Abstract

These are brief notes on an interesting derivation of the Born rule from the behaviour of Quantum states at Killing Horizons. They are rough, but nicely consistent with previous work on the thermal framework for collapse.

Born Rule from Redshift at Killing Horizons

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1 Consider

Consider the state

$$\rho = \sum_{n,m} a_n a_m^* e^{i(\omega_n - \omega_m)t} |n\rangle \langle m| \tag{1}$$

Imagine this state observed from the asymptotically flat infinity of a static spacetime. The state will be redshifted by

$$\rho = \sum_{n,m} a_n a_m^* e^{\pm i\sqrt{\xi\cdot\xi}(\omega_n - \omega_m)t} |n\rangle \langle m| \tag{2}$$

where $\xi = \partial_t$ is the relevant Killing vector field. Consider now the state around a Killing horizon with non-zero temperature. If the horizon has nonzero temperature then

$$\nabla(\xi \cdot \xi) = -2\kappa\xi \neq 0 \tag{3}$$

and so if $\xi \cdot \xi > 0$ on one side of the horizon, it must be < 0 on the other. Thus, just over the horizon relative to the observer, the state becomes

$$\rho = \sum_{n,m} a_n a_m^* e^{\pm\chi(\omega_n - \omega_m)t} |n\rangle \langle m| \tag{4}$$

where χ is the magnitude of the redshift factor. The state must be normalizable, and so we always choose $\pm$ such that the exponent is negative. This will become important in a moment, and is truthfully quite weak (I hope to justify it more mechanically later). Relative to the observer, it will take an infinite coordinate time for the state to cross the horizon, and so

$$\begin{aligned}
\lim_{t\to\infty} \rho &= \lim_{t\to\infty} \sum_{n,m} a_n a_m^* e^{-\chi|\omega_n - \omega_m|t} |n\rangle \langle m| \\
&= \sum_{n,m} a_n a_m^* \delta_{nm} |n\rangle \langle m| \\
&= \sum_{n,m} |a_n|^2 |n\rangle \langle n|
\end{aligned} \tag{5}$$

Therefore giving us the Born rule. This has essentially come from a thermalization procedure. Basically any non-unitary evolution will return the Born rule in this way, and event horizons happen to cause non-unitary evolution (or at least so the mathematics, and of course various well known paradoxes, suggests). The normalization bit was, however, quite strange. It is required for the state to make sense, but the mathematical framework is so loose that this potential lack of self-consistency may simply be an expression of the lack of quantum gravity. On the other hand, this hint of thermalization, which we saw in a previous writing about similar behaviours, does suggest some shenanigans with the arrow of time. Maybe when the energy reverses, the state is now moving backwards in time, and so time goes to negative infinity, rather than positive infinity. We do know that it is impossible to move along negative coordinate time insie black holes, so this would definitely make sense. Maybe this can be made even more concrete in a path integral formulation.