Multivariable logistic regression

GENERALIZED LINEAR MODELS IN PYTHON

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Multivariable setting

- Model formula

$$\text{logit}(y) = \beta_0 + \beta_1 x_1$$

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Multivariable setting

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$$\mathrm{logit}(y) = \beta_0 + \beta_1 x_1 + \beta_2 \boldsymbol{x_2} + \ldots + \beta_p \boldsymbol{x_p}$$

Multivariable setting

- Model formula

$$\text{logit}(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_p x_p$$

- In Python

```
model = glm('y ~ x1 + x2 + x3 + x4',
            data = my_data,
            family = sm.families.Binomial()).fit()
```

Example - well switching

```
formula = 'switch ~ distance100 + arsenic'
```

```
wells_fit = glm(formula = formula, data = wells,
                family = sm.families.Binomial()).fit()
```

```
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept      0.0027      0.079      0.035      0.972      -0.153       0.158
distance100   -0.8966      0.104     -8.593      0.000      -1.101      -0.692
arsenic        0.4608      0.041     11.134      0.000       0.380       0.542
==============================================================================
```

Example - well switching

```
                 coef    std err          z      P>|z|      [0.025      0.975]
--------------------------------------------------------------------------------
Intercept      0.0027      0.079      0.035      0.972      -0.153       0.158
distance100   -0.8966      0.104     -8.593      0.000      -1.101      -0.692
arsenic        0.4608      0.041     11.134      0.000       0.380       0.542
```

- Both coefficients are statistically significant
- Sign of coefficients logical
- A unit-change in `distance100` corresponds to a negative difference of 0.89 in the logit
- A unit-change in `arsenic` corresponds to a positive difference of 0.46 in the logit

Impact of adding a variable

- Impact of `arsenic` variable
- `distance100` changes from -0.62 to -0.89
- Further away from the safe well
  - More likely to have higher arsenic levels

```
                   coef    std err
----------------------------------
Intercept        0.0027      0.079
distance100     -0.8966      0.104
arsenic          0.4608      0.041
```

```
                   coef    std err
----------------------------------
Intercept        0.6060      0.060
distance100     -0.6291      0.097
```

Multicollinearity

- Variables that are **correlated** with other model variables

- Increase in standard errors of coefficients
  - Coefficients may not be statistically significant

[1] https://en.wikipedia.org/wiki/Correlation_and_dependence

Presence of multicollinearity?

What to look for?

- Coefficient is not significant, but variable is highly correlated with y
- Adding/removing a variable significantly changes coefficients
- Not logical sign of the coefficient
- Variables have high pairwise correlation

Variance inflation factor (VIF)

- Most widely used diagnostic for multicollinearity
  - Computed for each explanatory variable
  - How inflated the variance of the coefficient is
- Suggested threshold VIF > 2.5
- In Python

```
from statsmodels.stats.outliers_influence import variance_inflation_factor
```

Let's practice!

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Comparing models

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Deviance

- Formula

$$D = -2LL(\beta)$$

- Measure of error
- Lower deviance $\rightarrow$ better model fit
- Benchmark for comparison is the **null deviance** $\rightarrow$ intercept-only model
- Evaluate
  - Adding a random noise variable would, on average, decrease deviance by 1
  - Adding p predictors to the model deviance should decrease by more than p

Deviance in Python

```
                 Generalized Linear Model Regression Results
==============================================================================
Dep. Variable:                 switch   No. Observations:                 3020
Model:                            GLM   Df Residuals:                     3018
Model Family:                Binomial   Df Model:                            1
Link Function:                  logit   Scale:                          1.0000
Method:                          IRLS   Log-Likelihood:                -2038.1
Date:                Mon, 08 Apr 2019   Deviance:                       4076.2
Time:                        10:24:56   Pearson chi2:                 3.02e+03
No. Iterations:                     4   Covariance Type:             nonrobust
===============================================================================
                  coef    std err          z      P>|z|      [0.025      0.975]
-------------------------------------------------------------------------------
Intercept       0.6060      0.060     10.047      0.000       0.488       0.724
distance100    -0.6219      0.097     -6.383      0.000      -0.813      -0.431
===============================================================================
```

Compute deviance

- Extract null-deviance and deviance

```
# Extract null deviance
print(model.null_deviance)
```

```
4118.0992
```

```
# Extract model deviance
print(model.deviance)
```

```
4076.2378
```

- Compute deviance using log likelihood

```
print(-2*model.llf)
```

```
4076.2378
```

- Reduction in deviance by 41.86
- Including `distance100` improved the fit

Model complexity

- `model_1` and `model_2` , where
  - $L1 > L2$
  - Number of parameters higher in `model_2`
- `model_2` is **overfitting**

Let's practice!

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Model formula

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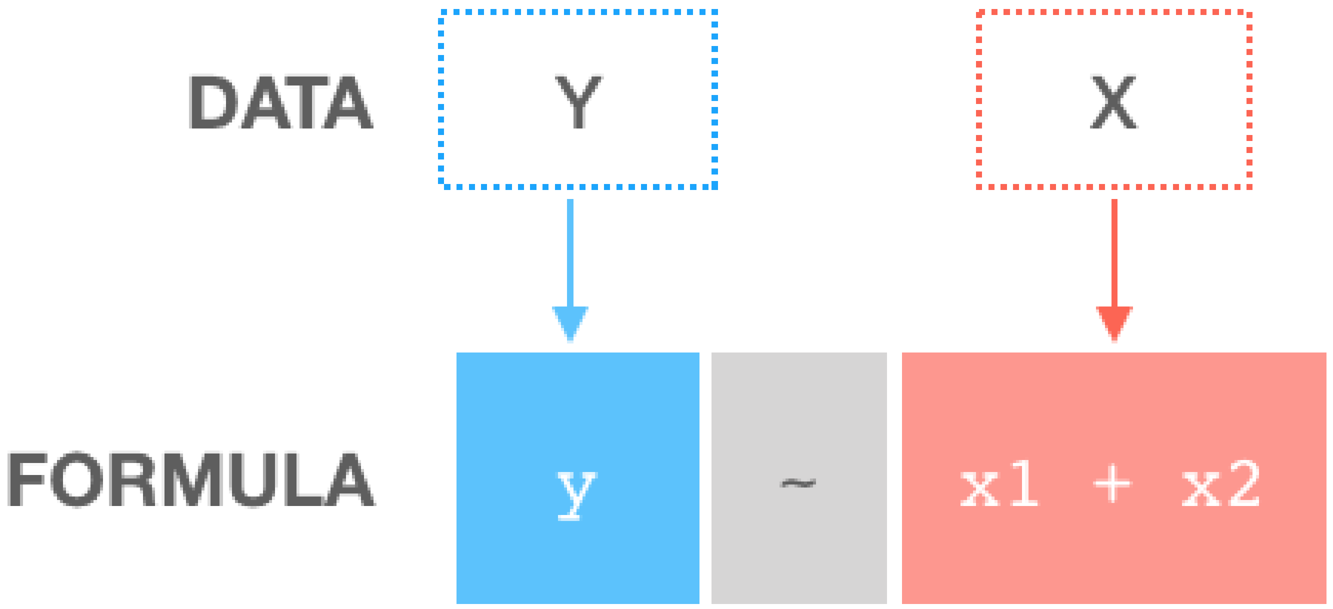
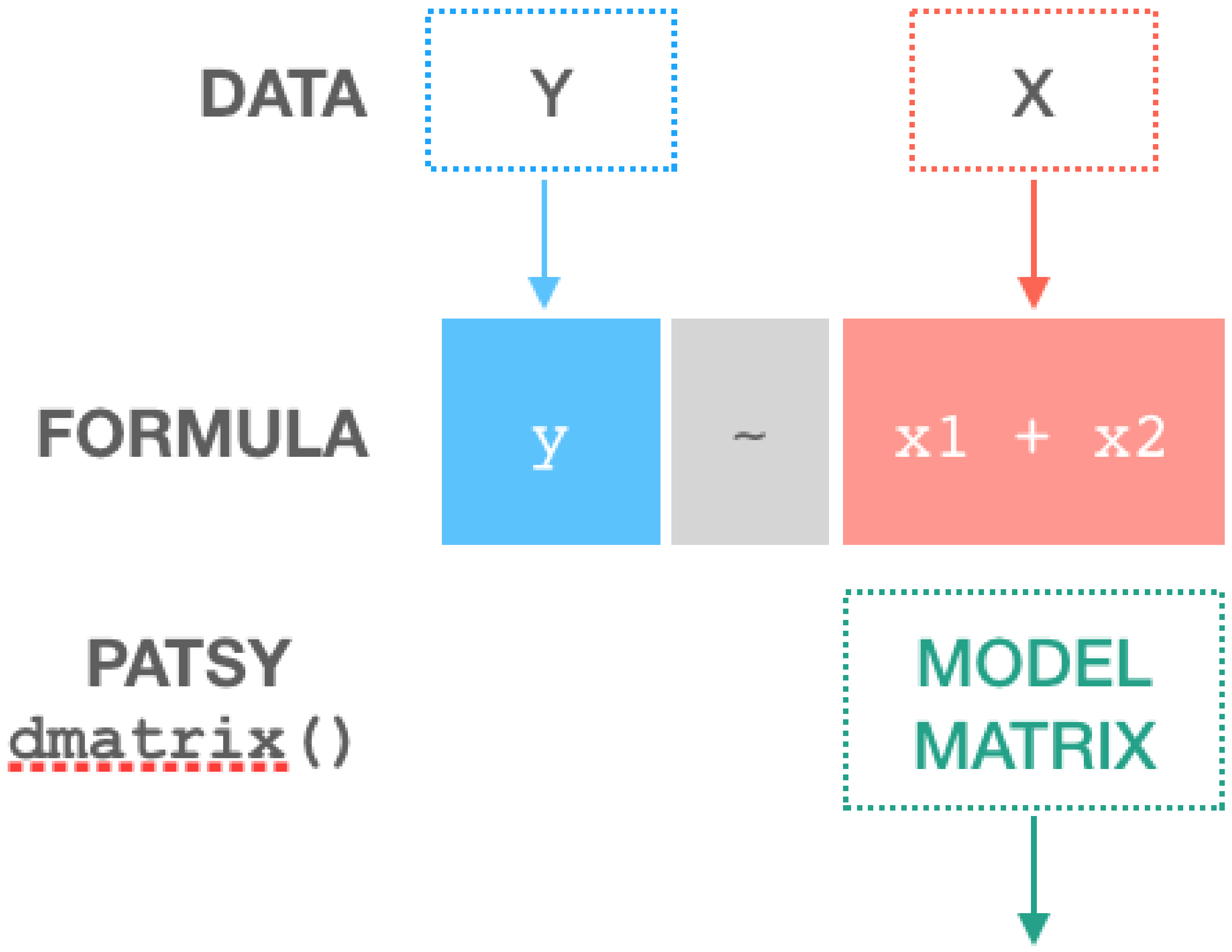
Formula and model matrix

DATA Y X

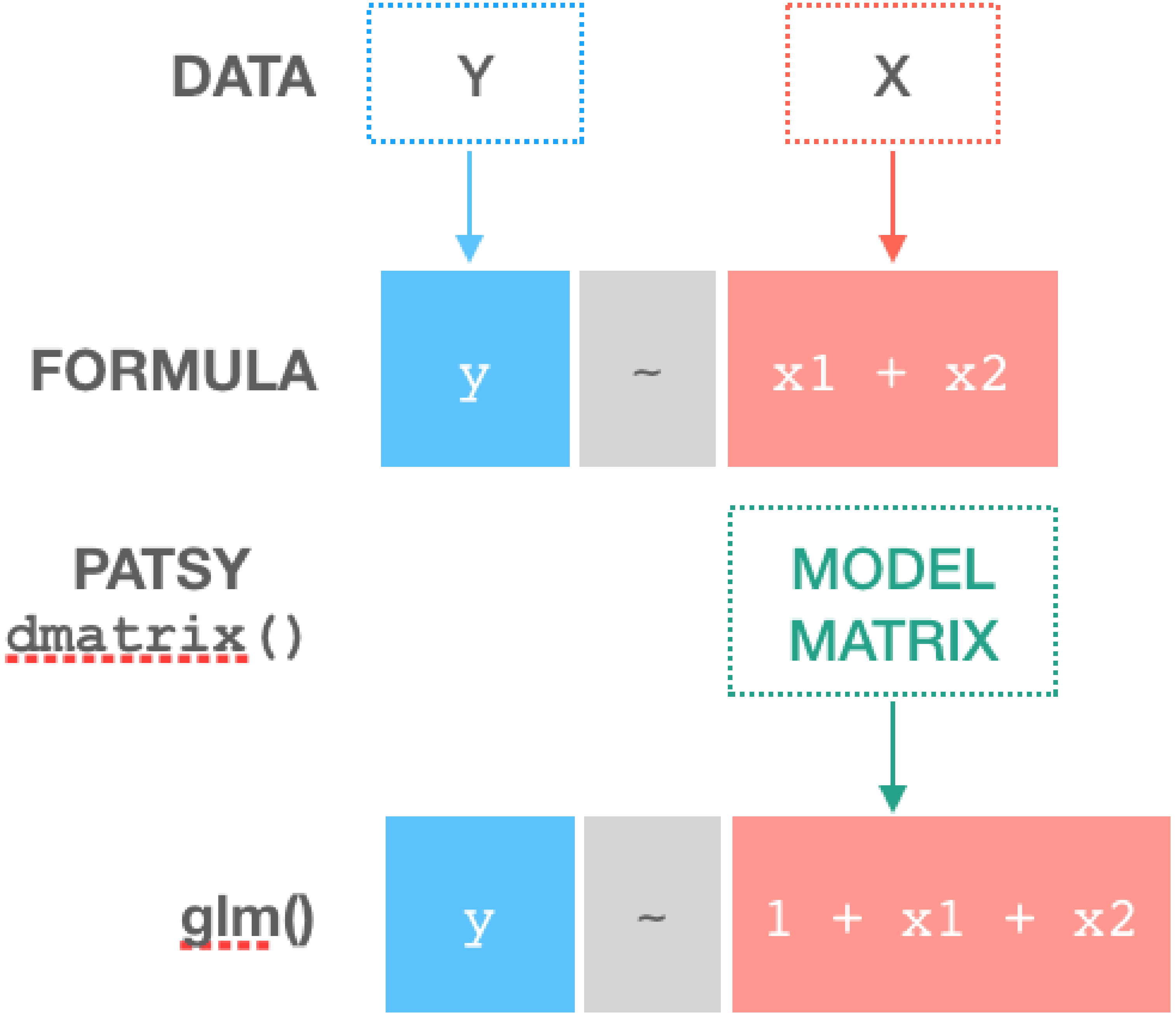
Formula and model matrix



Formula and model matrix



Formula and model matrix



Model matrix

- Model matrix: $y \sim \mathbf{X}$
- Model formula

```
'y ~ x1 + x2'
```

- Check model matrix structure

```
from patsy import dmatrix
dmatrix('x1 + x2')
```

```
   Intercept  x1  x2
           1   1   4
           1   2   5
           1   3   6
```

Variable transformation

```python
import numpy as np
```

```python
'y ~ x1 + np.log(x2)'
```

```python
dmatrix('x1 + np.log(x2)')
```

```
DesignMatrix with shape (3, 3)
  Intercept  x1  np.log(x2)
          1   1     1.38629
          1   2     1.60944
          1   3     1.79176
```

Centering and standardization

- Stateful transforms

```
'y ~ center(x1) + standardize(x2)'
```

```
dmatrix('center(x1) + standardize(x2)')
```

```
DesignMatrix with shape (3, 3)
  Intercept  center(x1)  standardize(x2)
          1          -1         -1.22474
          1           0          0.00000
          1           1          1.22474
```

Build your own transformation

```python
def my_transformation(x):
  return 4 * x
```

```python
dmatrix('x1 + x2 + my_transformation(x2)')
```

```
DesignMatrix with shape (3, 4)
  Intercept  x1  x2  my_transformation(x2)
          1   1   4                     16
          1   2   5                     20
          1   3   6                     24
```

Arithmetic operations

```
x1 = np.array([1, 2, 3])
x2 = np.array([4,5,6])

dmatrix('I(x1 + x2'))
```

```
DesignMatrix with shape (3, 2)
  Intercept  I(x1 + x2)
          1           5
          1           7
          1           9
```

```
x1 = [1, 2, 3]
x2 = [4,5,6]

dmatrix('I(x1 + x2)')
```

```
DesignMatrix with shape (6, 2)
  Intercept  I(x1 + x2)
          1           1
          1           2
          1           3
          1           4
          1           5
          1           6
```

Coding the categorical data

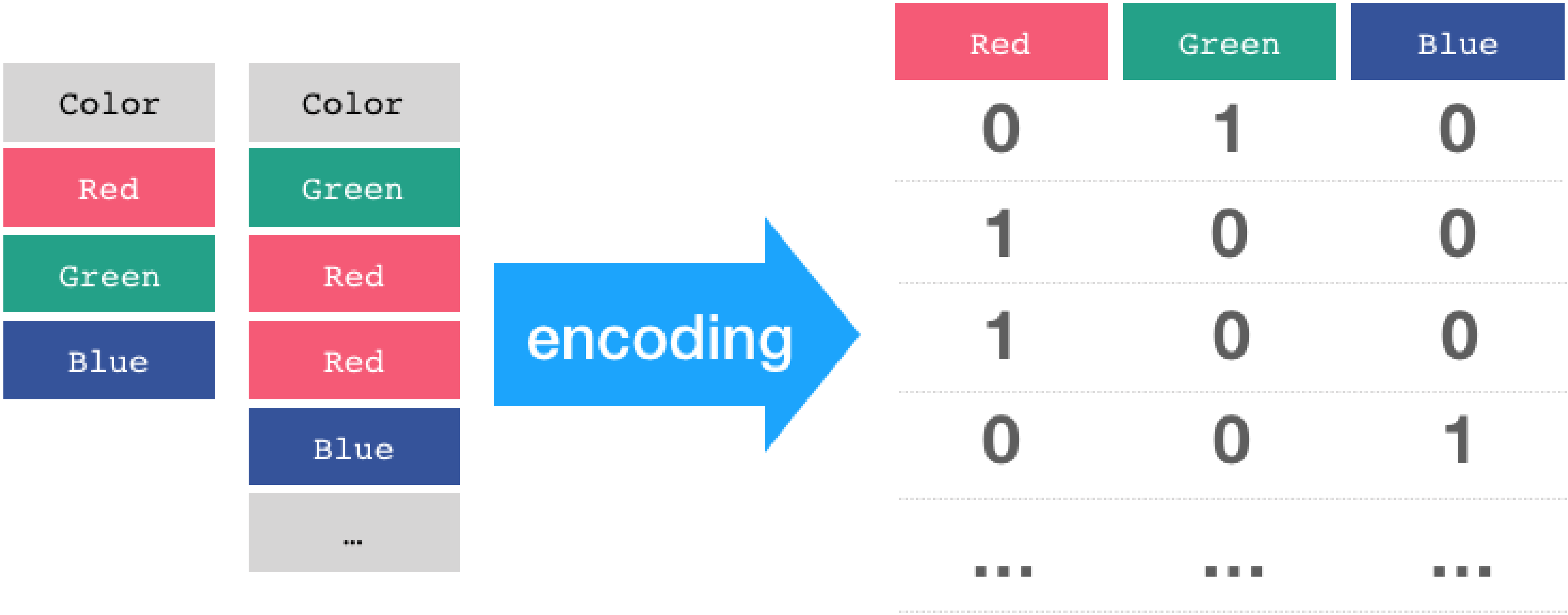
Color
Red
Green
Blue

Coding the categorical data

Color
Red
Green
Blue

Color
Green
Red
Red
Blue
...

Coding the categorical data



Patsy coding

- Strings and booleans are automatically coded
- Numerical $\rightarrow$ categorical
  - `C()` function
- Reference group
  - Default: first group
  - `Treatment`
  - `levels`

The C() function

- Numeric variable

```
dmatrix('color', data = crab)
```

```
DesignMatrix with shape (173, 2)
  Intercept  color
          1      2
          1      3
          1      1
  [... rows omitted]
```

- How many levels?

```
crab['color'].value_counts()
```

```
2    95
3    44
4    22
1    12
```

The C() function

- Categorical variable

```
dmatrix('C(color)', data = crab)
```

```
DesignMatrix with shape (173, 4)
  Intercept  C(color)[T.2]  C(color)[T.3]  C(color)[T.4]
          1              1              0              0
          1              0              1              0
          1              0              0              0
  [... rows omitted]
```

Changing the reference group

```
dmatrix('C(color, Treatment(4))', data = crab)
```

```
DesignMatrix with shape (173, 4)
  Intercept  C(color)[T.1]  C(color)[T.2]  C(color)[T.3]
          1              0              1              0
          1              0              0              1
          1              1              0              0
  [... rows omitted]
```

Changing the reference group

```
l = [1, 2, 3,4]
dmatrix('C(color, levels = l)', data = crab)
```

```
DesignMatrix with shape (173, 4)
  Intercept  C(color)[T.2]  C(color)[T.3]   C(color)[T.4]
          1              1              0               0
          1              0              1               0
          1              0              0               0
  [... rows omitted]
```

Multiple intercepts

```
'y ~ C(color)-1'
dmatrix('C(color)-1', data = crab)
```

```
DesignMatrix with shape (173, 4)
  C(color)[1]  C(color)[2]  C(color)[3]  C(color)[4]
            0            1            0            0
            0            0            1            0
            1            0            0            0
  [... rows omitted]
```

Let's practice!

GENERALIZED LINEAR MODELS IN PYTHON

Categorical and interaction terms

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Categorical variables

- Simple binary variable
  - `Yes, No`
- Nominal variables
  - Color: `red, green, blue`
- Ordinal variables
  - Levels of education: `Education1, Education2,...,Education4`

Analysis of covariance

- Explanatory variables
  - x_1: categorical (binary)
  - x_2: continuous
- Logistic model

$$\text{logit}(y = 1|X) = \beta_0 + \beta_1 x_1 + \beta_2 x_2$$

Analysis of covariance

- Explanatory variables
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Analysis of covariance

- Explanatory variables
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- Logistic model

$$\text{logit}(y=1|X) = \beta_0 + \beta_1 x_1 + \beta_2 x_2$$

- If $x_1 = 0$ then

$$\text{logit}(y=1|x_1=0, x_2) = \beta_0 + 0 + \beta_2 x_2$$

Analysis of covariance

- Explanatory variables
  - x_1: categorical (binary)
  - x_2: continuous
- Logistic model

$$\text{logit}(y=1|X) = \beta_0 + \beta_1 x_1 + \beta_2 x_2$$

- If $x_1 = 0$ then

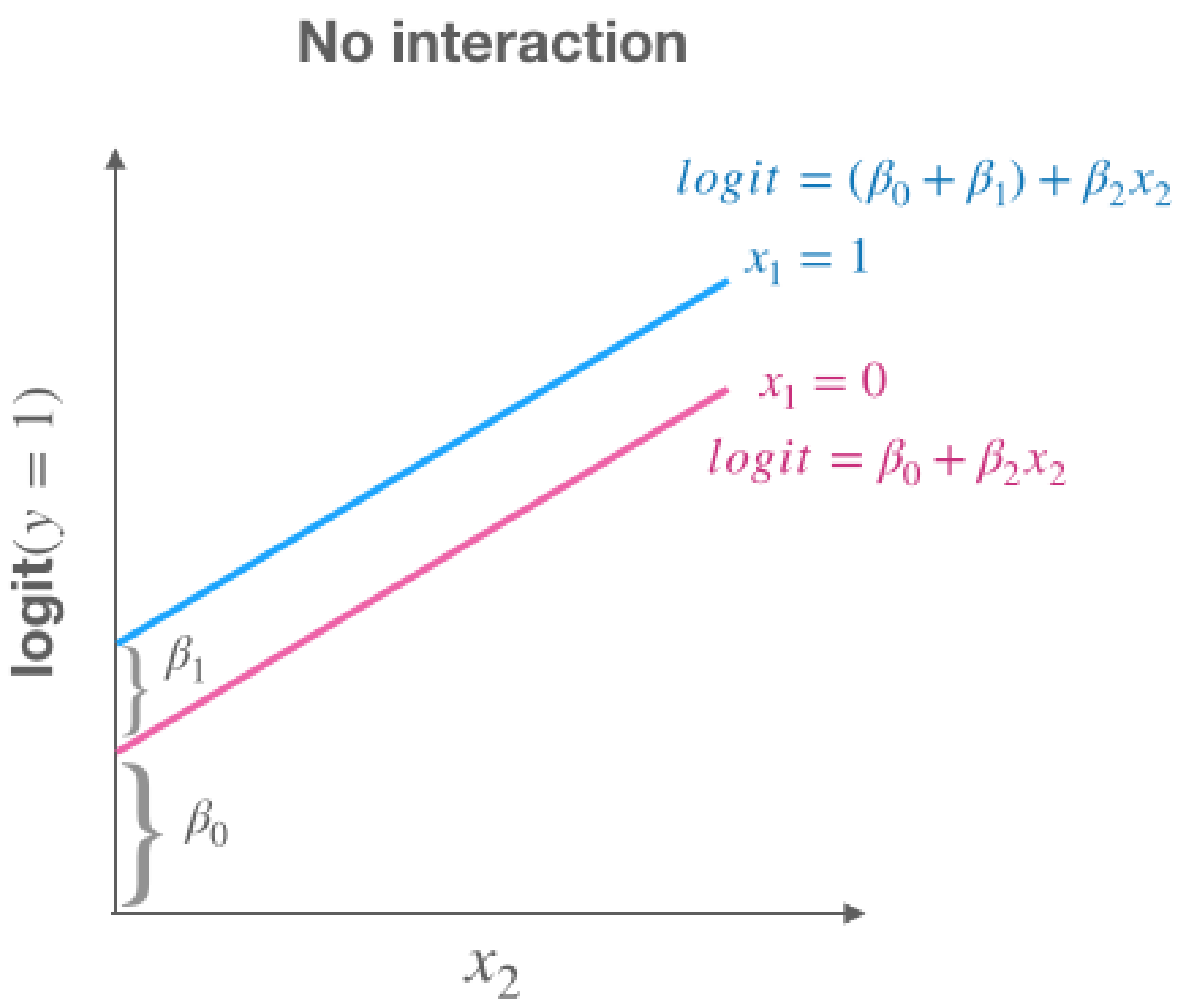
$$\text{logit}(y=1|x_1=0, x_2) = \beta_0 + 0 + \beta_2 x_2$$

- If $x_1 = 1$ then

$$\text{logit}(y=1|x_1=1, x_2) = \beta_0 + \beta_1 + \beta_2 x_2$$

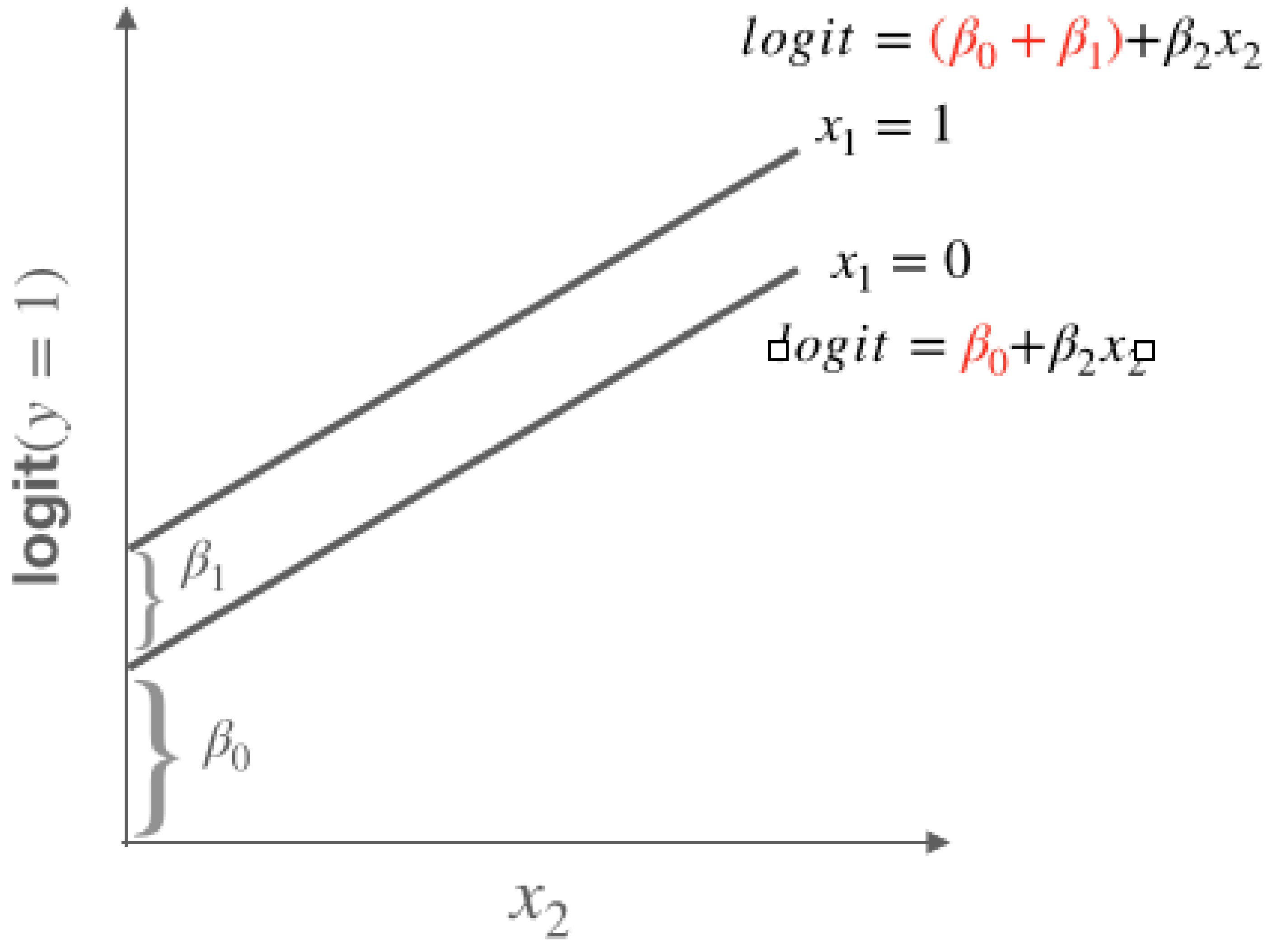
$$\text{logit}(y=1|x_1=1, x_2) = (\beta_0 + \beta_1) + \beta_2 x_2$$

Assumptions



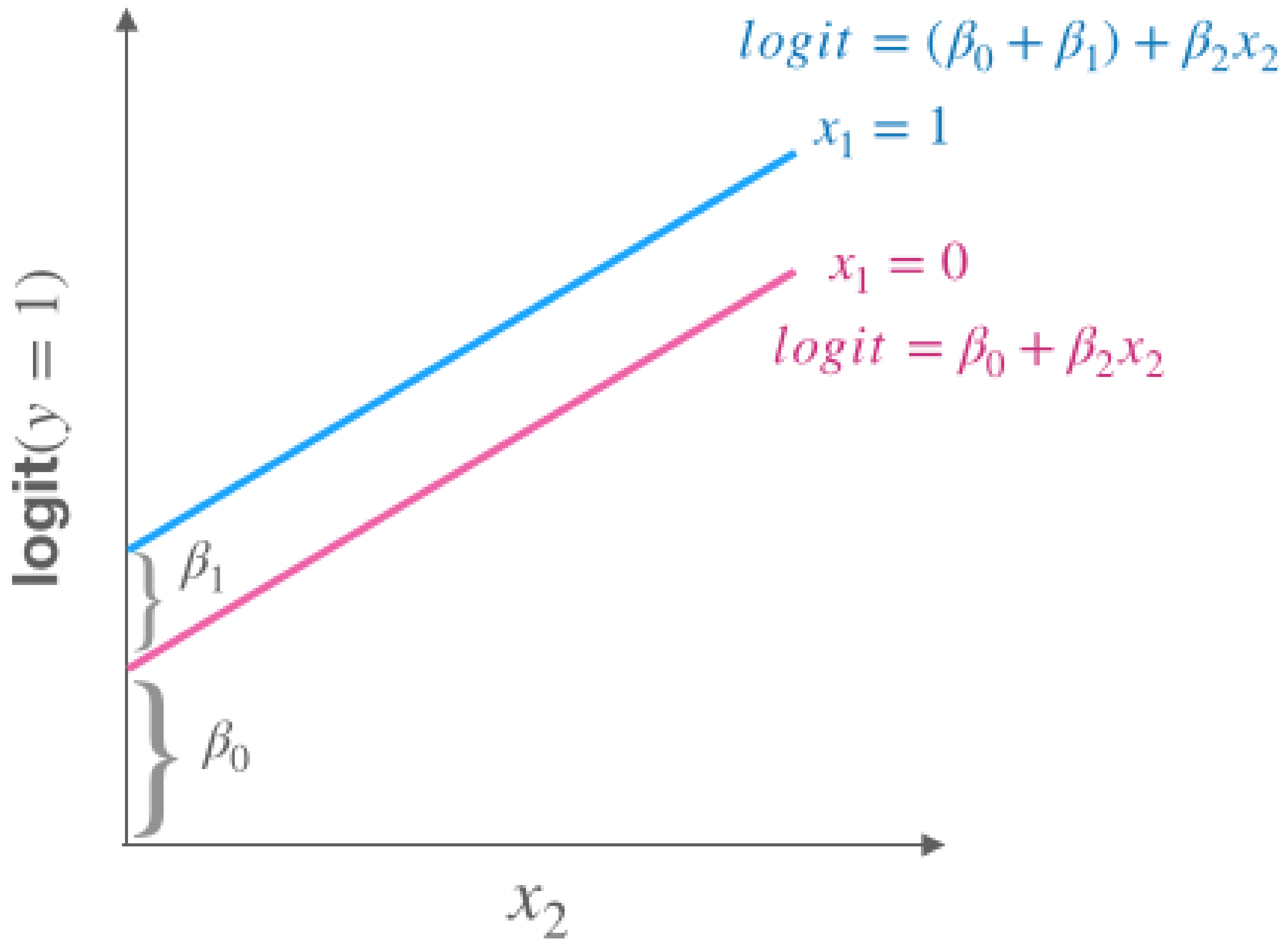
Assumptions

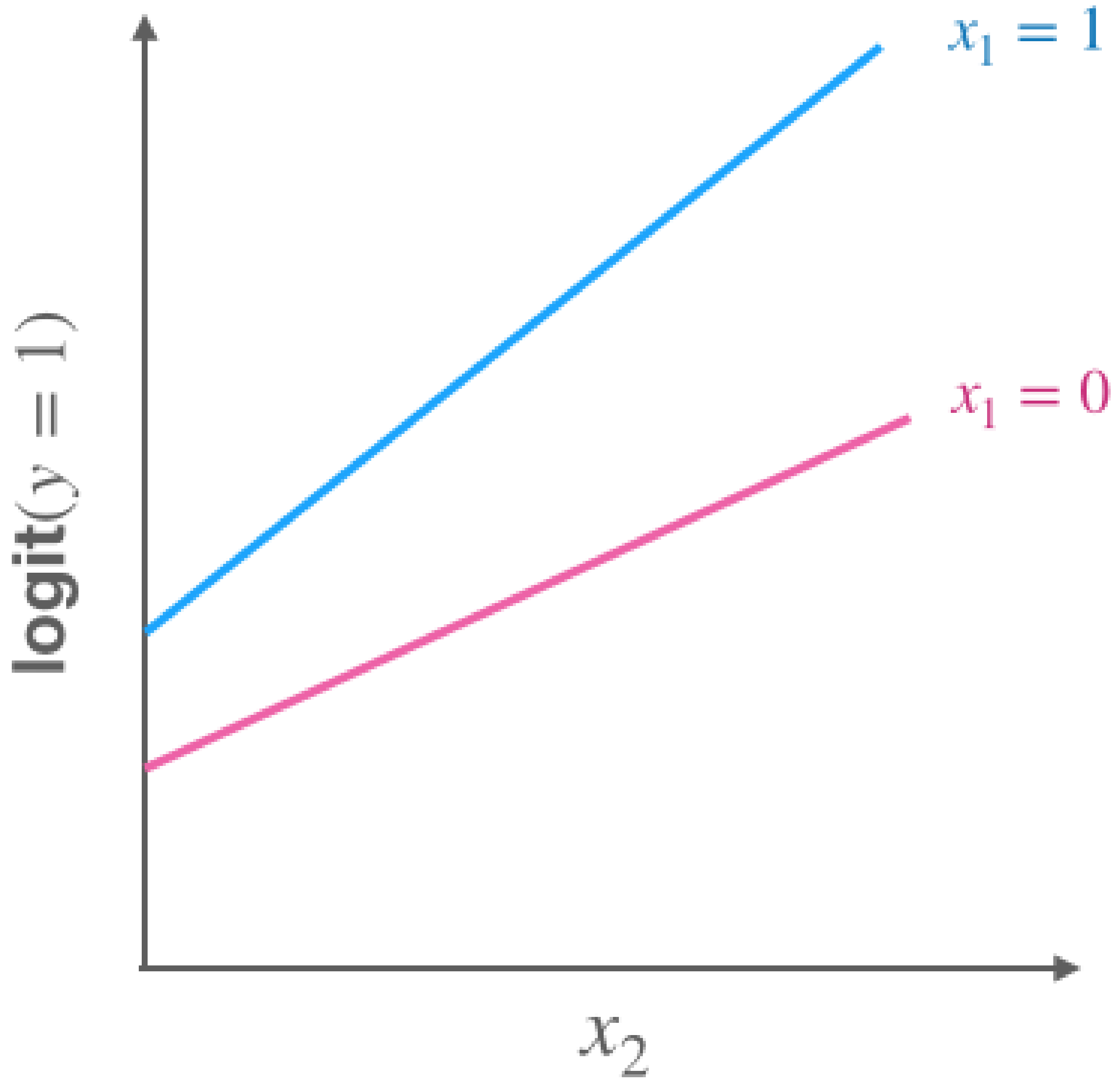
No interaction



Assumptions

No interaction





Interactions

- Not equal slopes $\rightarrow$ presence of **interaction**
- The effect of x_1 on y depends on the level of x_2 and vice versa
- Logistic model allowing for interactions

$$\text{logit}(y = 1|X) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \color{red}{\beta_3 x_1 x_2}$$

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$$\text{logit}(y = 1|X) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2$$

- If $x_1 = 0$ then

$$\text{logit}(y = 1|x_1 = 0, x_2) = \beta_0 + 0 + \beta_2 x_2 + 0$$

Interactions

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$$\text{logit}(y = 1|X) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2$$

- If $x_1 = 0$ then

$$\text{logit}(y = 1|x_1 = 0, x_2) = \beta_0 + \beta_2 x_2$$

- If $x_1 = 1$ then

$$\text{logit}(y = 1|x_1 = 1, x_2) = \beta_0 + \beta_1 + \beta_2 x_2 + \beta_3 x_2$$

$$\text{logit}(y = 1|x_1 = 1, x_2) = (\beta_0 + \beta_1) + (\beta_2 + \beta_3) x_2$$

Interactions

- Not equal slopes $\rightarrow$ presence of **interaction**
- The effect of x_1 on y depends on the level of x_2 and vice versa
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$$\text{logit}(y=1|X) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2$$

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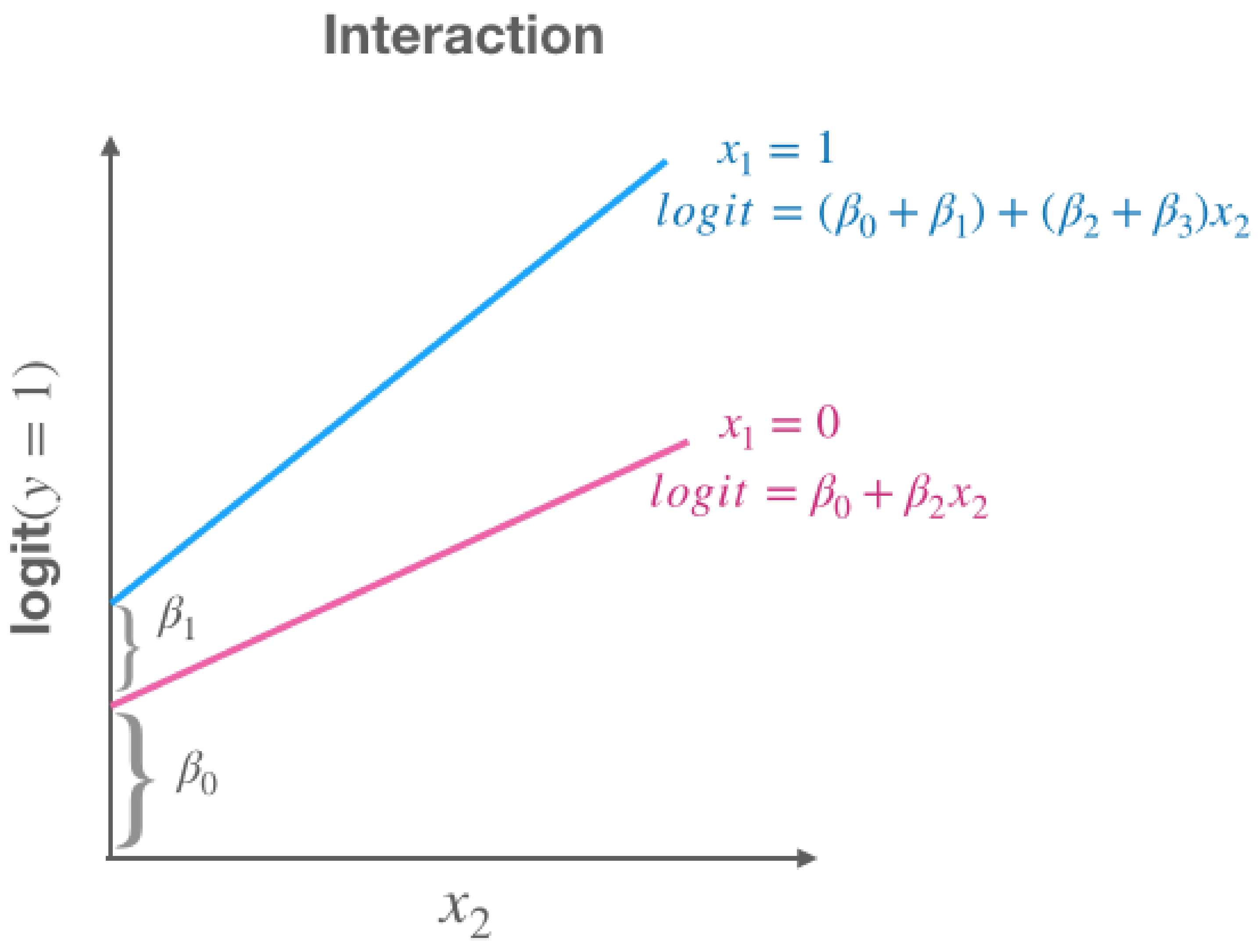
- If $x_1 = 1$ then

$$\text{logit}(y=1|x_1=1, x_2) = \beta_0 + \beta_1 + \beta_2 x_2 + \beta_3 x_2$$

$$\text{logit}(y=1|x_1=1, x_2) = (\beta_0 + \beta_1) + (\beta_2 + \beta_3) x_2$$

Visualizing interactions



Interactions allow for:

- intercept and slope different for x_1
- β_1: difference between the two intercepts
- β_3: difference between the two slopes

Interaction types

- binary $\times$ binary
- binary $\times$ categorical
- binary $\times$ continuous
- continuous $\times$ categorical
- continuous $\times$ continuous
- categorical $\times$ categorical
- more than 2 variable interactions

Let's practice!

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Congratulations!

GENERALIZED LINEAR MODELS IN PYTHON

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MODEL

- **Data** $\rightarrow$
- **Link function** $\rightarrow$
- **Model** $\rightarrow$
- **1-unit increase in** x $\rightarrow$

LOGISTIC REGRESSION

- Binary
- Logit
- $logit(y) = \beta_0 + \beta_1 x_1$
- increases log odds by β_1

LINEAR MODEL

- Continuous
- Identity
- $y = \beta_0 + \beta_1 x_1$
- increases y by β_1

POISSON REGRESSION

- Count
- Logarithm
- $log(\lambda) = \beta_0 + \beta_1 x_1$
- $\text{multiplies } \lambda$ by $exp(\beta_1)$

MAIN PYTHON FUNCTIONS

- Fit the model

  `statmodels` →

LINEAR MODEL

```
glm('y ~ x', data)
```

```
glm('y ~ x', data,
    family = sm.families.Gaussian())
```

LOGISTIC REGRESSION

```
glm('y ~ x', data,
    family = sm.families.Binomial())
```

POISSON REGRESSION

```
glm('y ~ x', data,
    family = sm.families.Poisson())
```

Next steps...

- DataCamp courses
- Excellent reference books
  - *Regression Modeling Strategies* by Frank E. Harrell, Jr.
  - *An Introduction to Categorical Data Analysis* by Alan Agresti
  - *Applied Predictive Modeling* by Max Kuhn and Kjell Johnson

Happy modeling!

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