Naive Bayes

Maximum Likelihood Estimation (MLE)

- Consider a sequence of coin flips, for example *(Coin)*

  (prior →) HHTTTTTTHTTTTTHTTHTT (5 times H, 15 times T)

- Which Pr(H) and Pr(T) are the most likely?
- Looks like Pr(H) = ¼ and Pr(T) = ¾ ...

$$\theta = \qquad P(T) = \frac{|T|}{|F|} = \frac{15}{20} = 3/4$$

(Handwritten right side:)

$$Pr(H) = Pr(T) = \tfrac{1}{2}$$
$$|\text{flips}| = 20$$
$$|H| = 5, \quad |T| = 15$$
$$P(H) = \frac{|H|}{|F|} = \frac{8}{20} = \tfrac{1}{4}$$

Conditional probabilities

- Let A and B be events in a probability space Ω
- Denote by Pr(A | B) the probability of A ∩ B in the space B

  (1) Pr(A | B) := Pr(A ∩ B) / Pr (B)

  (2) Pr(A | B) · Pr(B) = Pr (B | A) · Pr(A)

$$Pr(A|B) = \frac{Pr(B|A) \cdot P(A)}{P(B)}$$

(Handwritten right side:)

$$\Omega = \{1, 2, 3, 4, 5, 6\}$$
$$A = \{2, 4, 6\} \Rightarrow P(A) = \frac{|A|}{|\Omega|} = \frac{3}{6} = \tfrac{1}{2}$$
$$B = \{1, 2, 3\} \Rightarrow P(B) = \frac{|B|}{|\Omega|} = \frac{3}{6} = \tfrac{1}{2}$$
$$P(A|B) = \frac{|A \cap B|}{|B|} = \frac{1}{3} ✓$$

Probabilistic assumptions

- Underlying probability distributions:

  A distribution p_c over the classes ... where $\Sigma_c\, p_c = 1$

  For each c, a distr. p_{wc} over the words ... where $\Sigma_w\, p_{wc} = 1$

- Naïve Bayes assumes the following process for generating a document D with m words $W_1...W_m$ and class label C:

  Pick C=c with prob. p_c , then pick each word W_i=w with probability p_{wc} , independent of the other words

Learning phase

- For a **training set** T of objects, let:

  T_c = the set of documents from class c

  n_{wc} = #occurrences of word w in documents from T_c

  n_c = #occurrences of all words in documents from T_c

- We compute the following probabilities or likelihoods:

  (→) $p_c := |T_c| / |T|$ global likelihood of a class

  $p_{wc} := n_{wc} / n_c$ likelihood of a word for a class

(Handwritten right side:)

$$T = 90$$
$$\rightarrow T_{Horror} = 10$$
$$\rightarrow T_{Doc} = 25$$
$$\rightarrow T_{comedy} = 15$$
$$P_{Jupyter, Doc} = \frac{20}{80} = \tfrac{1}{4} = 0.25$$

Learning phase, example

- Consider Example 2 (artificial documents)

aba	A
baabaaa	A
bbaabbab	B
abbaa	A
abbb	B
bbbaab	B

Handwritten annotations:

$T_A = 3, \quad T_B = 3, \quad T = 6$

$P_A = P(T_A) = \frac{3}{6} = \frac{1}{2}, \quad P_B = \frac{1}{2}$

$n_{aA} = 10, \quad n_{bA} = 5, \quad n_A = 15$

$P_{aA} = \frac{10}{15} = \frac{2}{3}, \quad P_{bA} = \frac{5}{15} = \frac{1}{3}$

$n_{aB} = 6, \quad n_{bB} = 12, \quad n = 18$

$P_{aB} = \frac{6}{18} = \frac{1}{3}, \quad P_{bB} = \frac{12}{18} = \frac{2}{3}$

$\frac{16}{8} < \frac{24}{8}$

$(2) \quad (3)$

0.5

Prediction

- For a given document **d** we want to compute the probability of each class **c**, given document d:

  Pr(C=c | D=d)

- Using Bayes Theorem, we have:

  Pr(C=c | D=d) = Pr(D=d | C=c) · Pr(C=c) / Pr(D=d)

- Using our (naive) probabilistic assumptions, we have:

  $Pr(D=d \mid C=c) = Pr(W_1=w_1 \cap \ldots \cap W_m=w_m \mid C=c)$

  $= \Pi_{i=1,\ldots,m} Pr(W_i=w_i \mid C=c)$ A

  a, b

Prediction ... continued

- We thus obtain that

  Pr(C=c | D=d)

  $= \Pi_{i=1,\ldots,m} Pr(W_i=w_i \mid C=c) \cdot Pr(C=c) / Pr(D=d)$

  $= \Pi_{i=1,\ldots,m} P_{w_i c} \cdot P_c / Pr(D=d)$

- Note 1: for $\Pi_{i=1,\ldots,m} P_{w_i c}$ just take the p_{wc} for all words w in the document and multiply them (if a word w occurs multiple times, also take the factor p_{wc} multiple times)

- Note 2: the Pr(D=d) is the same for all classes c

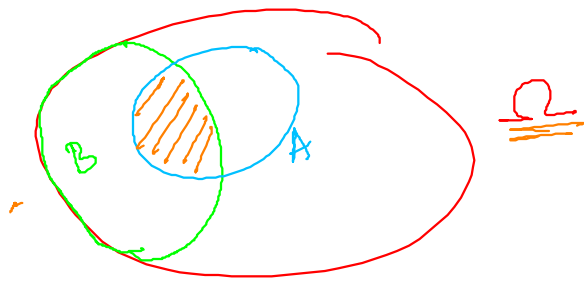# Prediction, example

- Consider Example 2 (artificial documents)

aba	A
baabaaa	A
bbaabbab	B
abbaa	A
abbb	B
bbbaab	B

- Let us predict the class for aab ... A or B ?

Handwritten annotations:

$P(A \mid aab) = P_{aA} \cdot P_{aA} \cdot P_{bA} \cdot P_A$

$= \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{1}{3} \cdot \frac{1}{2} = \frac{2}{27}$

$P(B \mid aab) = P_{aB} \cdot P_{aB} \cdot P_{bB} \cdot P_B$

$= \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{2}{3} \cdot \frac{1}{2} = \frac{1}{27}$

$P(A \mid aab) > P(B \mid aab) \rightarrow$ Predict A

$$P(A|B) = \frac{|A \cap B|}{|B|}$$

$$= \frac{|A \cap B| \, / \, |\Omega|}{|B| \, / \, |\Omega|} = \frac{Pr(A \cap B)}{Pr(B)}$$

$$P_{\text{Zombie, Horror}} = \frac{|Zombie|}{|class, word|} = \frac{5}{20} = \frac{1}{4} = 0.25$$