

CSE 15: Homework 3

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February 26, 2020

Knight and Knaves

1. One day a traveller was wandering around the island of Knights and Knaves, when he encountered two local inhabitants, P and Q. The traveller asked: “Is any of you a knave?”. P replied: “At least one of us is a knave”. Can you find out what P and Q are? If so, what are they? If not, explain why not, and what other

Answer: P is the knight, while Q is knave.

2. Later on, the traveller met two other locals, A and B. He asked whether either of them is a knight. A replied: “If B is a knave, then I am a knave too”. What are A and B?

Answer: A is a Knight and B is also a Knight

Logical Identities

Simplify the following propositions. Show all steps of your solutions.

1. $\neg(p \rightarrow (q \rightarrow p))$

- $\neg(\neg p \wedge (q \rightarrow p))$

- $p \wedge \neg(q \rightarrow p)$

- $p \wedge \neg(\neg q \vee p)$

- $p \wedge (q \wedge \neg p)$

- $p \wedge \neg p \wedge q$

- $p \wedge \neg p = \text{False}$

Answer: $\text{False} \wedge q = \text{False}$

2. $\neg((p \wedge q) \rightarrow (q \vee p))$

- $\neg(\neg(p \wedge q) \vee (q \vee p))$
- $(p \wedge q) \wedge \neg(q \vee p)$
- $(p \wedge q) \wedge \neg q \wedge \neg p$
- $(p \wedge \neg p) \wedge \neg q \wedge \neg q$

Answer: $\text{False} = \text{False}$

Logical Equivalences

Determine whether or not the following pairs of propositions are equivalent. Show all steps.

1. $p \rightarrow (q \rightarrow r)$ and $(p \wedge q) \rightarrow r$

q	r	p	$q \rightarrow r$	$p \wedge q$	$p \rightarrow (q \rightarrow r)$	$(p \wedge q) \rightarrow r$
F	F	F	T	F	T	T
F	F	T	T	F	T	T
F	T	F	T	F	T	T
F	T	T	T	F	T	T
T	F	F	F	F	T	T
T	F	T	F	T	F	F
T	T	F	T	F	T	T
T	T	T	T	T	T	T

Answer: The final two columns depicting the pair of propositions are equivalent to one another

2. $p \rightarrow (q \rightarrow r)$ and $(p \rightarrow q) \rightarrow r$

q	r	p	$q \rightarrow r$	$p \rightarrow q$	$p \rightarrow (q \rightarrow r)$	$(p \rightarrow q) \rightarrow r$
F	F	F	T	T	T	F
F	F	T	T	F	T	T
F	T	F	T	T	T	T
F	T	T	T	F	T	T
T	F	F	F	T	T	F
T	F	T	F	T	F	F
T	T	F	T	T	T	T
T	T	T	T	T	T	T

Answer: The pair of propositions are not equivalent as the final two columns show.

Logical Consequence

1. Jimmy is smart
Smart people are rich
Jimmy is rich

Answer: It is impossible for the conclusion to be false if we assume the current conditions to be true, therefore this is a valid argument

2. Islands are surrounded by water
Puerto Rico is surrounded by water
Puerto Rico is an island

Answer: It is not valid to label Puerto Rico as an island solely because it is surrounded by water, since being an island is a subset of being surrounded by water.