Artificial Intelligence
CS-401

Chapter # 04

Local Search & Optimization Problems

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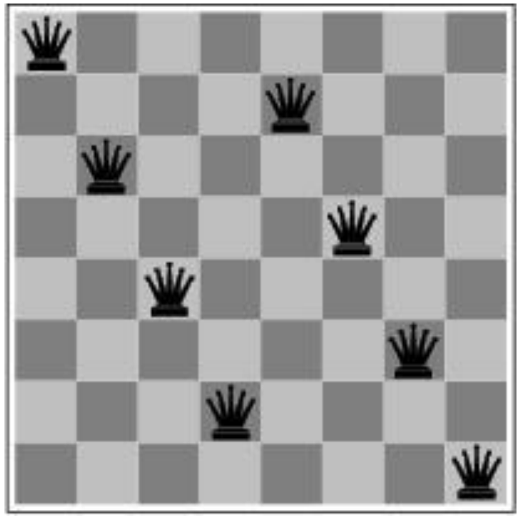
Outline

- Local search techniques and optimization
  - Hill-climbing
  - Simulated annealing
  - Local Beam Search
  - Genetic algorithms

Local search and optimization

- Previously: **systematic exploration** of search space.
  - Path to goal is solution to problem

- YET, for some problems **path is irrelevant**.
  - E.g 8-queens
  - Factory Floor Layout
  - Automatic programming
  - Integrated circuit design
  - Vehicle routing

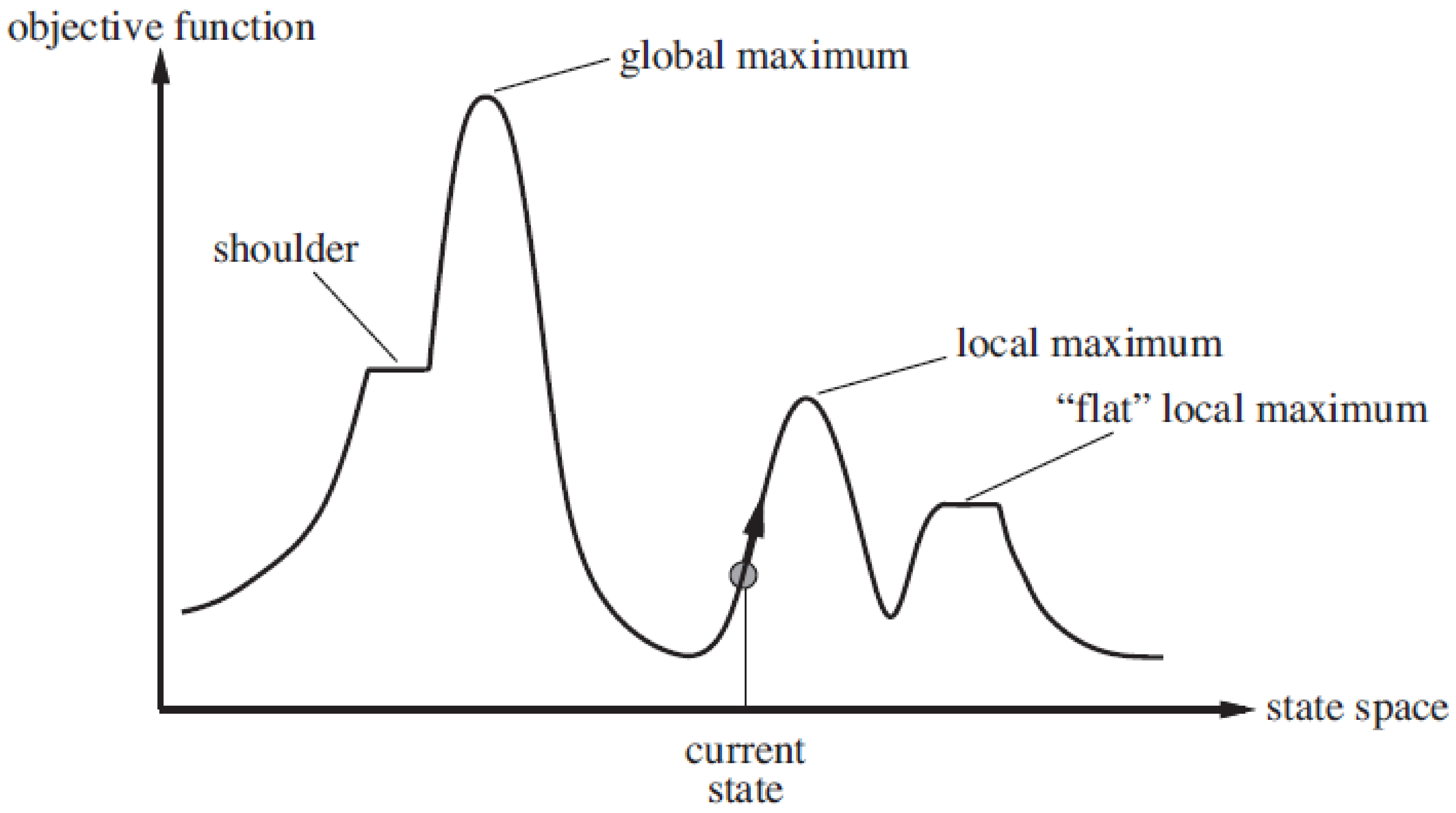


- For such problems different algorithms can be used
  - Local search

Local search and optimization

- Local search
  - Keep track of single current state
  - Move only to neighboring states
  - Ignore paths
- Advantages:
  - Use very little memory
  - Can often find reasonable solutions in large or infinite (continuous) state spaces.
- “Pure optimization” problems
  - All states have an objective function
  - Goal is to find **state with max (or min)** objective value
  - Some problems do not quite fit into path-cost/goal-state formulation e.g. nature provides **reproductive fitness**, Darwin evolution seems to optimize it.
  - Local search can do quite well on these problems.

“Landscape” of search



Hill-climbing search (1)

- “a loop that continuously moves in the direction of increasing value”
  - terminates when a peak is reached
  - Aka greedy local search

- Value can be either
  - Objective function value
  - Heuristic function value (minimized)

- Hill climbing **does not look ahead** of the immediate neighbors of the current state.

- Can **randomly** choose among the set of best successors, if multiple have the best value

- Characterized as “**trying to find the top of Mount Everest while in a thick fog**”

Hill-climbing search algorithm (1)

```
function HILL-CLIMBING( problem) return a state that is a local
    maximum
    input: problem, a problem
    local variables: current, a node.
                     neighbor, a node.

    current ← MAKE-NODE(INITIAL-STATE[problem])
    loop do
        neighbor ← a highest valued successor of current
        if VALUE [neighbor] ≤ VALUE[current]
        then return STATE[current]
        current ← neighbor
```

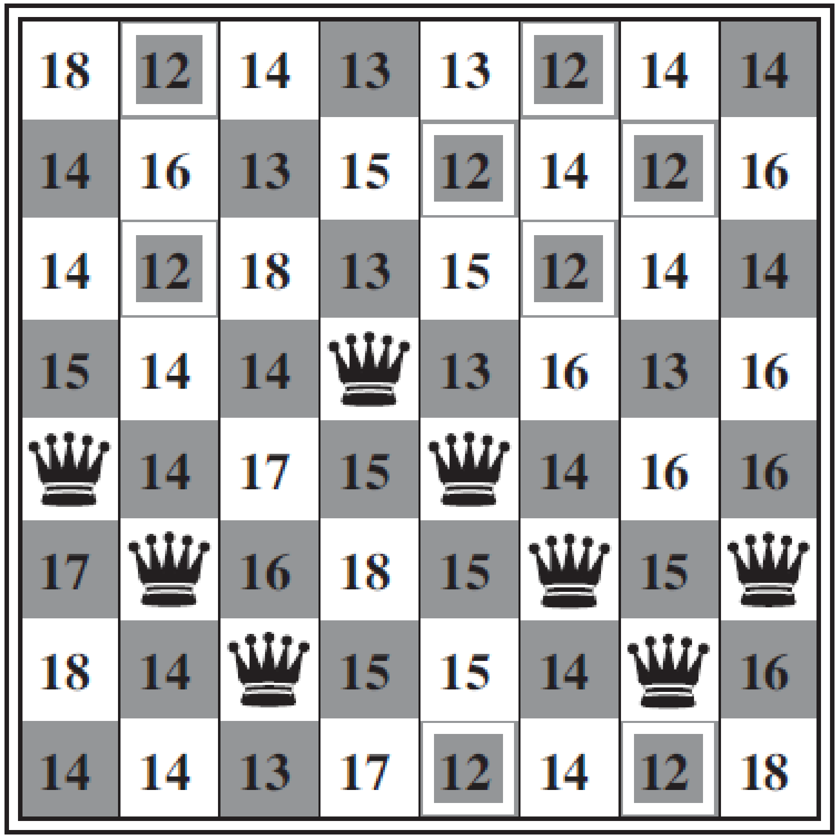
Hill-climbing example

- 8-queens problem, complete-state formulation
  - All 8 queens on the board in some configuration

- Successor function:
  - move a single queen to another square in the same column.

- Example of a heuristic function *h(n)*:
  - the number of pairs of queens that are attacking each other (directly or indirectly)
  - (so we want to minimize this)

Hill-climbing example

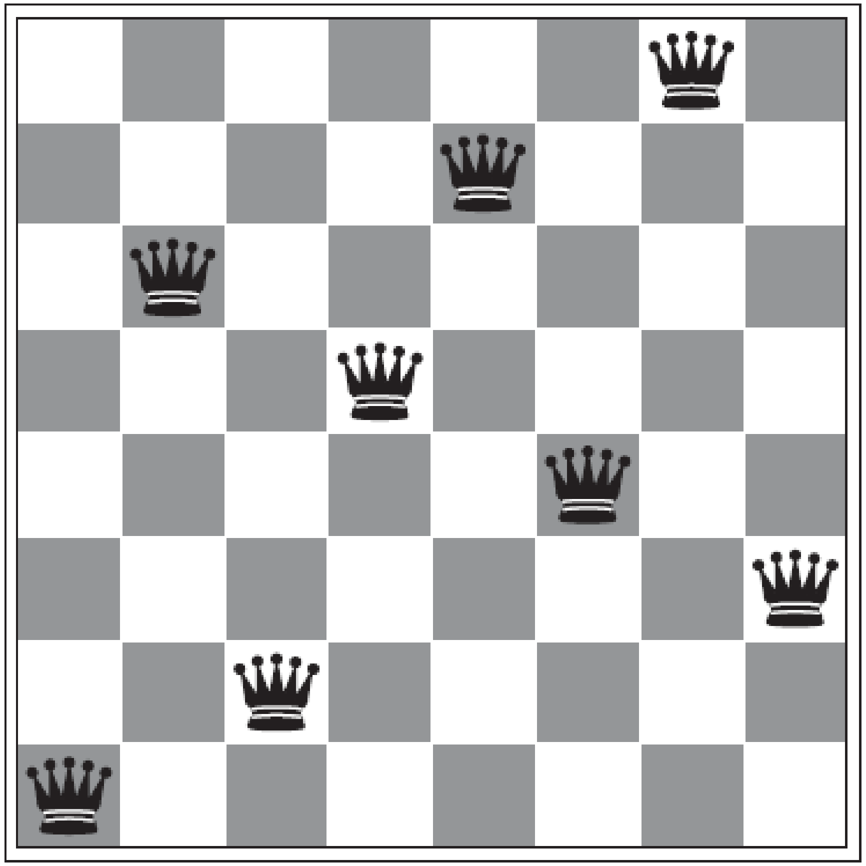


Current state: h=17

h is the number of queens attacking each other.

Shown is the h-value for each possible successor in each column. Best moves are marked.

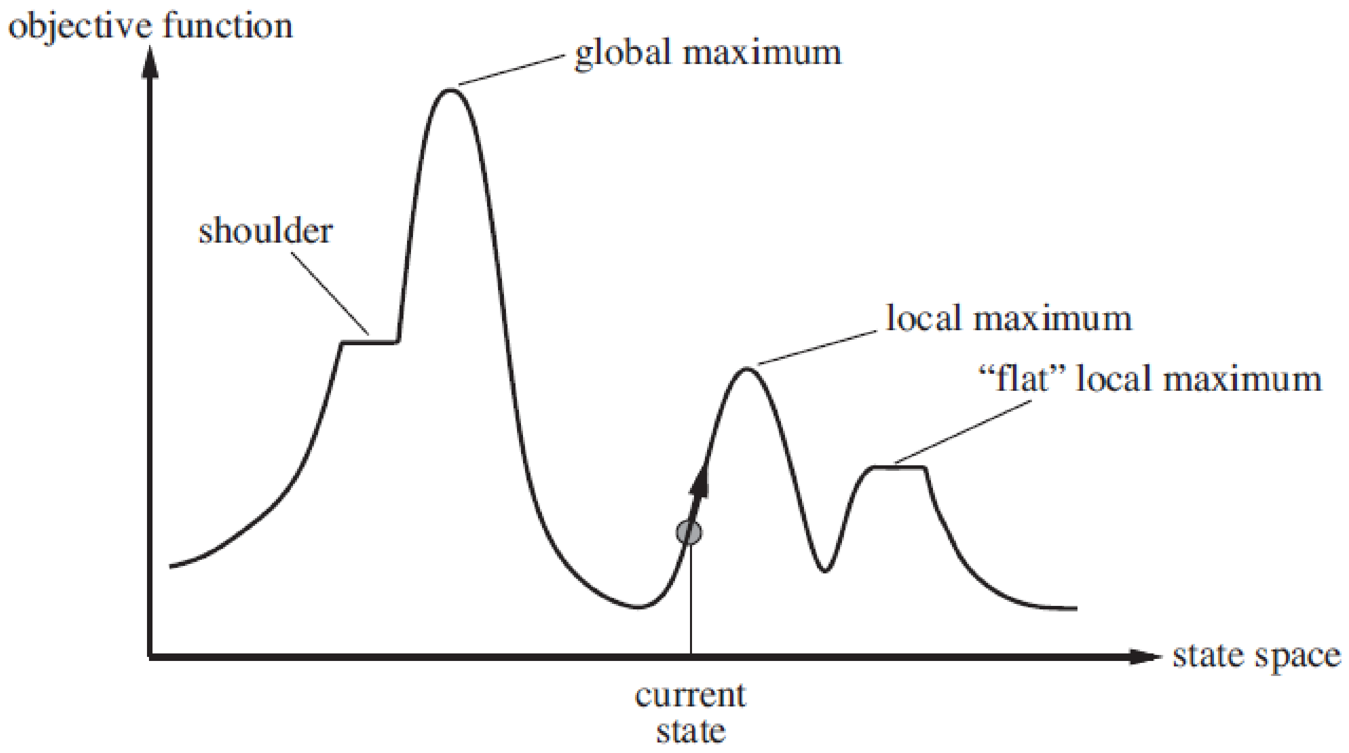
A local minimum for 8-queens



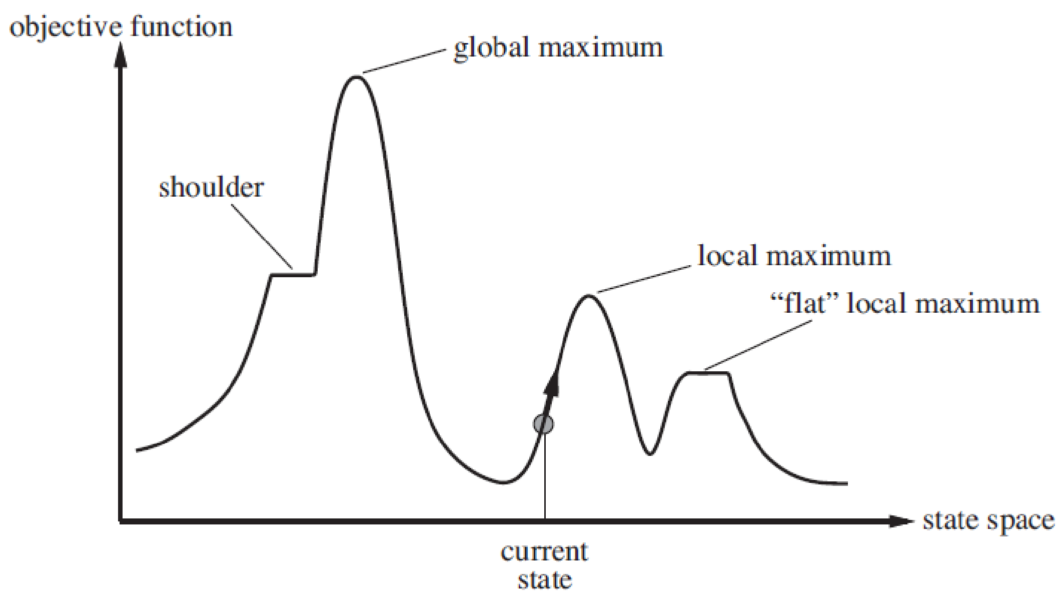
A local minimum in the 8-queens state space (h=1)

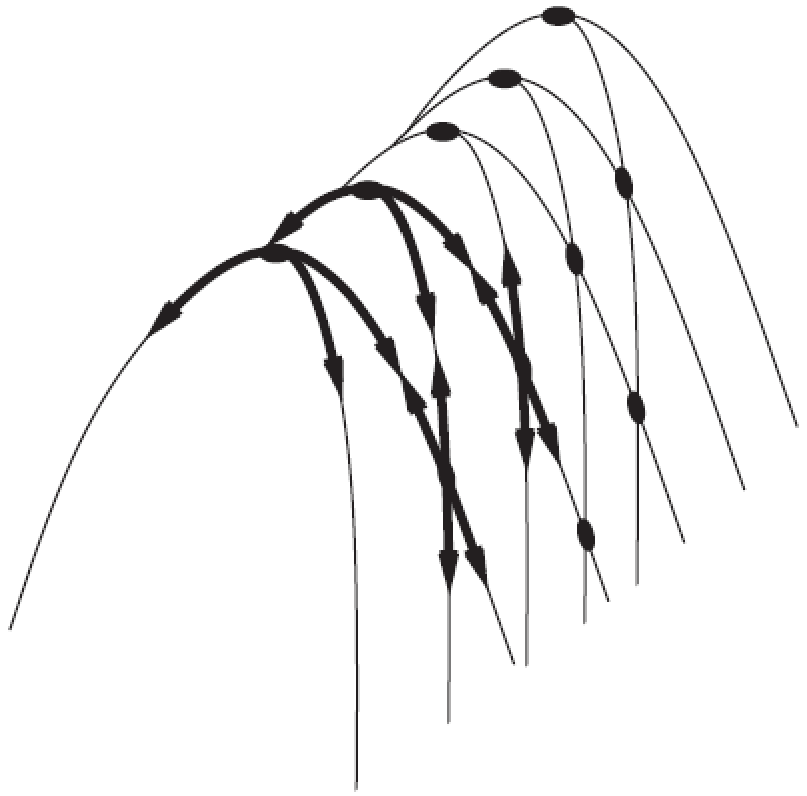
Problems: Hill climbing and local maxima

- **Local Maxima:** When local maxima exists, hill climbing is suboptimal.
- Simple (often effective) solution
  - Multiple random restarts



Problems: Other drawbacks





- **Ridge** = sequence of local maxima difficult for greedy algorithms to navigate (shown above right).

- **Plateau** = an area of the state space where the evaluation function is flat, shoulder region is a type of plateau.

Performance of hill-climbing on 8-queens (Statistics)

- Randomly generated 8-queens starting states…
- 14% the time it solves the problem
- 86% of the time it get stuck at a local minimum
- However…
  - Takes only 4 steps on average when it succeeds
  - And 3 on average when it gets stuck
  - (for a state space with ~17 million states)

Possible solution…sideways moves

- If no downhill (uphill) moves, allow sideways moves in hope that algorithm can escape
  - Need to place a limit on the possible number of sideways moves to avoid infinite loops
- For 8-queens
  - Now **allow sideways moves with a limit of 100**
  - Raises percentage of problem instances solved from 14 to 94%
  - However on average….
    - 21 steps for every successful solution
    - 64 for each failure

Hill-climbing variations

1. **Stochastic hill-climbing**
   - **Random selection** among the uphill moves.
   - The selection probability can vary with the **steepness of the uphill move.**
   - This usually converges slowly than **steepest ascent**.
2. **First-choice hill-climbing**
   - Stochastic hill climbing by **generating successors randomly until a better one is found**
   - Useful when there are a very large number of successors
3. **Random-restart hill-climbing**
   - Hill Climbing from randomly generated initial states
   - Tries to avoid getting stuck in local maxima.
   - Complete

Hill-climbing with random restarts

- Different variations
  - For each restart: run until termination v. Run for a fixed time
  - Run a fixed number of restarts or run indefinitely

- Analysis
  - Say each search has probability p of success then expected no of restarts required is: 1/p.
    - E.g., for 8-queens, p = 0.14 with no sideway moves

  - **Expected number of restarts?**

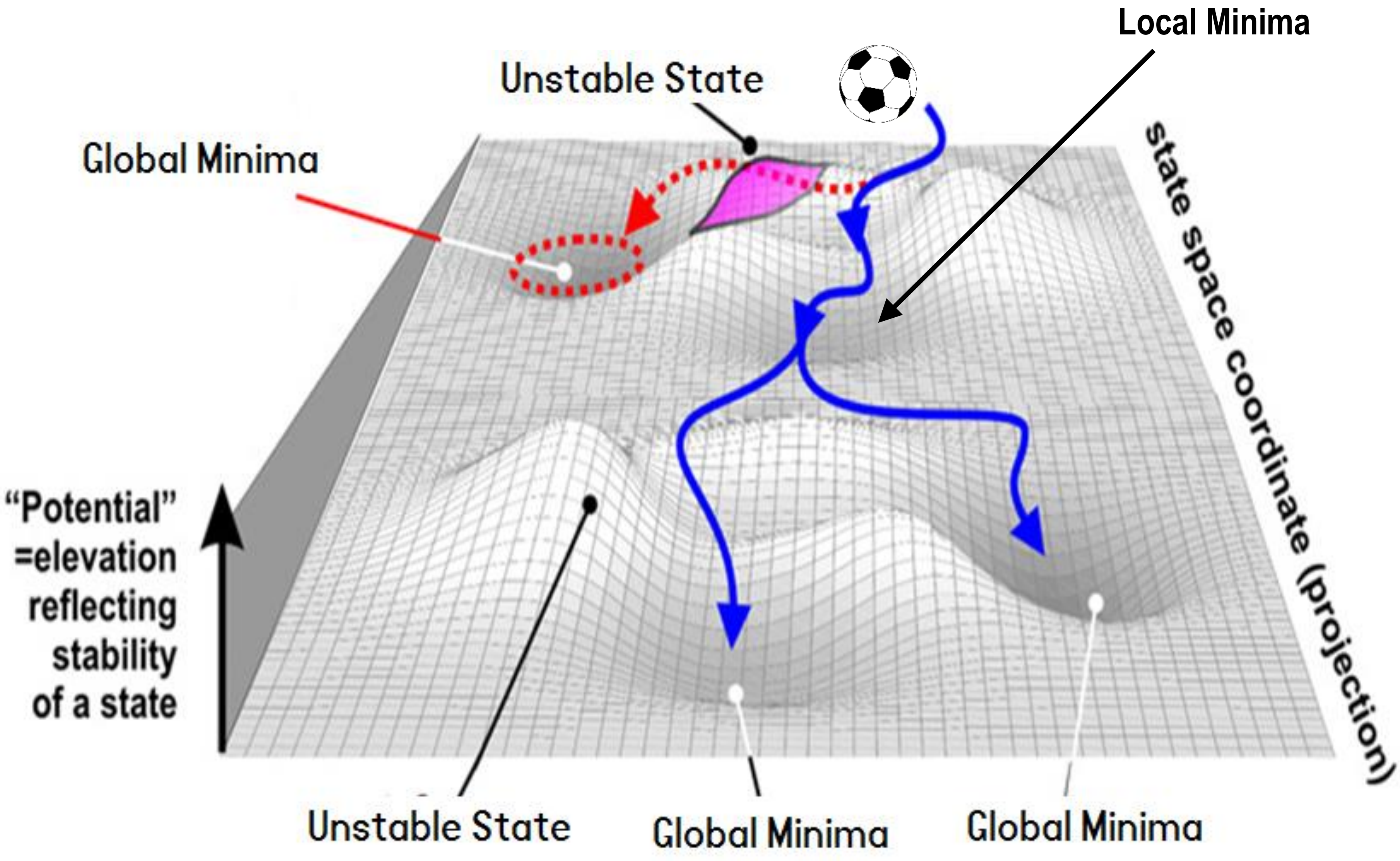
    Ans: 1/p
  - **Expected number of steps taken?**

    Ans: p x avg. of success + (1-p) avg. steps of failure

Simulated Annealing Search (2)

- **A Physical Analogy:**
  - imagine letting a ball roll downhill on the function surface
    - this is like hill-climbing (for minimization)
  - now imagine **shaking the surface, while the ball rolls, gradually reducing the amount of shaking**
    - this is like simulated annealing
- **Annealing** = physical process of cooling a liquid or metal until particles achieve a certain frozen crystal state
  - simulated annealing:
    - free variables are like particles
    - seek “low energy” (high quality) configuration
    - get this by slowly reducing temperature T, which particles move around randomly

Physical Interpretation of Simulated Annealing


Local Minima
Unstable State
Global Minima
"Potential" =elevation reflecting stability of a state
state space coordinate (projection)
Unstable State
Global Minima
Global Minima

Search using Simulated Annealing (2)

- Simulated Annealing = hill-climbing with non-deterministic search (i.e. randomness)

- Basic ideas (for maximization problems):
  - like hill-climbing identify the quality of the local improvements
  - instead of picking the best move, pick one randomly
  - say the **change** in objective function is Δ
  - if Δ is **positive**, then move to that state
  - **otherwise**:
    - move to this state with probability proportional to Δ
    - thus: worse moves (very large negative Δ) are executed less often
  - however, there is always a chance of escaping from local maxima
  - over time, make it less likely to accept locally bad moves
  - (Can also make the size of the move random as well, i.e., allow “large” steps in state space)

Simulated annealing (for maximization problem)

```
function SIMULATED-ANNEALING( problem, schedule) return a solution state
    input: problem, a problem
           schedule, a mapping from time to temperature
    local variables: current, a node.
                     next, a node.
                     T, a "temperature" controlling the probability of downward steps
           current ← MAKE-NODE(INITIAL-STATE[problem])
    for t ← 1 to ∞ do
           T ← schedule[t]
           if T = 0 then return current
           next ← a randomly selected successor of current
           ΔE ← VALUE[next] - VALUE[current]
           if ΔE > 0 then current ← next
           else current ← next only with probability $e^{\Delta E/T}$
```

The key equation of the algorithm is: $P_k = e^{\frac{\Delta E_k}{T}}$

More Details on Simulated Annealing

- Lets say there are 3 moves available, with changes in the objective function of **d1** = -0.1, **d2** = 0.5, **d3** = -5.
- (Let’s fix the temperature and let T = 1).
- pick a move randomly:
  - if d2 is picked, move there.
  - if d1 or d3 are picked, probability of move = exp(d/T)
  - move 1: prob1 = exp(-0.1) = 0.9,
    - i.e., 90% of the time we will accept this move
  - move 3: prob3 = exp(-5) = 0.05
    - i.e., 5% of the time we will accept this move

More Details on Simulated Annealing

- T = “temperature” parameter
  - high T => probability of “locally bad” move is higher
  - low T => probability of “locally bad” move is lower
  - typically, T is decreased as the algorithm runs longer
    - i.e., there is a “temperature schedule”

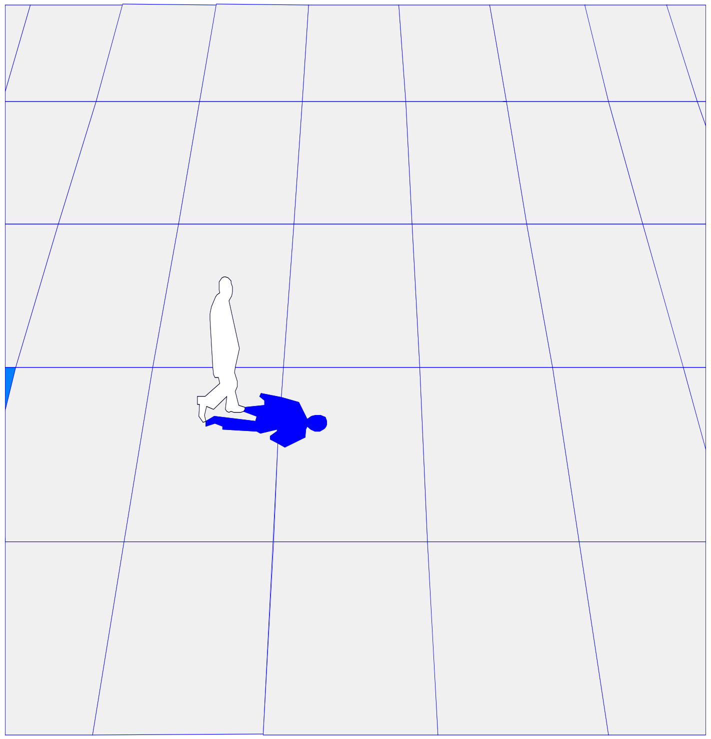
Simulated Annealing in Practice

- method proposed in 1983 by IBM researchers for solving **VLSI layout** problems (Kirkpatrick et al, *Science*, 220:671-680, 1983).
  - Very-large-scale integration (**VLSI**) is the process of creating an integrated circuit (IC) by combining thousands of transistors into a single chip. **VLSI** began in the 1970s when complex semiconductor and communication technologies were being developed. The microprocessor is a **VLSI** device.
  - theoretically will always find the global optimum (the best solution)
- useful for some problems, but can be very slow
    - slowness comes about because T must be decreased very gradually to retain optimality
  - In practice how do we decide the rate at which to decrease T? (this is a practical problem with this method)

Local beam search (3)

- Keep track of *k* states instead of one
  - Initially: *k* randomly selected states
  - Next: determine all successors of *k* states
  - If any of successors is goal → finished
  - Else **select *k* best from ALL successors** and repeat.
- Major difference with random-restart search
  - Information is shared among *k* search threads.
- Can suffer from lack of diversity.
  - Stochastic beam search
    - choose k successors at random proportional to state quality.

Genetic Algorithms (4)



"Genetic Algorithms are good at taking large, potentially huge search spaces and navigating them, looking for optimal combinations of things, solutions you might not otherwise find in a lifetime."

- Salvatore Mangano

Computer Design, May 1995

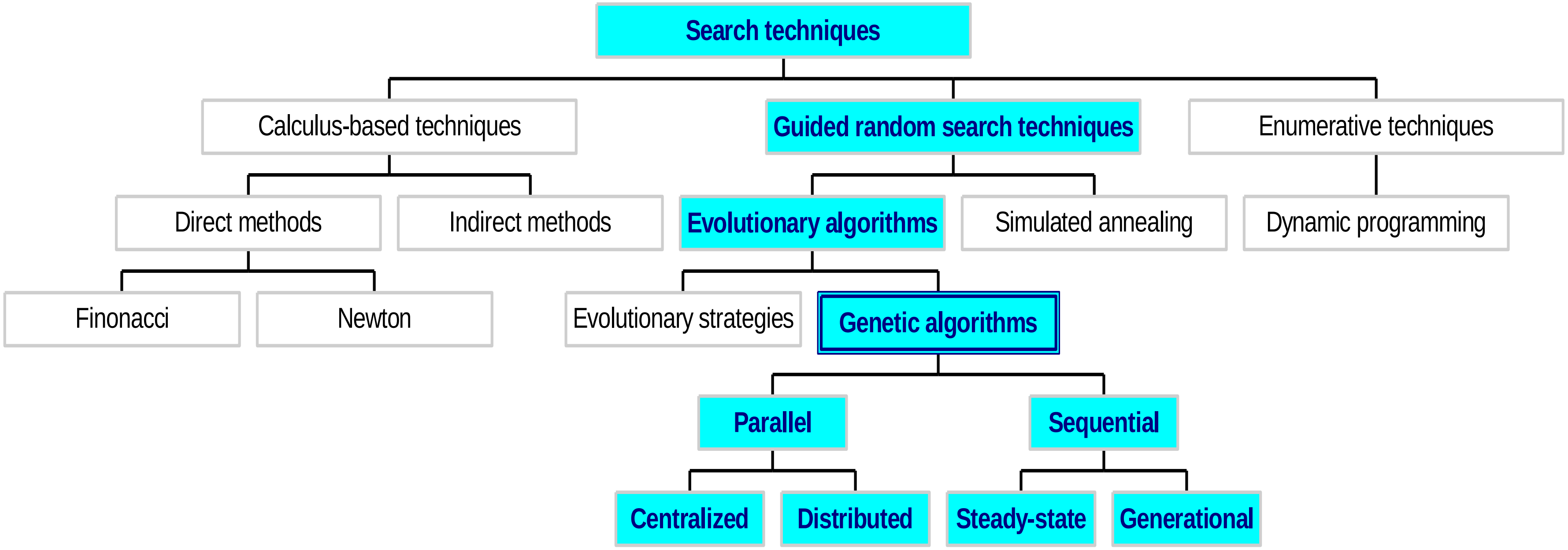
The Genetic Algorithm

- Directed search algorithms based on the mechanics of biological evolution
- Developed by John Holland, University of Michigan (1970’s)
    - ♦ To understand the adaptive processes of natural systems
    - ♦ To design artificial systems software that retains the robustness of natural systems

The Genetic Algorithm (cont.)

- Provide efficient, effective techniques for optimization and machine learning applications
- Widely-used today in business, scientific and engineering circles

Classes of Search Techniques



Components of a GA

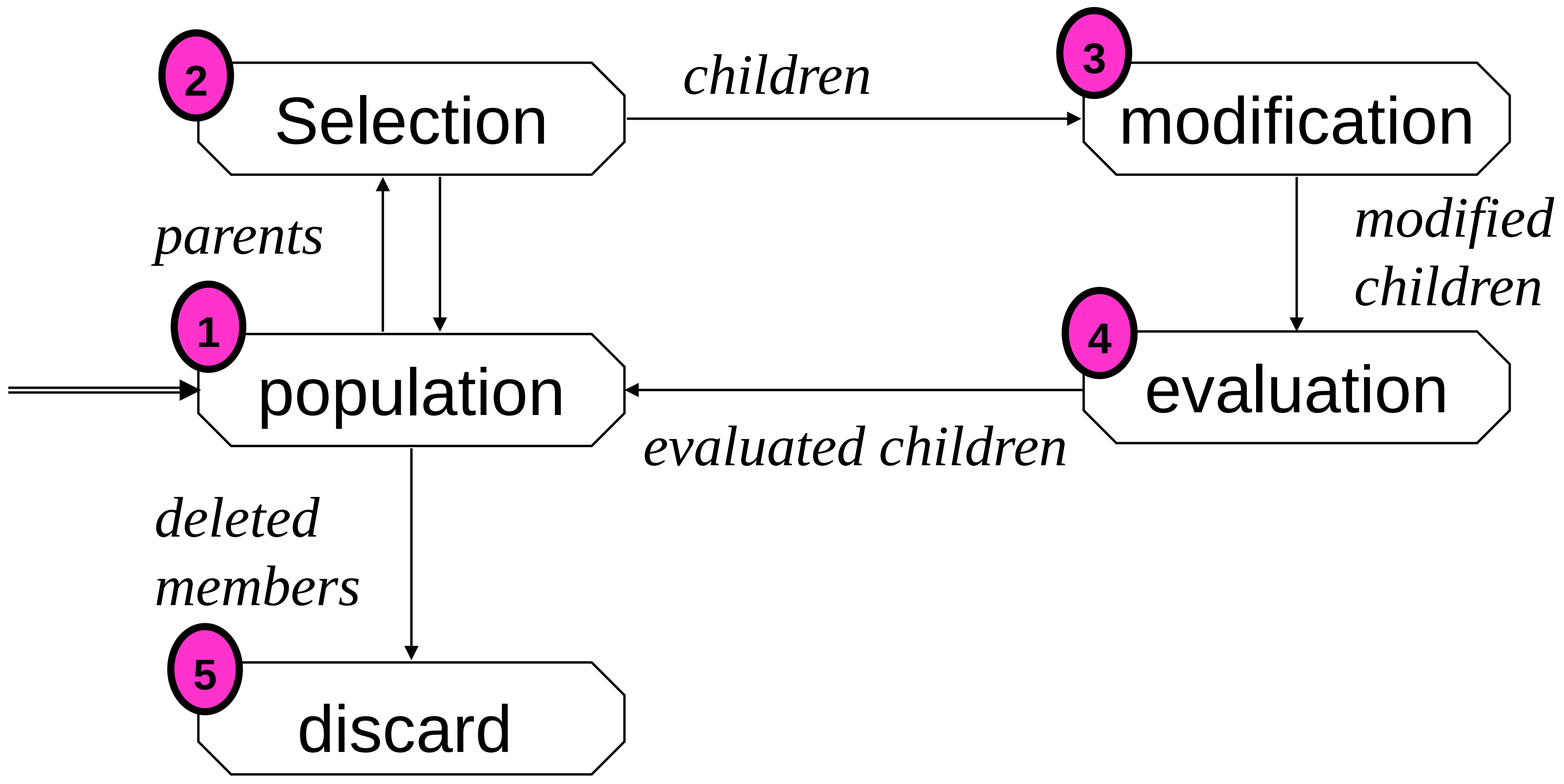
A problem to solve, and ...

- Encoding technique (*gene, chromosome*)
- Initialization procedure *(creation)*
- Evaluation function *(environment)*
- Selection of parents *(reproduction)*
- Genetic operators *(mutation, recombination)*
- Parameter settings *(practice and art)*

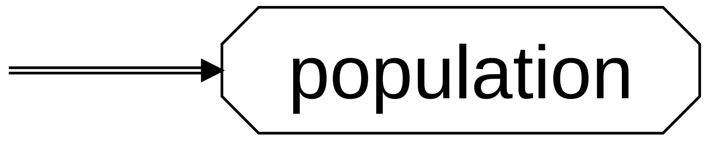
Simple Genetic Algorithm

```
{
    initialize population;
    evaluate population;
    while TerminationCriteriaNotSatisfied
    {
        select parents for reproduction;
        perform recombination and mutation;
        evaluate population;
    }
}
```

The GA Cycle of Reproduction



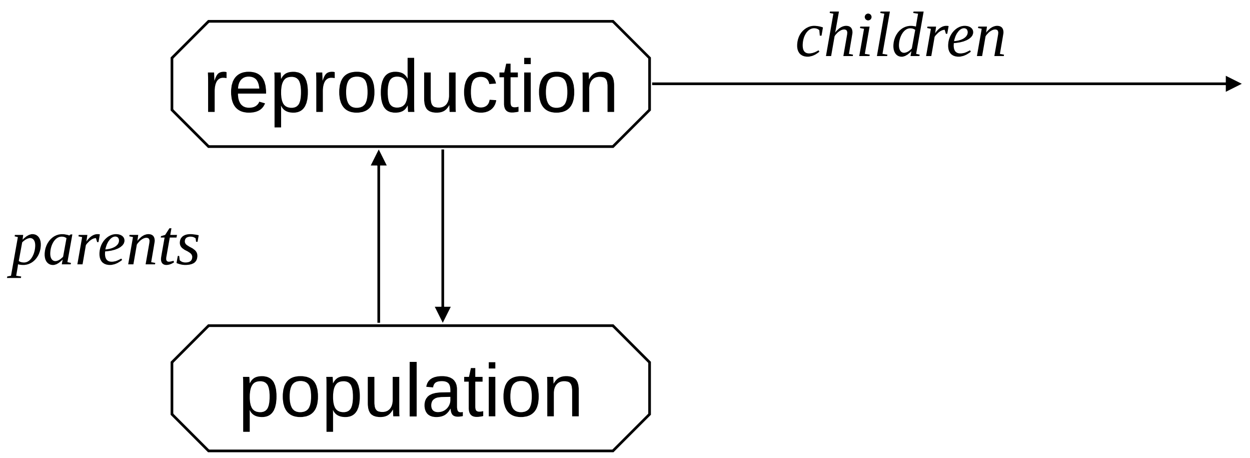
1- Population



Chromosomes could be:

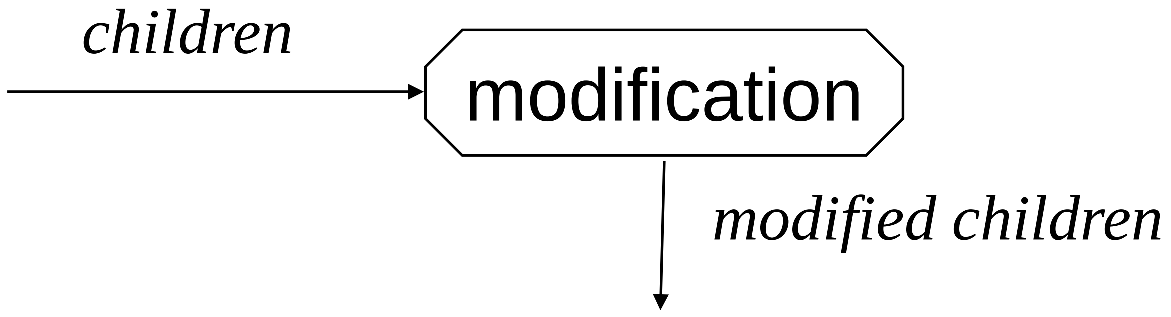
♦ Bit strings	(0101 ... 1100)
♦ Real numbers	(43.2 -33.1 ... 0.0 89.2)
♦ Permutations of element	(E11 E3 E7 ... E1 E15)
♦ Lists of rules	(R1 R2 R3 ... R22 R23)
♦ Program elements	(genetic programming)
♦ ... any data structure ...	

2- Reproduction/Selection



Parents are selected at random with selection chances biased in relation to chromosome evaluations.

3- Chromosome Modification



- Modifications are stochastically triggered
- Operator types are:
  - ♦ Mutation
  - ♦ Crossover (recombination)

Crossover: Recombination

$$\begin{array}{llcll} \text{P1} & (0\ 1\ \overset{*}{\boxed{1}}\ 0\ 1\ 0\ 0\ 0) & & (0\ 1\ \boxed{0}\ 0\ 1\ 0\ 0\ 0) & \text{C1} \\ & & \longrightarrow & & \\ \text{P2} & (1\ 1\ \boxed{0}\ 1\ 1\ 0\ 1\ 0) & & (1\ 1\ \boxed{1}\ 1\ 1\ 0\ 1\ 0) & \text{C2} \end{array}$$

Crossover is a critical feature of genetic algorithms:

- ♦ It greatly accelerates search early in evolution of a population
- ♦ It leads to effective combination of schemata (subsolutions on different chromosomes)

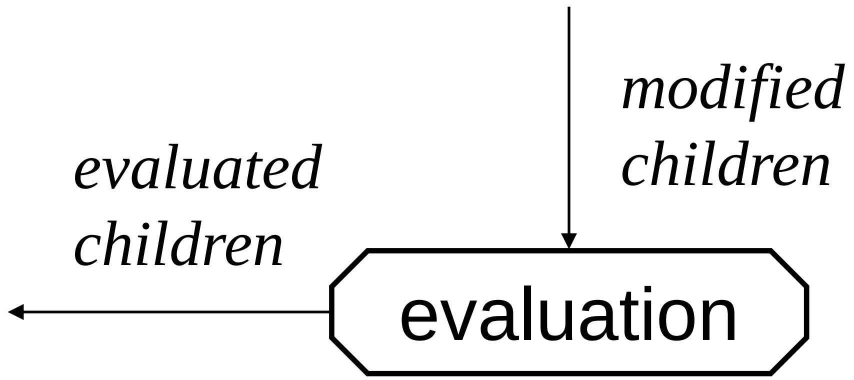
Mutation: Local Modification

Before:	(1 0 1 1 0 1 1 0)
After:	(0 1 1 0 0 1 1 0)

Before:	(1.38	-69.4	326.44	0.1)
After:	(1.38	-67.5	326.44	0.1)

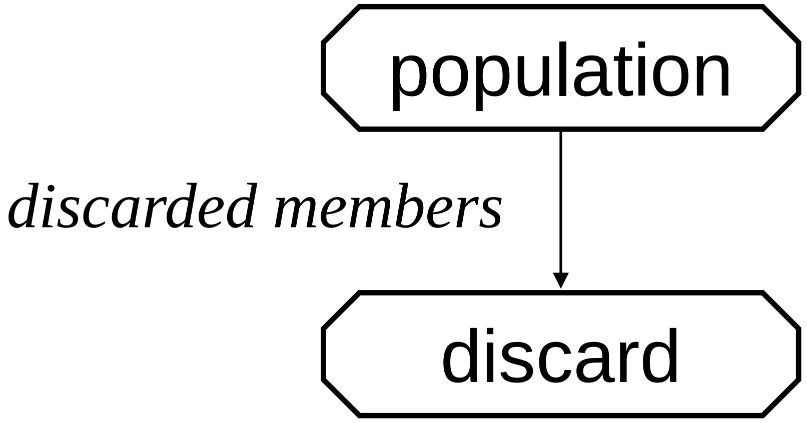
- Causes movement in the search space (local or global)
- Restores lost information to the population

4- Evaluation



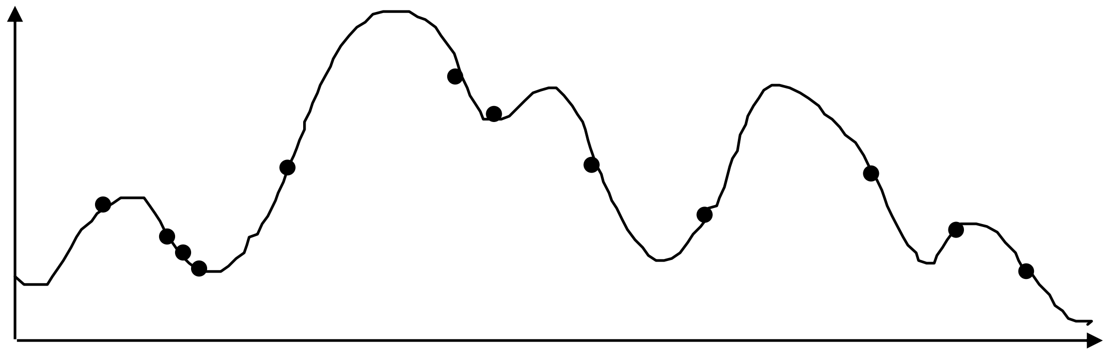
- The evaluator decodes a chromosome and assigns it a fitness measure
- The evaluator is the only link between a classical GA and the problem it is solving

5- Deletion

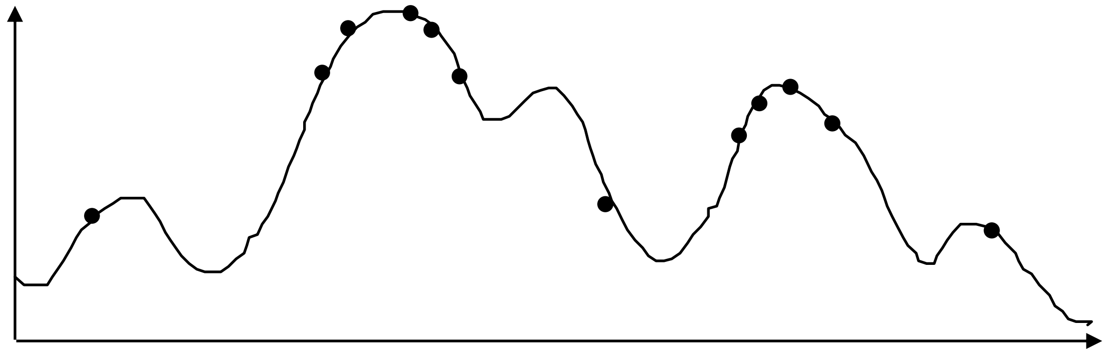


- *Generational* GA:
  entire populations replaced with each iteration
- *Steady-state* GA:
  a few members replaced each generation

An Abstract Example

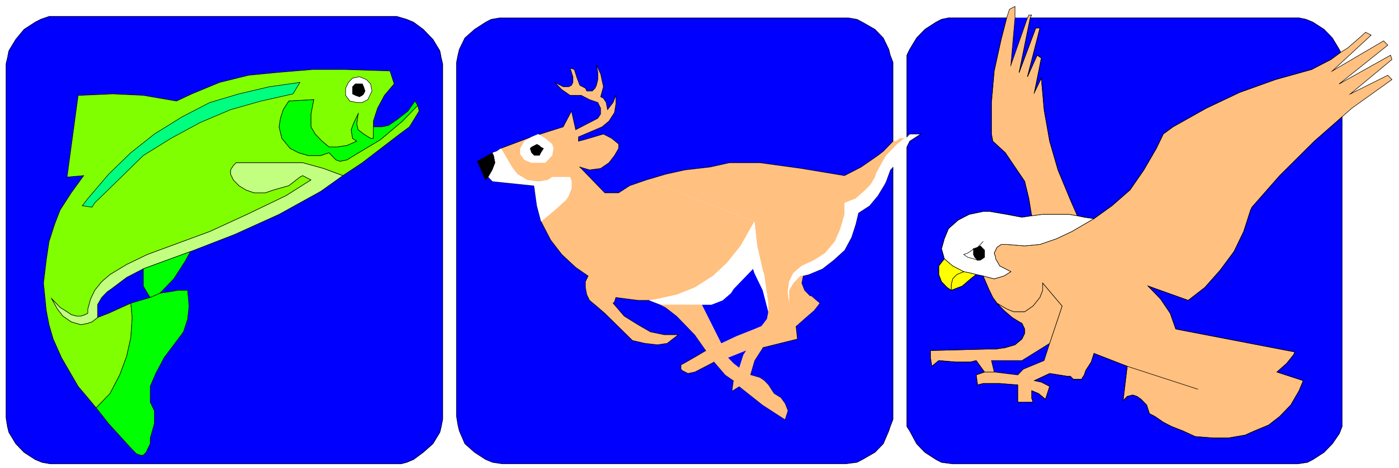


Distribution of Individuals in Generation 0



Distribution of Individuals in Generation N

A Simple Example



"The Gene is by far the most sophisticated program around."

- Bill Gates, *Business Week*, June 27, 1994

A Simple Example

The Traveling Salesman Problem:

Find a tour of a given set of cities so that

- ♦ each city is visited only once
- ♦ the total distance traveled is minimized

Representation & Selection

Representation is an ordered list of city numbers known as an *order-based* GA.

1) London	3) Dunedin	5) Beijing	7) Tokyo
2) Venice	4) Singapore	6) Phoenix	8) Victoria

```
CityList1     (3   5   7   2   1   6   4   8)
CityList2     (2   5   7   6   8   1   3   4)
```

Crossover

Crossover combines inversion and recombination:

$$\begin{array}{ll} & * * \\ \text{Parent1} & (3\quad 5\quad \boxed{7\quad 2\quad 1\quad 6}\quad 4\quad 8) \\ \text{Parent2} & (2\quad 5\quad 7\quad 6\quad 8\quad 1\quad 3\quad 4) \\ \hline \text{Child} & (5\quad 8\quad \boxed{7\quad 2\quad 1\quad 6}\quad 3\quad 4) \end{array}$$

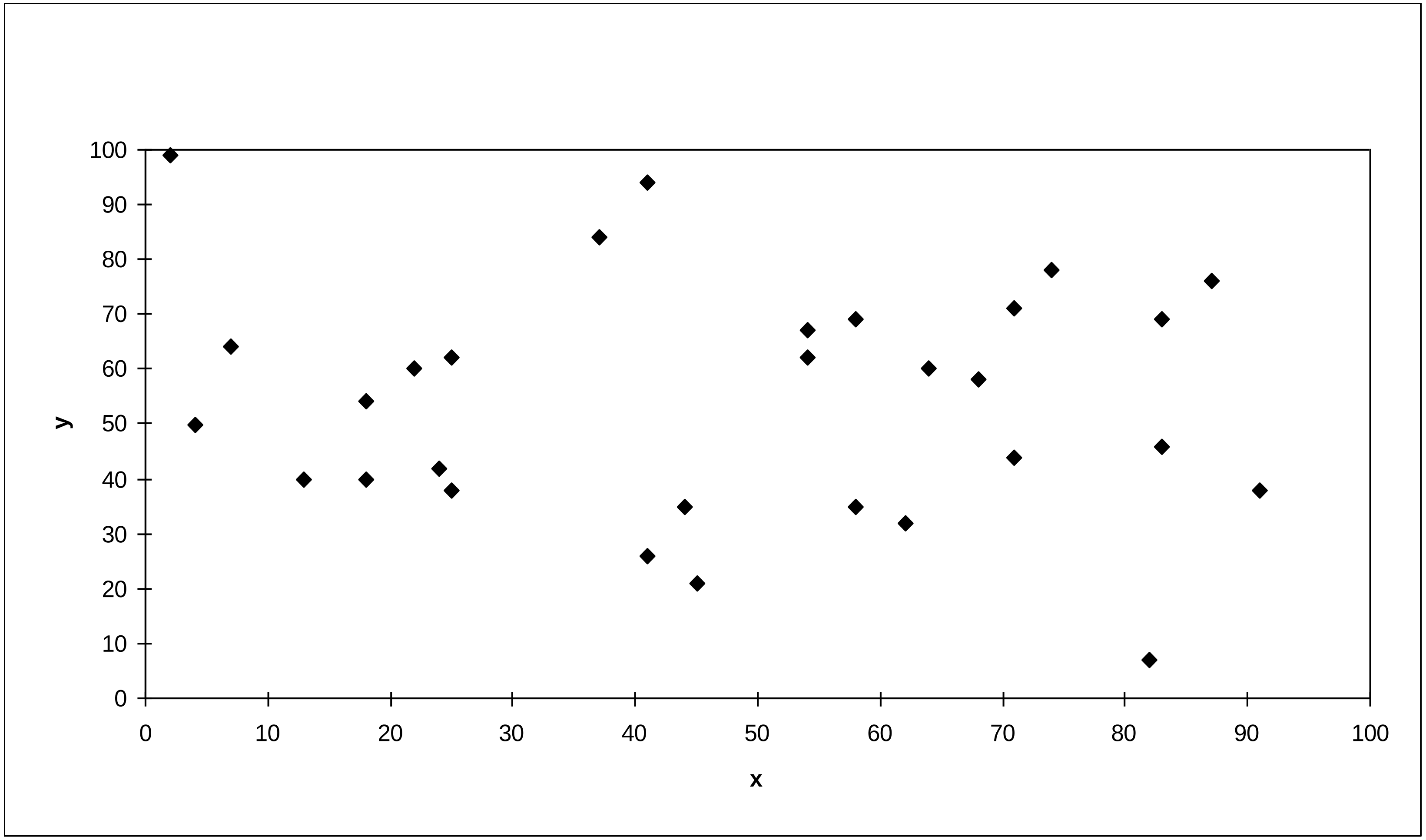
This operator is called the *Order1* crossover.

Mutation

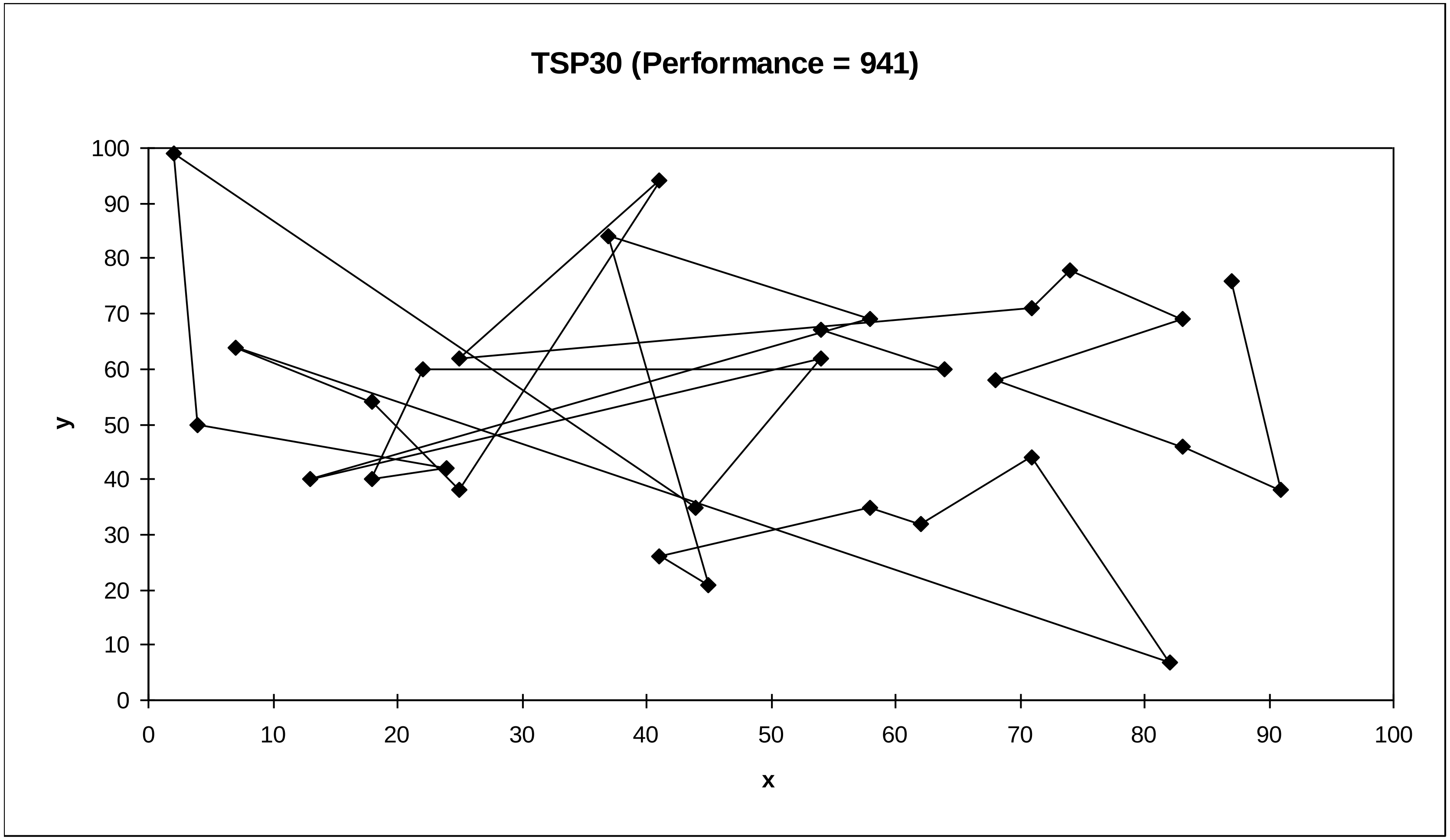
Mutation involves reordering of the list:

			*			*		
Before:	(5	8	7	2	1	6	3	4)
				$\longleftarrow$	$\longrightarrow$			
After:	(5	8	6	2	1	7	3	4)

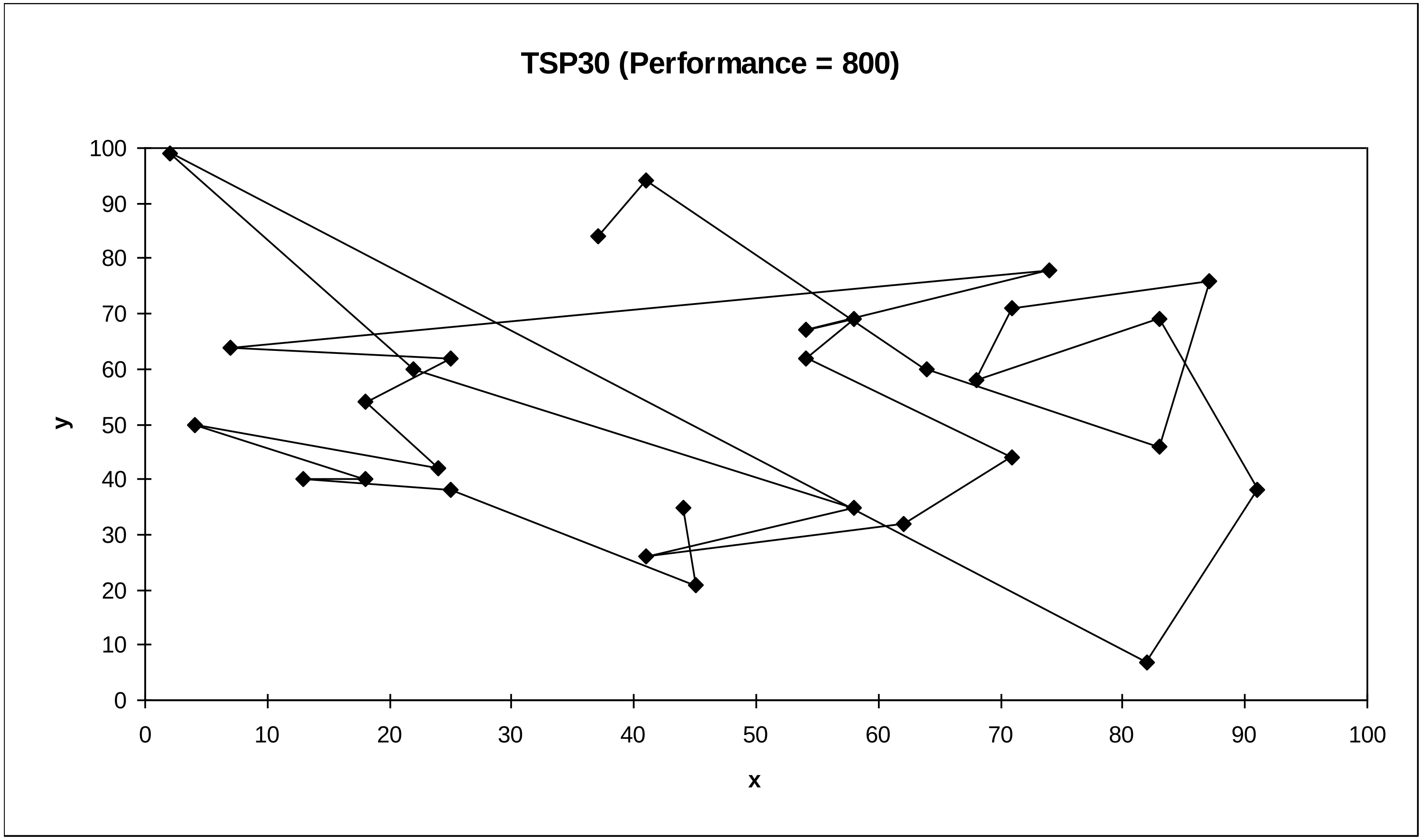
TSP Example: 30 Cities



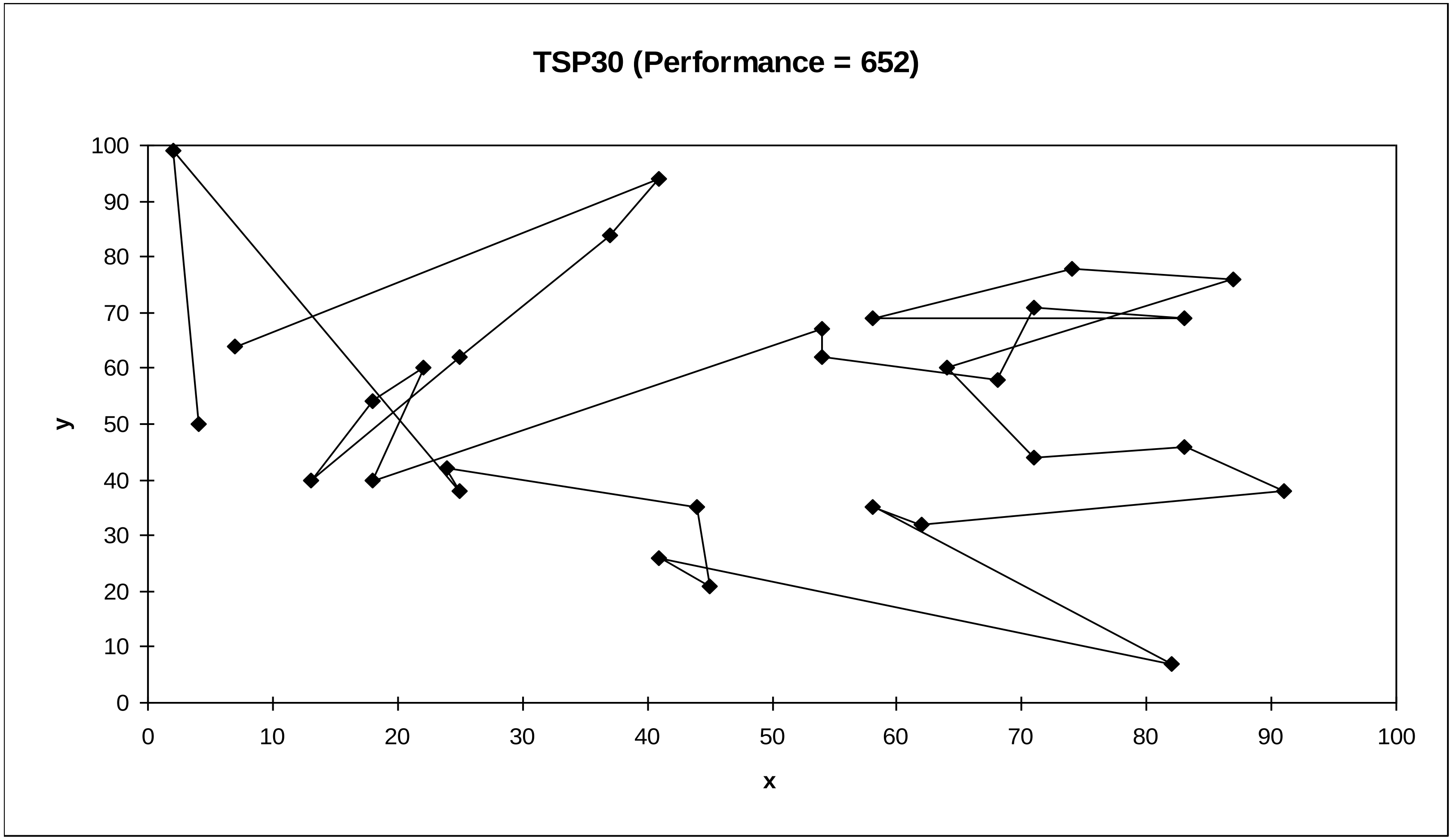
Solution $_i$ (Distance = 941)



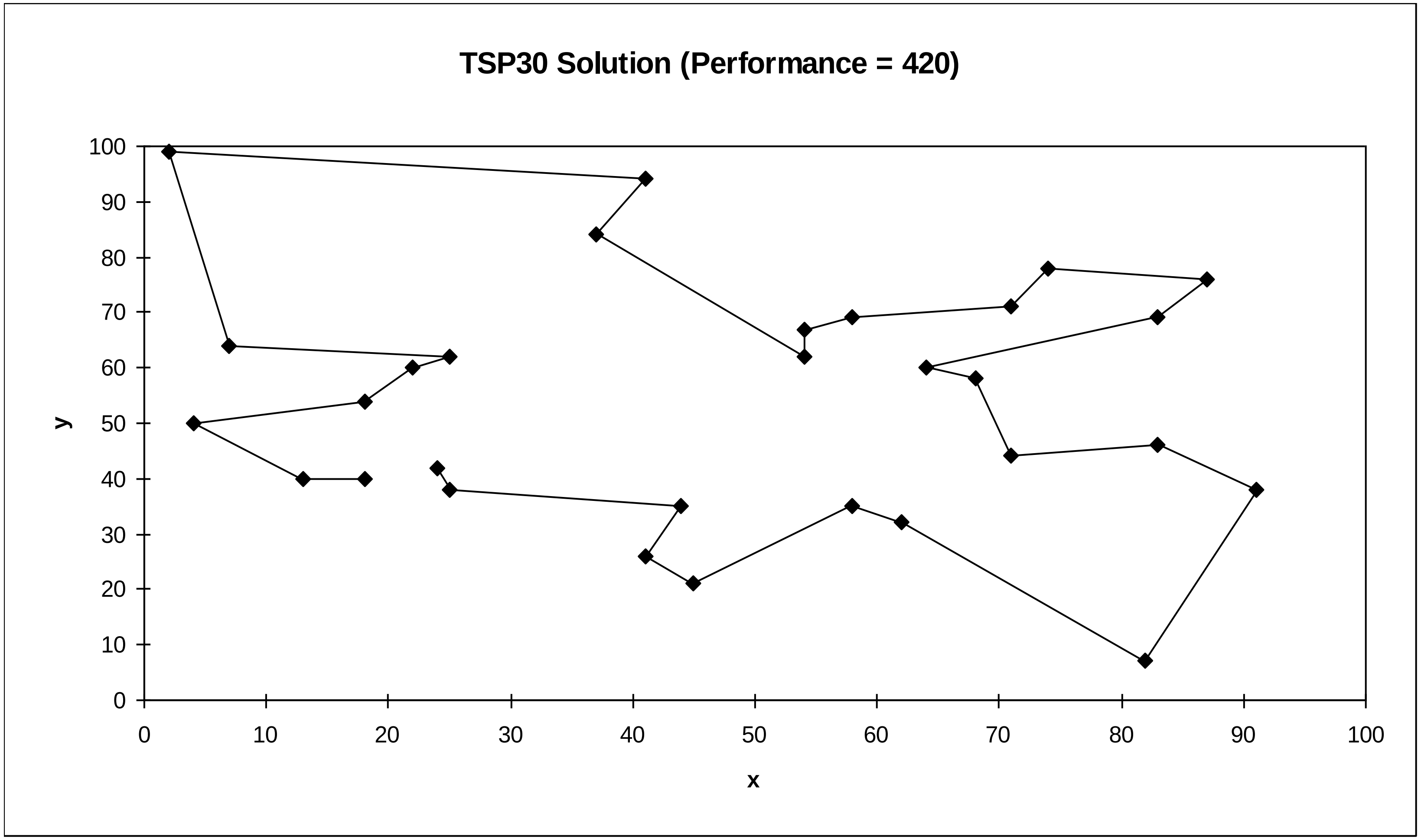
Solution $_j$(Distance = 800)



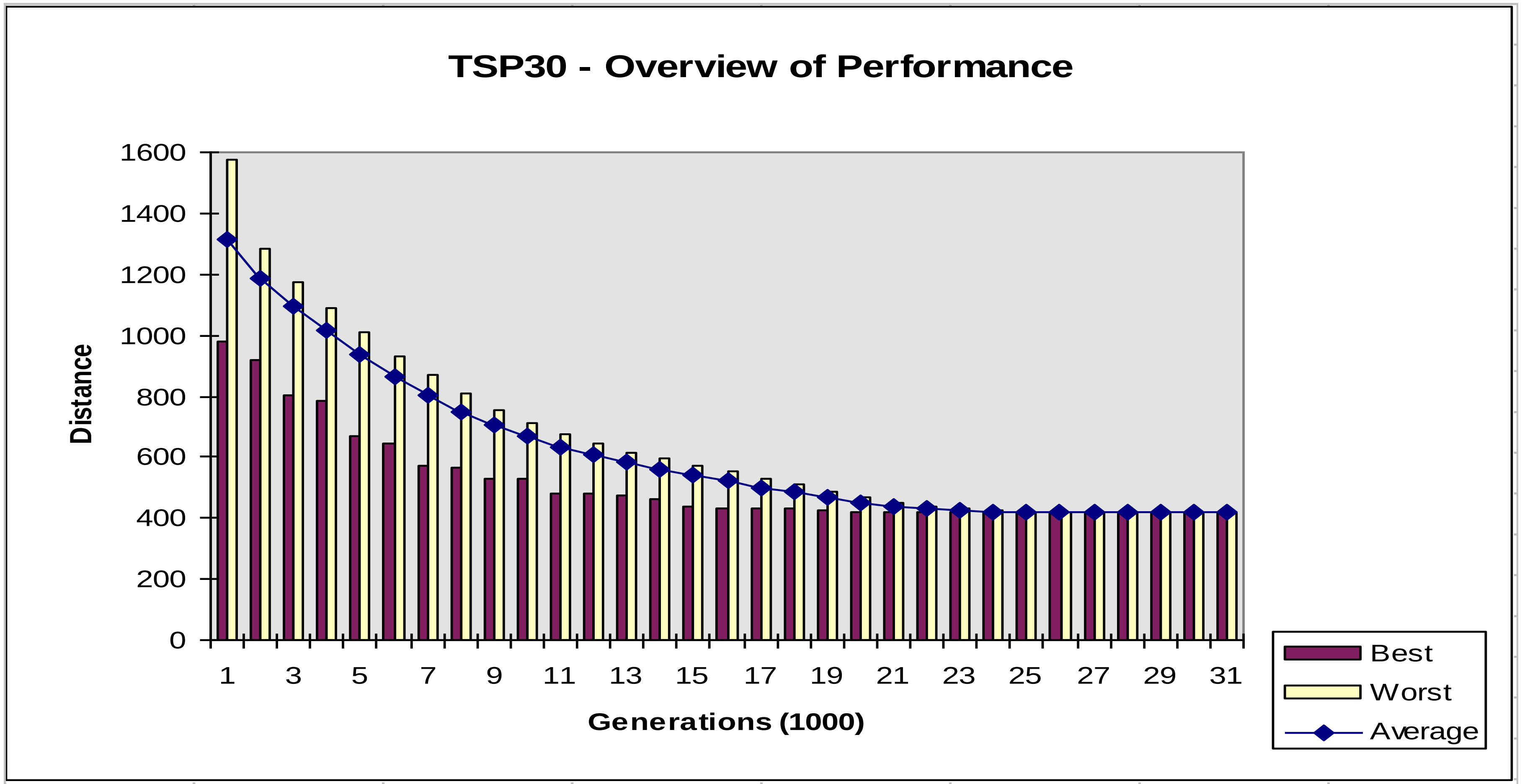
Solution $_k$(Distance = 652)



Best Solution (Distance = 420)



Overview of Performance



Genetic algorithms- 8 Queen Example

- A state is represented as a string over a finite alphabet (e.g. binary)
  - 8-queens
    - State = position of 8 queens each in a column

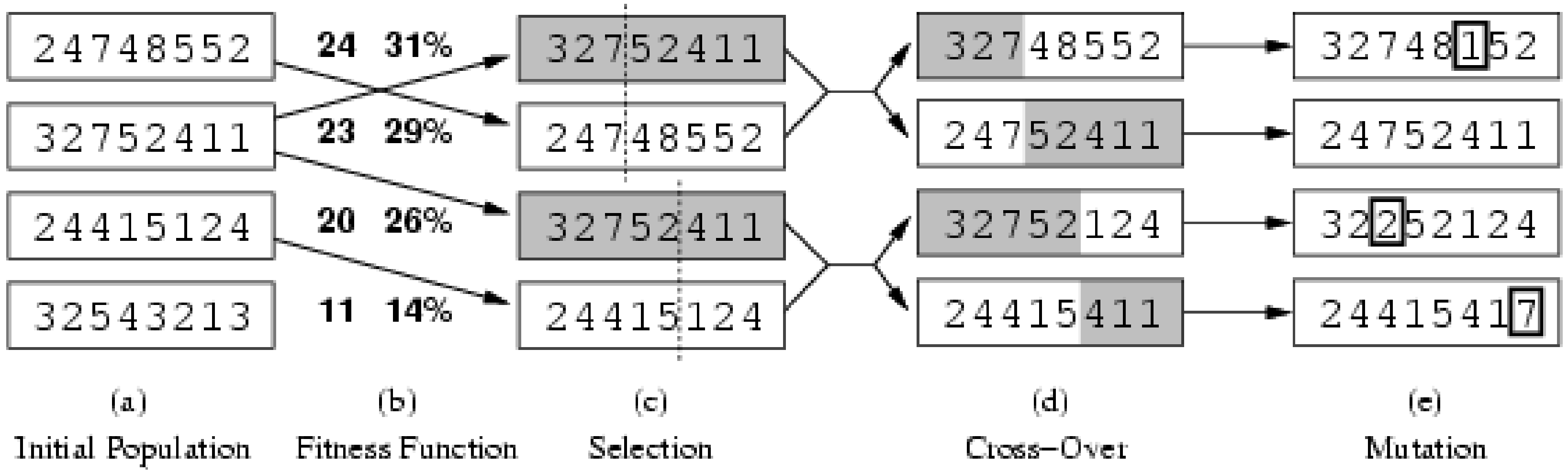
      => 8 x log(8) bits = 24 bits (for binary representation)
- **Start with *k* randomly generated states (population)**
- **Evaluation function (fitness function).**
  - Higher values for better states.
  - Opposite to heuristic function, e.g., # non-attacking pairs in 8-queens
  - Solution has a **value of 28**
- **Produce the next generation of states by “simulated evolution”**
  - **Random selection**
  - **Crossover**
  - **Random mutation**

Genetic algorithms

Fitness function: number of non-attacking pairs of queens (min = 0, max = 8 × 7/2 = 28)

24/(24+23+20+11) = 31%

23/(24+23+20+11) = 29% etc



4 states for 8-queens problem

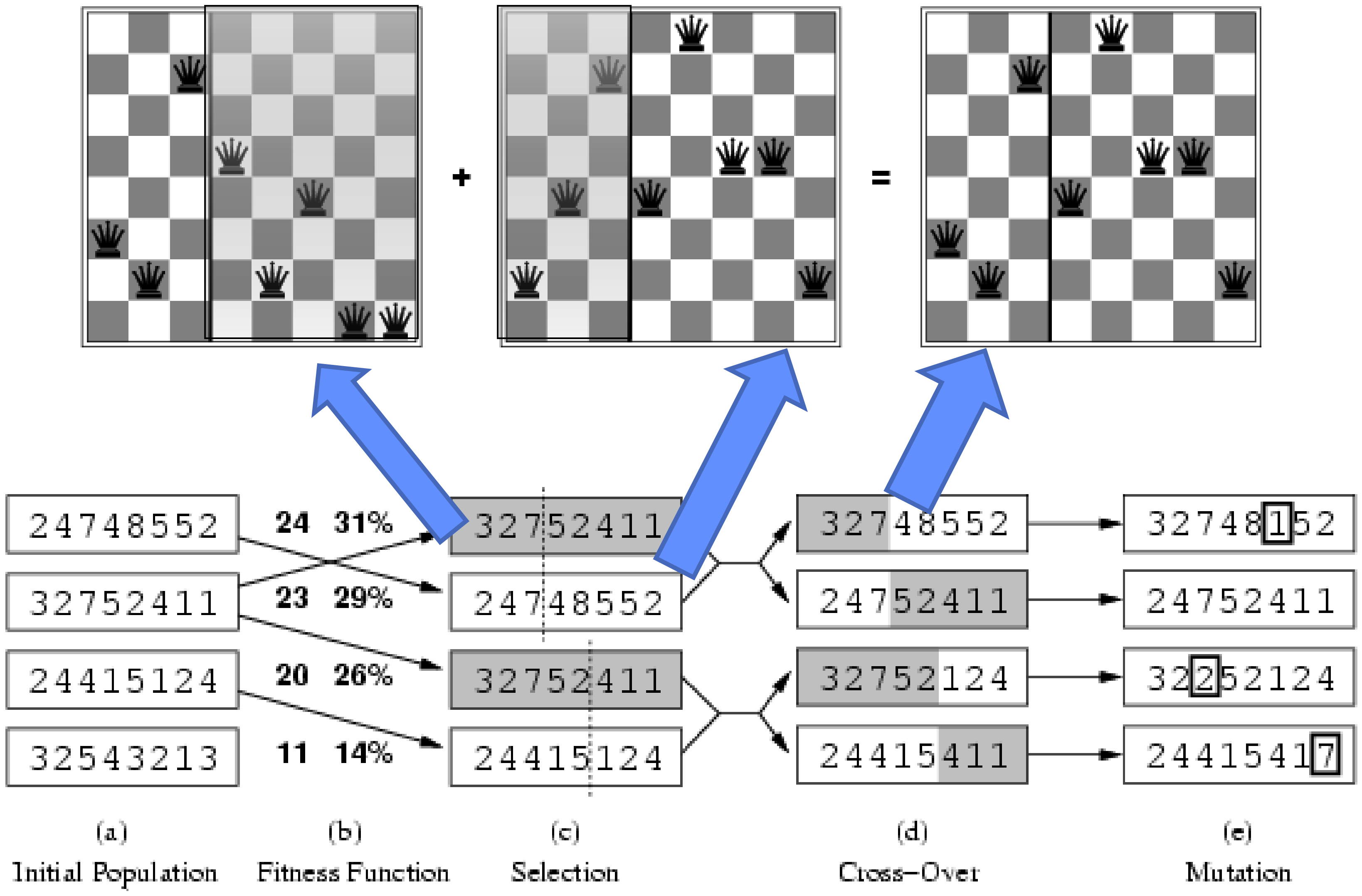
2 pairs of 2 states randomly selected based on fitness. Random crossover points selected

New states after crossover

Random mutation applied

Genetic algorithms

Like Simulated Annealing **Cross Over** takes **large** steps at the **beginning** of the search process while **small** steps when the population (individuals) are quite similar.



Genetic algorithm pseudocode

```
function GENETIC_ALGORITHM( population, FITNESS-FN) return an individual
    input: population, a set of individuals
           FITNESS-FN, a function which determines the quality of the individual
    repeat
           new_population $\leftarrow$ empty set
           loop for i from 1 to SIZE(population) do
                 x $\leftarrow$ RANDOM_SELECTION(population, FITNESS_FN)
                 y $\leftarrow$ RANDOM_SELECTION(population, FITNESS_FN)
                 child $\leftarrow$ REPRODUCE(x,y)
                 if (small random probability) then child $\leftarrow$ MUTATE(child )
                 add child to new_population
           population $\leftarrow$ new_population
    until some individual is fit enough or enough time has elapsed
    return the best individual
```

Issues for GA Practitioners

- Choosing basic implementation issues:
  - representation
  - population size, mutation rate, ...
  - selection, deletion policies
  - crossover, mutation operators
- Termination Criteria
- Performance, scalability
- Solution is only as good as the evaluation function (often hardest part)

Benefits of Genetic Algorithms

- Concept is easy to understand
- Modular, separate from application
- Supports multi-objective optimization
- Good for “noisy” environments
- Always an answer; answer gets better with time
- Inherently parallel; easily distributed

Benefits of Genetic Algorithms (cont.)

- Many ways to speed up and improve a GA-based application as knowledge about problem domain is gained
- Easy to exploit previous or alternate solutions
- Flexible building blocks for hybrid applications
- Substantial history and range of use