Digital Image Processing

Image Restoration:

Noise Removal

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Contents

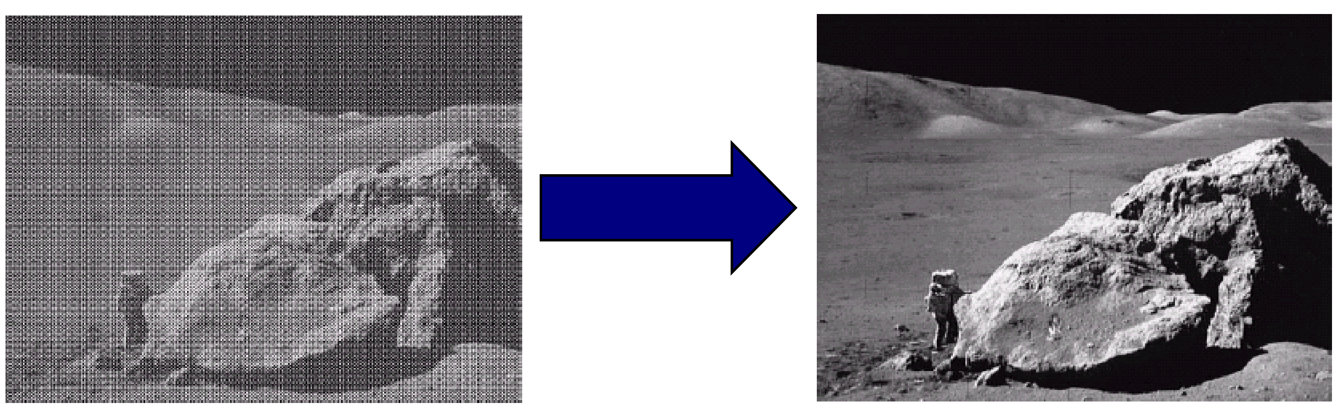
In this lecture we will look at image restoration techniques used for noise removal

- What is image restoration?
- Noise and images
- Noise models
- Noise removal using spatial domain filtering
- Periodic noise
- Noise removal using frequency domain filtering

What is Image Restoration?

Image restoration attempts to restore images that have been degraded

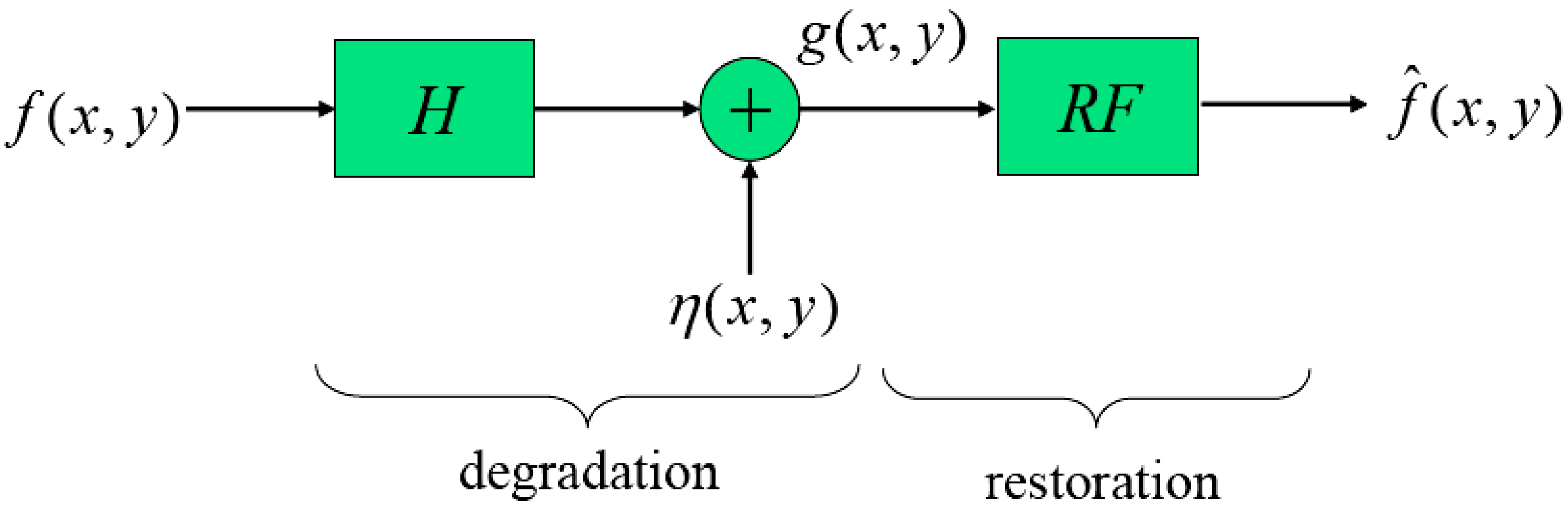
- **Identify** the **degradation process** and attempt to reverse it
- Similar to image enhancement, but more objective



What is Image Restoration?

- Goal of image restoration
  - Improve an image in some predefined sense
  - Difference with image enhancement ?
- Features
  - Image restoration v.s image enhancement
  - Objective process v.s. subjective process
  - A prior knowledge v.s heuristic process
  - A prior knowledge of the degradation phenomenon is considered
  - Modeling the degradation and apply the inverse process to recover the original image

Image Degradation/Restoration Model



- **When *H* is a LSI system**

$$g(x,y) = h(x,y) * f(x,y) + \eta(x,y)$$

$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

Noise and Images

The sources of noise in digital images arise during image acquisition (digitization) and transmission:

- **Noise from sensors**
  - Electronic circuits
  - Light level
  - Sensor temperature
- **Noise from environment**
  - Interference during Transmission
  - Lightening
  - Atmospheric disturbance
  - Other strong electric/magnetic signals

Noise Model

We can consider a noisy image to be modelled as follows:

$$g(x, y) = f(x, y) + \eta(x, y)$$

where $f(x, y)$ is the original image pixel, $\eta(x, y)$ is the noise term and $g(x, y)$ is the resulting noisy pixel

If we can estimate the model the noise in an image is based on this will help us to figure out how to restore the image

Noise probability density functions

- Since noise in pixels is random so image noises are taken as **random variables**
- Random variables have:
  - Probability density function (PDF)

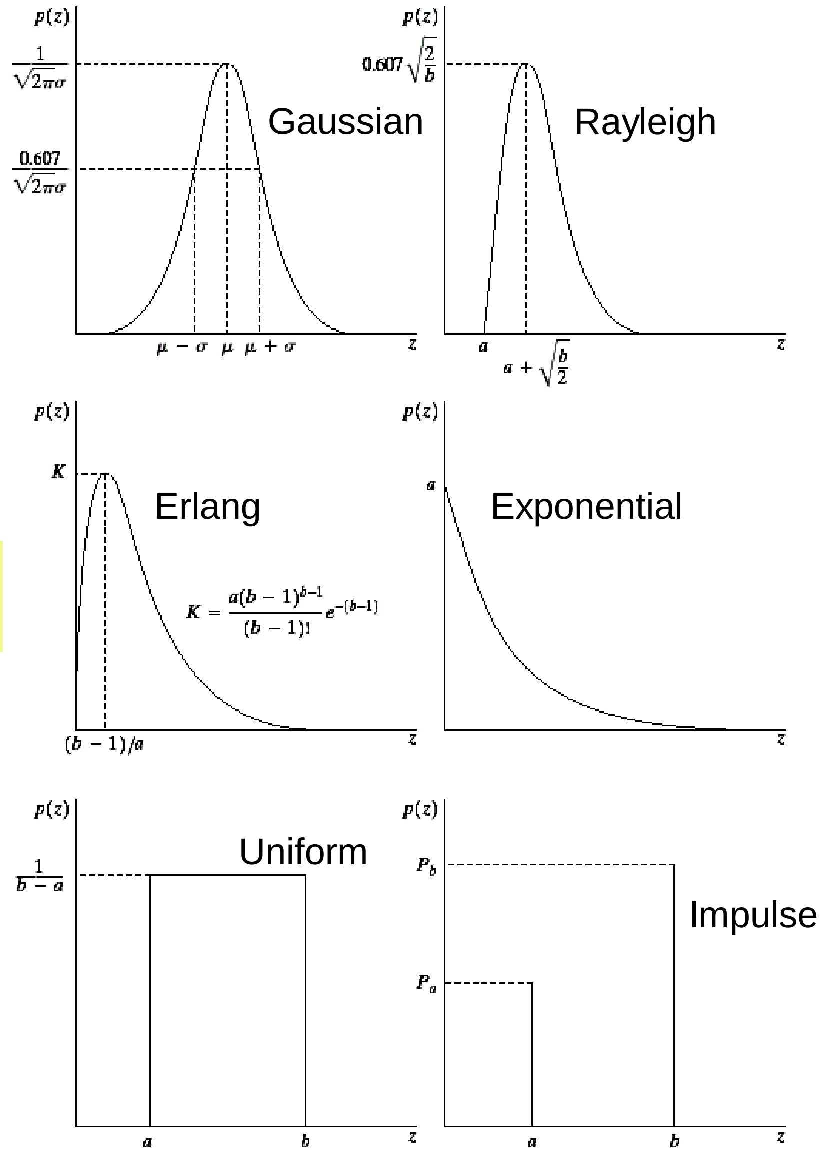
Famous Noise Models

There are many different models for the image noise term $\eta(x, y)$:

- Gaussian
  - Most common model
- Rayleigh

$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-(z-a)^2/b} & \text{for } z \ge a \\ = 0 & \text{for } z < a \end{cases}$$

- Erlang
- Exponential
- Uniform
- Impulse
  - *Salt and pepper* noise



Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Gaussian noise

- Math. Tractability (convenience) in spatial and frequency domain
- **Electronic circuit** noise and **sensor** noise follows Gaussian distribution i.e.,

$$p(z) = \frac{1}{\sqrt{2\pi\sigma}} e^{-(z-\mu)^2/2\sigma^2}$$

mean

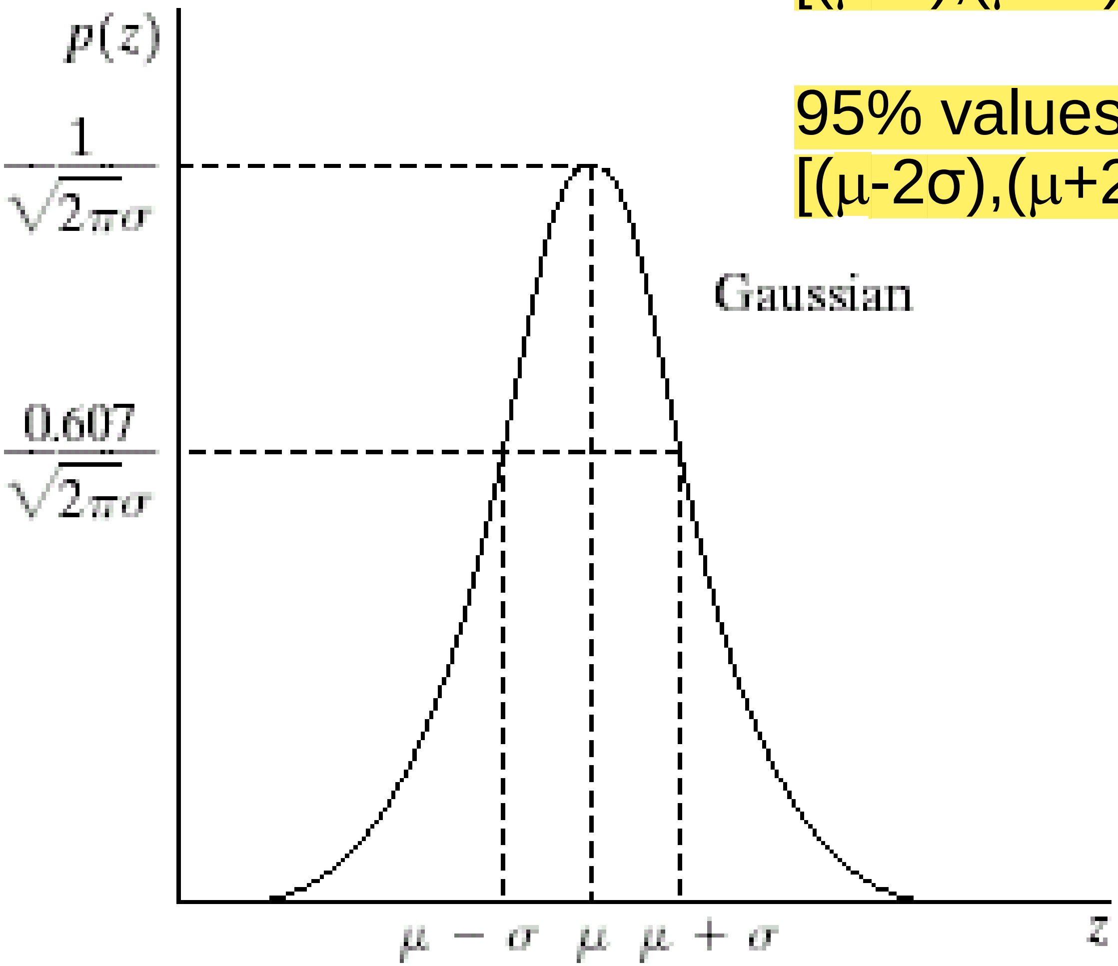
variance

Note: $$\int_{-\infty}^{\infty} p(z)dz = 1$$

Gaussian noise (PDF)

70% values of z fall in the range
[(μ-σ),(μ+σ)]

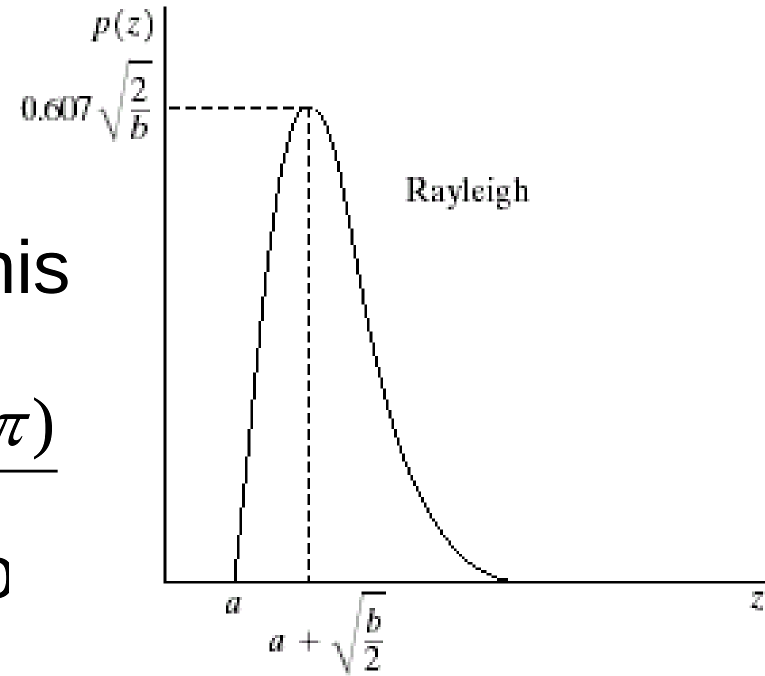
95% values of z fall in the range
[(μ-2σ),(μ+2σ)]



Rayleigh Noise

- **Rayleigh** noise

$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-(z-a)^2/b} & \text{for } z \geq a \\ 0 & \text{for } z < a \end{cases}$$



– The mean and variance of this density are given by

$$\mu = a + \sqrt{\pi b/4} \text{ and } \sigma^2 = \frac{b(4-\pi)}{4}$$

– a and b can be obtained thro mean and variance

– Noise in **MRI and sea bed images** follows Rayleigh distribution

Erlang Noise

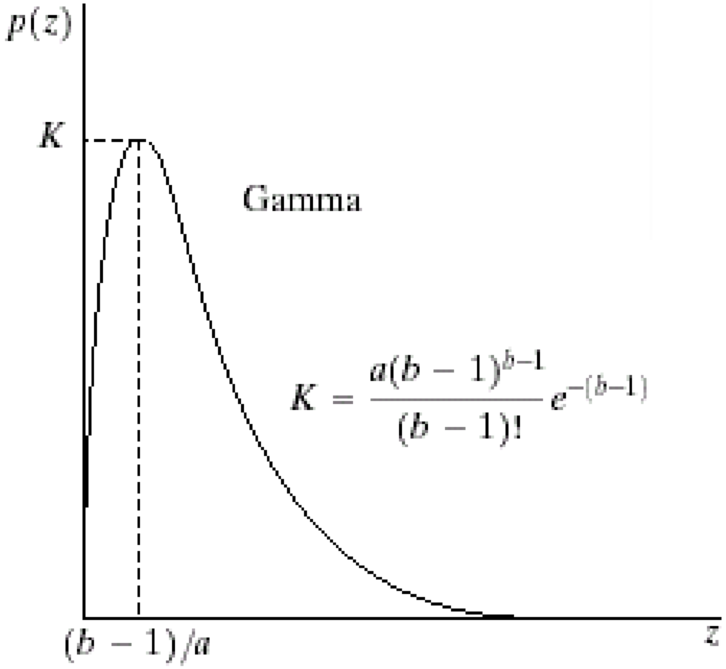
- **Erlang (Gamma)** noise

$$p(z) = \begin{cases} \dfrac{a^b z^{b-1}}{(b-1)!} e^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases}$$

  – The mean and variance of this density are given by

$$\mu = b/a \text{ and } \sigma^2 = \frac{b}{a^2}$$

  – a and b can be obtained through mean and variance



Exponential Noise

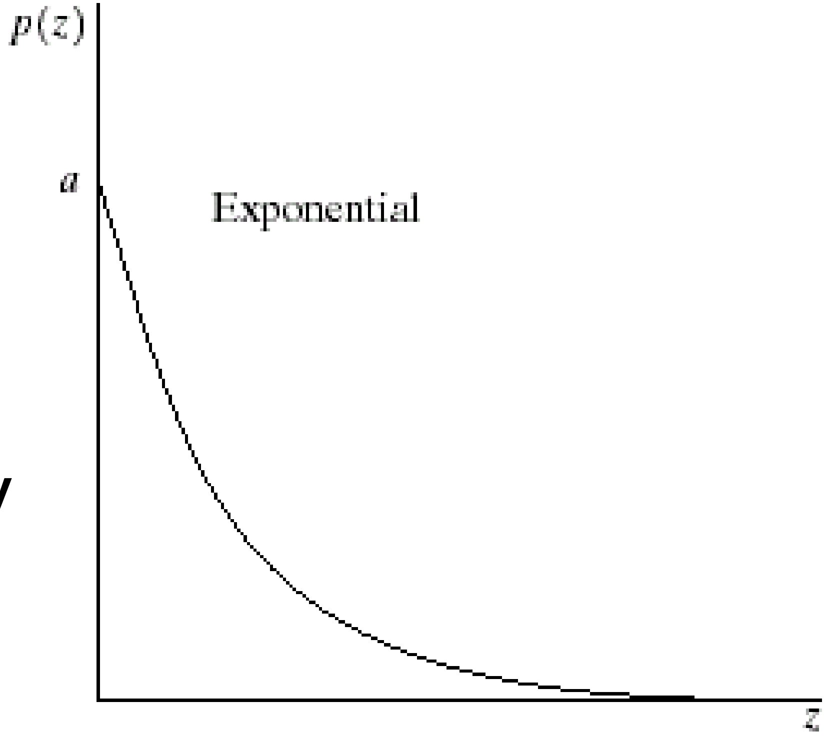
- **Exponential** noise

$$p(z) = \begin{cases} ae^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases}$$

  - The mean and variance of this density are given by

$$\mu = 1/a \ \text{ and } \sigma^2 = \frac{1}{a^2}$$

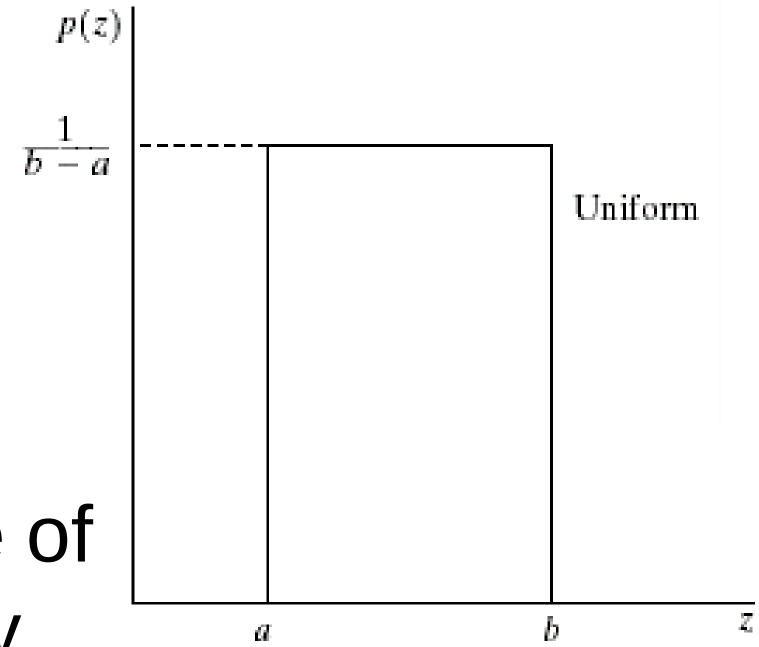
  - Special case pf Erlang PDF with b=1



Uniform Noise

- **Uniform** noise

$$p(z) = \begin{cases} \dfrac{1}{b-a} & \text{if } a \le z \le b \\ 0 & \text{otherwise} \end{cases}$$



  – The mean and variance of this density are given by

$$\mu = (a+b)/2 \text{ and } \sigma^2 = \frac{(b-a)^2}{12}$$

  – Used to Model **Quantization** noise in images

Impulse (salt-and-pepper) noise

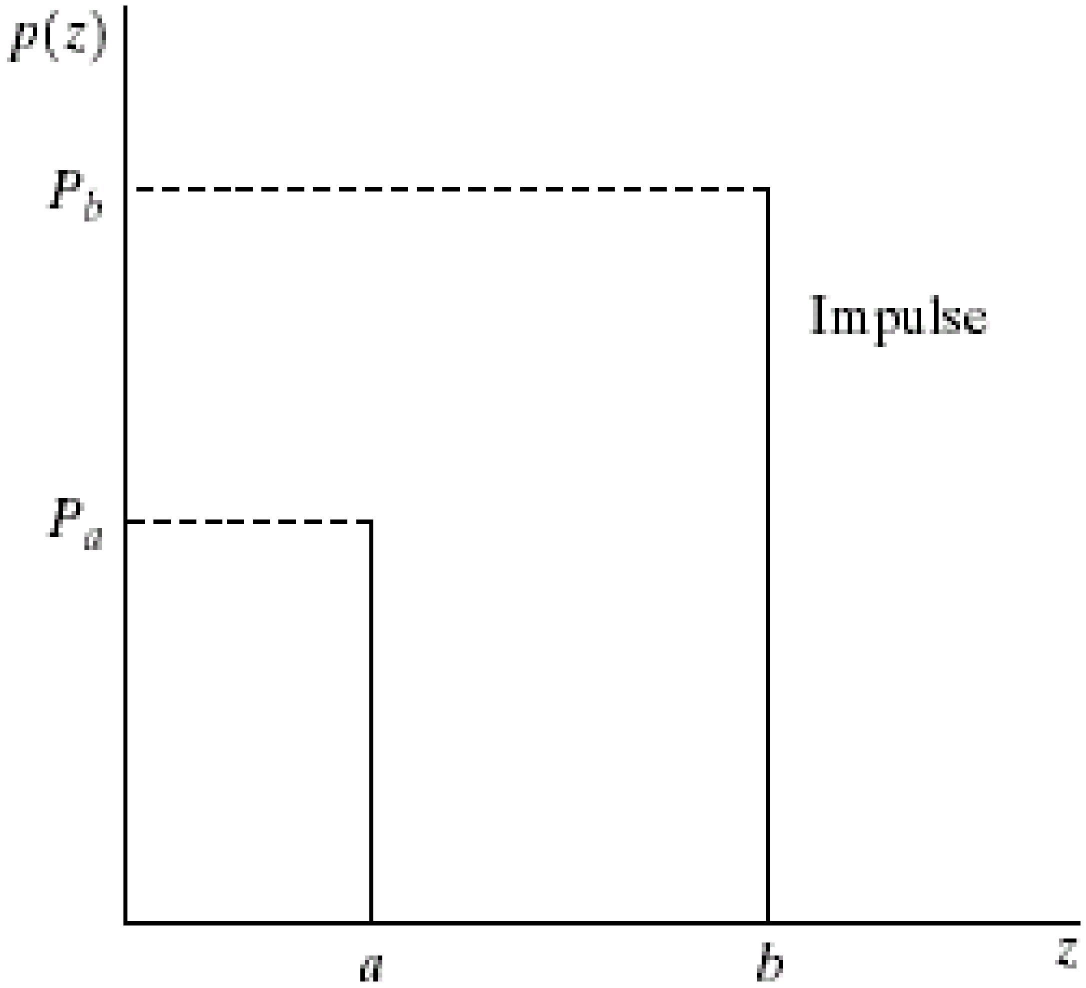
- Quick transients, such as faulty switching during imaging

$$p(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases}$$

If either P_a or P_b is zero, it is called *unipolar.* Otherwise, it is called bipoloar.

- In practice, impulses are usually stronger than image signals. Ex., a=0(black) and b=255(white) in 8-bit image.

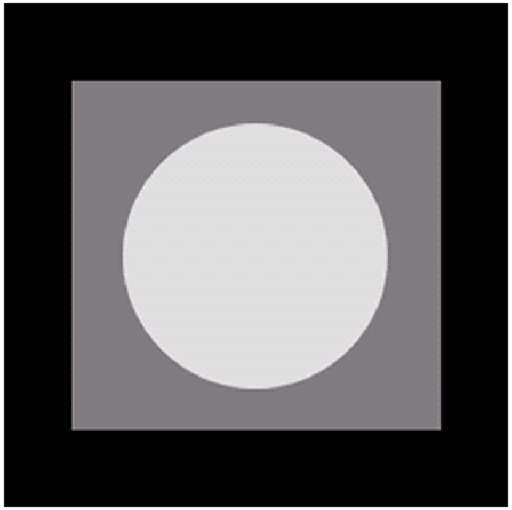
Impulse (salt-and-pepper) noise PDF



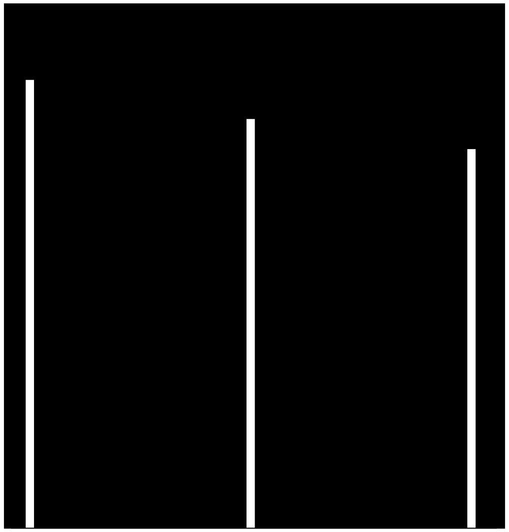
Noise Example

The test pattern to the right is ideal for demonstrating the addition of noise

The following slides will show the result of adding noise based on various models to this image

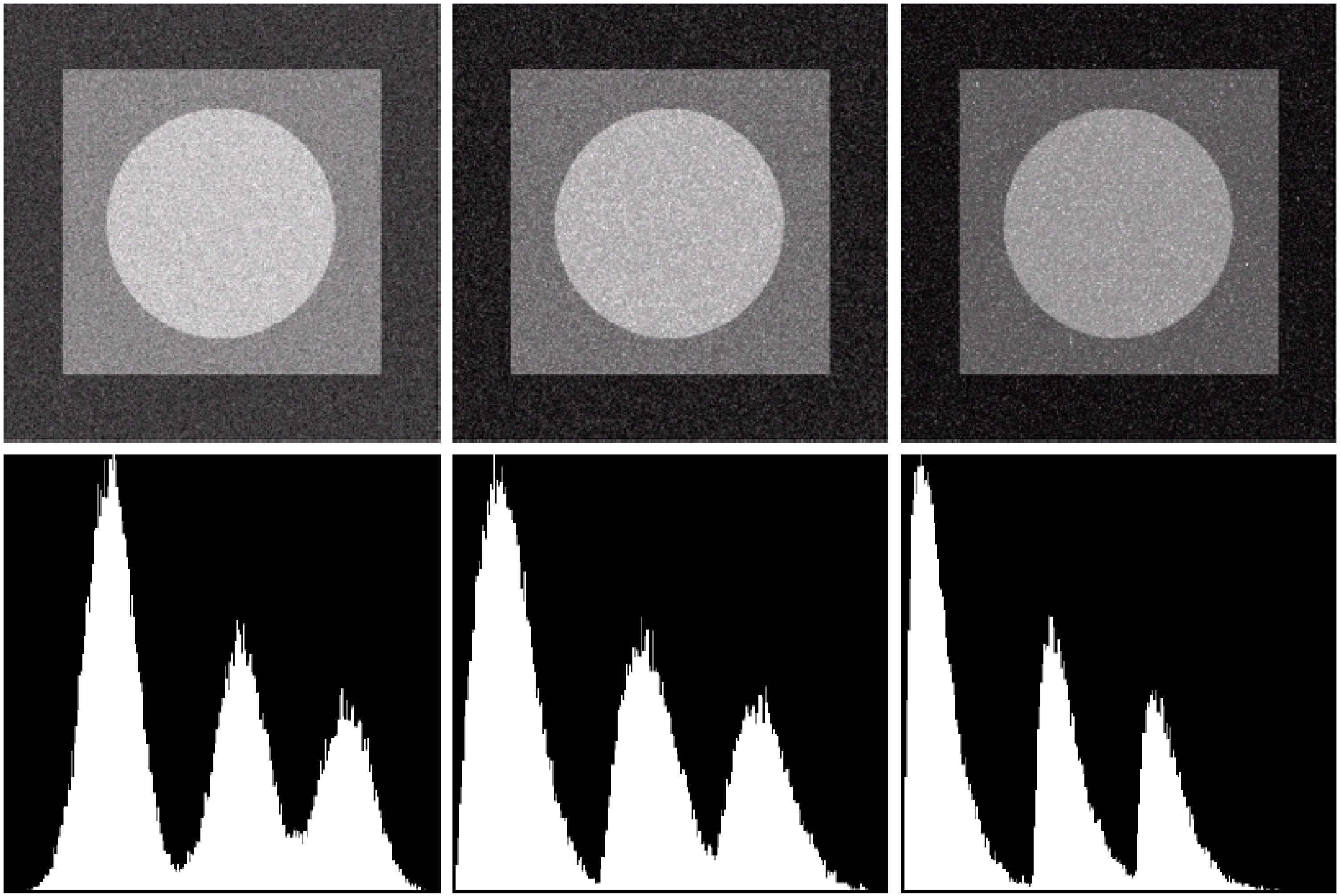


Image



Histogram

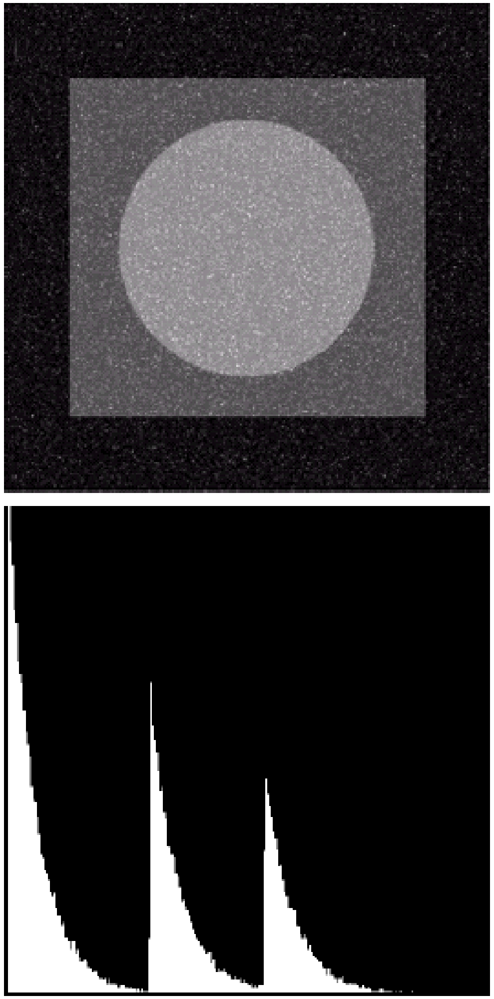
Noise Example (cont…)



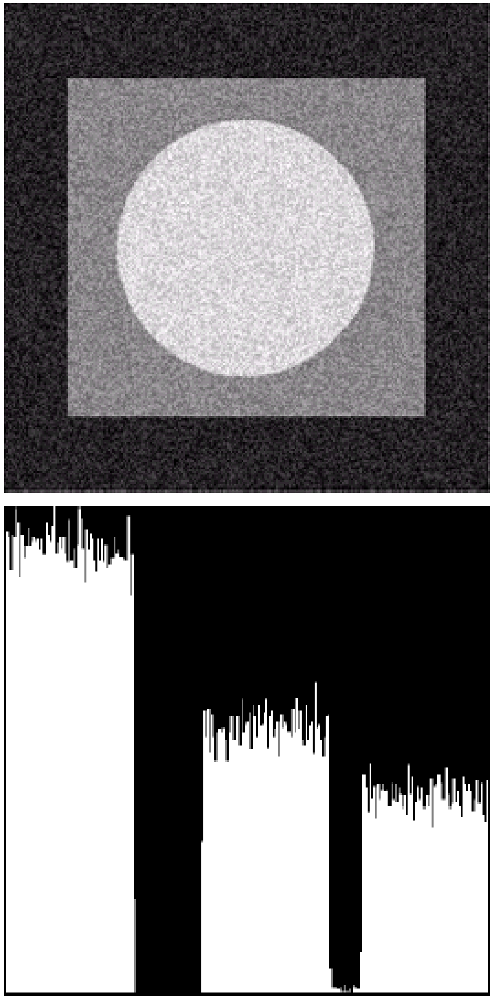
Gaussian Rayleigh Erlang

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

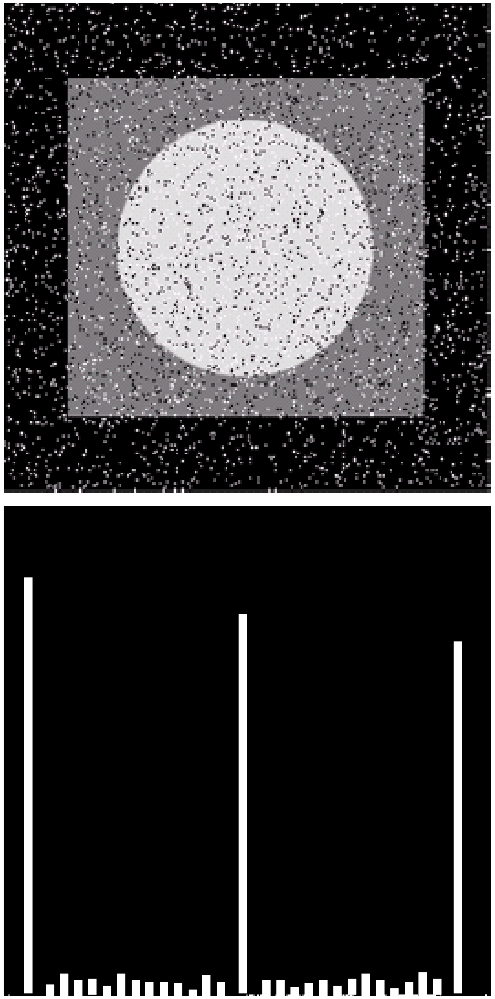
Noise Example (cont…)



Exponential



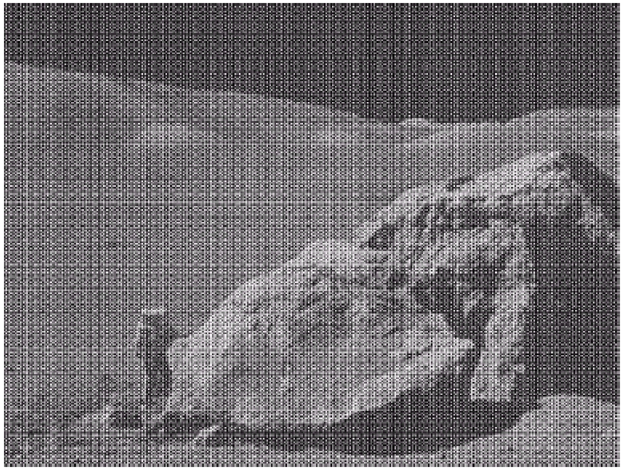
Uniform

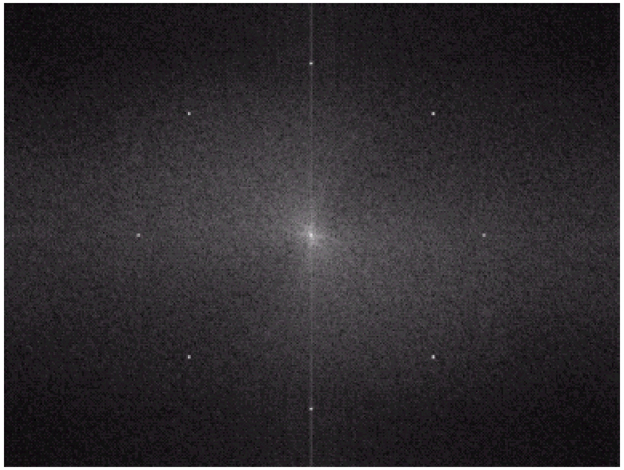


Impulse

Periodic Noise

- Arises typically from **electrical or electromechanical interference** during image acquisition
- It can be observed by visual inspection both in the spatial domain and frequency domain
- The only spatially dependent noise will be considered





Estimation of Noise Parameters

- Periodic noise
  - Parameters can be estimated by inspection of the spectrum
- Noise PDFs
  - From sensor specifications
  - If imaging sensors are available, capture a set of images of plain environments
  - If only noisy images are available, parameters of the PDF involved can be estimated from **small patches** of constant regions of the noisy images

Estimation of Noise Parameters

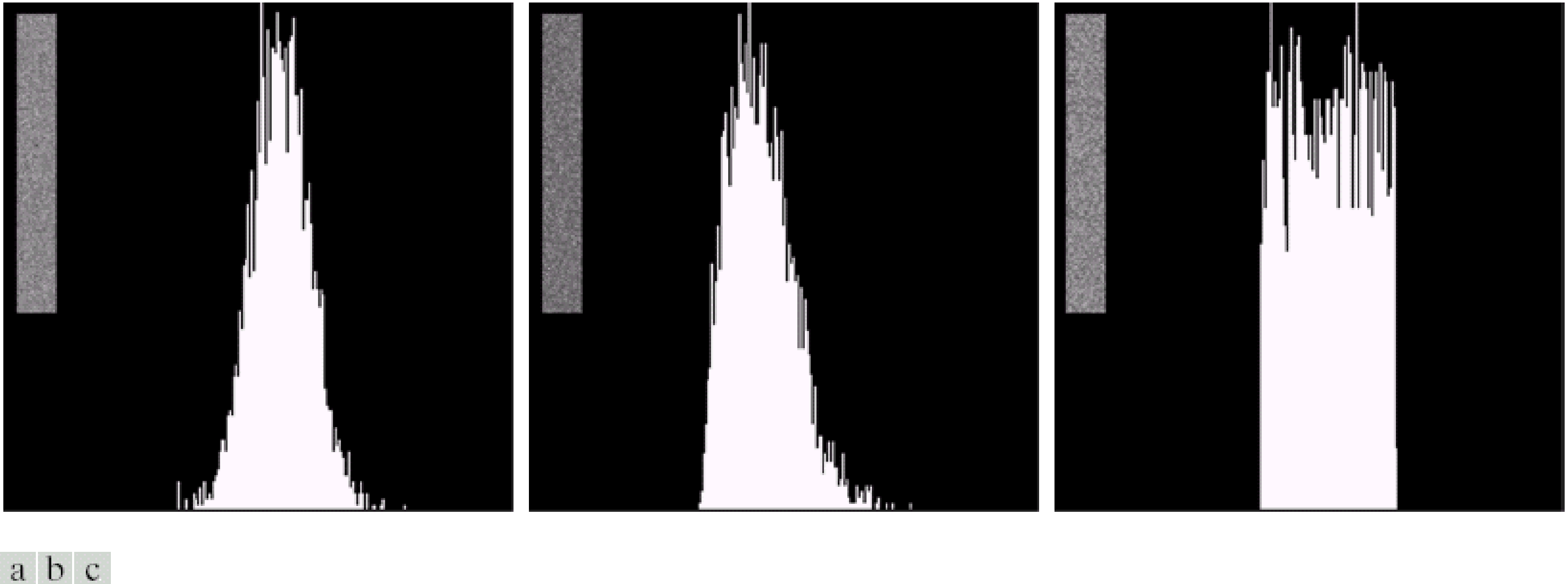


FIGURE 5.6 Histograms computed using small strips (shown as inserts) from (a) the Gaussian, (b) the Rayleigh, and (c) the uniform noisy images in Fig. 5.4.

- In most cases, only mean and variance are to be estimated
  - Others can be obtained from the estimated mean and variance

$$\hat{\mu} = \frac{1}{N_S} \sum_{(x_i, y_i) \in S} z(x_i, y_i)$$

$$\hat{\sigma}_0^{\,2} = \frac{1}{N_S} \sum_{(x_i, y_i) \in S} (z(x_i, y_i) - \mu)^2$$

Restoration by Spatial Filtering

We can use **spatial filters** of different kinds to remove different kinds of noise.

The ***arithmetic mean*** filter is a very simple one and is calculated as follows:

$$\hat{f}(x, y) = \frac{1}{mn} \sum_{(s,t) \in S_{xy}} g(s,t)$$

Blurs the image to remove noise.

Well **suited for random noise like Gaussian or Uniform noise**.

Other Means

There are different kinds of mean filters all of which exhibit slightly different behaviour:

- Geometric Mean (better results than mean)
- Harmonic Mean
- Contraharmonic Mean

Other Means (cont…)

There are other variants of the mean which can give different performance

Geometric Mean:

$$\hat{f}(x, y) = \left[\prod_{(s,t) \in S_{xy}} g(s,t) \right]^{\frac{1}{mn}}$$

Achieves similar smoothing to the arithmetic mean, but **tends to lose less image detail** hence results in sharper image. Also like mean filter well suited for **random noise**.

Noise Removal Examples

Original Image

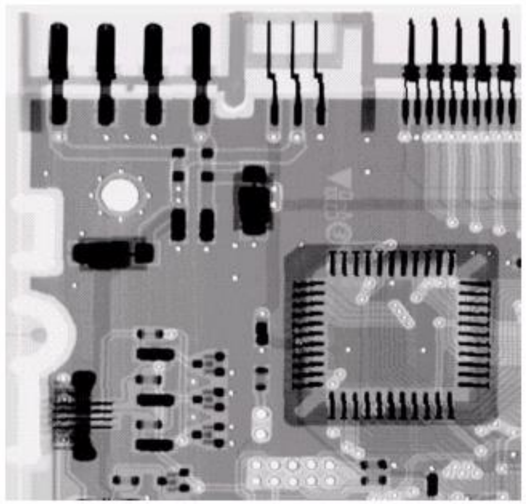
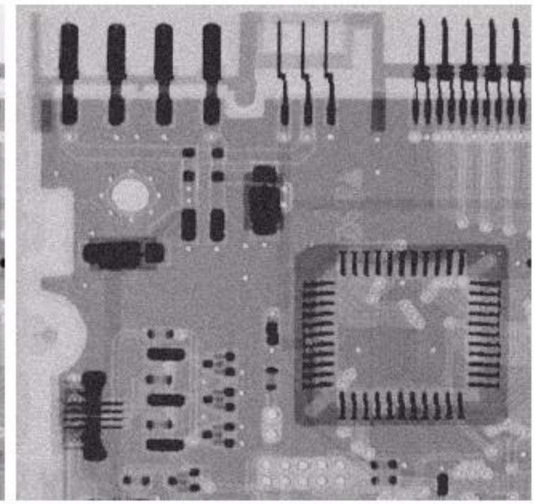
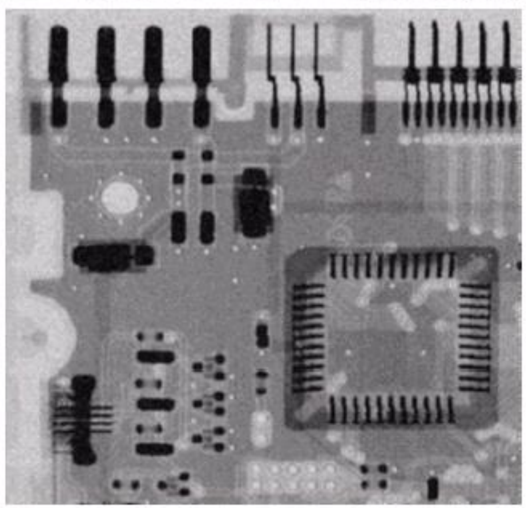


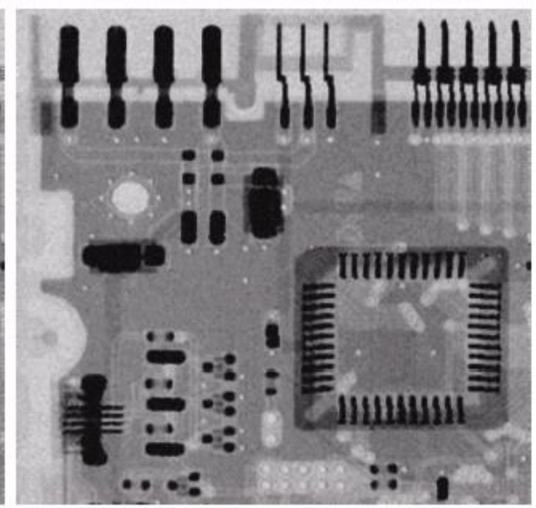
Image Corrupted By Gaussian Noise



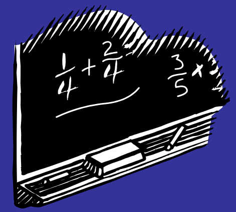
After A 3*3 Arithmetic Mean Filter



After A 3*3 Geometric Mean Filter



Images taken from Gonzalez & Woods, Digital Image Processing (2002)



Other Means (cont…)

Harmonic Mean:

$$\hat{f}(x, y) = \frac{mn}{\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

Works well for **salt noise**, but **fails** for **pepper noise**.

Also does well for other kinds of noise such as **Gaussian noise**.

Other Means (cont…)

Contraharmonic Mean:

$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

Q is the *order* of the filter and adjusting its value changes the filter's behaviour

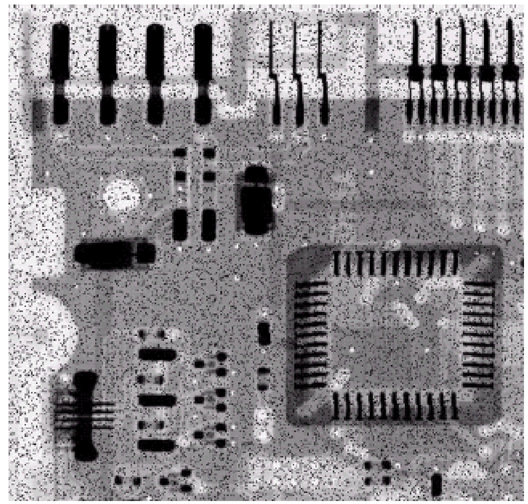
Positive values of Q eliminate **pepper** noise

Negative values of Q eliminate **salt** noise

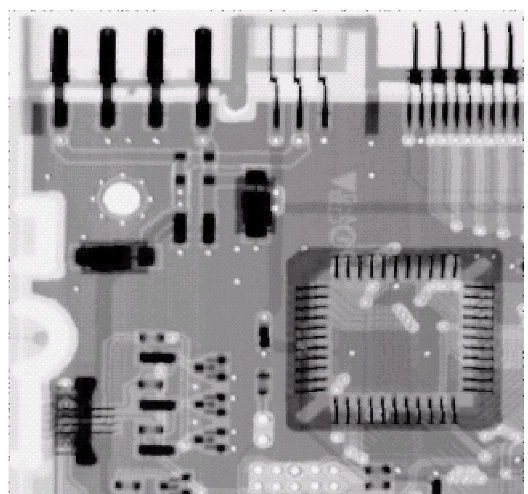
Noise Removal Examples (cont…)

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Image Corrupted By Pepper Noise



Result of Filtering Above With 3*3 Contraharmonic Q=1.5



Noise Removal Examples (cont…)

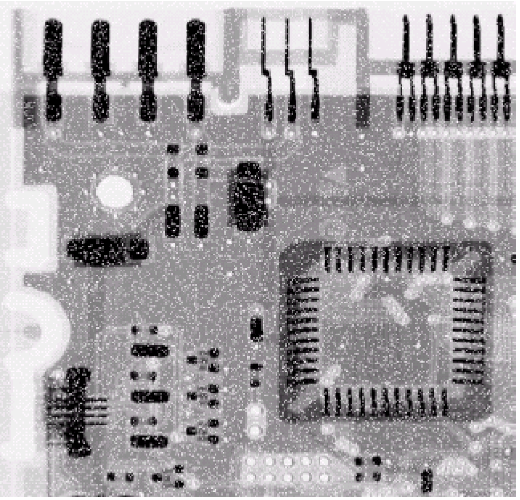
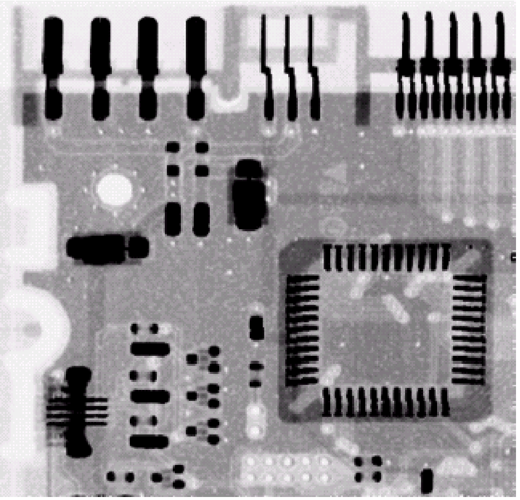


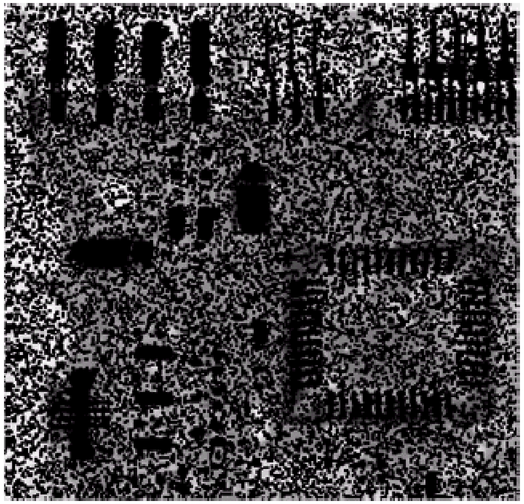
Image Corrupted By Salt Noise



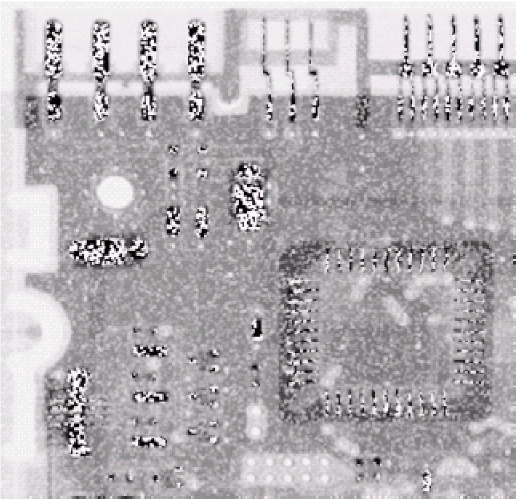
Result of Filtering Above With 3*3 Contraharmonic Q=-1.5

Contraharmonic Filter: Here Be Dragons

Choosing the wrong value for Q when using the contraharmonic filter can have drastic results



Negative value of Q



Positive value of Q

Order Statistics Filters

Spatial filters that are based on ordering the pixel values that make up the neighbourhood operated on by the filter

Useful spatial filters include

- Median filter
- Max and min filter
- Midpoint filter
- Alpha trimmed **mean** filter
- Adaptive Filtering

We Prefer Sharp Images!!

Median Filter

Median Filter:

$$\hat{f}(x, y) = \underset{(s,t) \in S_{xy}}{median}\{g(s,t)\}$$

Excellent at noise removal, without the smoothing effects that can occur with other smoothing filters

Particularly good when **salt and pepper noise** is present

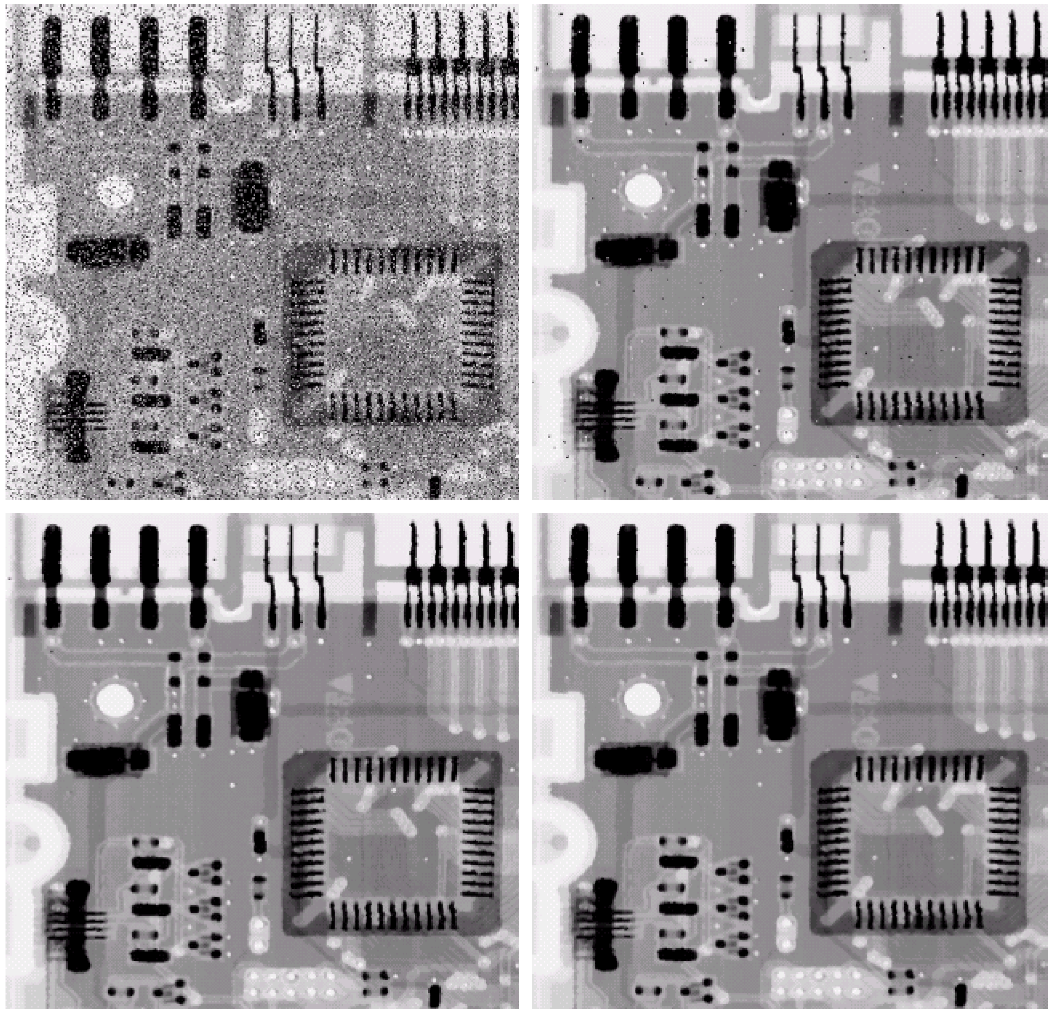
Noise Removal Examples

Image Corrupted By Salt And Pepper Noise

Result of 1 Pass With A 3*3 Median Filter

Result of 2 Passes With A 3*3 Median Filter

Result of 3 Passes With A 3*3 Median Filter



Max and Min Filter

Max Filter:

$$\hat{f}(x, y) = \max_{(s,t) \in S_{xy}} \{g(s,t)\}$$

Min Filter:

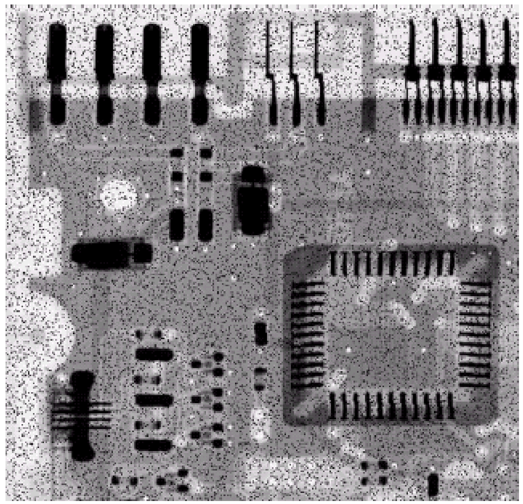
$$\hat{f}(x, y) = \min_{(s,t) \in S_{xy}} \{g(s,t)\}$$

Max filter is good for pepper noise and min is good for salt noise

Min & Max also do Dilation and Erosion respectively.

Noise Removal Examples (cont…)

Image Corrupted By Pepper Noise



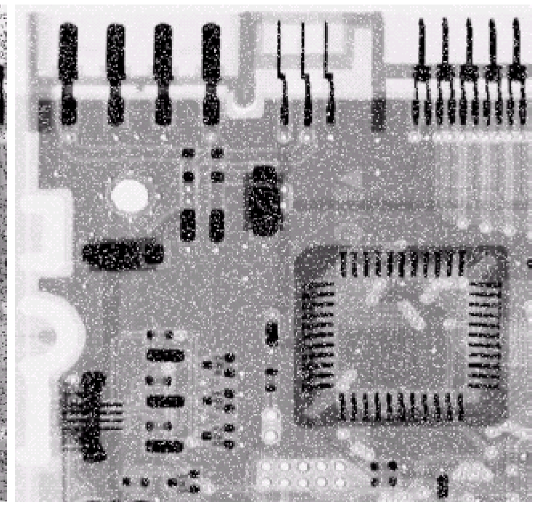
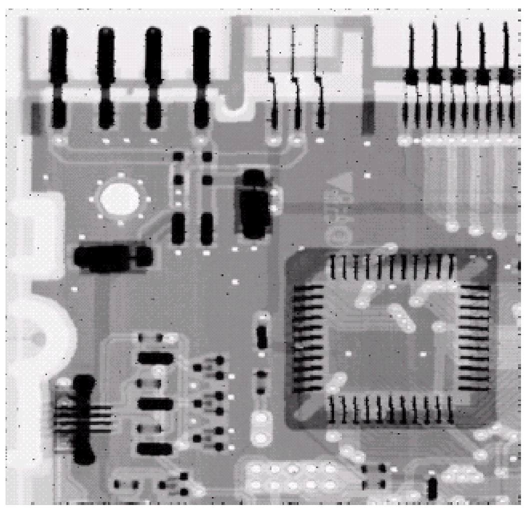
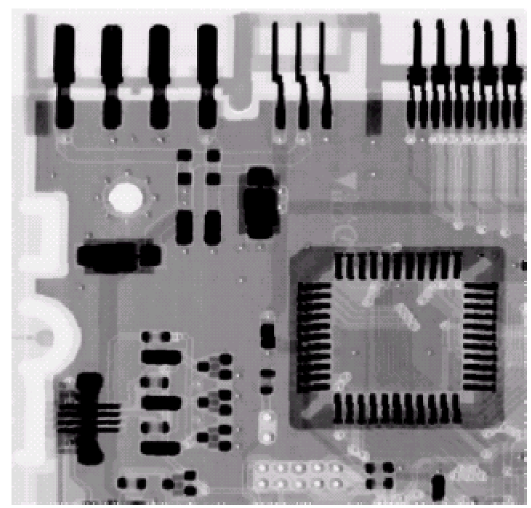


Image Corrupted By Salt Noise

Result Of Filtering Above With A 3*3 Max Filter





Result Of Filtering Above With A 3*3 Min Filter

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Midpoint Filter

Midpoint Filter:

$$\hat{f}(x,y) = \frac{1}{2}\left[\max_{(s,t)\in S_{xy}}\{g(s,t)\} + \min_{(s,t)\in S_{xy}}\{g(s,t)\}\right]$$

Good for random **Gaussian and uniform** noise.

Tends to produce unwanted artifacts in the image.

Alpha-Trimmed Mean Filter

Alpha-Trimmed Mean Filter:

$$\hat{f}(x,y) = \frac{1}{mn-d} \sum_{(s,t)\in S_{xy}} g_r(s,t)$$

We can delete the $d/2$ lowest and $d/2$ highest grey levels

So $g_r(s, t)$ represents the remaining $mn - d$ pixels

Suitable for situations involving <u>**multiple types of noise,**</u> such as combination of Gaussian and Salt & Pepper noise.

With $d=0$, ATM Filter becomes Mean filter.

With $d=mn\text{-}1$, ATM Filter becomes Median filter.

Noise Removal Examples (cont…)

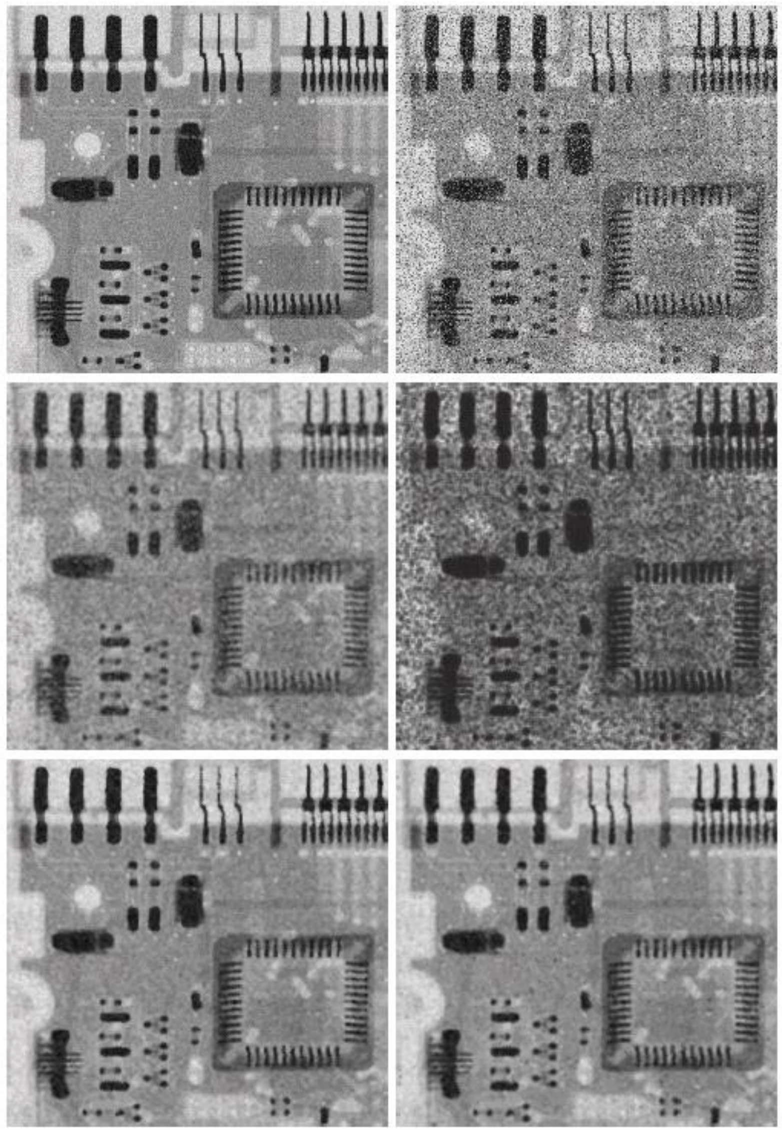


Image Corrupted By Uniform Noise

Image Further Corrupted By Salt and Pepper Noise

Filtered By 5*5 Arithmetic Mean Filter

Filtered By 5*5 Geometric Mean Filter

Filtered By 5*5 Median Filter

Filtered By 5*5 Alpha-Trimmed Mean Filter

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Adaptive Filters - (intelligent!)

The filters discussed so far are applied to an entire image without any regard for how image characteristics vary from one point to another.

The behaviour of **adaptive filters** changes depending on the **characteristics** of the image inside the filter region.

We will take a look at the **adaptive median filter.**

Adaptive Local Noise Reduction Filter

1. If σ_η^2 is zero, the filter should return simply the value of $g(x, y)$. This is the trivial, zero-noise case in which $g(x, y)$ is equal to $f(x, y)$.
2. If the local variance is high relative to σ_η^2, the filter should return a value close to $g(x, y)$. A high local variance typically is associated with edges, and these should be preserved.
3. If the two variances are equal, we want the filter to return the arithmetic mean value of the pixels in S_{xy}. This condition occurs when the local area has the same properties as the overall image, and local noise is to be reduced simply by averaging.

An adaptive expression for obtaining $\hat{f}(x, y)$ based on these assumptions may be written as

$$\hat{f}(x, y) = g(x, y) - \frac{\sigma_\eta^2}{\sigma_L^2}\left[g(x, y) - m_L\right]. \tag{5.3-12}$$

Adaptive Median Filtering

The median filter performs relatively well on impulse noise as long as the spatial density of the impulse noise is not large (i.e., $p<0.20$)

The adaptive median filter can handle much more spatially dense impulse noise, and also performs some smoothing for non-impulse noise.

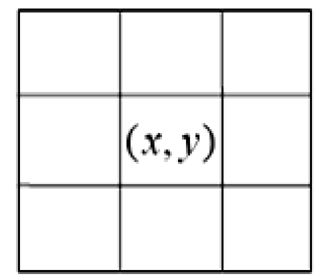
The key insight in the adaptive median filter is that the filter **size changes depending on the characteristics of the image.**

Adaptive Median Filtering (cont…)

Remember that filtering looks at each original pixel image in turn and generates a new filtered pixel

First examine the following notation:

- z_{min} = minimum grey level in S_{xy}
- z_{max} = maximum grey level in S_{xy}
- z_{med} = median of grey levels in S_{xy}
- z_{xy} = grey level at coordinates (x, y)
- S_{max} =maximum allowed size of S_{xy}



S_{xy}

Adaptive Median Filtering (cont…)

Level A: $A1 = z_{med} - z_{min}$

$A2 = z_{med} - z_{max}$

If $A1 > 0$ and $A2 < 0$, Go to level B

Else increase the window size

If window size $\leq S_{max}$ repeat level A

Else output z_{med}

z_{med} is not impulse noise, since: $z_{min} < z_{med} < z_{max}$

Level B: $B1 = z_{xy} - z_{min}$

$B2 = z_{xy} - z_{max}$

If $B1 > 0$ and $B2 < 0$, output z_{xy}

Else output z_{med}

z_{xy} is not impulse noise, since: $z_{min} < z_{xy} < z_{max}$

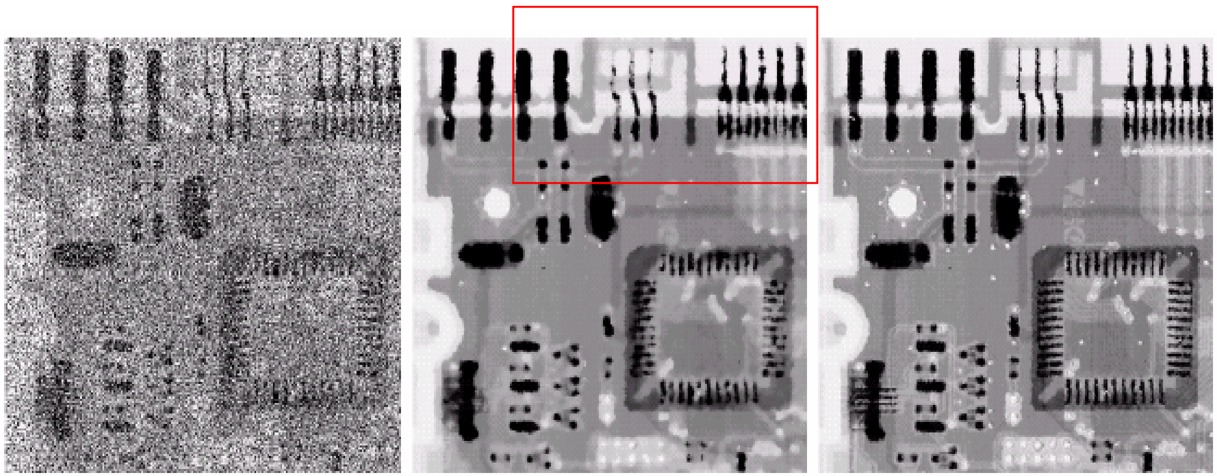
Adaptive Median Filtering (cont…)

The key to understanding the algorithm is to remember that the adaptive median filter has three purposes:

1. Remove impulse noise
2. Provide smoothing of other noise
3. Reduce distortion (excessive thining or thickening of object boundaries).

Adaptive Filtering Example

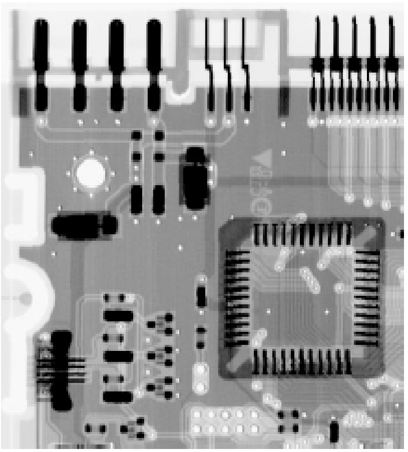
- **An example of results by two filters**



a b c

7×7

$S_{max} = 7 \times 7$



Original image

FIGURE 5.14 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.25$. (b) Result of filtering with a 7×7 median filter. (c) Result of adaptive median filtering with $S_{max} = 7$.

Noise removed effectively but significant loss of details

Noise removed effectively and preserving details and sharpness