Rules of Probability

On a table, there are a total of 30 distinct books: 9 math books, 10 physics books, and 11 chemistry books.

What is the probability of getting a book that is not a math book?

These events are "mutually exclusive".

$$P(\overline{M}) = P(H) + P(C) \qquad \text{no overlap!}$$

$$= \frac{10}{30} + \frac{11}{30} = 21/30$$

This is the "sum rule of probability". (Handling OR)

Generalize to include events that are <u>not</u> mutually exclusive:

A fair 20-sided dice is rolled.

What is the probability that the roll is an even number or prime number or both?

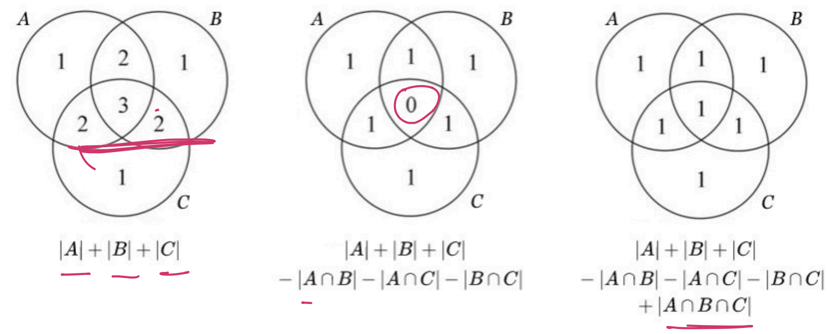
$$P(E \cup R) = P(E) + P(R) - \overbrace{P(E \cap R)}^{\text{Double counted}}$$

$$= \frac{10}{20} + \frac{8}{20} - \frac{1}{20}$$

2
3
5
7
11
13
17
19

$$= \frac{17}{20}$$

Inclusion Exclusion Principle:



Again, the number in each subset represents how many times that subset is counted. Here, the overlap of all three subsets must be added back in due to it being removed after the second step.

Product rule: (Handling AND)

Two coin flips: Probability of both being heads:

$$P(HH) = P(H) * P(H)$$

Independent events: Knowing one has occured doesn't change the probability of the other!

Conversely if $P(A) * P(B) = P(A \cap B)$

then A and B are independent

Example: 20-sided dice is rolled. Probability that the number is even and prime.

$$P(E) = \frac{10}{20} \qquad P(R) = \frac{8}{20} \qquad P(E \cap R) = \frac{1}{20}$$

E and R are dependent

$$P(E \cap R) \neq P(E) \cdot P(R)$$

$$\frac{1}{20} \neq \frac{10}{20} \cdot \frac{8}{20}$$

$$0.05 \neq 0.2$$

So, if we know one, the probability of the other changes.

$P(E) = \frac{1}{2} = 0.5$ given no other information.

If you are told that the number was a prime

$P(E) = \frac{1}{8} = 0.125$ (less likely now!)

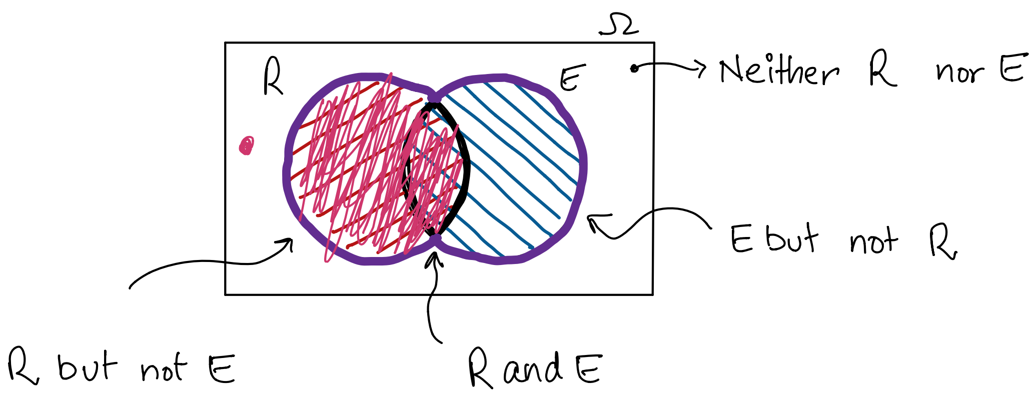
⊛ Notation and Intuition:

$P(E \mid R)$

probability of event

given that

this event is already known to have occurred!



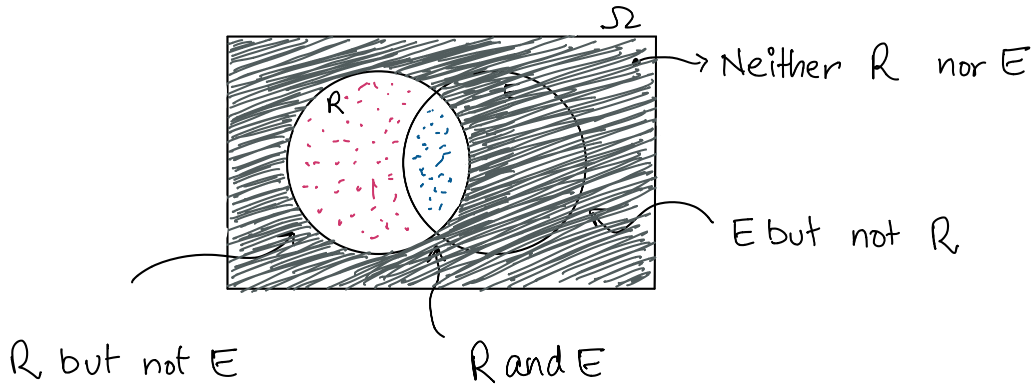
Recall the axioms of Probability

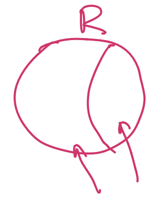
The two axioms of Probability:

— must lie in : [0 — 1]

— Sum of all events must be 1

R is already known to have occurred!

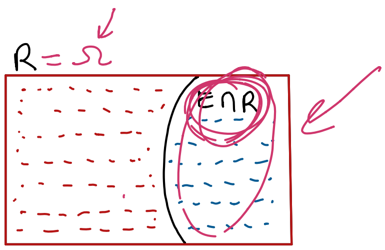




- R is our new "universe". There is no "not R".
- R is definite — P(R) has to be 1.
- But P(R) was $8/20 = 0.4$
  - our math does not work!
  - Rescale everything so that P(R) becomes 1.

$$P(R) \text{ becomes } \frac{P(R)}{P(R)} = \frac{0.4}{0.4} = 1$$

$$P(R|R) = 1$$



$$P(E \cap R) = \frac{1}{20} = 0.05$$

$$P(E|R) = \frac{P(E \cap R)}{P(R)} = \frac{0.05}{0.4} = 0.125$$

$$\boxed{P(A \mid B) = \frac{P(A \cap B)}{P(B)}}$$

Conditional Probability

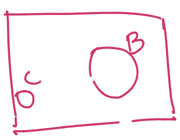
$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Prob of A and B both happening

Prob of A given that B has occurred

normalized so that axioms of probability still hold

$P(\_\_ \mid B) \rightsquigarrow$ ⊛ You are transported to a universe where 'B' is true.



$p(c \mid B) = 0$

Normalization is only necessary to make axioms of probability work

Otherwise:

$\frac{0.125}{1}$ is same as $\frac{0.05}{0.4}$

$P(\Omega)$

$P(\Omega)$ ~~X~~

Both are ratios!