Implications of The Assumption of Parametric Time on Wavefunction Collapse and A Resulting Analogy With Killing Horizon Thermodynamics

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Abstract

Two basic assumptions are applied here which enable us to probe the nature of wavefunction collapse. Firstly, we expect all levels of physics to be described in terms of systems in states. Secondly, time is parametric and not associated with an operator. From these assumptions we argue the existence of a thermodynamic nonunitary operator governing collapse, and analyze its behaviour. We then note that the property of wavefunction collapse to conceal a systems previous states from an observer is similar to the concealment of the past of an infalling system at a future Killing Horizon. This leads to the reinterpretation of the behaviour of the redshift factor at a horizon in a stationary spacetime as a manifestation of wavefunction collapse, and so Hawking radiation as a consequence of the same phenomenon.

1 Introduction

It is well known that wavefunction collapse stands out as a fundamental incompleteness to Quantum Mechanics [1, 8, 9]. Aside from being so far unrigorously described in terms of underlying equations of motion, it bears numerous unphysical properties, such as instantaneousness, nonlocality, and information loss [4, 3]. Although there have been many attempts to describe collapse, none have been very well agreed upon. The most succesful such description has been decoherence, which fails to explain nonlocality or derive the born rule, and is not generally considered a complete description.

In this writing we will not attempt to fully describe the nature of collapse, but instead deduce its basic properties from the time evolution of physical entities. Ultimately, we find that collapse is characterized by a shift from Quantum Mechanical evolution governed by the Hamiltonian H to Statistical Mechanical evolution governed by a Heat operator Q. We then associate the thermodynamic, information concealing nature of collapse with that of gravitational event horizons, such as to identify $Q \propto \mathcal{L}_\xi$ where ξ is a horizon generating killing field. We will use $\hbar = k = c = 1$ and signature $(+ - --)$.

2 Time Evolution

There are two natural assumptions on which the following derivations will be dependent. The first and most basic is that at all levels of description systems should be realizable as "states". Perhaps these states are fundamentally a set of quantities which can be measured with some accuracy or certainty, perhaps they are collections of single bits, perhaps they are only definable in terms of their interactions with eachother. Exactly what states are will not be important, as long as they carry a "dynamics" which can in some sense be observed by an observer system. The meaning of "dynamics" of course depends on the nature of time in the relevant framework. Inspired by failures concerning the definition of a time operator, as well as the success of Quantum Field Theory, we shall assume time to be continuous and parametric in nature. The meaning of "dynamics" is therefore trivial to define, and can be analyzed in terms of familiar tools.

Let us assume we have a system in state $|\psi(t_0)\rangle$ at proper time t_0, and that the state evolves in some deterministic way. This can be expressed as

$$U(t)\,|\psi(t)\rangle = |\psi(t_0 + t)\rangle \tag{1}$$

The nature of time ensures

$$U(t_1)U(t_2) = U(t_1 + t_2) \tag{2}$$

and therefore the time-evolution operator forms a group, which by a simple variation on Stone's Theorem can be written as

$$U(t) = e^{iHt} \tag{3}$$

where we are now working in a linear representation of that group (with no loss of generality). Imagine now that the state undergoes some process, and that there exists an observer that does not know which process it undergoes. The process must still occur in a finite time, and express itself in reality as a transformation on the state of the system. Depending on what the observer knows of the process, it would see the state undergo $A(t)\,|\psi(t)\rangle$ with probability $p(A, |\psi\rangle)$

$$A(t) = e^{iGt} \tag{4}$$

Experimentation tells us that U is always unitary, and that for an instantaneous measurement M, wavefunction collapse occurs with probability [7]

$$p(M) = \mathrm{Tr}(\rho P) \tag{5}$$

where ρ is the density matrix, and $P = M^\dagger M$ the probability operator. Interestingly, if $M = U$, then Eqn. (5) just evaluates to 1, ie. an observer knows the outcome with certainty. This motivates the conclusion that the probability a state undergoes a specific evolution $A(t)$ is $p(M = A)$.

If we write

$$G = H + iQ \tag{6}$$

$$\implies A(t) = e^{iHt-Qt} \tag{7}$$

then $Q = 0$ implies $p(A) = 1$. Again, through experimentation, we know that if $Q = 0$ then H is the Hamiltonian and represents the energy of the system. It is also well known that Quantum and Statistical Mechanics are related to eachother via a Wick rotation, such that if we have e^{iHt} in QM, then we expect something like e^{-Ht} in SM. This hints that perhaps Q is an energy in a SM framework. Furthermore, $Q \neq 0$ implying nondeterminism associates Q with a loss of information, or inaccessible information. The Statistical Mechanical energy associated with inaccessible information is just heat, and so we conclude that Q is the heat of the system, and that wavefunction collapse is a deterministic Quantum system becoming Statistical. Such a system would obey

$$\delta S = \beta \delta Q \tag{8}$$

where β is the inverse temperature, and S is the entropy. Because Q is an operator, these must be operators as well, so really what we mean is $\mathrm{Tr}(\rho\beta)$ and $\mathrm{Tr}(\rho S)$ are the expected temperature and entropy. Note, these quantities are dealing with probabilities about the system at a later time after some evolution. Thus, S would not be the Von Neumann entropy operator $\ln \rho$ dealing with an ensemble of states, but instead $S = \ln P$ dealing with an ensemble of *histories*. Substituting this into Eqn. (8)

$$P^{-1}\delta P = \beta \delta Q \tag{9}$$

where the specific ordering of the operators is not necessarily knowable. Given $P = A^\dagger A$ we get

$$\begin{aligned}
\delta P &= A^\dagger \delta A + \delta A^\dagger A \\
&= itA^\dagger A\delta G + (itA\delta G)^\dagger A \\
&= it\big(P\delta G - \delta G^\dagger P\big) \\
1 &= it\Big(\frac{\delta H}{\delta P}P - P\frac{\delta H}{\delta P}\Big) - t\Big(P\frac{\delta Q}{\delta P} + \Big(\frac{\delta Q}{\delta P}\Big)^\dagger P\Big)
\end{aligned} \tag{10}$$

The LHS has no imaginary part, so the first term vanishes. Then using Eqn.(9)

$$1 = -t(\beta^{-1} + (\beta^{-1})^\dagger) \tag{11}$$

and so the real part of the temperature is inversely proportional to the time elapsed by the process. An eigenstate $|\psi_i\rangle$ of G with eigenvalue $E_i + iq_i$ would then evolve according to $A(t)$ with probability

$$\begin{aligned} p &= \mathrm{Tr}\big(|\psi_i\rangle\langle\psi_i| A^\dagger A\big) \\ &= \mathrm{Tr}\Big(|\psi_i\rangle\langle\psi_i| e^{i(-E_i+iq_i)t} e^{i(E_i+iq_i)t}\Big) \\ &= e^{-2q_i t} \end{aligned} \tag{12}$$

Using Eqn.(11)

$$p = \exp\left(\frac{q_i}{\mathrm{Re}(T)}\right) \tag{13}$$

which is very similar to Boltzmann's distribution. Although seemingly profound, this is simply because Quantum Mechanically $\delta\mathcal{S} \approx E\delta t$ (for action $\mathcal{S}$) , while thermodynamically $\delta S \approx \beta\delta Q$, which itself arose from the identification of e^{-Qt} as a thermodynamic evolution operator. These calculations thus ultimately come from the fact that, again, we are dealing with an ensemble of histories, such that time elapsed where an observer is uncertain about some system becomes something like that systems inverse temperature; that is, Eqn. (13) is nothing more than a consistency of these ideas, and the real physics lies in Eqn. (11). In fact, Eqn.(13) isn't even perfectly Boltzmann: if $P = A$ rather than $P = A^\dagger A$, then $\mathrm{Re}(T) \to T$, and we would have the normal formula. Perhaps then Eqn.(13) *is* interesting in that it hints that demanding real temperatures implies the Born rule, although this is not very strong, and not worth dwelling on without a futher understanding of these ideas.

More interesting than these thermodynamic analyses is the form of Eqn.(12), which tells us that evolutions occuring over a longer time, and with higher energy, are less likely. All possible evolutions will broadly obey these relations, and so to ensure normalization as time passes, more histories must become possible. This is exactly as we would expect: the more time that passes where we do not know how a system evolves, the less certainty we have about its state. More precisely, the normalization requires

$$\int \mathcal{D}G A^\dagger A = \int \mathcal{D}G e^{-iG^\dagger t} e^{iGt} = 1 \tag{14}$$

where $\mathcal{D}G$ refers to the integral over all possible values of the matrix G. It is obvious that this cannot be solved without introducing a time dependency into G, such that we should instead use

$$\int \mathcal{D}G e^{-i\int dt G^\dagger} e^{i\int dt G} = 1 \tag{15}$$

But even so the LHS will have a time dependency which must vanish. The issue here is that we have not been sufficiently selective about which measurements we use. Take for example the typical instantaneous measure of energy in terms of projection operators $P_i = |\psi_i\rangle \langle\psi_i|$. Here it is the sum over all P_i which is normalized, not the sum over all possible matrices which can be formed in the basis of energy eigenstates $|\psi_i\rangle$. Thus we expect something like

$$\sum_i e^{-i\int dtG_i^\dagger} e^{i\int dtG_i} = 1 \tag{16}$$

however most generally have

$$\int \mathcal{D}G f(G, t_1, t_2) e^{-i\int dtG^\dagger} e^{i\int dtG} = 1 \tag{17}$$

where f is essentially a degeneracy with support on allowed histories only, and t_1 and t_2 are the limits of the integrals. The general decay of the nonunitary part of the evolution operators then demands that as time passes, more futures gain support, again as expected.

3 Wavefunction Collapse and Killing Horizons

In the previous section it was shown that if we treat time as we normally would during wavefunction collapse, we should achieve some sort of thermodynamic behaviour in the evolution of our quantum state emerging from a statistics on the ensemble of possible histories of that state. In particular, we identified the energy like term associated with this evolution with a heat on the basis of its association with information concealment.

Something similar is observed at killing horizons, where there is a local thermalization of states, and information is concealed over the boundary. This comparison quite naturally leads to the idea that event horizons cause a non-unitary, wavefunction collapse like event. In [6], T. Jacobson shows that treating the boost current of a locally static killing horizon like a heat flow, and applying the first thermodynamic law, retrieves Einstein's field equations; this shows us we can associate heat flow with a flow of matter along the killing field ξ normal to and generating the horizon. A wavefunction evolving like this obeys

$$\psi' = (1 + i\delta t\xi^\mu\partial_\mu)\psi \tag{18}$$

integrating over this

$$\psi' = e^{i\int dt\xi^\mu\partial_\mu}\psi \tag{19}$$

The group of pullbacks ϕ_t generated by ξ will give rise to the conserved quantity $m\xi_\mu\dot{x}^\mu(\tau)$ for a mass m particle moving along geodesic $x(\tau)$. Thus $H = i\xi^\mu\partial_\mu$ or, more generally, $H = i\mathcal{L}_\xi$. Wavefunctions have no well-defined geodesics, but it is sensical to do $m\dot{x}^\mu \to k^\mu$ is the wavector, and so

$$i\mathcal{L}_\xi\psi = \xi_\mu k^\mu\psi \tag{20}$$

A stationary spacetime will be equipped with a killing field ξ. If we choose coordinates such that $\xi^\mu = (1,0,0,0)$ then the scalar solution becomes

$$\psi = e^{ig_{tt}\omega t} \tag{21}$$

where $k^t = \omega$. A stationary wavefunction will have $k = \xi/|\xi|$ and so

$$\begin{aligned}\psi &= e^{i\frac{\xi\cdot\xi}{|\xi|}t} \\ &= e^{i\sqrt{\xi\cdot\xi}t}\end{aligned} \tag{22}$$

At a killing horizon $\xi \cdot \xi = 0$, such that the norm squared will either change sign or have an extremum. If there is an extremum then

$$n^\mu \nabla_\mu (\xi^\nu \xi_\nu) = 0 \tag{23}$$

where n^μ is some vector transverse to the horizon satisfying $n^\mu \xi_\mu = 1$. Using the Killing Equation it can be shown that

$$\nabla_\mu (\xi^\nu \xi_\nu) = -2\kappa \xi_\mu \tag{24}$$

where κ is the surface gravity at the horizon. Therefore for horizons with temperature, the redshift factor changes from real to imaginary, and the wavefunction solution changes from wave-like and oscillating to Boltzmann-like and decaying. We therefore have at horizon boundaries 1. concealment of information, 2. thermalization, and 3. transition to non-unitarity, which are the exact properties describing collapse from the previous section. This naturally implies $H = i\mathcal{L}_\xi$ on one side of the horizon, and $H \to Q = \mathcal{L}_\xi$ on the other. In the case of Schwarzschild

$$\psi = e^{i\left(1-\frac{2M}{r}\right)\omega t} \tag{25}$$

for $r > 2M$ and

$$\psi = e^{\left(1-\frac{2M}{r}\right)\omega t} \tag{26}$$

for $r < 2M$. This solution decays towards the singularity, and emerges from the horizon as a wave, thus predicting black hole evaporation. Interestingly, the decaying side causes a time asymmetry in that $\omega < 0$ results in a solution diverging towards the singularity. This time asymmetry is entirely unobservable to anything outside the black hole

4 Conclusion and Discussion

There were three key insights to these derivations. Firstly, even if physics is very different at the level of collapse, the Universe should still be describable in terms of states. Secondly, those states, even when adopting statistical behaviour with respect to some observer (ie. even as that observer lacks information about the future of those states), should evolve in time. Thirdly , the concealment of information in time by collapse is similar, or perhaps identical, to the concealment of information by killing horizons. From the first two points we derived a nonunitary operator to describe collapse, and said that if energy governs time evolution, and collapse is simply a concealed time evolution, then the energy associated with collapse should be a heat. Using the last point we then identified this wavefunction thermodynamics with horizon thermodynamics, and so argued that wavefunctions evolve along killing fields, and collapse when the norm of that killing field vanishes. We then applied this to a Schwarzschild black hole and found that it predicted black hole evaporation.

Although we call these insights, they could just as well be called assumptions. Indeed, the first two are very basic, however it would be foolish to turn our backs on decades of philosophical debate by so freely taking for granted the nature of time, and the phenomenological interpretations of the quantum mechanical state. Nontheless, answering philosophical questions like this concretely (if possible) can only come down to experiment, and the evaluation of physical theories which build off specific answers to those questions. While we have no means for experiment, it is in our interest to apply philosophies which have been most straightforward, unmarred, and successful so far; even the axioms of quantum mechanics have not been tested more deeply than these, and so we maintain them.

Of course, maintaining them so puts us in an awkward position, where treating the quantum state in a not-quite hidden variables fashion leads us to something akin to an ensemble interpretation of quantum mechanics, both long dead interpretations. The differentiating trick was that the ensemble is one of histories, and the hidden variable was the future. Mathematically this led to the association between time and inverse temperature, a not unfamiliar association from Thermal Field Theory. This relationship is clarified when considering that as time passes a larger selection of futures is available for the system, such that time passing is naturally equated to a distribution change in the ensemble. Regardless, this equivalence feels a bit strange, and requires more structure to understand its appearance. Perhaps in a proper statistics of histories, or something similar.

With regards to the last insight, we are essentially working off analogy. But, given the unexpected success of the very literal treatment of thermodynamics in deeper physics [2, 6, 5], it would seem foolish to dismiss these coincidences as just so. Indeed, information concealment and the arrow of time are phenomena observed exclusively with regards to entropy, wavefunction collapse, and horizon dynamics, and non-locality specifically with regards to ER and EPR. It is thus the opinion of the author that these behaviours are somehow deeply inter-dependent, and their relationship should be further studied, as they have been here.

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