Ordinal Logit and Fractional Regression: Analysis Options for Ordered and Continuous Bounded Variables

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Overview of the Presentation

Review options for analyzing dependent variables that are continuous over a bounded range or involve three or more ranked categories.

- **Consider pros and cons of OLS regression**
- **Consider pros and cons of Binary Logit regression**
- **Consider pros and cons of Multinomial Logit Regression**
- **Consider pros and cons of Ordered Logit Regression**
- **Consider pros and cons of Fractional Logit Regression**
- **Note related procedures such as Beta Regression**
- **Review selected results from analyses performed using demonstration programs.**

Assumptions

This presentation presumes familiarity with:

- **Multiple regression analysis**
- **Logit regression analysis**
- **Multinomial Logit regression analysis**

Also

- **Odds ratio transformations of proportions (OR= P/(1-P)**
- **Logit transformations of odds ratios (L = ln(OR))**
- **Inverse logit to odds ratio transforms ($OR = e^{L}$)**
- **Inverse odds ratio to prop transform (P = OR/(1+OR))**

Dependent Variables – Level of Measurement

Nominal Dependent Variables
- Distinctive outcomes can be identified
- Theory and research question make distinctions meaningful
- Notions of distance or even rank order are not valid

Ordinal Dependent Variables
- Adds the ability to make valid rank order distinctions
- Weak notions of "distance" thresholds justify rank distinctions
- Strong notions of a "distance" scale are not valid

Interval Dependent Variables (Integer & Continuous)
- Adds the ability to make valid distinctions on a "distance" scale
- True zero and relative distance distinctions are not valid

Ratio Dependent Variables (Continuous)
- Adds true zero
- Adds the ability to make relative scale "distance" distinctions

Dependent Variables – Bounded vs. Unbounded

Unbounded - In principle, lower and upper values are unbounded

Income, log-odds ratios, zero-centered scaled variables

Single Bounded – Lower or upper values are bounded (capped)

Odds ratios, open-ended count variables such as years of education, number of children, soccer goals, years since immigration, etc.

Double Bounded – Lower and upper values are bounded (capped)

All ordinal variables (e.g., income quintile, low-medium-high, etc.), Likert style scales, SES indices, inequality measures (e.g., Gini), segregation indices (e.g., Dissimilarity), proportions and rates (e.g., poverty rate)

Bounds can be intrinsic (lower boundary for number of children) or bounds can be imposed by measurement procedures (e.g., placing top and bottom codes on income) for practical reasons, to protect confidentiality, etc.

Selected Analysis Methods by Type of DV

	Nominal	Ordinal	Interval[a]	Ratio
	Bounded Dependent Variables			
Crosstab and X^2	√√√	√	---	---
Binary Logit Reg.	√√√	√	---	---
OLS Lin Prob Reg.	√	---	---	---
Multinomial Logit	√√√	√	---	---
Ordinal Logit Reg.	---	√√√	---	---
Count Reg.	---	---	√√√[b]	---
OLS Regression	---	√	√	√
Fractional Reg.	√√√	√	√√√	√√√
	Unbounded Dependent Variables			
OLS Regression	---	---	√√√	√√√

"a" includes both integer (count) and continuous variables; "b" integer (count); "---" does not apply; "√" feasible, but suboptimal; "√√√" feasible and optimal.

Summary Remarks on Selected Analysis Methods

This section provides brief remarks on pros and cons of analysis methods for different types of dependent variables

Analysis Methods – Dichotomous DV's

Cross-tabulation and Chi Square

- **Pros: Easy to implement**
- **Con1: No estimates of effects**
- **Con2: Multivariate analysis is difficult**

Linear probability regression (OLS using 0,1 DV)

- **Pros: Easy to interpret effects; easy to implement**
- **Con1: OLS assumptions for error term are not met**
- **Con2: Incorrectly assumes effects are linear & additive**

Logit regression

- **Pro1: Assumptions for error term are met**
- **Pro2: Correctly assumes effects are nonlinear & nonadditive**
- **Con1: Effects are harder to interpret**
- **Con2: Harder to implement and requires larger samples**

Analysis Methods – Polytomous DV's

Cross-tabulation and Chi Square

- **Pros: Easy to implement**
- **Cons: Multivariate analysis is difficult**

OLS regression – No options

Multinomial Logistic Regression

- **Pro1: Assumptions for error term are met**
- **Pro2: Correctly assumes effects are nonlinear & nonadditive**
- **Con1: Effects are very hard to interpret**
- **Con2: Harder to implement and requires larger samples**

Analysis Methods – Ordinal DV's

OLS Regression Using "Rank" (1, 2, 3, etc.) as DV

- **Pros: Easy to interpret and easy to implement**
- **Con1: OLS assumptions for error term are not met**
- **Con2: Incorrectly assumes effects are linear & additive**
- **Con3: Incorrectly assumes equal "distance" between ranks**

Multinomial Logit regression

- **Pro1: Assumptions for error term are met**
- **Pro2: Correctly assumes effects are nonlinear & nonadditive**
- **Con1: Effects are unnecessarily complicated and hard to interpret**
- **Con2: Harder to implement and requires larger samples**

Analysis Methods – Ordinal DV's (continued)

Fractional Regression (scale ranks to fall in 0-1 range)

- **Pro1: Assumptions for error term are met**
- **Pro2: Correctly assumes effects are nonlinear & nonadditive**
- **Con1: Parsimonious and easier to interpret**
- **Con2: Harder to implement and requires larger samples**
- **Con3: Incorrectly assumes equal "distance" between ranks**

Ordinal Logit Regression

- **Pro1: Assumptions for error term are met**
- **Pro2: Correctly assumes effects are nonlinear & nonadditive**
- **Pro3: Effects are parsimonious and easier to interpret (in comparison to multinomial logit regression)**
- **Pro4: Ordinality assumption can be tested (ordered logit is "nested" under multinomial logit regression)**
- **Cons: Harder to implement and requires larger samples**

Analysis Methods – Unbounded Interval DV's

OLS Regression Using "Raw Scores"

- **Pro1: Easy to interpret and easy to implement**
- **Pro2: OLS assumptions for error term are appropriate**
- **Pro3: Assumptions of linear, additive effects are appropriate**
- **Cons: Limited to usual OLS concerns**

Other Methods

- **Not indicated unless OLS assumptions are violated**
- **Next logical alternatives are to consider extensions of OLS regression such as Weight Least Squares regression, Robust regression, and Bootstrapped OLS regression**

Concerns – Double Bounded Continuous DV's

OLS regression assumes the dependent variable is "unbounded" on both the lower and upper ends of the relevant scale

- **This follows directly from the OLS assumptions that the errors of prediction (e_i) at any combination of values on the independent variables (X's) are: (a) normally distributed and (b) have uniform (equal) variance**
- **OLS errors for bounded variables do not meet this assumption; errors are non-normal and have unequal variance.**
- **Significance tests are compromised**

OLS regression assumes effects are linear and additive

- **This is inappropriate**
- **It can lead to larger errors of prediction, especially near the lower and upper boundaries of the DV**
- **In the extreme, can lead to predictions that are impossible (out-of-bounds)**

Analysis Methods – Double Bounded Continuous DV's

OLS Regression Using "Raw Scores" or, alternatively, Using a 0-1 Transform (to highlight bounds)

- **Pro1: Easy to interpret and easy to implement**
- **Pro2: In some cases, results can be fairly robust to mild violations of assumptions. For example, OLS analysis of SES scores coded 1-99 with predicted values in range 20-80.**
- **Con1: OLS assumptions for error term are not met**
- **Con2: Incorrectly assumes effects are linear & additive**

OLS Regression Using Logit (or Similar) Transform for "Raw Scores" Converted to 0-1 Range

- **Pro1: Easy to interpret and easy to implement**
- **Pro2: Appropriately assumes effects are nonlinear & nonadditive**
- **Con1: The logit transform is undefined at the boundary values (0 & 1) and results can be sensitive to arbitrary choices for how boundary values are scored**
- **Con2: Models the mean of logits, not the mean of "raw scores"**

Analysis Methods – Double Bounded Cont. DV's (II)

Fractional Regression (DV scaled to fall in 0-1 range)

- **Pro1: Assumptions for error term are met**
- **Pro2: Appropriately assumes effects are nonlinear & nonadditive**
- **Pro3: Especially flexible for modeling proportions**
- **Pro4: Directly models the mean of the DV (not the mean of a logit transform of the DV)**
- **Cons: Harder to implement, requires larger samples, uses quasi-likelihood theory (not maximum likelihood)**

Beta Regression (DV scaled to fall in 0-1 range)

- **Pros: Same as fractional regression**
- **Con1: Harder to implement, requires larger samples, uses quasi-likelihood theory (not maximum likelihood)**
- **Con2: The model is complicated – it involves simultaneously estimating separate parameters for effects on both the mean and the dispersion in the distribution**

Hands on with Ordinal Logit Regression – I

When is Ordinal Logit Regression relevant?

- **The DV has 3 or more categories**
  - **If only 2 categories, use regular logit regression**
- **The DV has rank order quality**
  - **If rank order is questionable, use multinomial logit regression**
- **The Distances between ranks are not equal**
  - **If distances are approximately equal, consider fractional regression**

What is the contrast with Multinomial Logit Regression?

- **Ordinal regression is simpler and more parsimonious, only one effect parameter for each X**
- **Ordinal logit regression "nests" under multinomial logit, so the assumption of simpler effects can be tested**

Hands on with Ordinal Logit Regression – II

What is the contrast with OLS Regression with Rank Scores for Y?

- **OLS assumptions for the error term are not met. So, significance tests are compromised.**
- **OLS incorrectly assumes effects are linear and additive. So, predictions can be flawed and even fall "out-of-bounds".**
- **OLS imposes an arbitrary assumption of equal distance between ranks. This can lead to incorrect predictions of the distribution of cases over categories of Y (the DV).**

What is the contrast with Binary Logit Regression?

- **One can "collapse" the ordinal variable into a simple dichotomy to make binary logit regression feasible.**
- **The advantage is the model is easier to interpret.**
- **The cost is potentially important loss of information regarding the dependent variable. Generally speaking, this is undesirable.**

Hands on with Ordinal Logit Regression – III

What is the contrast with Fractional Regression with Continuous Scores for Y over the 0-1 Range?

- **FR incorrectly assumes distances between categories are known.**
- **FR assumes scores on Y can be continuous. This is inappropriate when the ordinal variable has fewer than 10 categories.**
- **FR incorrectly assumes equal precision in measuring distinctions on Y across the full range of Y.**

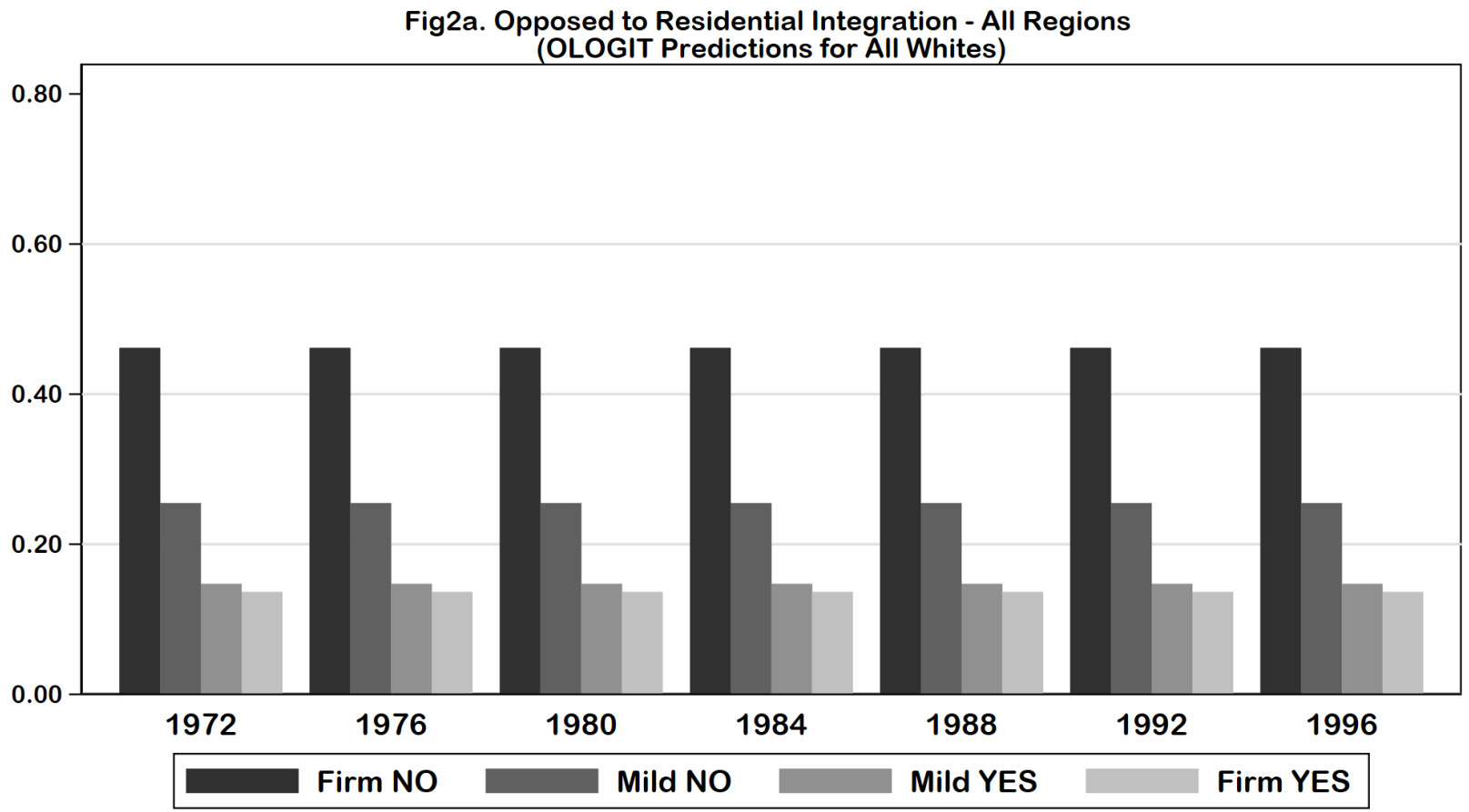
Hands on with Ordinal Logit Regression – IV

The Logic of Ordered Logit Regression

- **As with multinomial logit regression, the dependent variable is the relative distribution of cases across categories of Y.**
  - **In graphical terms, the DV is the shape of the histogram for Y across combinations of values on X's.**
- **A baseline histogram is captured with "cut-point" coefficients.**
  - **The cut-point coefficients predict the shape of the histogram for Y when X's are at 0.**
  - **With no X's in the model, the cut-point coefficients are the logits for the ratio of cases above and below the cut point.**
- **Effects of X's shift the shape of the predicted histogram.**
  - **Coefficients for X's increment or decrement the cut points coefficients; this shifts all bars in the predicted histogram.**
  - **Negative effects shift the relative frequency distribution to the left (thus increasing left-most bars).**
  - **Positive effects shift the relative frequency distribution to the right (thus increasing right-most bars)**

White Opposition to Residential Integration

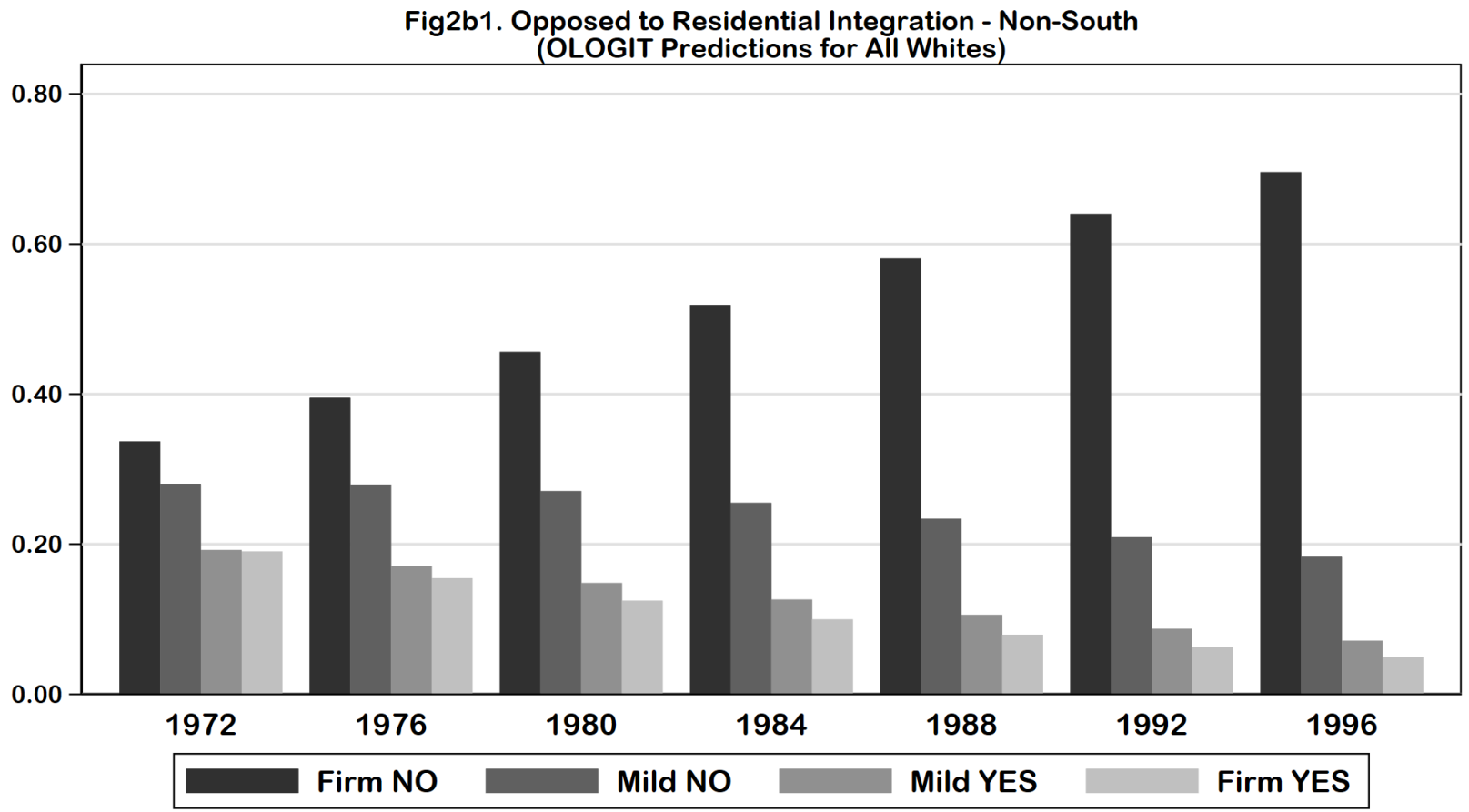
No Effects Over Time (4-Year Intervals), Histogram Bars are Constant



Source: National Opinion Research Center General Social Survey. Notes: Sample consists of White adults.

White Opposition to Residential Integration

Negative Effect Over Time (4-Year Intervals), Histogram Bars Shift Left



Source: National Opinion Research Center General Social Survey. Notes: Sample consists of White adults.

From Tabulation to MLOGIT Relative Risk Coefficients

```
. tab oppint4    // simple tabulation of dependent variable

Okay to Oppose Integration
                    Freq.     Percent        Cum.
------------+-----------------------------------
    Firm-No |      3,801       46.16       46.16
    Mild-No |      2,099       25.49       71.65
   Mild-Yes |      1,212       14.72       86.36
   Firm-Yes |      1,123       13.64      100.00
------------+-----------------------------------
      Total |      8,235      100.00

. mlogit oppint4 , base(1) rr

Multinomial logistic regression                         Number of obs =  8,235
                                                        LR chi2(0)    =   0.00
                                                        Prob > chi2   =      .
Log likelihood = -10367.648                             Pseudo R2     = 0.0000

     oppint4 |  Rel. risk   Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
Firm_No      |  (base outcome)
Mild_No      |
       _cons |   .5522231   .0150171   -21.84   0.000     .5235608    .5824545
Mild_Yes     |
       _cons |   .3188635   .0105185   -34.65   0.000     .2988999    .3401604
Firm_Yes     |
       _cons |   .2954486   .0100346   -35.90   0.000     .2764214    .3157855
------------------------------------------------------------------------------

Note: Mild No _cons = 0.5522 = 2099/3801
```

From Tabulation to MLOGIT Logit Coefficients

```
. mlogit oppint4 , base(1)

Multinomial logistic regression                         Number of obs =  8,235
                                                        LR chi2(0)    =   0.00
                                                        Prob > chi2   =      .
Log likelihood = -10367.648                             Pseudo R2     = 0.0000

------------------------------------------------------------------------------
     oppint4 | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
Firm_No      |  (base outcome)
Mild_No      |
       _cons |  -.5938031   .0271939   -21.84   0.000    -.6471021   -.5405042
Mild_Yes     |
       _cons |  -1.142992   .0329874   -34.65   0.000    -1.207646   -1.078338
Firm_Yes     |
       _cons |  -1.219261   .0339641   -35.90   0.000    -1.285829   -1.152692
------------------------------------------------------------------------------
Note: Mild No _cons = -0.5938 = ln(0.5522) = ln(2099/3801)
```

From Tabulation to OLOGIT Logit Coefficients

```
. tab oppint4    // simple tabulation of dependent variable

Okay to Oppose Integration
                      Freq.     Percent        Cum.
--------------+-----------------------------------
    Firm-No Y1|      3,801       46.16       46.16
    Mild-No Y2|      2,099       25.49       71.65
   Mild-Yes Y3|      1,212       14.72       86.36
   Firm-Yes Y4|      1,123       13.64      100.00
--------------+-----------------------------------
        Total |      8,235      100.00

Baseline cut point odds ratios calculated from frequency distribution
     0.85724 cut1 odds ratio = (Y1N) / (Y2N+Y3N+Y4N)  logit cut1 = -0.15404
     2.52677 cut2 odds ratio = (Y1N+Y2N) / (Y3N+Y4N)  logit cut2 =  0.92694
     6.33304 cut3 odds ratio = (Y1N+Y2N+Y3N) / (Y4N)  logit cut3 =  1.84578

. ologit oppint4

Ordered logistic regression                             Number of obs =  8,235
Log likelihood = -10367.648                             Pseudo R2     = 0.0000

------------------------------------------------------------------------------
     oppint4 | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
       /cut1 |  -.1540379   .0221047                     -.1973624   -.1107135
       /cut2 |   .9269405   .0244491                      .8790212    .9748598
       /cut3 |    1.84578   .0321104                     1.782845    1.908715
------------------------------------------------------------------------------
```

MLOGIT & OLOGIT Baseline Predictions (No X's)

MLOGIT and OLOGIT predictions are the same for baseline models that have no predictors (no X variables).

Predictions from mlogit (model with no X's)

```
. table , stat(mean pv1z_y1 pv1z_y2 pv1z_y3 pv1z_y4 ) nformat(%9.4f)

------------------------------
Pr(oppint4==Firm_No)  |  0.4616
Pr(oppint4==Mild_No)  |  0.2549
Pr(oppint4==Mild_Yes) |  0.1472
Pr(oppint4==Firm_Yes) |  0.1364
------------------------------
```

Predictions from ologit (model with no X's)

```
. table , stat(mean pv2a_y1 pv2a_y2 pv2a_y3 pv2a_y4) nformat(%8.4f)

-----------------------
Pr(oppint4==1) |  0.4616
Pr(oppint4==2) |  0.2549
Pr(oppint4==3) |  0.1472
Pr(oppint4==4) |  0.1364
-----------------------
```

Parsimony of OLOGIT vs MLOGIT – I

```
. mlogit oppint4 south , base(1)


Multinomial logistic regression                         Number of obs =  8,235
                                                        LR chi2(3)    = 193.68
                                                        Prob > chi2   = 0.0000
Log likelihood = -10270.81                              Pseudo R2     = 0.0093

------------------------------------------------------------------------------
     oppint4 | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
Firm_No      |  (base outcome)
-------------+----------------------------------------------------------------
Mild_No      |
       south |   .4419306   .0598196     7.39   0.000     .3246863    .5591748
       _cons |  -.7202567   .0325748   -22.11   0.000    -.7841022   -.6564113
-------------+----------------------------------------------------------------
Mild_Yes     |
       south |   .5116254   .0711859     7.19   0.000     .3721036    .6511472
       _cons |  -1.293165   .0401642   -32.20   0.000    -1.371886   -1.214445
-------------+----------------------------------------------------------------
Firm_Yes     |
       south |   .9367234   .0709243    13.21   0.000     .7977142    1.075733
       _cons |   -1.53871   .0443336   -34.71   0.000    -1.625602   -1.451818
------------------------------------------------------------------------------
```

Parsimony of OLOGIT vs MLOGIT – II

```
. ologit oppint4 south ,

Ordered logistic regression                       Number of obs =  8,235
                                                  LR chi2(1)    = 184.57
                                                  Prob > chi2   = 0.0000
Log likelihood = -10275.361                       Pseudo R2     = 0.0089

------------------------------------------------------------------------------
     oppint4 | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
       south |   .5963232   .0438793    13.59   0.000     .5103214     .682325
-------------+----------------------------------------------------------------
       /cut1 |   .0258228   .0259769                     -.0250909    .0767365
       /cut2 |   1.125163   .0289257                      1.06847    1.181857
       /cut3 |   2.056144    .036175                     1.985242    2.127045
------------------------------------------------------------------------------
```

MLOGIT & OLOGIT Predictions Now May Differ

Predictions from mlogit (model with one predictor - South)

```
. table , stat(mean pv1x_y1m pv1x_y2m pv1x_y3m pv1x_y4m ) nformat(%9.4f)

------------------------------
Pr(oppint4==Firm_No)  |  0.4616
Pr(oppint4==Mild_No)  |  0.2549
Pr(oppint4==Mild_Yes) |  0.1472
Pr(oppint4==Firm_Yes) |  0.1364
------------------------------
```

Predictions from ologit (model with one predictor - South)

```
. table , stat(mean pv1x_y1o pv1x_y2o pv1x_y3o pv1x_y4o ) nformat(%9.4f)

------------------------
Pr(oppint4==1) |  0.4615
Pr(oppint4==2) |  0.2546
Pr(oppint4==3) |  0.1473
Pr(oppint4==4) |  0.1366
------------------------
```

A Technical Aside RE: Stata's Ordinal Logit Routine

The "Notes" document for ordered logit discusses a technical issue concerning how the Stata implementation of the ologit procedure "parameterizes" the coefficients reported by the model.

The issue is relevant when one uses ologit results to generate predicted values by manual calculation.

In brief, the calculations are slightly different from what one might expect. Nothing is incorrect. However, the model parameterization is different from what researcher might expect (but mathematically equivalent). As a result, the calculations need to be implemented in a particular way to get correct results.

Yes. This is a somewhat esoteric issue. But it helps avoid confusion regarding how to generate predictions using results from ologit regressions.

OLOGIT vs MLOGIT – Tests and Model Choice

The "Notes" document discusses the following points.

- **The OLOGIT model "nests" under the MLOGIT model.**
- **So, one can test the assumption of "fixed" ordinality by comparing OLOGIT and MLOGIT models with the same X's.**

In large samples, the MLOGIT results may test out as a statistically significant improvement over OLOGIT results.

Should MLOGIT results be automatically preferred in this situation? Not necessarily.

- **OLOGIT results are much more parsimonious and easier to interpret.**
- **MLOGIT can capitalize on idiosyncratic patterns and "overfit" the effects. Ask three questions.**
- **Are you confident the differences to hold up in new analyses?**
- **Do the differences have important substantive implications?**
- **Do you have a sound theory to make sense of the differences?**

OLOGIT Prediction Success

How does one assess OLOGIT model fit?

- **Pseudo R2 statistics are well named. They really are "VERY pseudo"; they are not comparable to R2 in OLS regression.**
- **Remember, the DV is a relative frequency distribution. The notion of explained "variance" does not really apply.**
- **Dissimilarity between observed and predicted distributions provides some insight into whether predictions are "good" (see the "Notes" document for discussion).**
- **BIC and AIC are also options to consider.**

Think carefully before placing weight on model fit statistics.

- **Tests of theories do not hinge on model fit. They hinge on direction and magnitude of effects of particular X's.**
- **Model fit is mainly relevant for accuracy of prediction.**
- **Ill-conceived models can produce better predictions than well-conceived models. But we would not accept them.**

Fractional Regression Overview – I

Fractional regression models the mean of Y as following a logistic "S" curve such that predicted means stay in the range 0-1.

Fractional regression can be a good option to consider when:

- **Scores for the dependent variable are continuous and are bounded in a limited range.**
- **The classic case is proportions.**
- **Many other bounded variables can be converted to the range of 0-1 (by applying a simple transformation formula).**

OLS Regression (e.g., Linear Probability Models) are Inferior

- **OLS assumptions for errors of prediction are not met. So, significance tests are unsound.**
- **OLS assumes effects are linear and additive. This is unsound and can lead to inaccurate and even impossible (e.g., out-of-bounds) predictions.**

Fractional Regression Overview – II

An Attractive Practical Quality

- **When OLS regression is "okay", fractional regression will near-exactly reproduce the OLS results for predictions and significance tests.**
- **Thus, fractional regression will not lead to misleading findings (in comparison to OLS) when modeling bounded dependent variables.**
- **In contrast, OLS regression can easily lead to misleading findings.**

Implication

- **Compare fractional regression and OLS regression to validate OLS inferences.**
- **FR is always technical superior and, importantly, makes it clear that effects should be understood as nonlinear and non-additive.**
- **But OLS results can be easier to explain.**

Fractional Regression Overview – III

Fractional Regression models are estimated using the GLM (General Linear Model) framework. The framework can fit a wide range of models. The model involves specifying two items.

- **The relevant distributional model.**
  - **In the case of FR, it is the binomial model (variation in Y values is between 0 and 1 inclusive).**
- **The "link" function for the path of the mean.**
  - **In the case of FR, the link function is the "logit" (or probit) function.**

Under this specification,

- **Predicted means for Y will take values between 0 and 1.**
- **Predictions will follow a logistic "S" curve.**
- **Errors of prediction at any point on the prediction curve will follow a binomial distribution (not a normal distribution).**

Fractional Regression Overview – IV

Fractional regression models are fit by he method of quasi-maximum likelihood estimation (QMLE).

- **This is a more flexible (less restrictive) version of maximum likelihood estimation (MLE) methods.**
- **QMLE maximizes a function that is similar to the log likelihood function of MLE. However, QMLE makes fewer strong assumptions about the specification of the model (e.g., the form of the distribution of errors around the estimate of the predictions).**

Pros and Cons of QMLE Estimates

- **QMLE estimates have the desirable properties of being consistent and asymptotically normal (in large samples, as with maximum likelihood).**
- **However, QMLE estimates are less efficient than comparable MLE estimates (i.e., they exhibit greater sample-to-sample variability).**

Fractional Regression Overview – V

In some cases, the disadvantage in efficiency of QMLE estimates of model parameters (e.g., b's) may be modest and standard approaches to statistical inference for maximum likelihood estimates can be used.

- **That is, it may be okay to use standard errors calculated using analytic formulas that rest on assumptions that may not be met.**

More generally one should take a conservative approach and assume standard tests are too optimistic (standard errors based on analytic formulas are too small and generate too many false positives for statistical significance).

Stata's fracreg procedure follows this recommended approach.

- **Accordingly, it estimates "robust standard errors" by default.**
- **The fracreg procedure also provides the option to use bootstrap methods to obtain standard errors.**

The GLM and FRAGREG Implementations in Stata

The following two Stata commands will implement fractional regression and generate identical results.

```
glm y x1 x2 , family(binomial) link(logit) vce(robust)
fracreg logit y x1 x2 , vce(robust)
```

The fracreg command can be described as a convenient command for automating the glm estimation procedure.

Modeling a Continuous Version of OPPINT

The example analysis here models OPPINT. The values of OPPINT are continuous over the range 0-1. The value of 1 indicates opposition to integration. (See the "Notes" documents for details.)

We will compare results obtained when modeling this DV using OLS regression and fractional regression (FR).

First OLS with no X variables

```
. reg oppint

      Source |       SS           df       MS      Number of obs   =     8,235
-------------+----------------------------------   F(0, 8234)      =      0.00
       Model |           0         0           .   Prob > F        =         .
    Residual |  829.075143     8,234  .100689233   R-squared       =    0.0000
-------------+----------------------------------   Adj R-squared   =    0.0000
       Total |  829.075143     8,234  .100689233   Root MSE        =    .31732

------------------------------------------------------------------------------
      oppint |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       _cons |    .342631   .0034967    97.99   0.000     .3357766    .3494855
------------------------------------------------------------------------------
```

The OLS regression constant of 0.3426 indicates the average score on OPPINT.

Modeling a Continuous Version of OPPINT

Next Fractional Regression (FR) with no X variables

```
. fracreg logit oppint

Iteration 0:   log pseudolikelihood =  -6272.352
   ... (iterations omitted to save space)
Iteration 3:   log pseudolikelihood = -5293.1717

Fractional logistic regression                  Number of obs     =      8,235
Log pseudolikelihood = -5293.1717               Pseudo R2         =     0.0000

------------------------------------------------------------------------------
             |               Robust
      oppint |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       _cons |  -.6515912   .0155247   -41.97   0.000    -.6820191   -.6211633
------------------------------------------------------------------------------
```

The FR regression constant of -0.6516 indicates the logit (log odds) for the predicted average score on OPPINT. The inverse logit transformation of this value yields the raw score prediction of 0.3426 (based on OR = $e^{-0.6516}$ = 0.5212 and proportion (0-1) score = OR/(1+OR) = 0.5212/1.5212 = 0.3426.

Expanded OLS Regression Analysis

```
. reg oppint year7 xyear7 south male educ6 xeduc6 i.age3

      Source |       SS           df       MS      Number of obs   =     8,235
-------------+----------------------------------   F(8, 8226)      =    214.50
       Model |  143.048245         8  17.8810306   Prob > F        =    0.0000
    Residual |  685.738503     8,226  .083362327   R-squared       =    0.1726
-------------+----------------------------------   Adj R-squared   =    0.1718
       Total |  828.786748     8,234  .100654208   Root MSE        =    .28873

------------------------------------------------------------------------------
      oppint |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       year7 |  -.0282988   .0019899   -14.22   0.000    -.0321995   -.0243981
      xyear7 |  -.0143587    .003611    -3.98   0.000    -.0214372   -.0072803
       south |   .1316764   .0155079     8.49   0.000     .1012769    .1620758
        male |   .0064825   .0064131     1.01   0.312    -.0060887    .0190538
       educ6 |  -.0548333   .0030728   -17.84   0.000    -.0608567   -.0488099
      xeduc6 |   -.000167    .005127    -0.03   0.974    -.0102173    .0098833
             |
        age3 |
      30-59  |   .0568588   .0073587     7.73   0.000      .042434    .0712837
      60-99  |   .1286406   .0087472    14.71   0.000      .111494    .1457872
             |
       _cons |   .4635363   .0105831    43.80   0.000     .4427908    .4842819
------------------------------------------------------------------------------
```

See "Notes" document for details.

Expanded Fractional Regression Analysis

```
. fracreg logit oppint year7 xyear7 south male educ6 xeduc6 i.age3

Fractional logistic regression                  Number of obs     =      8,235
                                                Wald chi2(8)      =    1579.91
                                                Prob > chi2       =     0.0000
Log pseudolikelihood = -4970.9657               Pseudo R2         =     0.0611

------------------------------------------------------------------------------
             |               Robust
      oppint |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       year7 |  -.1397249   .0096108   -14.54   0.000    -.1585617   -.1208882
      xyear7 |  -.0483953   .0174826    -2.77   0.006    -.0826605   -.0141301
       south |   .4937653   .0752815     6.56   0.000     .3462162    .6413143
        male |   .0271416    .030895     0.88   0.380    -.0334115    .0876946
       educ6 |    -.26878   .0148411   -18.11   0.000    -.2978679   -.2396921
      xeduc6 |   .0270013   .0242389     1.11   0.265     -.020506    .0745087
             |
        age3 |
      30-59  |   .2689436   .0357766     7.52   0.000     .1988226    .3390645
      60-99  |   .5824124   .0426284    13.66   0.000     .4988623    .6659625
             |
       _cons |  -.0831893   .0513144    -1.62   0.105    -.1837638    .0173852
------------------------------------------------------------------------------
```

See "Notes" document for details.

Comparing OLS Results with FR Results – I

In this case, the OLS and FR results appear to be fairly close.

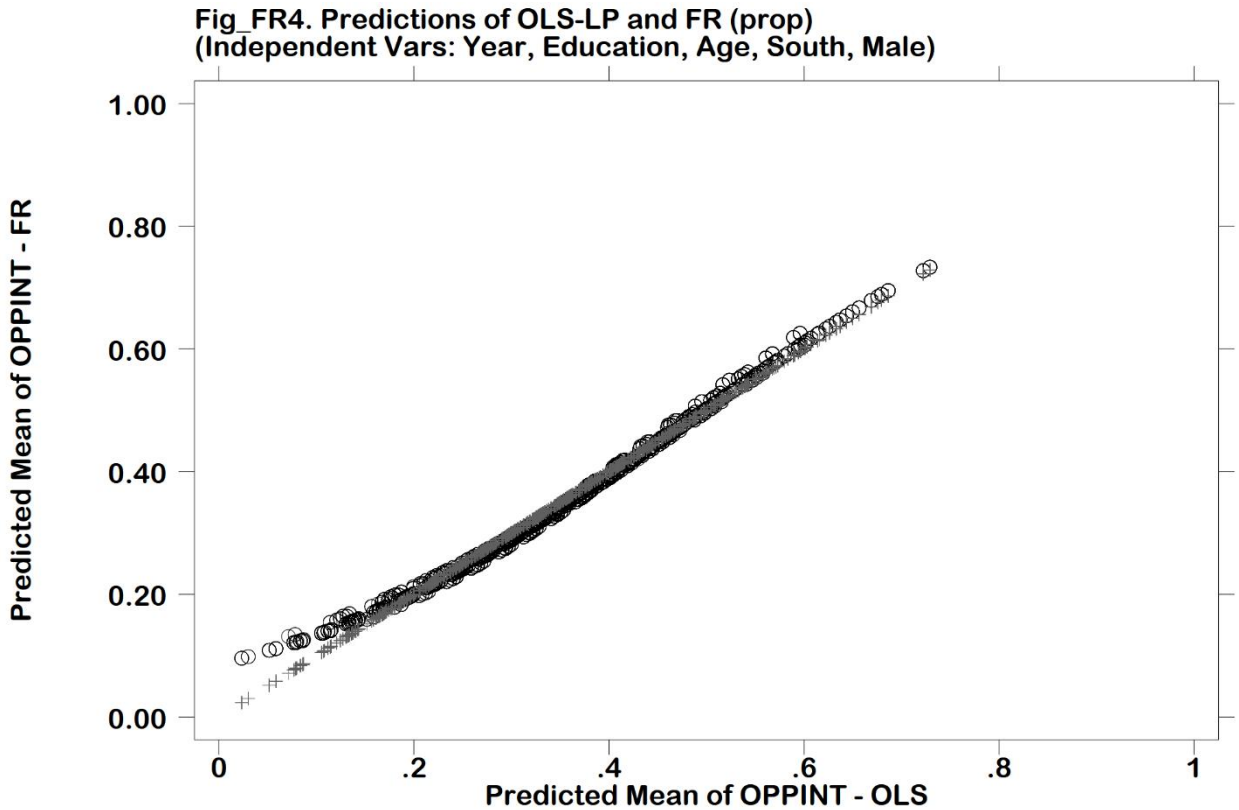
That is, they tend to agree on the sign (direction) of the effects of the indepependent variables an on reported statistical significance.

The OLS t-tests are less valid than the FR Z-tests. But the OLS t tests and the fracreg Z-tests do not disagree in any major ways.

One advantage of OLS is that effects are easier to interpret. But the OLS effects are potentially misleading because a more accurate description of effects requires adopting a nonlinear, nonadditive frame of reference (based on the settings of other X's).

OLS errors will be largest when X's occur in combinations that put predictions near the upper or lower boundaries for Y. This is evident in the following graph of OLS predictions.

Comparing OLS Results with FR Results – II



Larger differences between OLS and FR predictions occur when predicted values of Y are near the boundaries of 0 and 1. This is because the OLS model incorrectly assumes linear, additive effects.

Comparing OLS Results with FR Results – III

Differences between OLS Regression and Fractional Regression will be more dramatic in other situations.

To illustrate, the graphs below compare OLS and FR predictions for research in progress by Fossett and Crowell.

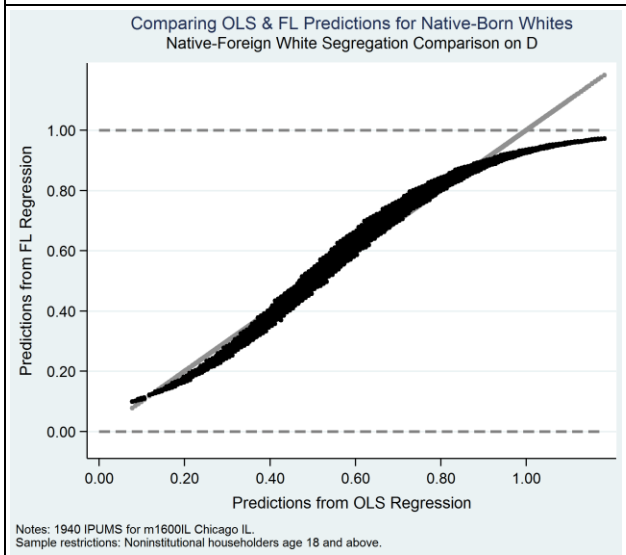
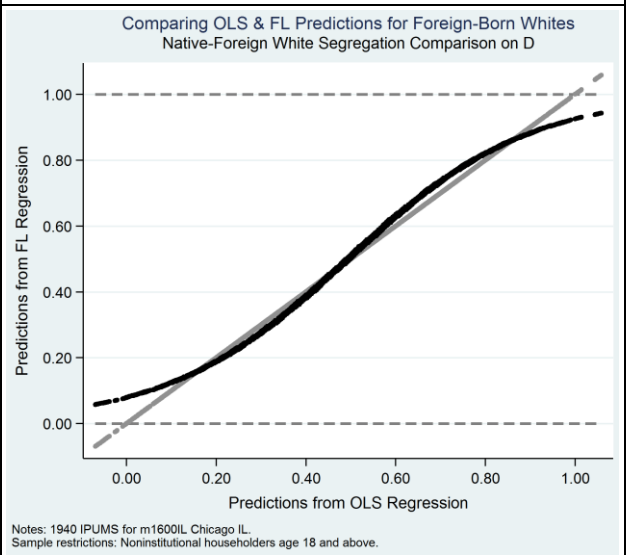
The research is investigating how residential attainment processes shape residential segregation in metropolitan areas in 1940. The models involved are micro-level regressions predicting contact with native-born whites at the level of neighborhoods.

Following standard practice, "contact" is measured by proportion native-born white in the neighborhood. The predictors (X's) include: nativity (US-born), age, education, income, gender, presence of one or more foreign-born household members.

The takeaway point is that the OLS predictions are clearly inferior when contact approaches boundaries. In many cases the predictions are "out of bounds" by large amounts.

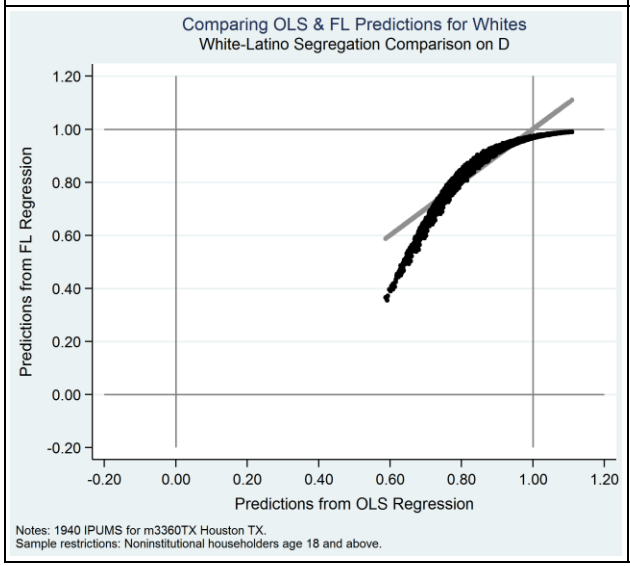
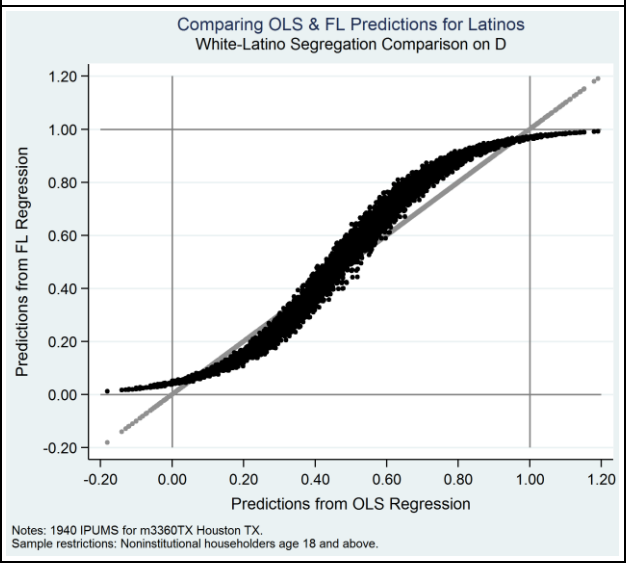
Comparing OLS Results with FR Results – IV

White-Minority Segregation – Predicting Co-Residence with Whites

Native-Born Whites in Chicago IL 1940	Foreign-Born Whites in Chicago IL 1940
	

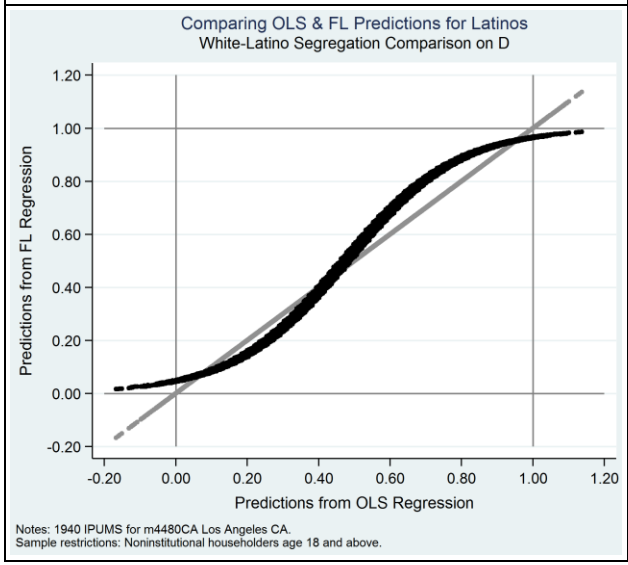
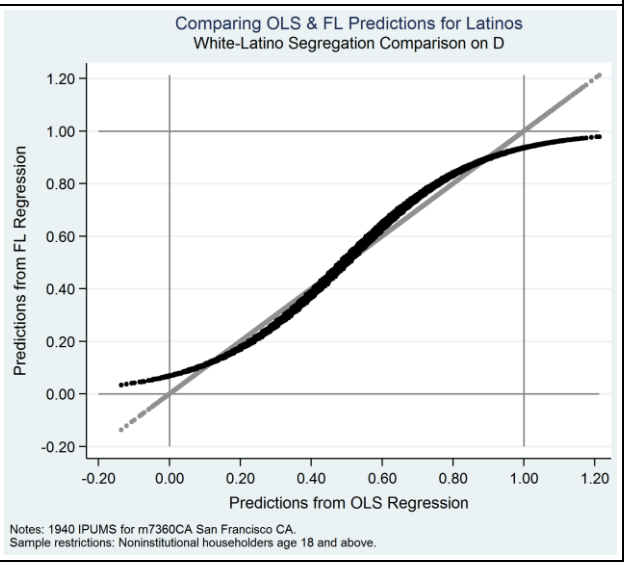
Comparing OLS Results with FR Results – V

White-Minority Segregation – Predicting Co-Residence with Whites

Blacks in Houston TX 1940	Latinos in Houston TX 1940
	

Comparing OLS Results with FR Results – VI

White-Minority Segregation – Predicting Co-Residence with Whites

Latinos in Los Angeles CA 1940	Latinos in San Francisco CA 1940
	

End of Slides

Thank you for your attention.