,y) = Voltage(n);
        end
    end

    figure(1)
    surf(sol)
    axis tight
    title("Voltage Surface plot using the numerical method in Two
        Dimensions")
    xlabel("X position")
    ylabel("Y position")
    zlabel("Voltage")

    %variables to be used in our analytical solution
    a = ny;
    b = nx/2;

    x2 = linspace(-nx/2,nx/2, 50);
    y2 = linspace(0,ny,ny);

    [i,j] = meshgrid(x2,y2);

    sol2 = sparse(ny,nx);

    %iterating to create a summation of the infinite series (finite in
    this
    %case)

    for n = 1:2:600

        sol2 = (sol2 + (cosh(n*pi*i/a).*sin(n*pi*j/a))./(n*cosh(n*pi*b/
a)))));
        figure(2)
        surf(x2,y2,(4/pi)*sol2)
        title("Voltage Surface plot using the analytical method in Two
            Dimensions")
        xlabel("X position")
        ylabel("Y position")
    end
end

```

```

        xlabel("Voltage")
        axis tight
        view(-130,30);
        pause(0.001)
    end
%the end

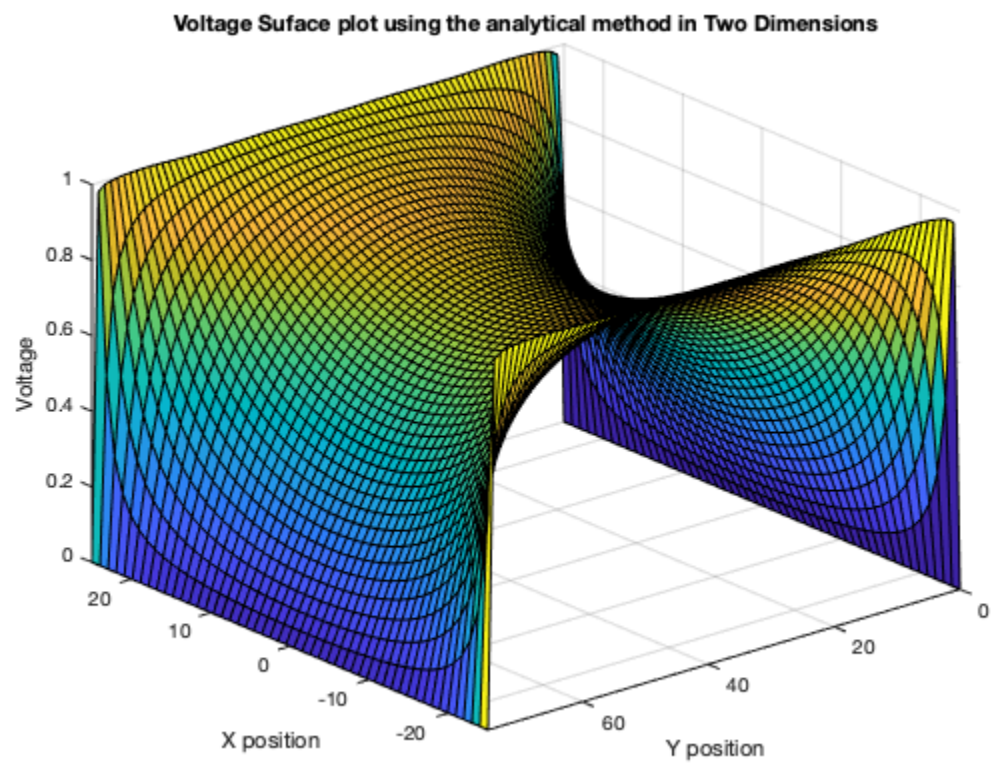
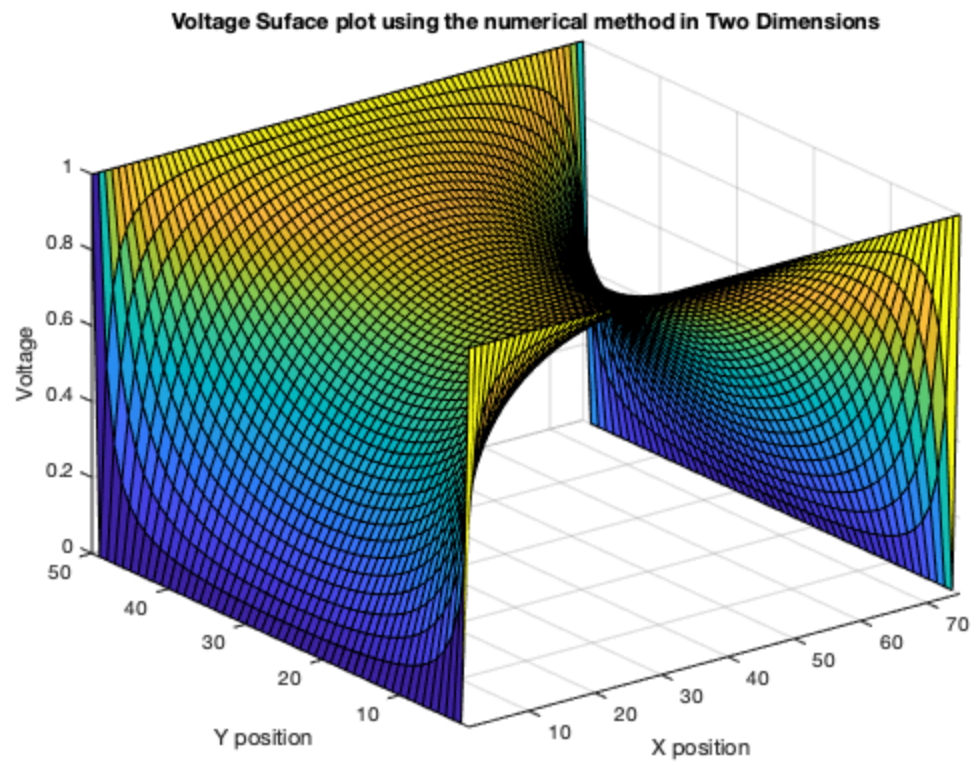
%CONCLUSIONS

%The solution from a series does approach the solution that was
    created
%using the FD method. Note that that we were capped at 600 iterations
%because of the fact that this series equation contained the terms
    cosh and sinh.
%This is the maximum number of iterations that could be used to
    recreate
%the FD solution. When I iterate above 600 the plot no longer looks
    like
%the true solution. This is because the cosh and sinh values approach
%infinity around this value, which increases the error in the
    solution, so
%we should stop at 600 iterations for best results.

%Through judging the results obtained using both methods, I would like
    to
%compare and contrast the strengths and weaknesses of both methods. It
%seems that numerical solutions would be an applicable means of
    finding a
%solution, given that the information you are feeding it is not too
%complicated. It is a method that will work given you have the right
%computing power to handle the equations you throw into it. For very
%complex equations, the hardware one uses may not be able to handle
    it.

%The analytical method, on the other hand, is better (quicker) at
    competing
%simpler equations, and is the method of choice when dealing with
%relatively small data sets (simpler equations). The limitations,
    however,
%can be surmised by observing this part of the assignment. Certain
%iteration values may cause a breakdown in the equation which limits
    it's
%reliable accuracy. One must understand the limits of the equation to
    avoid
%these possible pitfalls.

```



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```
%Assignment 2 Part 2 A%
%ELEC 4700
%Umeh Ifeanyi
%101045786
```

```
%Part 2 a) in this part of the assignment, we are setting up
%5 surface plots: sigma, voltage, the x and y components of the
  electric
%field and finally the current density plot. Comments are littered
%throughout the code to aid in the understanding in my process.
```

```
%setting up variables just like part 1
nx = 50;
ny = (3/2)*nx;
G = sparse(nx*ny);
Op = zeros(1, nx*ny);
```

```
Sigmatrrix = zeros(nx, ny);    % a sigma matrix is required for this
  part
Sig1 = 1;                      % sigma value given outside the box
Sig2 = 10^-2;                  % sigma value given inside the box
```

```
%The box will be difined using a 1x4 matrix containing it's dimensions
box = [nx*2/5 nx*3/5 ny*2/5 ny*3/5];
```

```
for i = 1:nx
    for j = 1:ny
        if i > box(1) && i < box(2) && (j < box(3) || j > box(4))
            Sigmatrrix(i, j) = Sig2;
        else
            Sigmatrrix(i, j) = Sig1;
        end
    end
end
```

```
% Filling in G matrix with corresponding bottleneck conditions
for x = 1:nx
    for y = 1:ny
        n = y + (x-1)*ny;
        nposx = y + (x+1-1)*ny;
        nnegx = y + (x-1-1)*ny;
        nposy = y + 1 + (x-1)*ny;
```

```

nnegy = y - 1 + (x-1)*ny;

if x == 1

    G(n, :) = 0;
    G(n, n) = 1;
    Op(n) = 1;

elseif x == nx

    G(n, :) = 0;
    G(n, n) = 1;
    Op(n) = 0;

elseif y == 1

    G(n, nposx) = (Sigmatrix(x+1, y) + Sigmatrix(x,y))/2;
    G(n, nnegx) = (Sigmatrix(x-1, y) + Sigmatrix(x,y))/2;
    G(n, nposy) = (Sigmatrix(x, y+1) + Sigmatrix(x,y))/2;
    G(n, n) = -(G(n,nposx)+G(n,nnegx)+G(n,nposy));

elseif y == ny

    G(n, nposx) = (Sigmatrix(x+1, y) + Sigmatrix(x,y))/2;
    G(n, nnegx) = (Sigmatrix(x-1, y) + Sigmatrix(x,y))/2;
    G(n, nnegy) = (Sigmatrix(x, y-1) + Sigmatrix(x,y))/2;
    G(n, n) = -(G(n,nposx)+G(n,nnegx)+G(n,nnegy));

else

    G(n, nposx) = (Sigmatrix(x+1, y) + Sigmatrix(x,y))/2;
    G(n, nnegx) = (Sigmatrix(x-1, y) + Sigmatrix(x,y))/2;
    G(n, nposy) = (Sigmatrix(x, y+1) + Sigmatrix(x,y))/2;
    G(n, nnegy) = (Sigmatrix(x, y-1) + Sigmatrix(x,y))/2;
    G(n, n) = -(G(n,nposx)+G(n,nnegx)+G(n,nposy)+G(n,nnegy));

end
end
end

% Sigma(x,y) Surface Plot
figure(1)
surf(Sigmatrix);
xlabel("X position")
ylabel("Y position")
zlabel("Sima")
axis tight
view([40 30]);
title("Sigma Surface Plot in the X and Y Planes")

Voltage = G\Op';

```

```

sol = zeros(ny, nx, 1);
for i = 1:nx
    for j = 1:ny
        n = j + (i-1)*ny;
        sol(j,i) = Voltage(n);
    end
end

%V(x,y) Surface Plot
figure(2)
surf(sol)
axis tight
xlabel("X position")
ylabel("Y position")
zlabel("Voltage")
view([40 30]);
title("Voltage Surface Plot with Given Bottleneck Conditions")

%The electric field can be derived from the surface voltage using a
%gradient

[elec_x, elec_y] = gradient(sol);

%X component of electric field surface plot
figure(3)
surf(-elec_x)
axis tight
xlabel("X position")
ylabel("Y position")
zlabel("Electric Field")
view([40 30]);
title("The Surface Plot of the X-component of the Electric Field")

%Y component of electric field surface plot
figure(4)
surf(-elec_y)
axis tight
xlabel("X position")
ylabel("Y position")
zlabel("Electric Field")
view([40 30]);
title("The Surface Plot of the Y-component of the Electric Field")

%J, the current density, is calculated by multiplying sigma and the
%electric field together. Combining the x and y matrices, a surface plot
is
%derived by surfing this matrix.

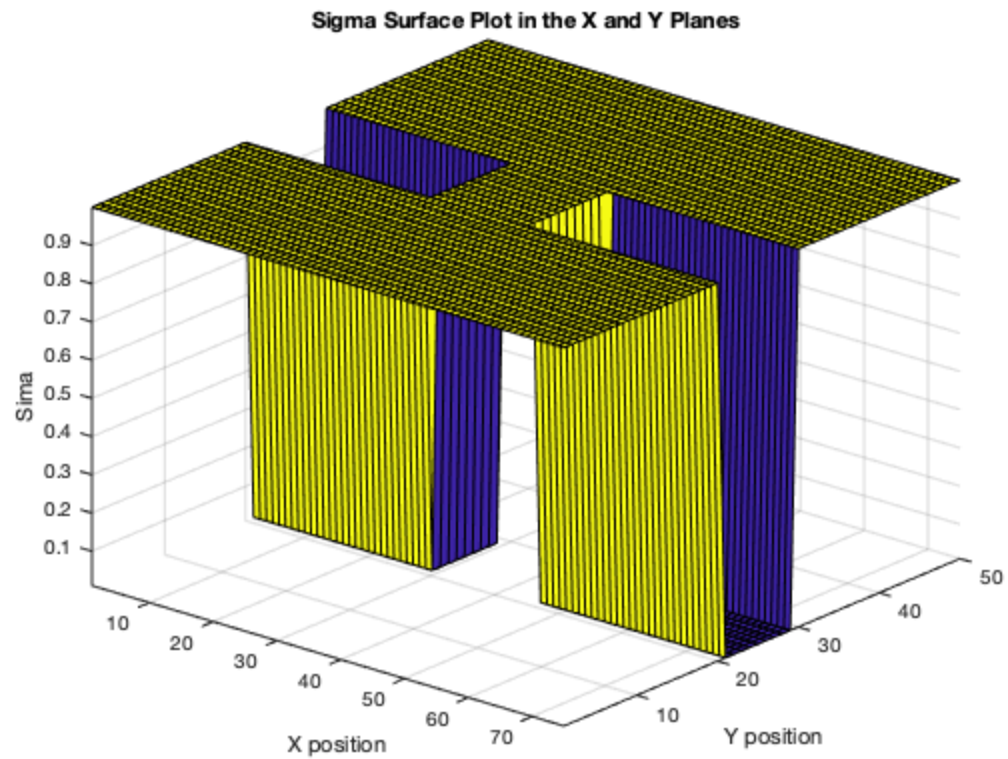
J_x = Sigmatrix'.*elec_x;
J_y = Sigmatrix'.*elec_y;
J = sqrt(J_x.^2 + J_y.^2);

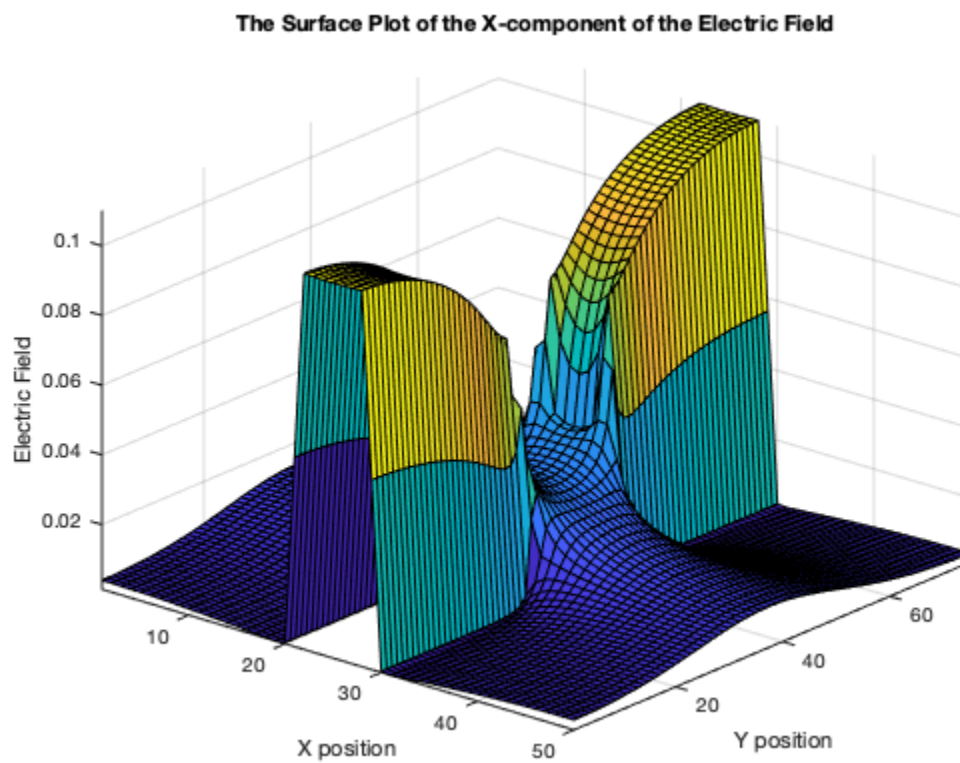
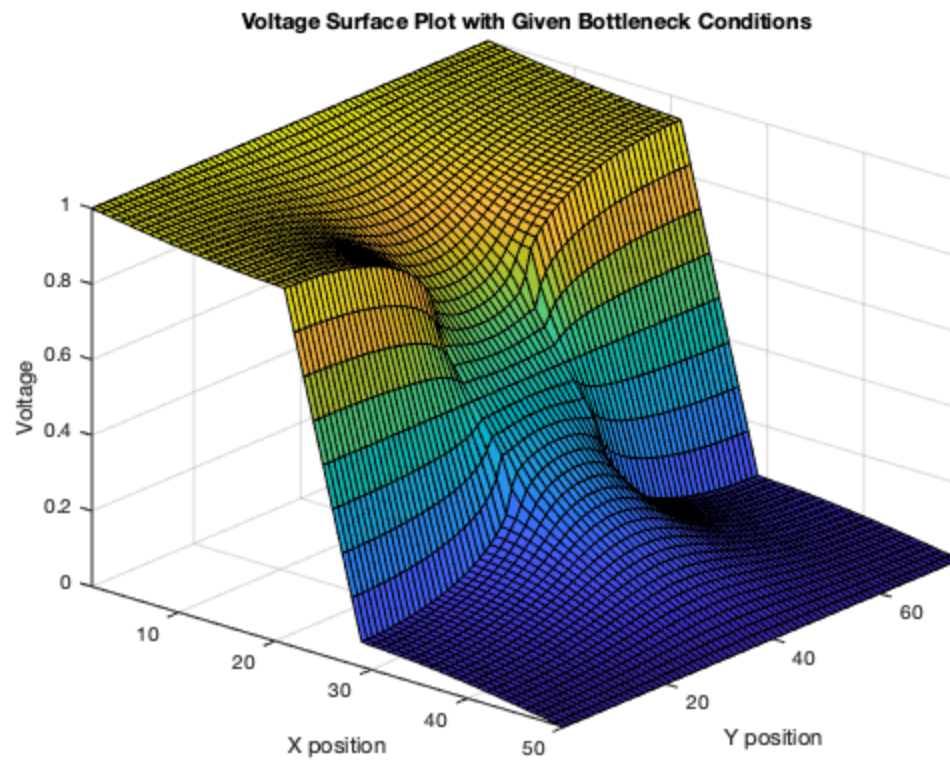
%J(x,y) Surface Plot
figure(5)

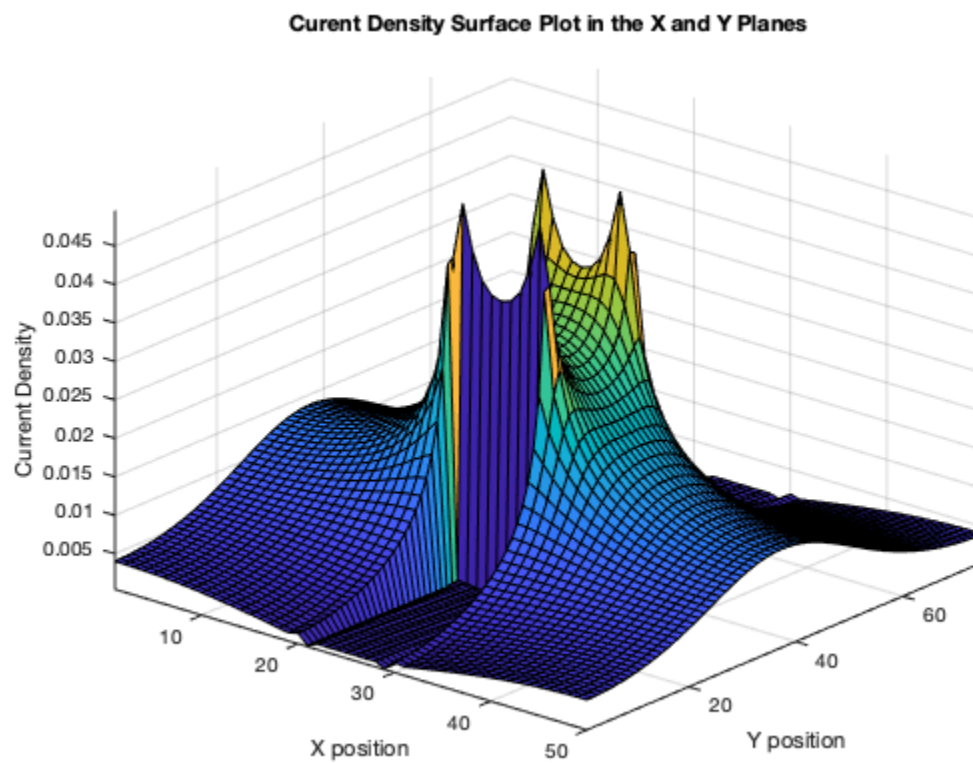
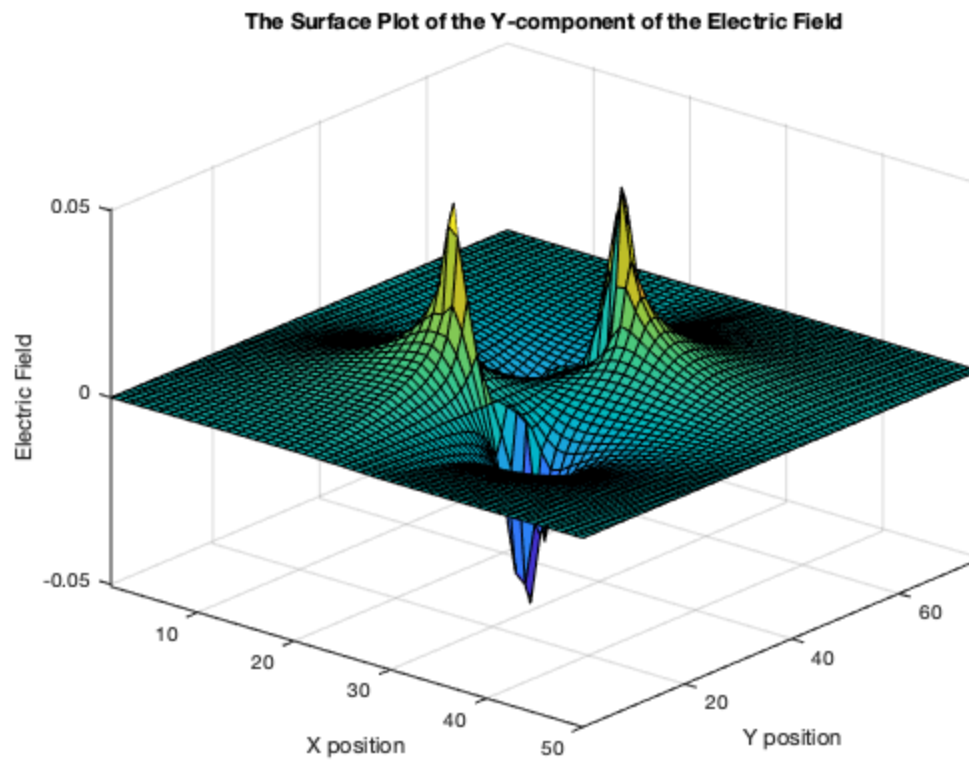
```

```
surf(J)
axis tight
xlabel("X position")
ylabel("Y position")
zlabel("Current Density")
view([40 30]);
title("Curent Density Surface Plot in the X and Y Planes")

%the end%
```







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```

%Assignment 2 Part 2 B
%ELEC 4700
%Umeh Ifeanyi
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% In this part of the assignment we are
% investigating mesh density. To do this we will start at a mesh size
% multiple
% of 10, and incrementally increase this size to observe the effect on
% the
% current density.

%nx will be incrementally increased using this loop. Note that
% meshsize
% replaces nx in this code.
for meshsize = 10:10:100

    %multiplying these values by the respective meshsize
    ny = (3/2)*meshsize;
    G = sparse(meshsize*ny);
    Op = zeros(1, meshsize*ny);

    Sigmatrix = zeros(ny, meshsize);           % The sigma matrix with nx
    set to meshsize
    Sig1 = 1;                                   % sigma value given outside
    the box
    Sig2 = 10^-2;                               % sigma value given inside
    the box

    %bottleneck conditions with meshsize replacing nx
    box = [meshsize*2/5 meshsize*3/5 ny*2/5 ny*3/5];

    %Filling in G matrix
    for x = 1:meshsize

        for y = 1:ny

            n = y + (x-1)*ny;

            if x == 1
                G(n, :) = 0;
                G(n, n) = 1;
                Op(n) = 1;

            elseif x == meshsize
                G(n, :) = 0;
                G(n, n) = 1;
                Op(n) = 0;

            elseif y == 1

```

```

        if x > box(1) && x < box(2)
            G(n, n) = -3;
            G(n, n+1) = Sig2;
            G(n, n+ny) = Sig2;
            G(n, n-ny) = Sig2;

        else

            G(n, n) = -3;
            G(n, n+1) = Sig1;
            G(n, n+ny) = Sig1;
            G(n, n-ny) = Sig1;

        end

    elseif y == ny

        if x > box(1) && x < box(2)

            G(n, n) = -3;
            G(n, n+1) = Sig2;
            G(n, n+ny) = Sig2;
            G(n, n-ny) = Sig2;

        else

            G(n, n) = -3;
            G(n, n+1) = Sig1;
            G(n, n+ny) = Sig1;
            G(n, n-ny) = Sig1;

        end

    else

        if x > box(1) && x < box(2) && (y < box(3) || y >
box(4))

            G(n, n) = -4;
            G(n, n+1) = Sig2;
            G(n, n-1) = Sig2;
            G(n, n+ny) = Sig2;
            G(n, n-ny) = Sig2;

        else

            G(n, n) = -4;
            G(n, n+1) = Sig1;
            G(n, n-1) = Sig1;
            G(n, n+ny) = Sig1;
            G(n, n-ny) = Sig1;

        end

```

```

        end
    end
end

%Just like in part a), except using different meshsizes
for Length = 1 : meshsize

    for Width = 1 : ny

        if Length >= box(1) && Length <= box(2)
            Sigmatrix(Width, Length) = Sig2;

        else

            Sigmatrix(Width, Length) = Sig1;

        end

        if Length >= box(1) && Length <= box(2) && Width >= box(3)
        && Width <= box(4)

            Sigmatrix(Width, Length) = Sig1;

        end

    end
end

Voltage = G\Op';

sol = zeros(ny, meshsize, 1);

for i = 1:meshsize

    for j = 1:ny

        n = j + (i-1)*ny;
        sol(j,i) = Voltage(n);

    end
end

%electric field found using gradient of voltage
[elec_x, elec_y] = gradient(sol);

%current density is sigma times electric field
J_x = Sigmatrix.*elec_x;
J_y = Sigmatrix.*elec_y;
J = sqrt(J_x.^2 + J_y.^2);

%plotting current density vs mesh size
figure(1)
hold on

```

```

    if meshsize == 10

        Curr = sum(J, 1);
        Currtot = sum(Curr);
        Currold = Currtot;
        plot([meshsize, meshsize], [Currold, Currtot])

    end
    if meshsize > 10

        Currold = Currtot;
        Curr = sum(J, 2);
        Currtot = sum(Curr);
        plot([meshsize-10, meshsize], [Currold, Currtot])
        xlabel("Meshsize")
        ylabel("Current Density")

    end

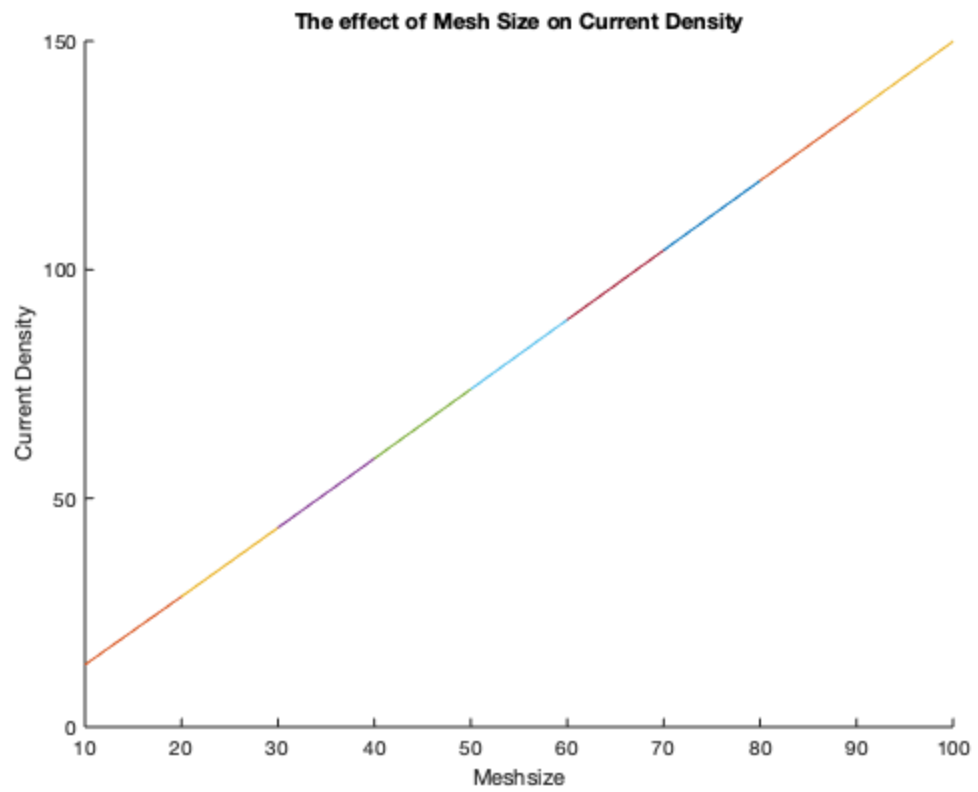
    title("The effect of Mesh Size on Current Density")

end

%the end%

%DISCUSSION%

%Analyzing the results of the plot, we see that meshsize and current
%density are proportional; an increase in meshsize leads to an
increase in
%current density, which is to be expected.
```



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```
%Assignment 2 Part 2 B
%ELEC 4700
%Umeh Ifeanyi
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%In part 2c) we are investigating narrowing of the bottle neck, this
    is
%done by changing the y valyes of the box that we used in parts a and
    b.
%Again, we are going to loop through values to multiply the y values
    and
%observe the effects this has on current. stay tuned for more

for bottleneck = 0.1:0.01:0.9

    %setting up variable matrices like in part 1
    nx = 50;
    ny = nx*3/2;
    G = sparse(nx*ny);
    Op = zeros(1, nx*ny);

    Sigmatrix = zeros(ny, nx); % a sigma matrix is required for this
part
    Sig1 = 10^-2;                % sigma value given outside the box
    Sig2 = 1;                    % sigma value given inside the box

    %The bottleneck is incrementally "narrowed" by modifying the y
values
    %of the box
    box = [nx*2/5 nx*3/5 ny*bottleneck ny*(1-bottleneck)];

    %filling in the G matrix
    for i = 1:nx

        for j = 1:ny

            n = j + (i-1)*ny;

            if i == 1

                G(n, :) = 0;
                G(n, n) = 1;
                Op(n) = 1;

            elseif i == nx

                G(n, :) = 0;
                G(n, n) = 1;
                Op(n) = 0;

            elseif j == 1
```

```

        if i > box(1) && i < box(2)

            G(n, n) = -3;
            G(n, n+1) = Sig1;
            G(n, n+ny) = Sig1;
            G(n, n-ny) = Sig1;

        else

            G(n, n) = -3;
            G(n, n+1) = Sig2;
            G(n, n+ny) = Sig2;
            G(n, n-ny) = Sig2;

        end

    elseif j == ny

        if i > box(1) && i < box(2)
            G(n, n) = -3;
            G(n, n+1) = Sig1;
            G(n, n+ny) = Sig1;
            G(n, n-ny) = Sig1;

        else

            G(n, n) = -3;
            G(n, n+1) = Sig2;
            G(n, n+ny) = Sig2;
            G(n, n-ny) = Sig2;

        end

    else

        if i > box(1) && i < box(2) && (j < box(3) || j >
box(4))

            G(n, n) = -4;
            G(n, n+1) = Sig1;
            G(n, n-1) = Sig1;
            G(n, n+ny) = Sig1;
            G(n, n-ny) = Sig1;

        else

            G(n, n) = -4;
            G(n, n+1) = Sig2;
            G(n, n-1) = Sig2;
            G(n, n+ny) = Sig2;
            G(n, n-ny) = Sig2;

        end

```

```

        end
    end
end

for Length = 1 : nx

    for Width = 1 : ny

        if Length >= box(1) && Length <= box(2)
            Sigmatrix(Width, Length) = Sig1;

        else
            Sigmatrix(Width, Length) = Sig2;

        end

        if Length >= box(1) && Length <= box(2) && Width >= box(3)
&& Width <= box(4)

            Sigmatrix(Width, Length) = Sig2;

        end
    end
end

Voltage = G\Op';

sol = zeros(ny, nx, 1);

for i = 1:nx

    for j = 1:ny

        n = j + (i-1)*ny;
        sol(j,i) = Voltage(n);

    end
end

%The electric field can be derived from the surface voltage using
a
%gradient
[elec_x, elec_y] = gradient(sol);

%J, the current density, is calculated by multiplying sigma and
the
%electric field together.

J_x = Sigmatrix.*elec_x;
J_y = Sigmatrix.*elec_y;
J = sqrt(J_x.^2 + J_y.^2);

```

```
%plotting bottleneck vs current
figure(1)
hold on

if bottleneck == 0.1

    Curr = sum(J, 2);
    Currtot = sum(Curr);
    Currold = Currtot;
    plot([bottleneck, bottleneck], [Currold, Currtot])

end

if bottleneck > 0.1

    Currold = Currtot;
    Curr = sum(J, 2);
    Currtot = sum(Curr);
    plot([bottleneck-0.01, bottleneck], [Currold, Currtot])
    xlabel("Bottleneck");
    ylabel("Current Density");

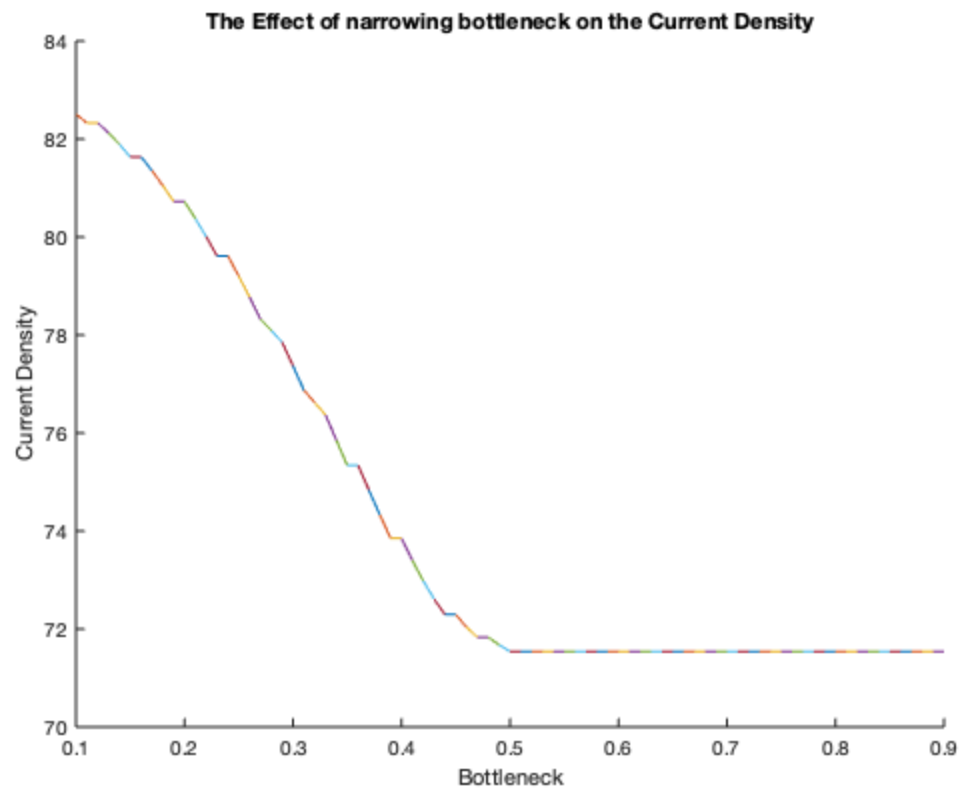
end

title("The Effect of narrowing bottleneck on the Current Density")

end

%DISCUSSION%

%Observing the plot, we see that narrowing the bottleneck
incrementally
%leads to a decrease in the current value, However, after a certain
point,
%when the value of narrowing reaches 0.5, the current stagnates and
does
%not decrease any more and stays fixed at about 71.5. Note that the
%relationship is not a linear decrease, but resembles an exponential
%decrease before current density stops decreasing.
```

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%In Part 2d) of the assignment, we are observing the effect of varying
%sigma on the current density. Like in parts c) and d), we are
iterating
%through a loop using different sigma values and plotting sigma vs
current
%density, and then drawing a conclusion from the plot.

```
for sigma = 1e-2:1e-2:0.9
```

```
    %setting up variable matrices like in part 1
```

```
    nx = 50;
```

```
    ny = nx*3/2;
```

```
    G = sparse(nx*ny);
```

```
    Op = zeros(1, nx*ny);
```

```
    Sigmatrix = zeros(ny, nx);
```

```
    % a sigma matrix is
```

```
    required for this part
```

```
    Sig1 = 1;
```

```
    % sigma value given outside
```

```
    the box
```

```
    Sig2 = sigma;
```

```
    % sigma inside box will be
```

```
    modified
```

```
    %bottleneck remains the same this time.
```

```
    box = [nx*2/5 nx*3/5 ny*2/5 ny*3/5];
```

```
    for x = 1:nx
```

```
        for y = 1:ny
```

```
            n = y + (x-1)*ny;
```

```
            if x == 1
```

```
                G(n, :) = 0;
```

```
                G(n, n) = 1;
```

```
                Op(n) = 1;
```

```
            elseif x == nx
```

```
                G(n, :) = 0;
```

```
                G(n, n) = 1;
```

```
                Op(n) = 0;
```

```
            elseif y == 1
```

```
                if x > box(1) && x < box(2)
```

```
                    G(n, n) = -3;
```

```
                    G(n, n+1) = Sig2;
```

```

        G(n, n+ny) = Sig2;
        G(n, n-ny) = Sig2;

    else

        G(n, n) = -3;
        G(n, n+1) = Sig1;
        G(n, n+ny) = Sig1;
        G(n, n-ny) = Sig1;

    end

elseif y == ny

    if x > box(1) && x < box(2)
        G(n, n) = -3;
        G(n, n+1) = Sig2;
        G(n, n+ny) = Sig2;
        G(n, n-ny) = Sig2;

    else

        G(n, n) = -3;
        G(n, n+1) = Sig1;
        G(n, n+ny) = Sig1;
        G(n, n-ny) = Sig1;

    end

else

    if x > box(1) && x < box(2) && (y < box(3) || y >
box(4))

        G(n, n) = -4;
        G(n, n+1) = Sig2;
        G(n, n-1) = Sig2;
        G(n, n+ny) = Sig2;
        G(n, n-ny) = Sig2;

    else

        G(n, n) = -4;
        G(n, n+1) = Sig1;
        G(n, n-1) = Sig1;
        G(n, n+ny) = Sig1;
        G(n, n-ny) = Sig1;

    end

end

end

end

```

```

for Length = 1 : nx

    for Width = 1 : ny

        if Length >= box(1) && Length <= box(2)
            Sigmatrix(Width, Length) = Sig2;

        else

            Sigmatrix(Width, Length) = Sig1;

        end

        if Length >= box(1) && Length <= box(2) && Width >= box(3)
            && Width <= box(4)

                Sigmatrix(Width, Length) = Sig1;

            end

        end

    end

Voltage = G\Op';

sol = zeros(ny, nx, 1);

for x = 1:nx

    for y = 1:ny

        n = y + (x-1)*ny;

        sol(y,x) = Voltage(n);

    end

end

[elec_x, elec_y] = gradient(sol);

J_x = Sigmatrix.*elec_x;
J_y = Sigmatrix.*elec_y;
J = sqrt(J_x.^2 + J_y.^2);

figure(1)
hold on
if sigma == 0.01
    Curr = sum(J, 2);
    Curr_tot = sum(Curr);
    Curr_old = Curr_tot;

```

```

        plot([sigma, sigma], [Currold, Currtot])
    end
    if sigma > 0.01
        Currold = Currtot;
        Curr = sum(J, 2);
        Currtot = sum(Curr);
        plot([sigma-0.01, sigma], [Currold, Currtot])
        xlabel("Sigma")
        ylabel("Current Density")
    end
    title("The Effect of varying the sigma value on Current Density")

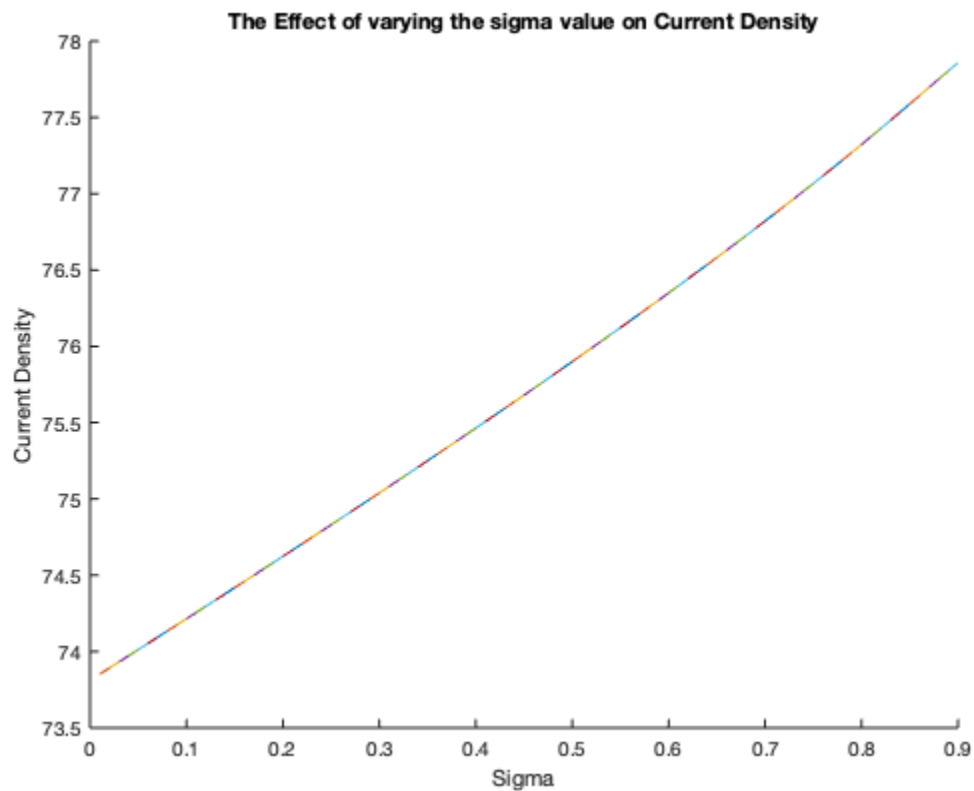
end

%the end%

%DISCUSSION%

%From the plot it is noticed that sigma and current density are
%proportional; an increase in sigma leads to an increase in current
%density. This relationship is linear, which is to be expected from
the
%formula  $J = \sigma \times \text{electric field}$ .

```



Published with MATLAB® R2019b