Theory of Automata

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Lecture 4

Languages Associated with Regular Expressions

Regular Languages

- The language generated by any *regular expression* is called a **regular language**.

- It is to be noted that if r_1, r_2 are regular expressions, corresponding to the languages L_1 and L_2 then the languages generated by $r_1 + r_2$, r_1r_2 (or $r_2 r_1$) and r_1*(or r_2*) are also regular languages.

Note

- It is to be noted that if L_1 and L_2 are expressed by r_1 and r_2, respectively then the language expressed by

1. $r_1 + r_2$, is the language $L_1 + L_2$ or $L_1 \cup L_2$
2. $r_1\ r_2$, , is the language $L_1\ L_2$, of strings obtained by prefixing every string of L_1 with every string of L_2
3. r_1^*, is the language L_1^*, of strings obtained by concatenating the strings of L, including the null string.

Rules for language association with R.E’s

- Rule 1:
  - The language associated with the regular expression that is just a single letter is that one-letter word alone, and the language associated with Λ is just {Λ}, a one-word language.

Rules for language association with R.E's

- Rule 2:

  If r_1 is a regular expression associated with the language L_1 and r_2 is a regular expression associated with the language L_2, then:

  (i) The regular expression $(r_1)(r_2)$ is associated with the product L_1L_2, that is the language L_1 times the language L_2:

  $\text{language}(r_1r_2) = L_1L_2$

Rules for language association with R.E's

- Rule 2:

  (ii) The regular expression $r_1 + r_2$ is associated with the language formed by the union of L_1 and L_2:

  $$\text{language}(r_1 + r_2) = L_1 + L_2$$

  (iii) The language associated with the regular expression $(r_1)^*$ is L_1^*, the Kleene closure of the set L_1 as a set of words:

  $$\text{language}(r_1^*) = L_1^*$$

Finite Languages Are Regular

Theorem 5:

- If L is a finite language (a language with only finitely many words), then L can be defined by a regular expression.
- In other words, all finite languages are regular.

Finite Languages Are Regular

- ***Proof:***
- Let L be a finite language. To make one regular expression that defines L, we insert plus signs between all the words in L e.g.

  L = {baa,abbba,bababa} is baa+abbba+bababa

- Note:
  - This algorithm only works for finite languages because an infinite language would become a regular expression that is infinitely long, which is forbidden.

Finite Automata

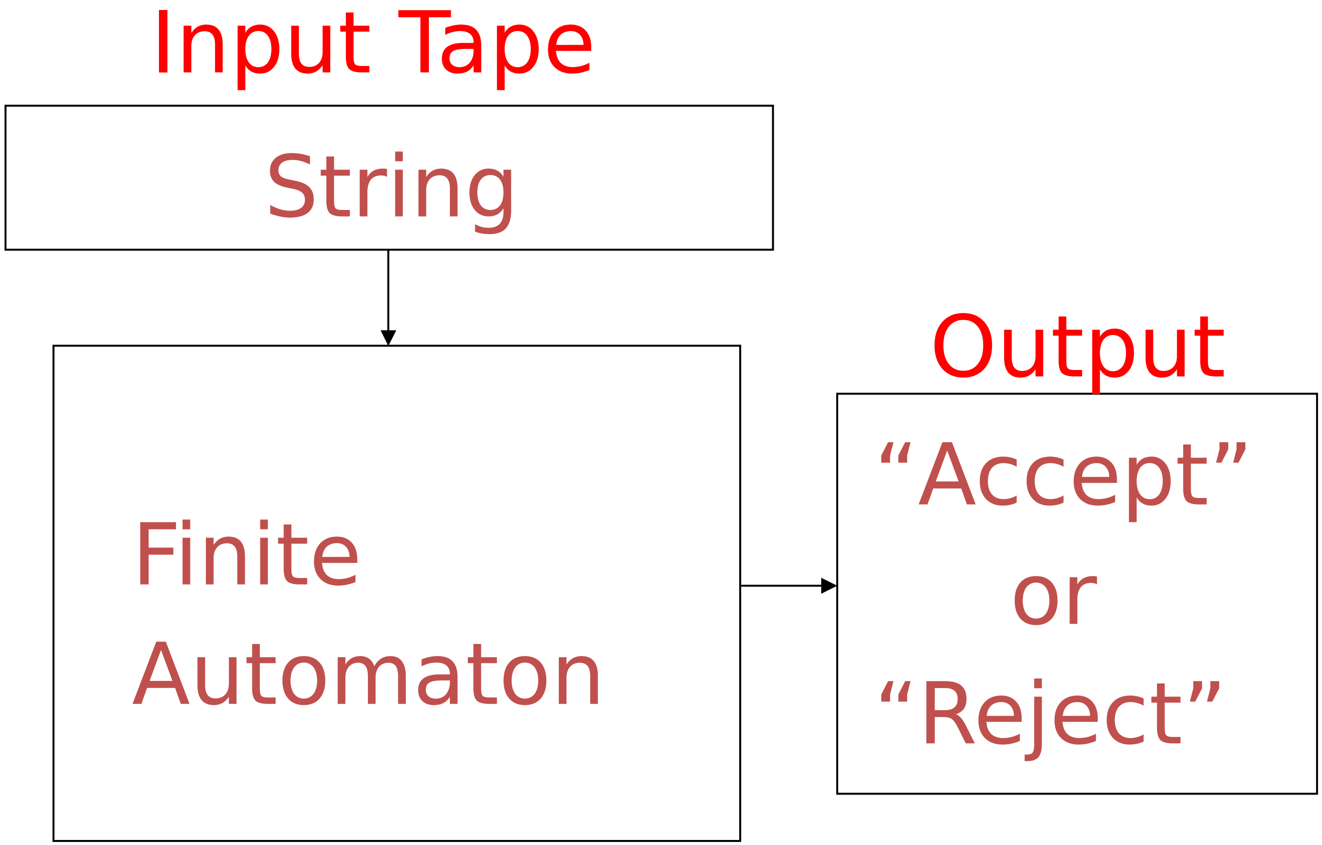
Finite Automata

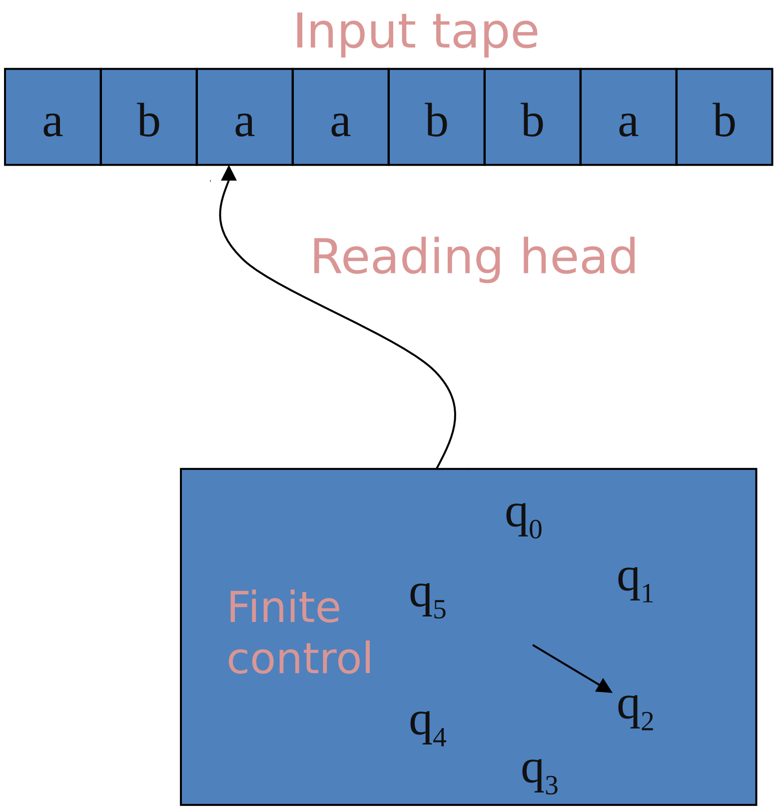
- Also known as **deterministic finite automaton (DFA), Finite State Automata (FSA) or simply FA**
- It is a simple language **recognition device**.
- It is called deterministic because their operation is completely determined by their input.
- Strings are fed into the device by means of an **input tape**, which is divided into squares, with one symbol inscribed in each tape square.

Finite Automata

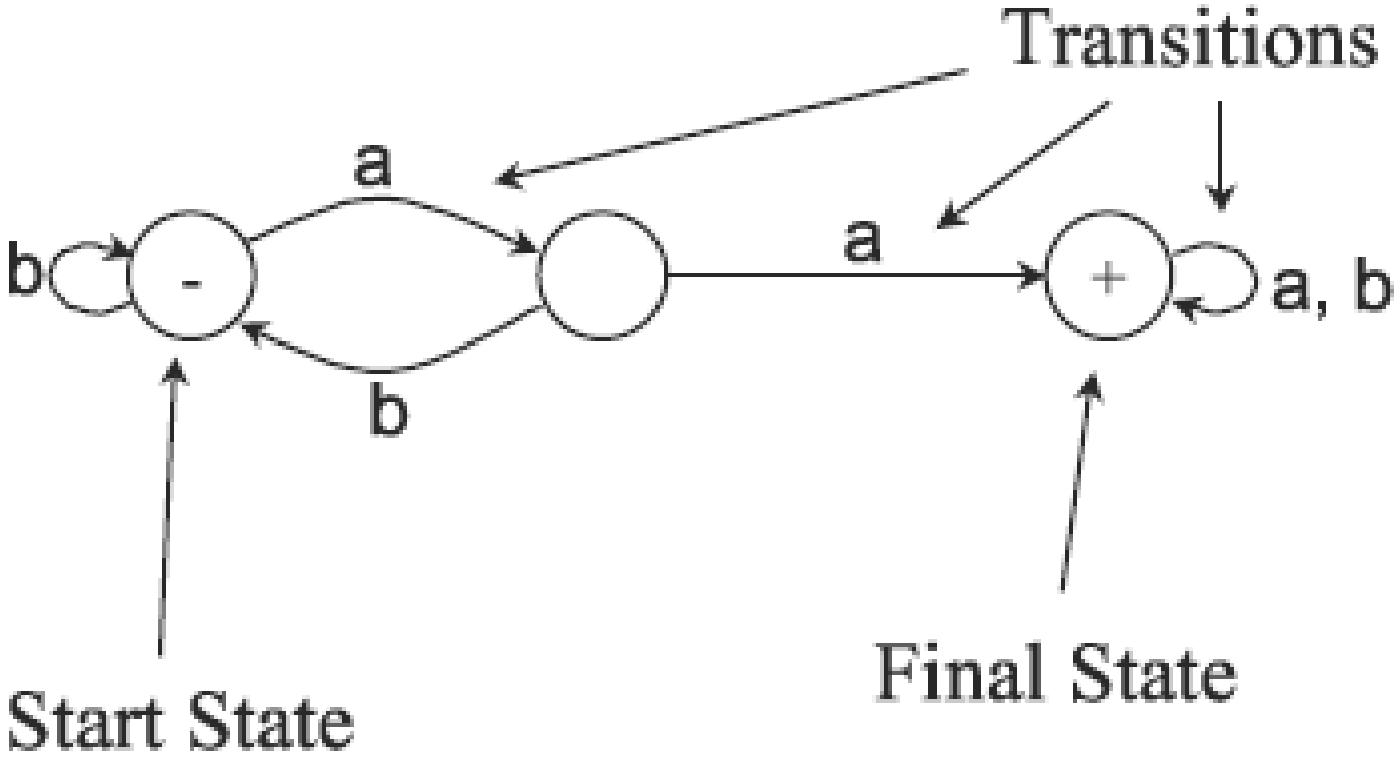
- It is used to determine whether a word does or does not belong to a Regular Language
- User for defining a Regular Language
- Used in Lexical Analyzers

Deterministic Finite Automaton

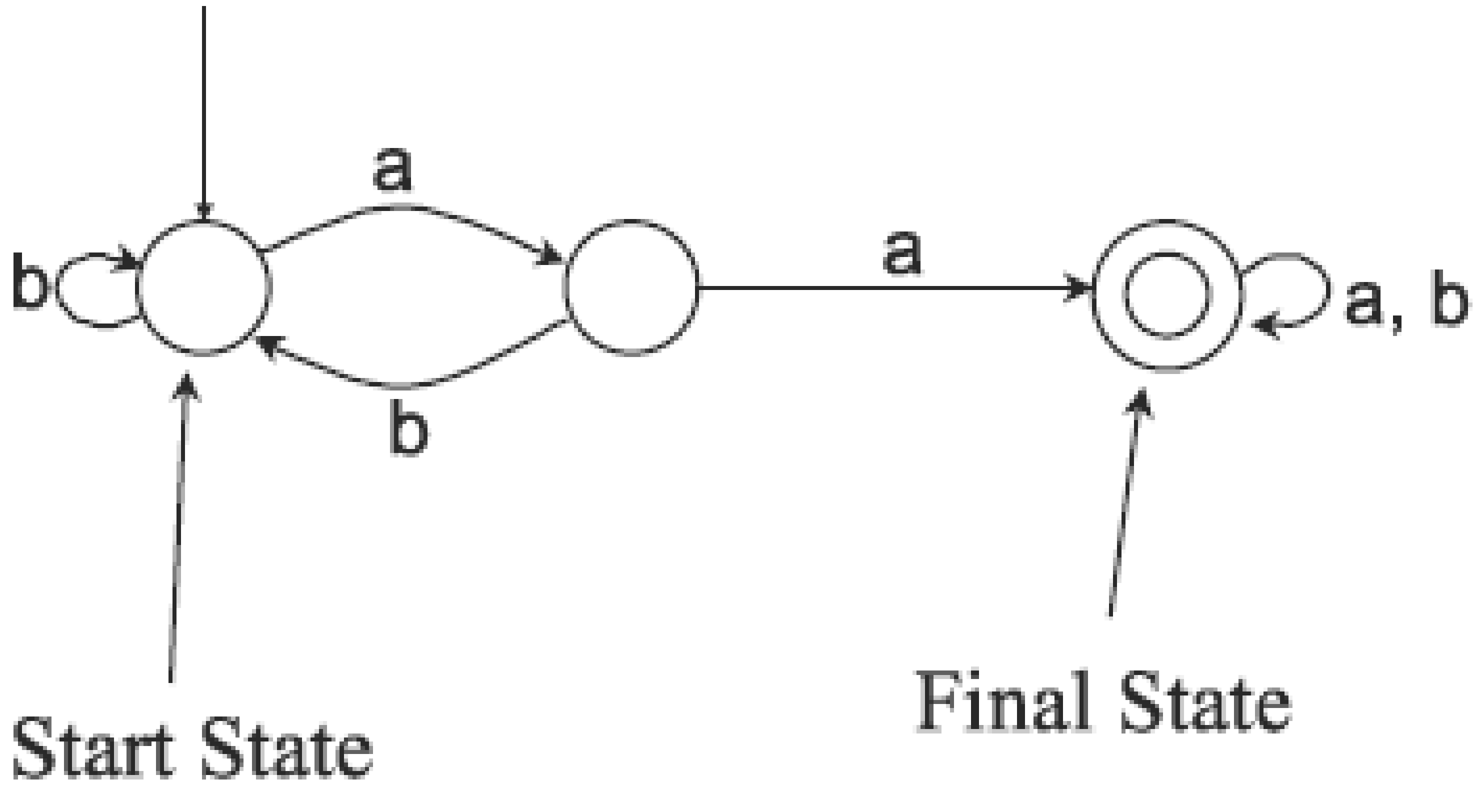



Input tape
a
b
a
a
b
b
a
b
Reading head
Finite control
q_0
q_1
q_5
q_2
q_4
q_3

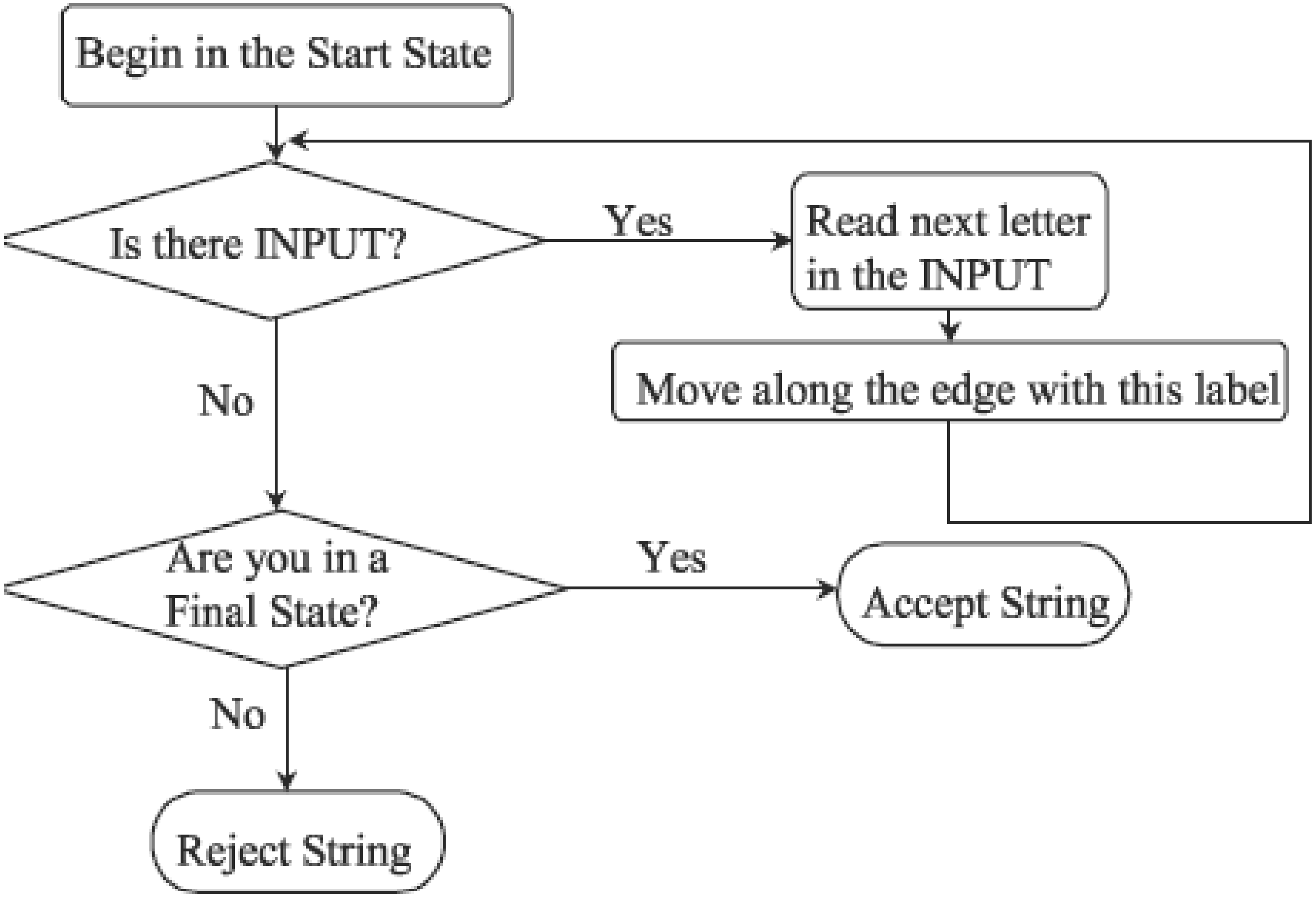
Notation



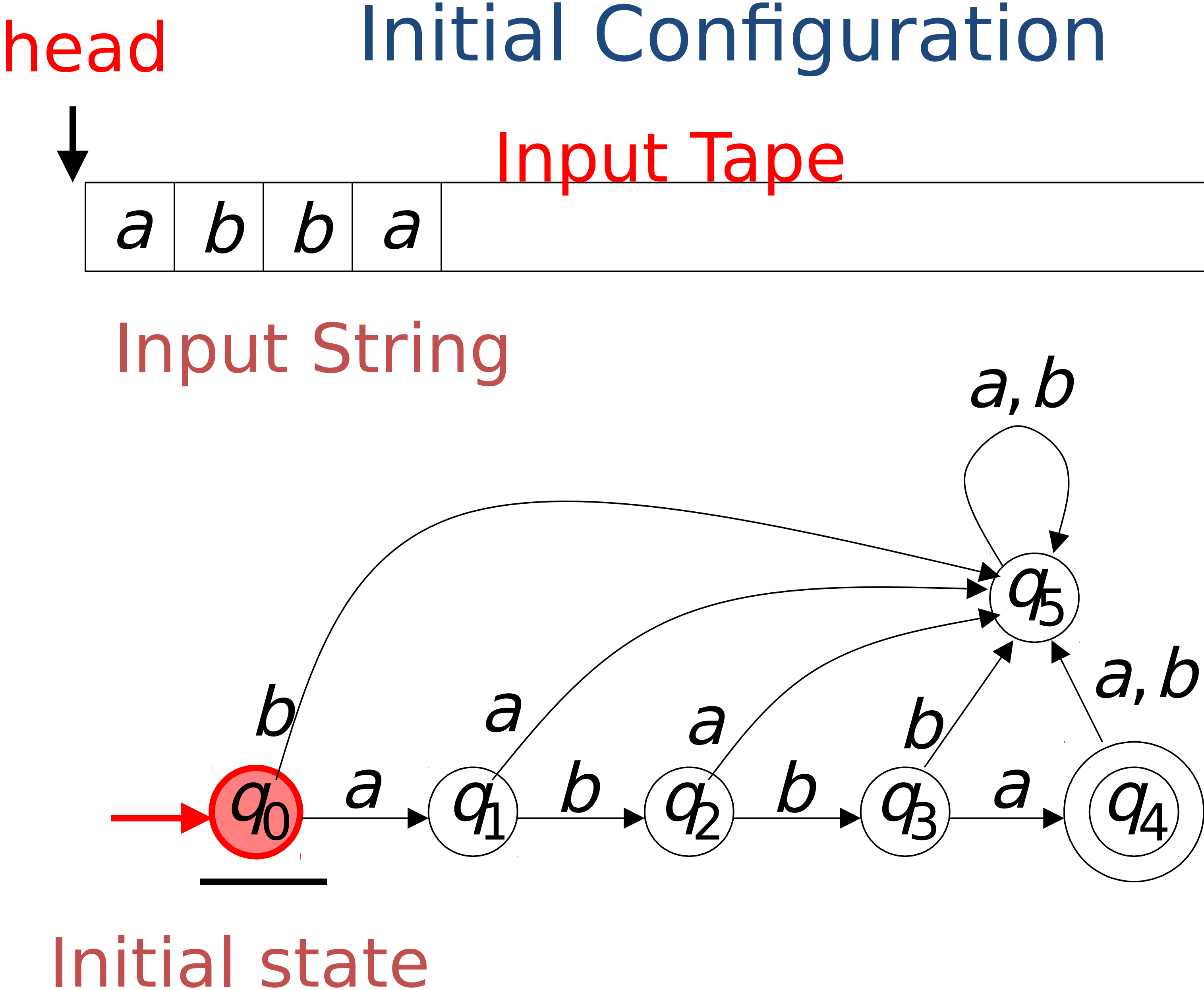
Alternative Notation



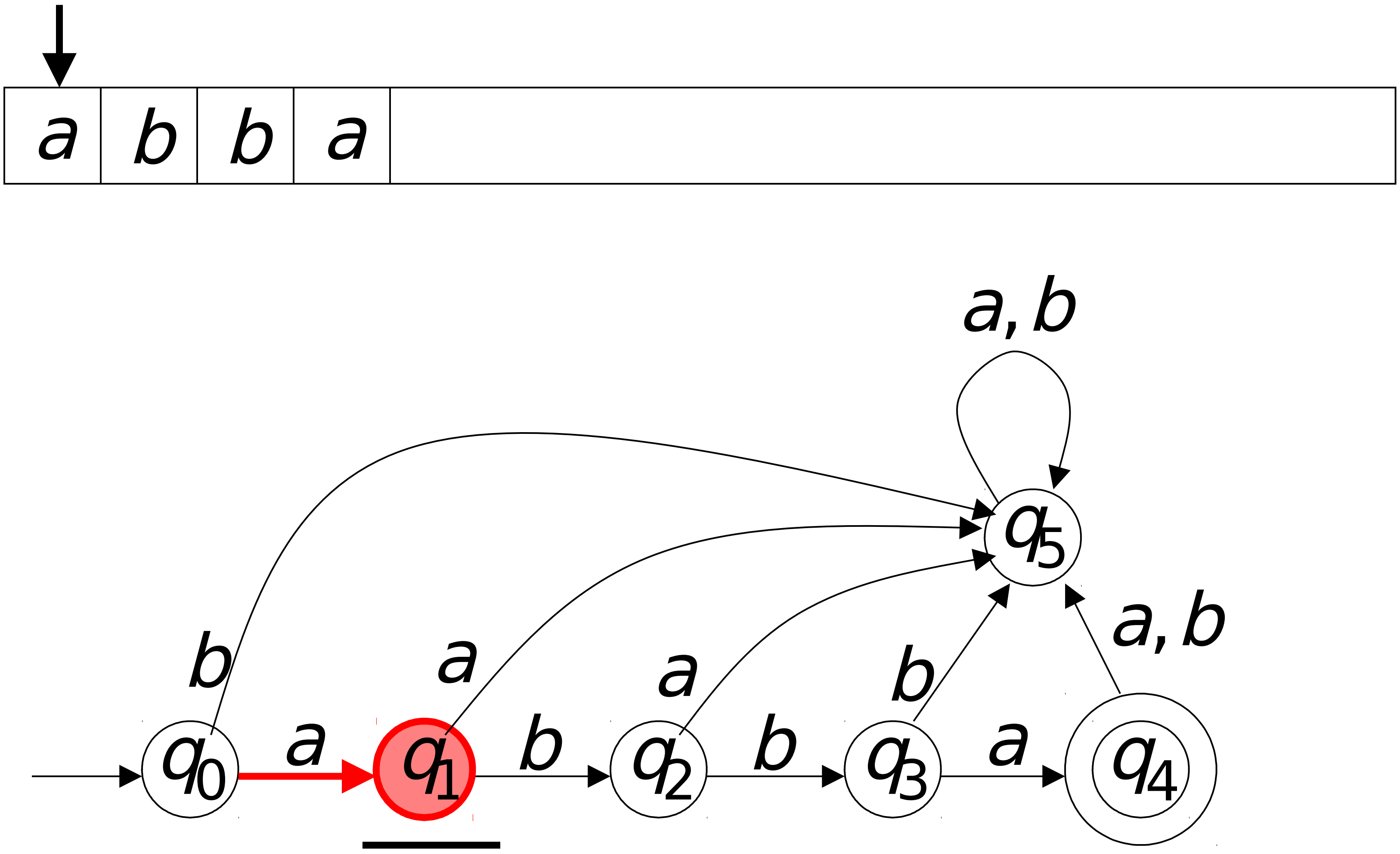
Flow Chart of FA

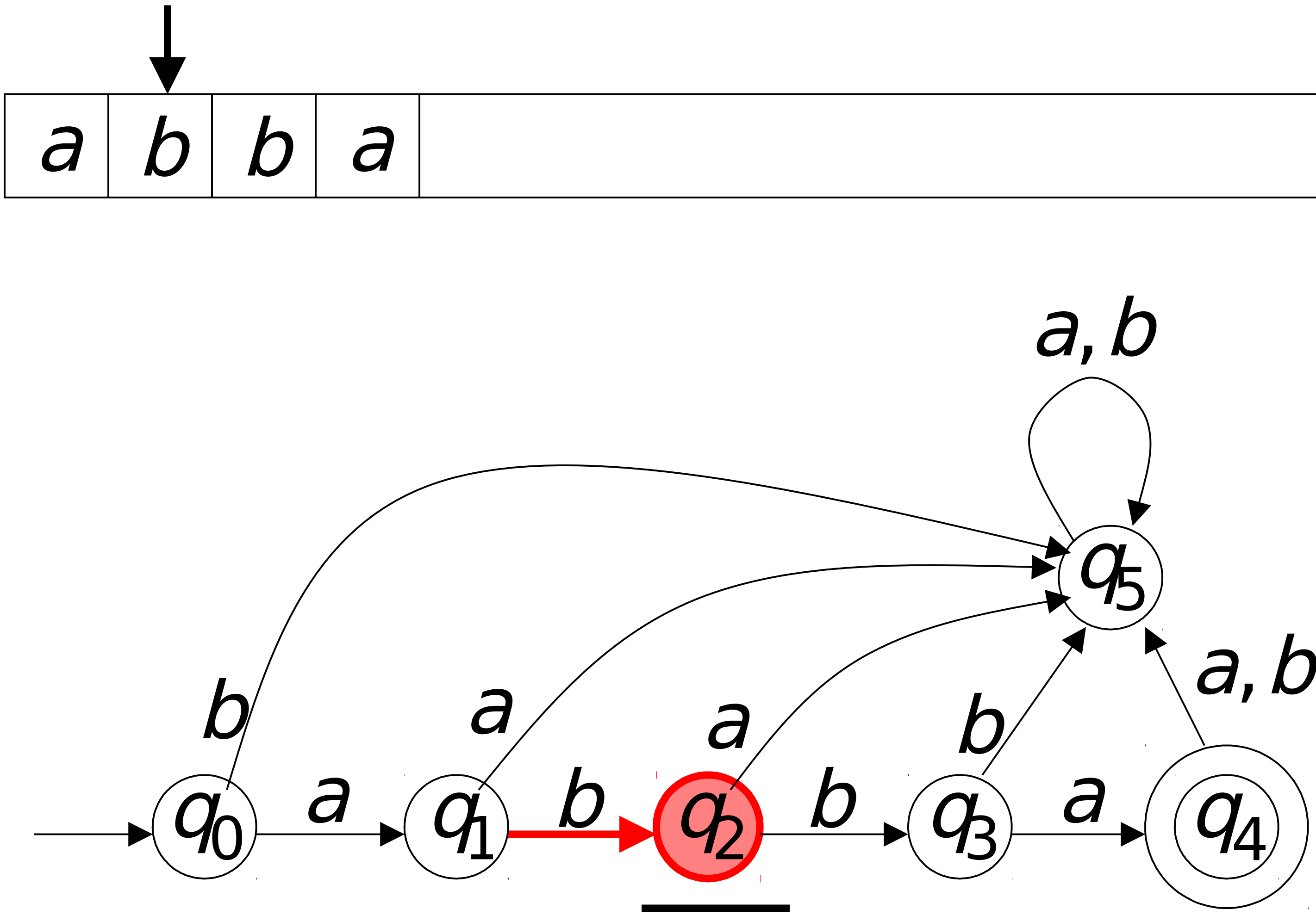


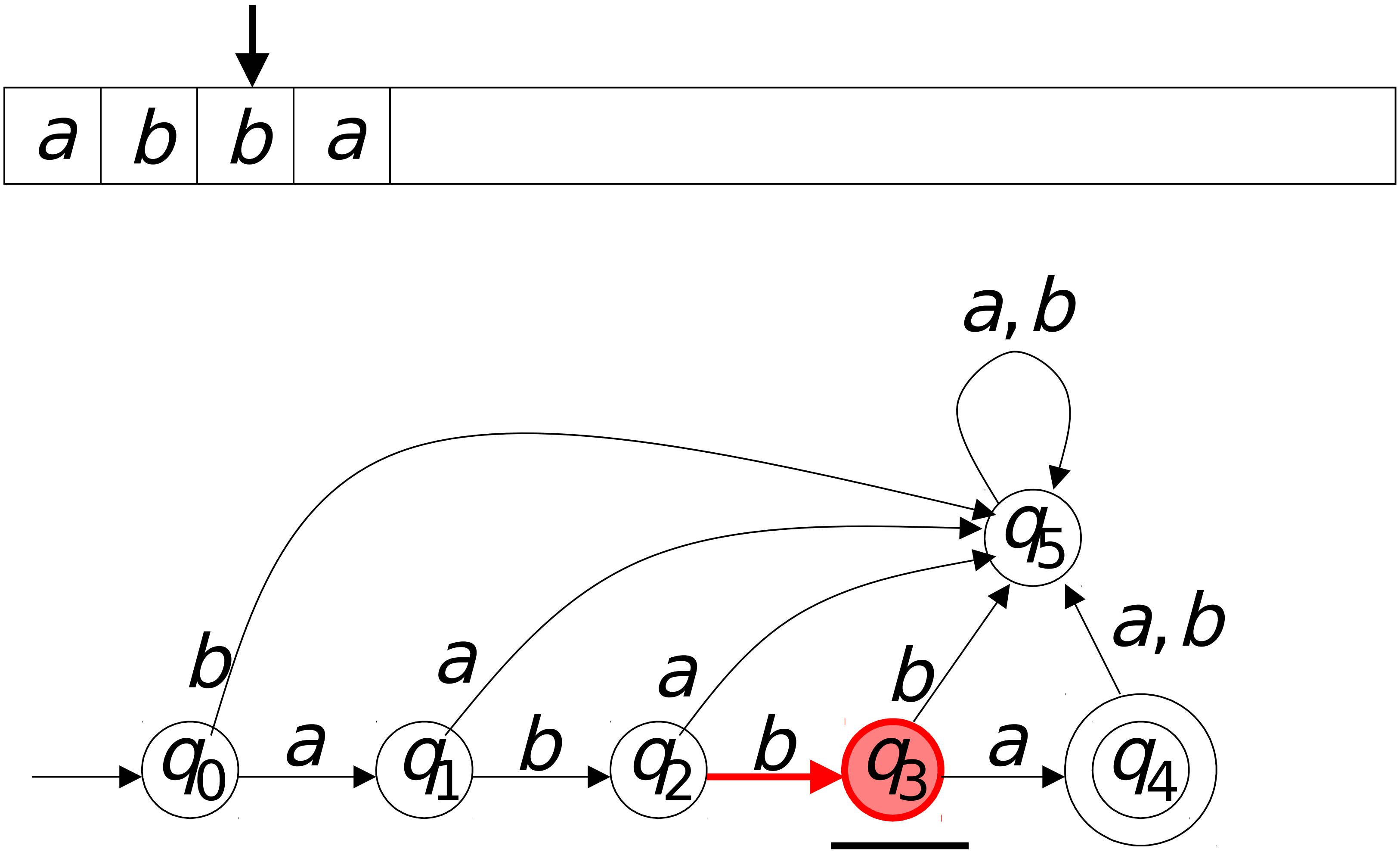
Initial Configuration



Scanning the Input

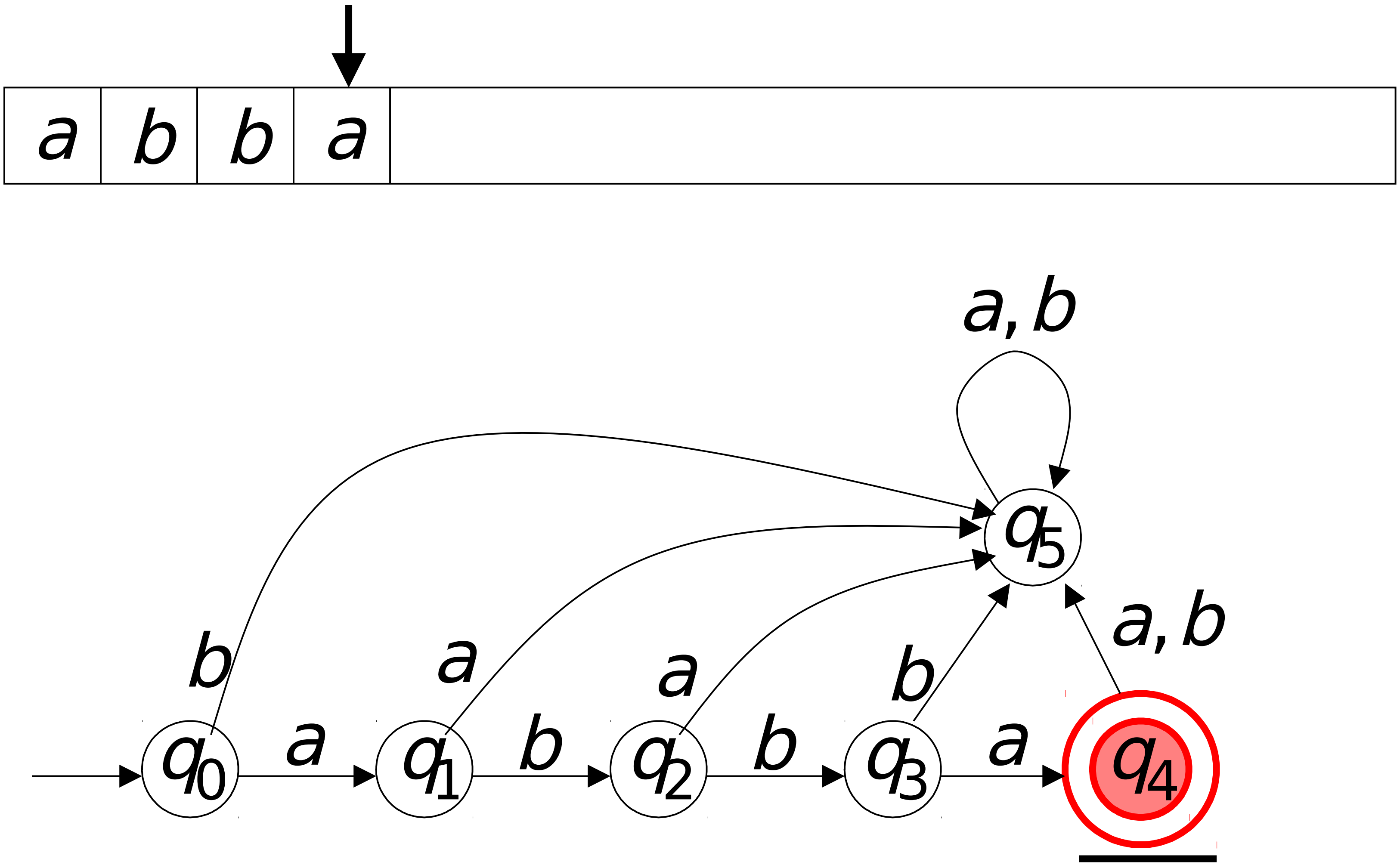



a b b a
a, b
b
a
a
b
b
a
a, b
q_0
q_1
q_2
q_3
q_4
q_5



a
b
b
a
a, b
b
a
a
b
b
a
a, b
q_0
q_1
q_2
q_3
q_4
q_5

Input finished

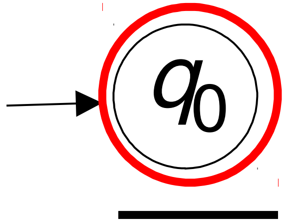


accept

Empty Tape

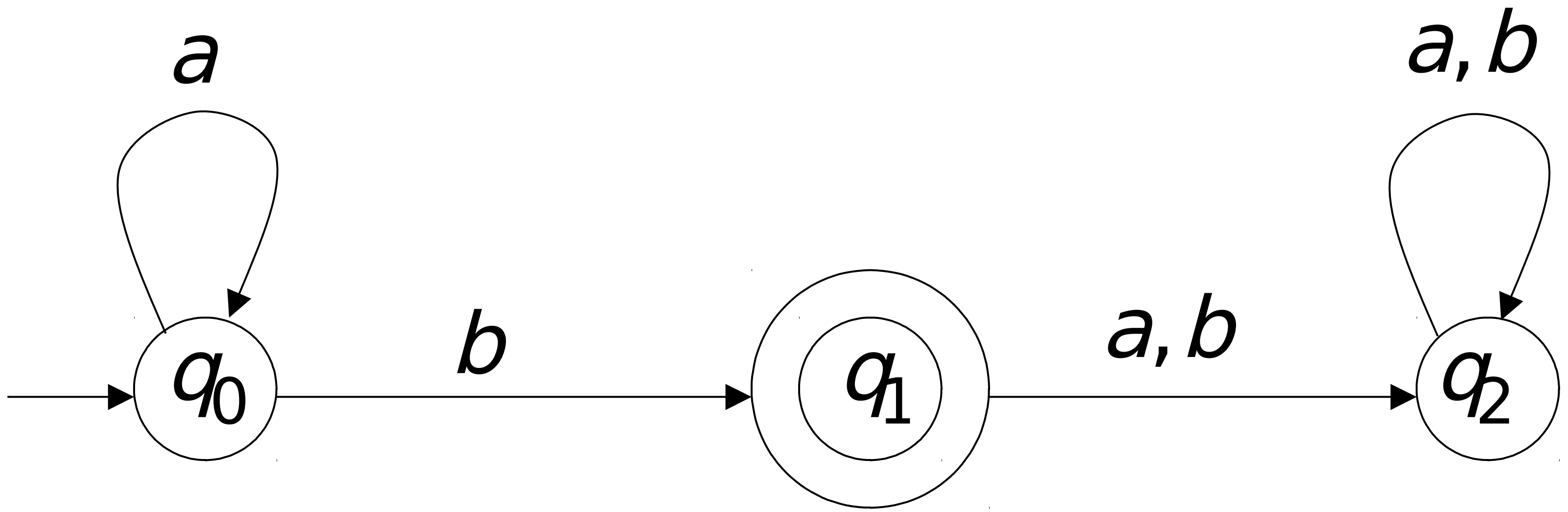
(λ)

Input Finished



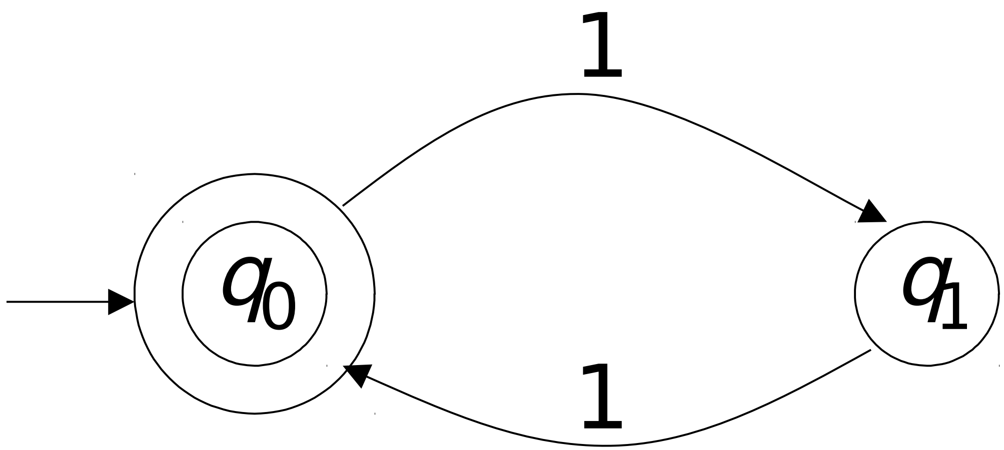
accept

Language Accepted: $L = \{a^n b : n \geq 0\}$



Another Example

Alphabet: $\Sigma = \{1\}$



Language Accepted:

$$EVEN = \{x : x \in \Sigma^* \text{ and } x \text{ is even}\}$$
$$= \{\lambda, 11, 1111, 111111, \ldots\}$$

Formal Definition

Deterministic Finite Automaton (DFA)

$$M = (Q, \Sigma, \delta, q_0, F)$$

Q : set of states

Σ : input alphabet

δ : transition function

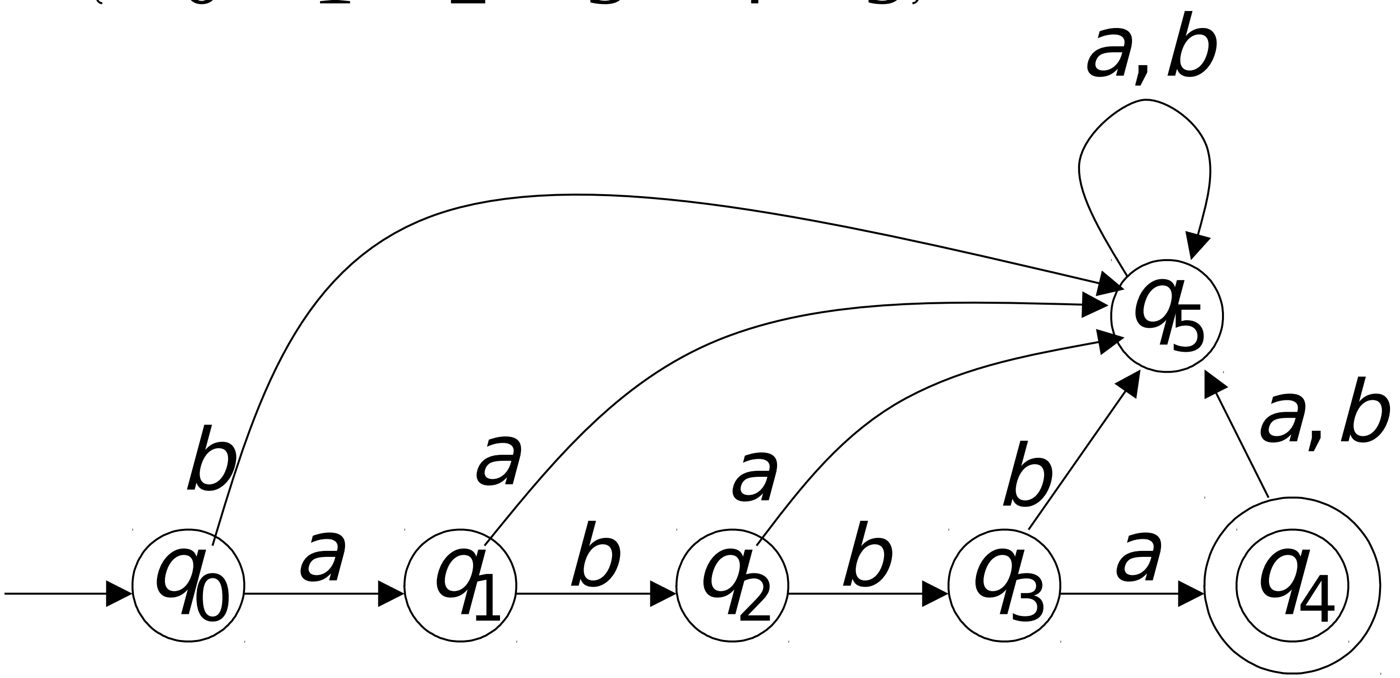
q_0 : initial state (one)

F : set of accepting states

Set of States Q

Example

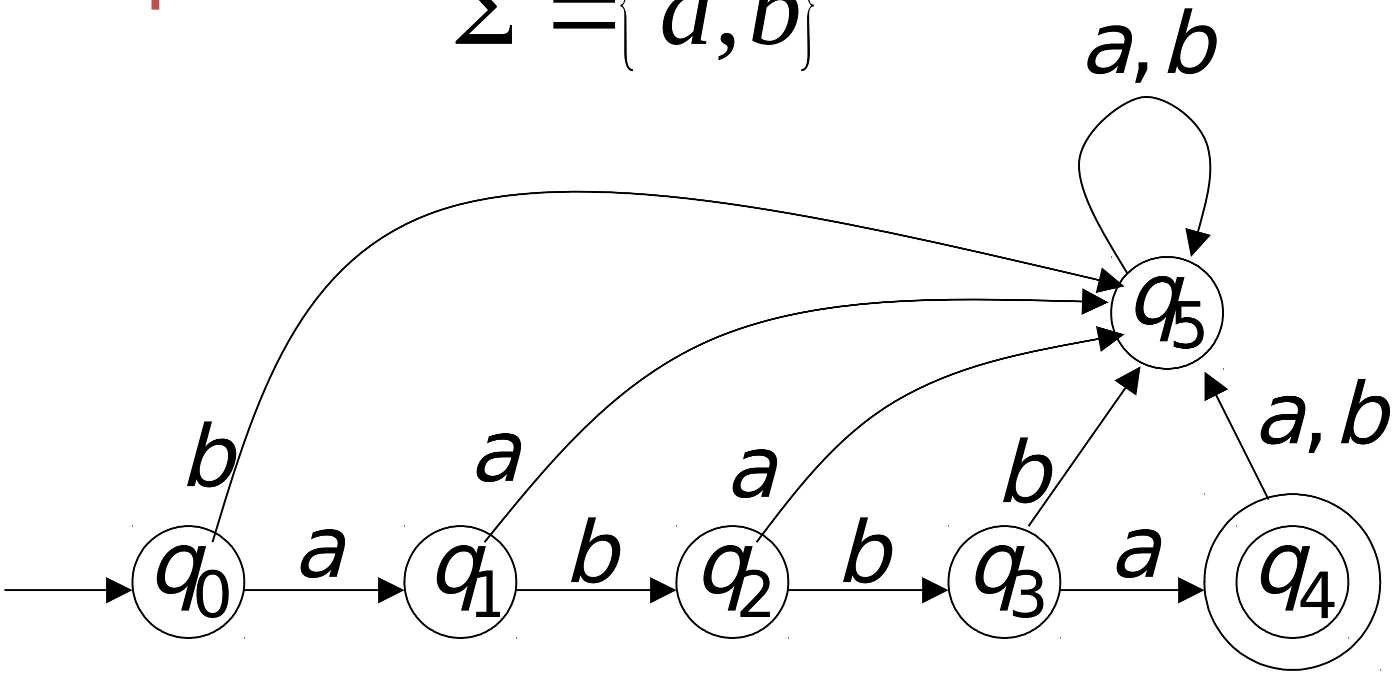
$$Q = \{ q_0, q_1, q_2, q_3, q_4, q_5 \}$$



Input Alphabet Σ

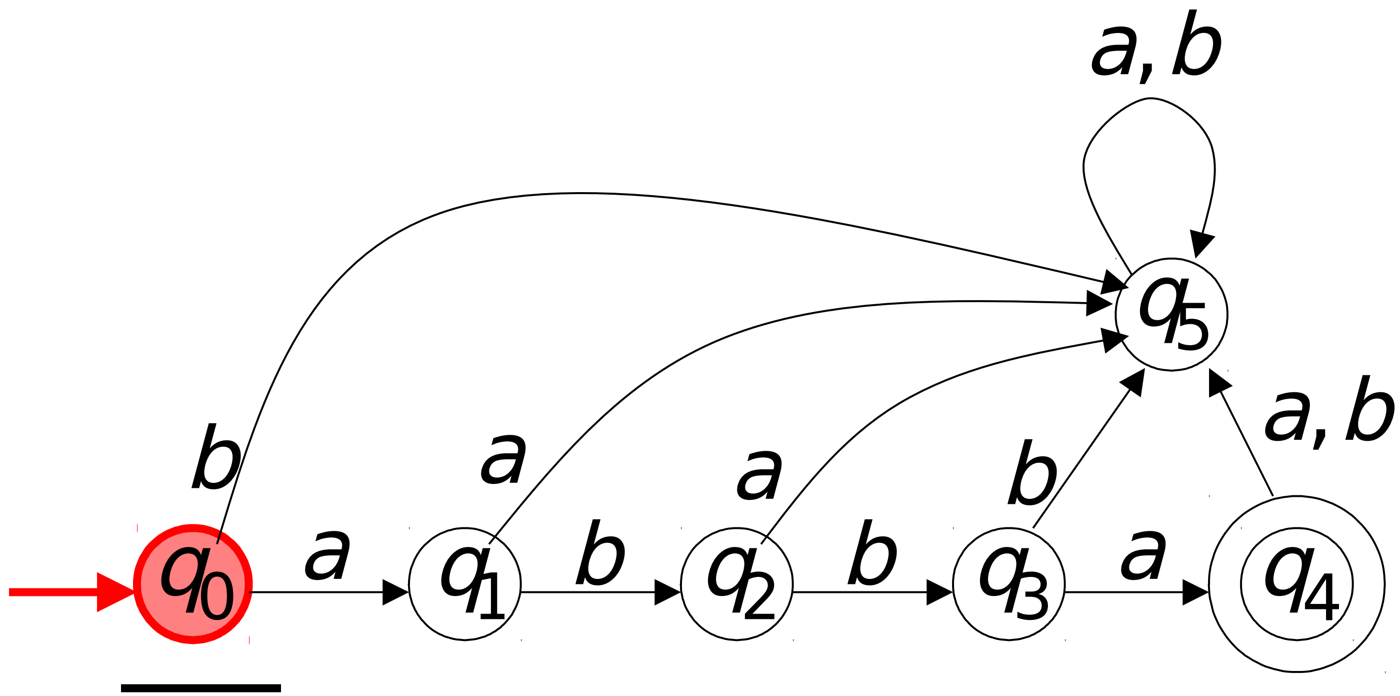
Example

$$\Sigma = \{a, b\}$$



Initial State q_0

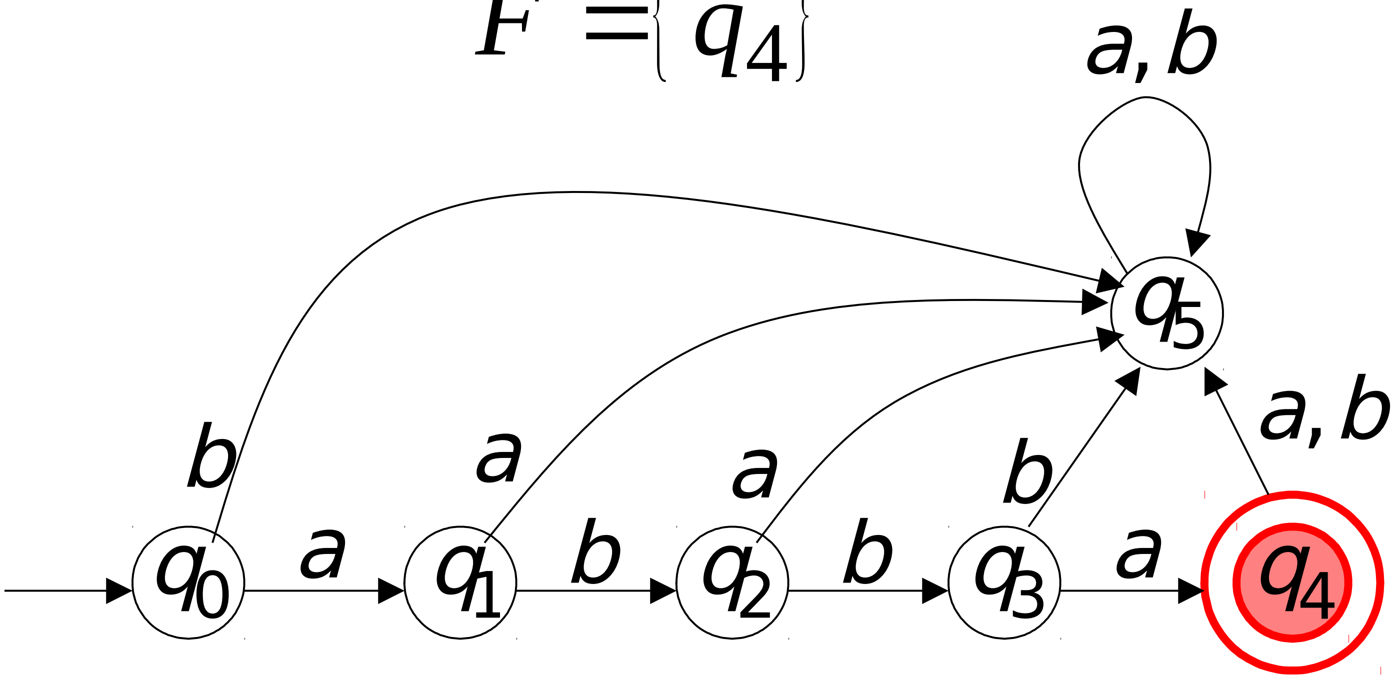
Example



Set of Accepting States $F \subseteq Q$

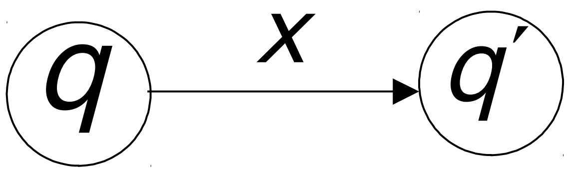
Example

$$F = \{ q_4 \}$$



Transition Function $\delta : Q \times \Sigma \rightarrow Q$

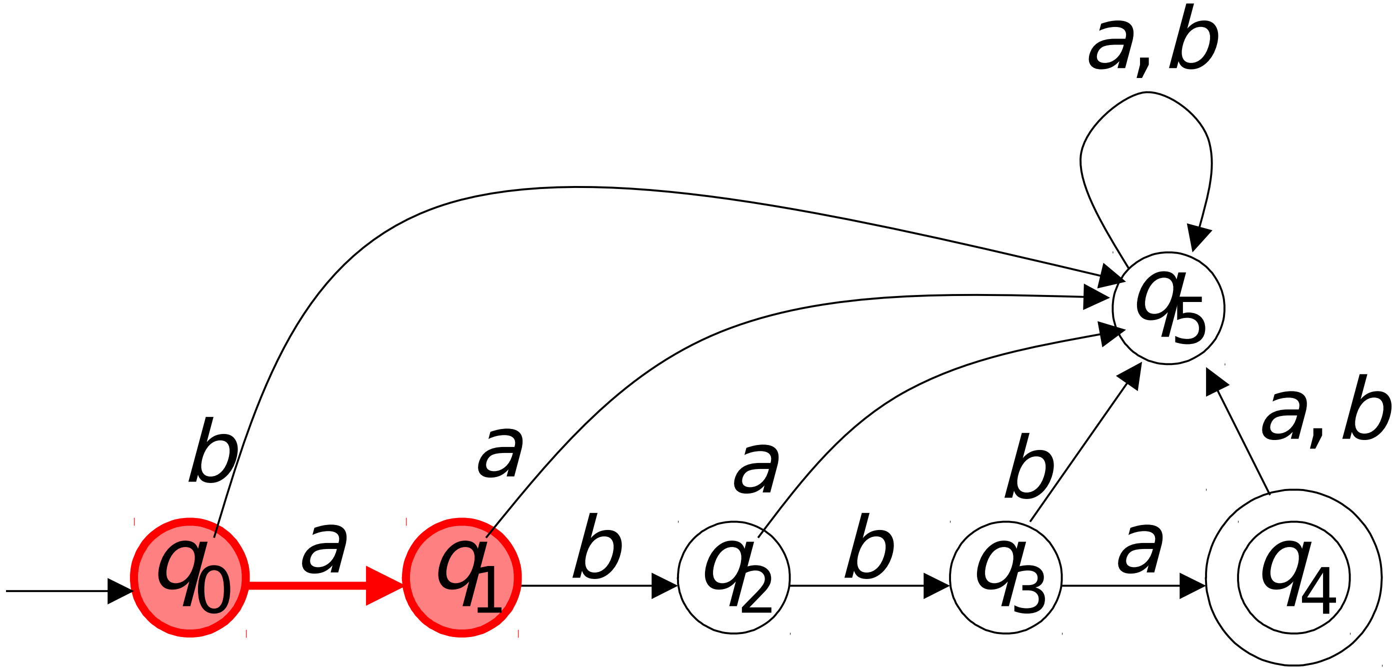
$$\delta(q, x) = q'$$



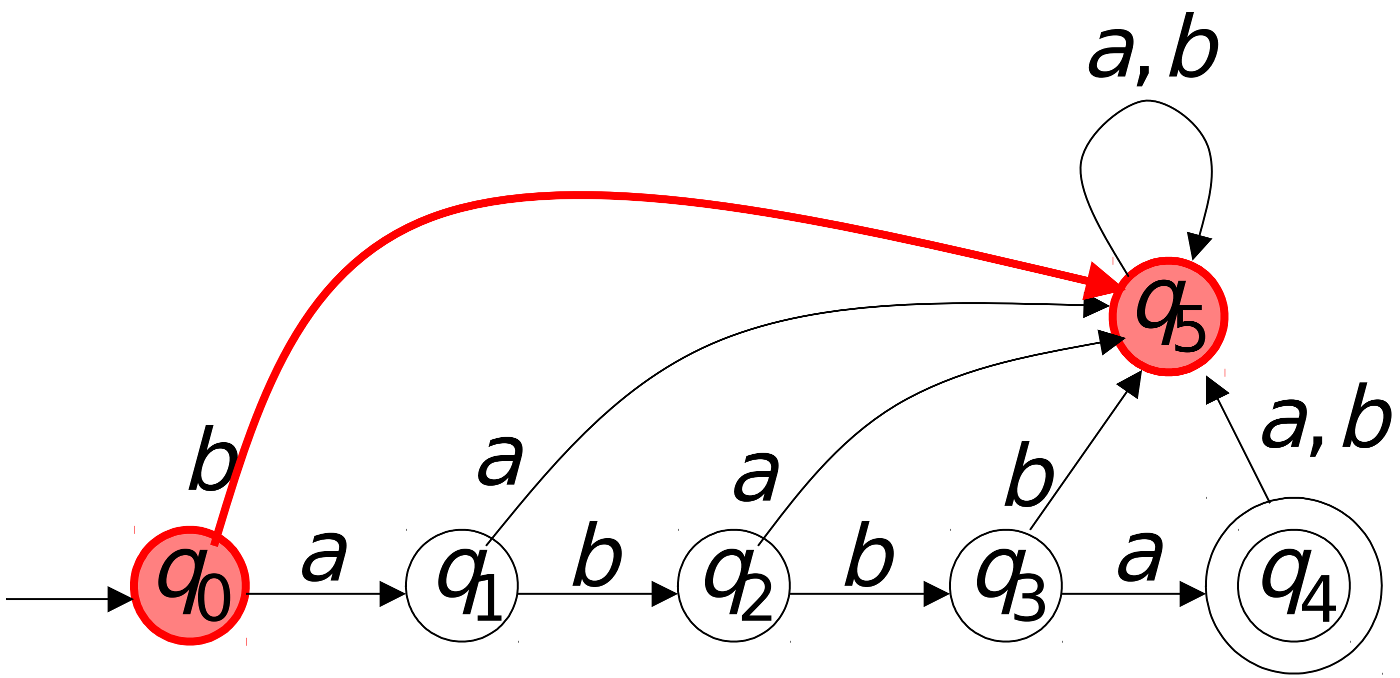
Describes the result of a transition from state q with symbol x

Example:

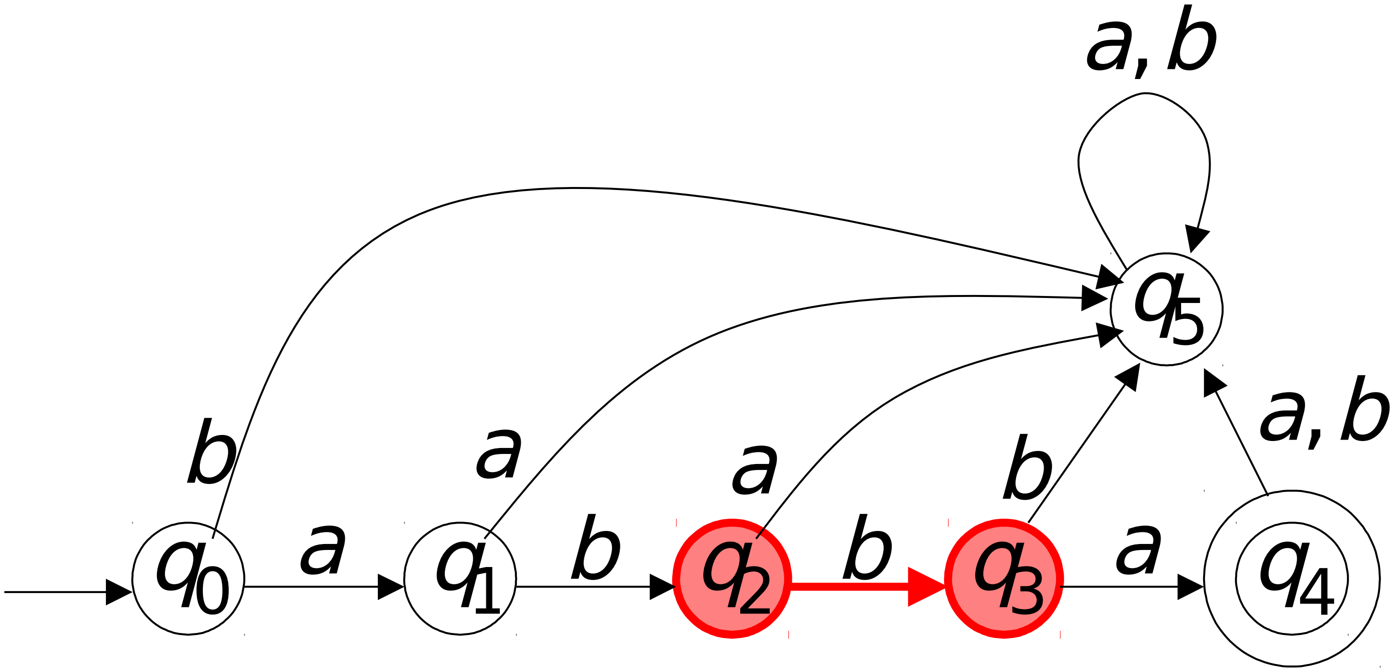
$$\delta(q_0, a) = q_1$$



$$\delta(q_0, b) = q_5$$



$$\delta(q_2, b) = q_3$$

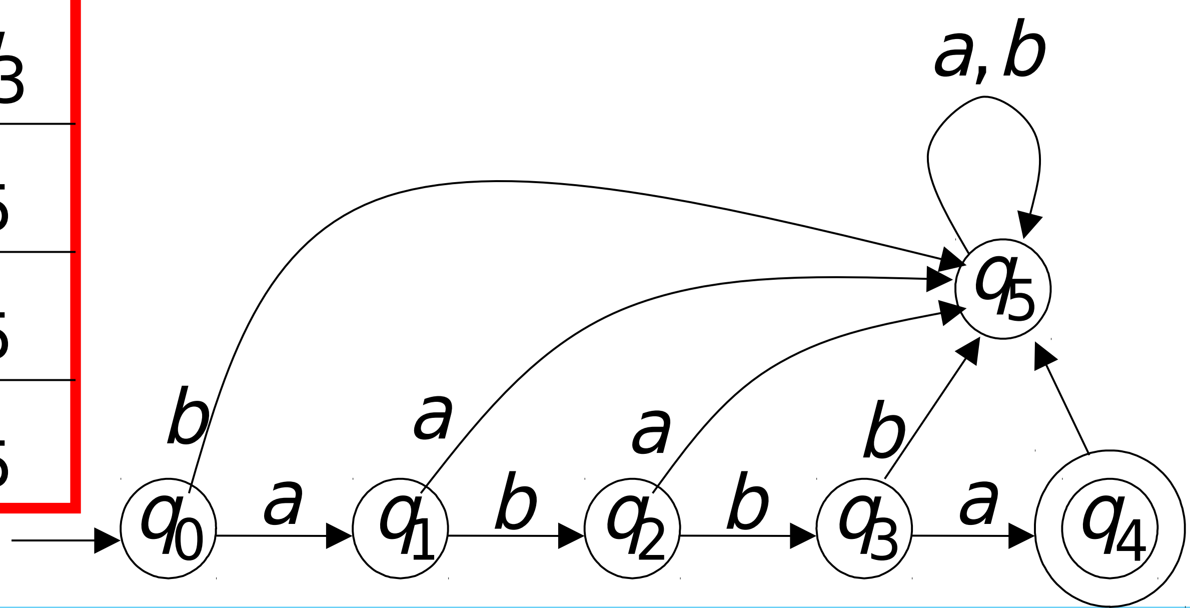


Transition Table for δ

symbols

states

δ	a	b
q_0	q_1	q_5
q_1	q_5	q_2
q_2	q_5	q_3
q_3	q_4	q_5
q_4	q_5	q_5
q_5	q_5	q_5



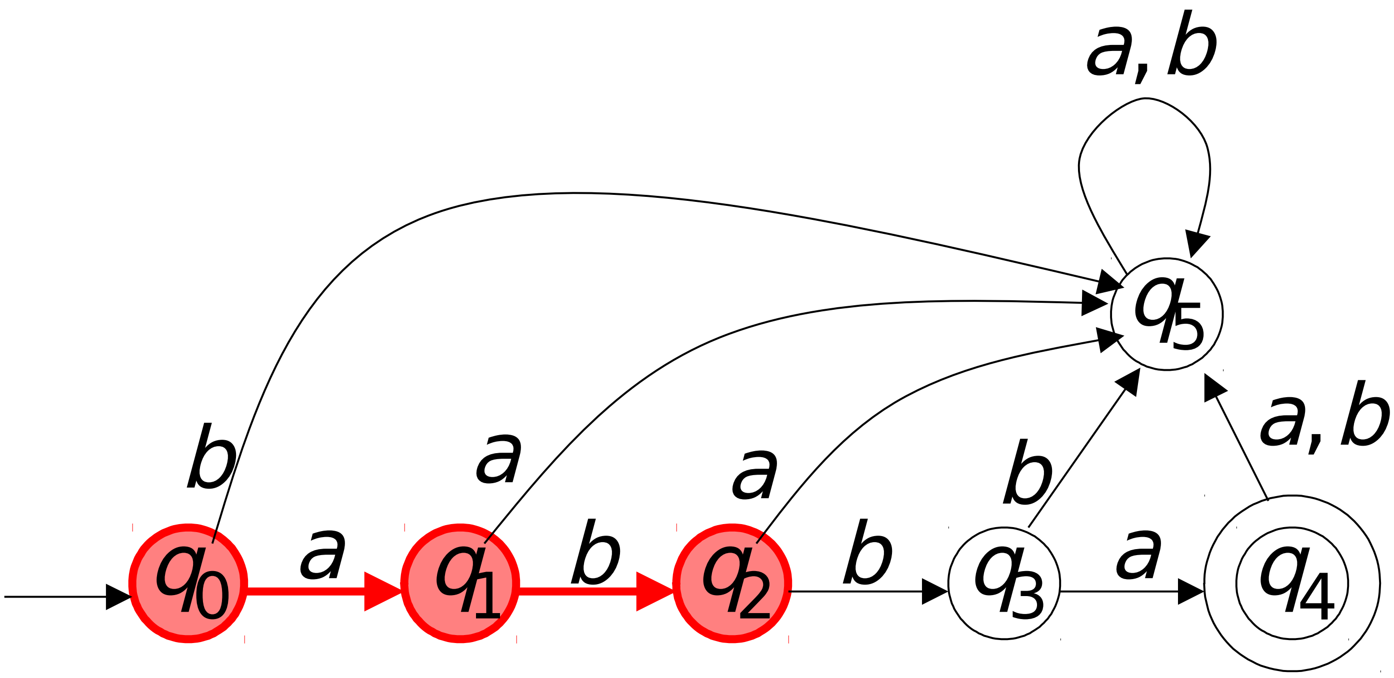
Extended Transition Function

$$\delta^*: Q \times \Sigma^* \to Q$$

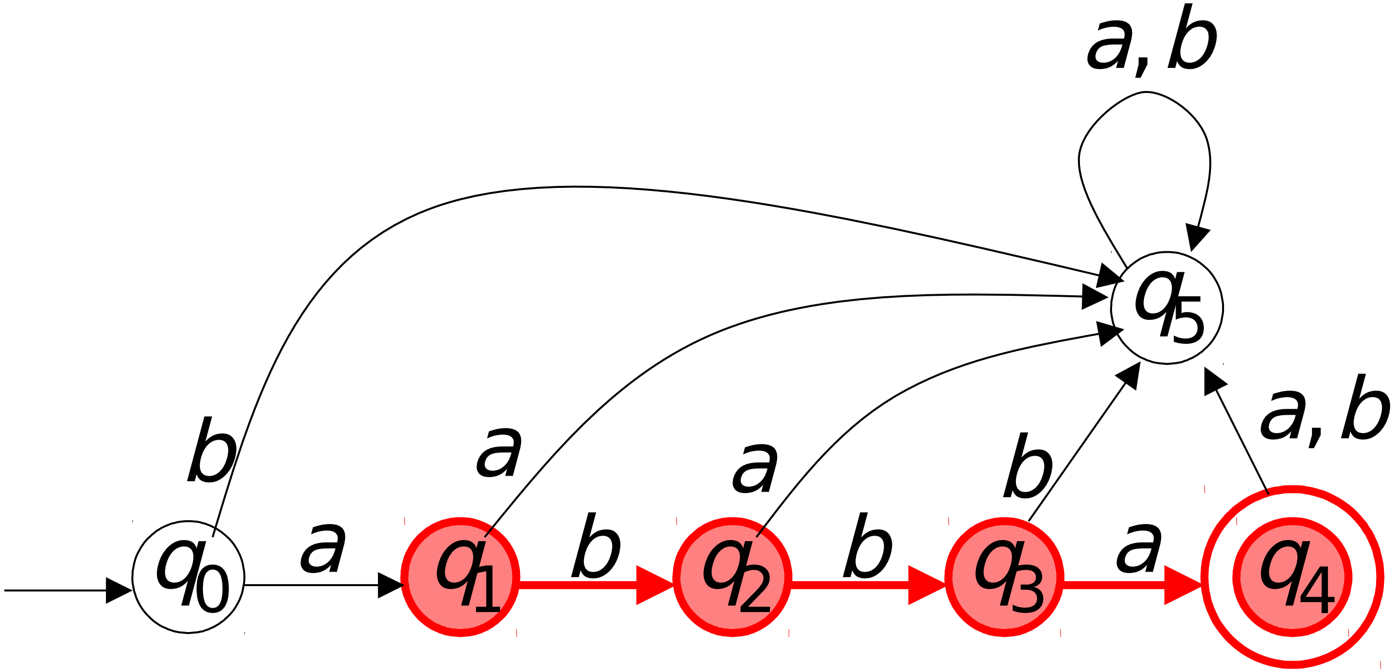
$$\delta^*(q, w) = q'$$

Describes the resulting state after scanning string w from state q

Example: $\delta^*(q_0, ab) = q_2$



$$\delta^*(q_1, bba) = q_4$$



Special case:

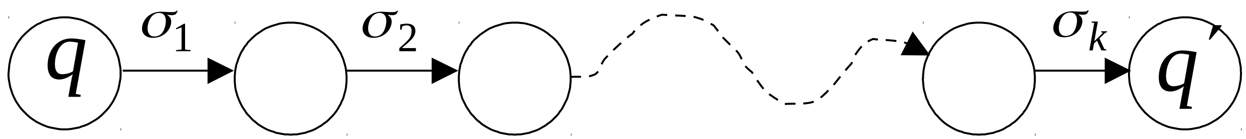
for any state q

$$\delta^*(q, \lambda) = q$$

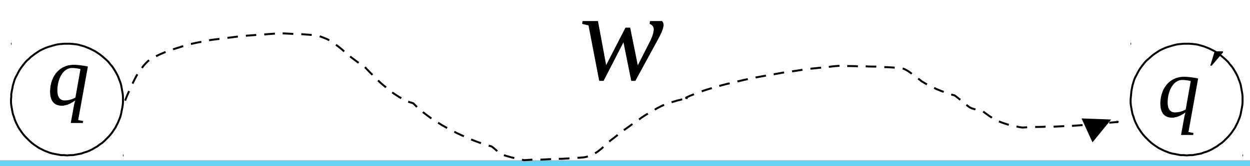
In general: $\delta^*(q,w) = q'$

implies that there is a walk of transitions

$$w = \sigma_1 \sigma_2 \cdots \sigma_k$$



states may be repeated



Language Accepted by DFA

Language of DFA M :

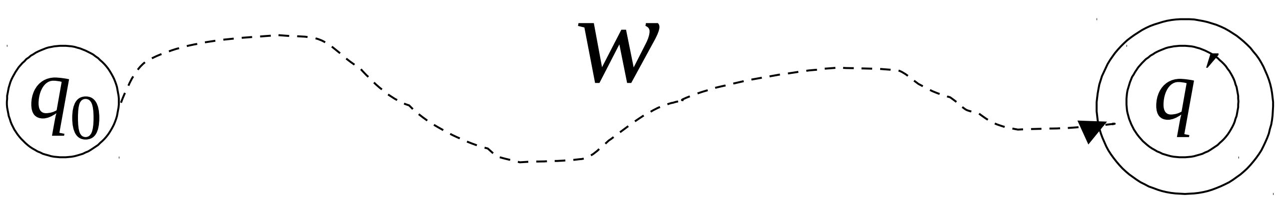
it is denoted as $L(M)$ and contains all the strings accepted by M

We say that a language L' is accepted (or recognized) by DFA M if $L(M) = L'$

For a DFA $M = (Q, \Sigma, \delta, q_0, F)$

Language accepted by M :

$$L(M) = \{ w \in \Sigma^* : \delta^*(q_0, w) \in F \}$$



$q' \in F$

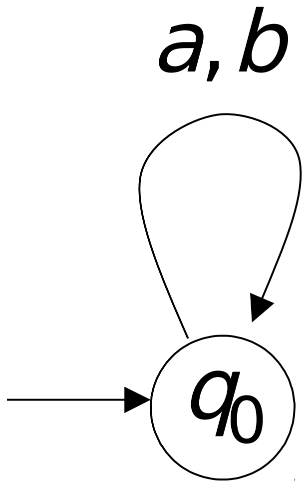
Language rejected by M :

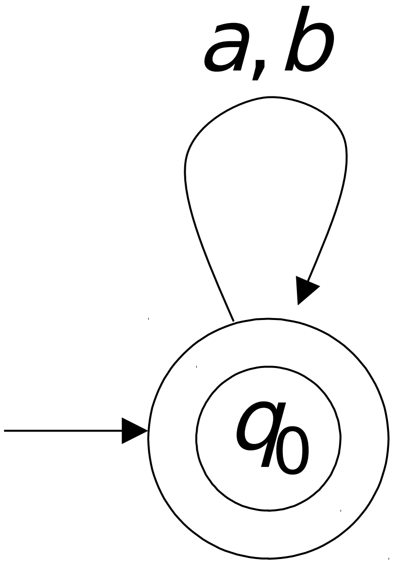
$$\overline{L(M)} = \left\{ w \in \Sigma^* : \delta^*(q_0, w) \notin F \right\}$$

$$q_0 \xrightarrow{w} q' \qquad q' \notin F$$

More DFA Examples

$$\Sigma = \{a, b\}$$





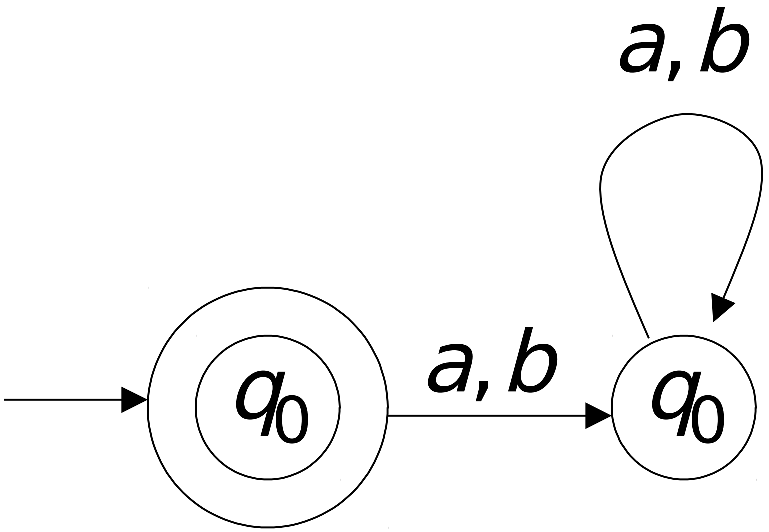
$$L(M) = \{\ \}$$

$$L(M) = \Sigma^*$$

Empty language

All strings

$$\Sigma = \{a, b\}$$



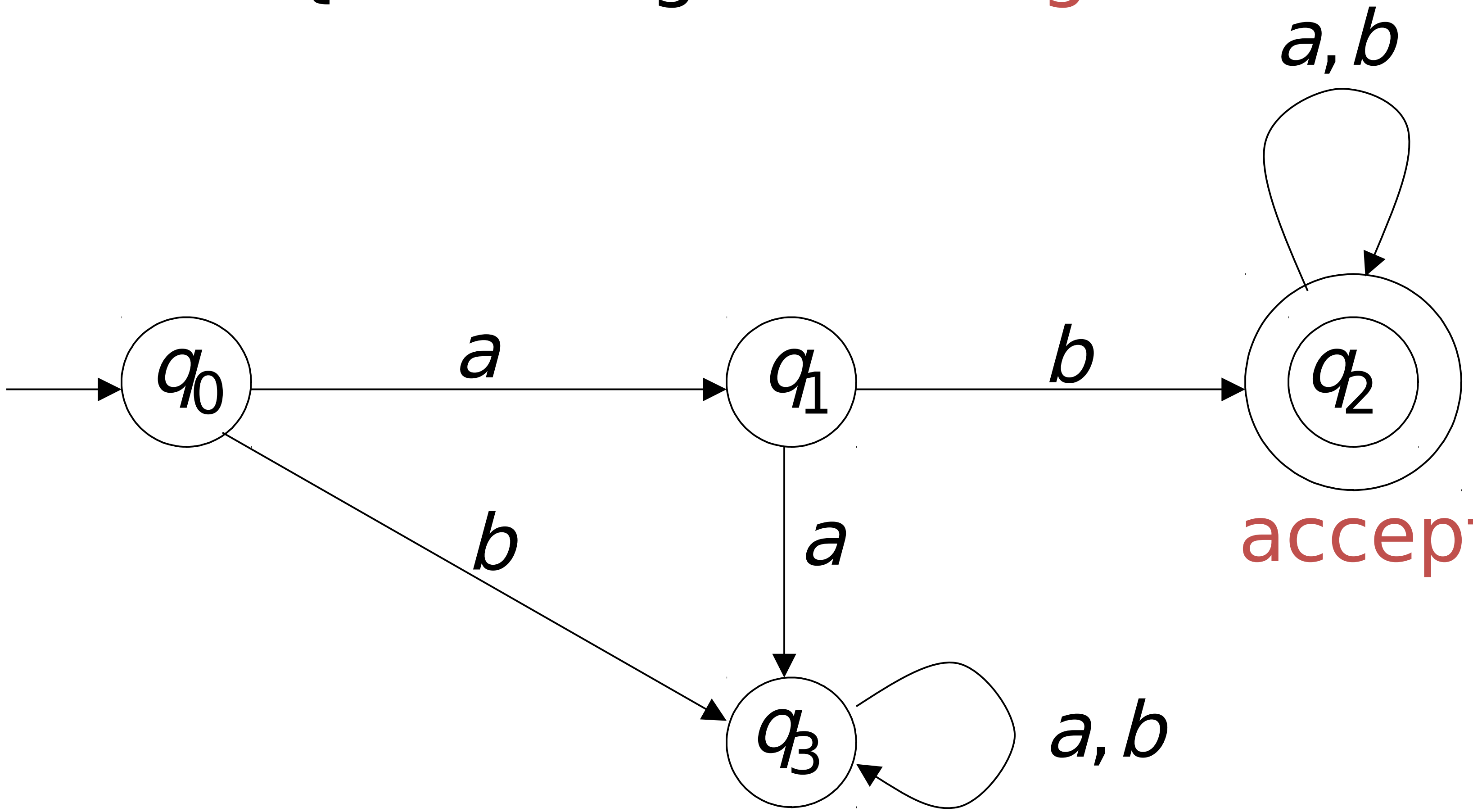
$$L(M) = \{\lambda\}$$

Language of the empty string

$$\Sigma = \{a, b\}$$

$L(M) =$ { all strings with prefix ab }

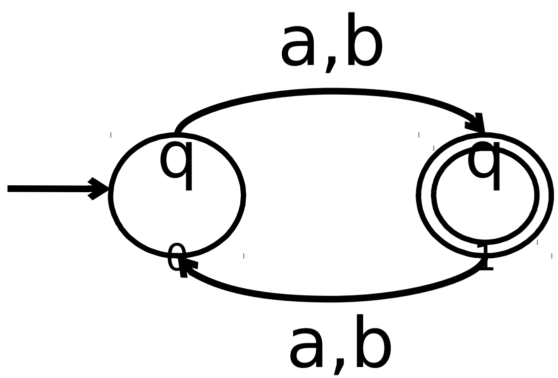
$=$ { all strings that begins with ab}



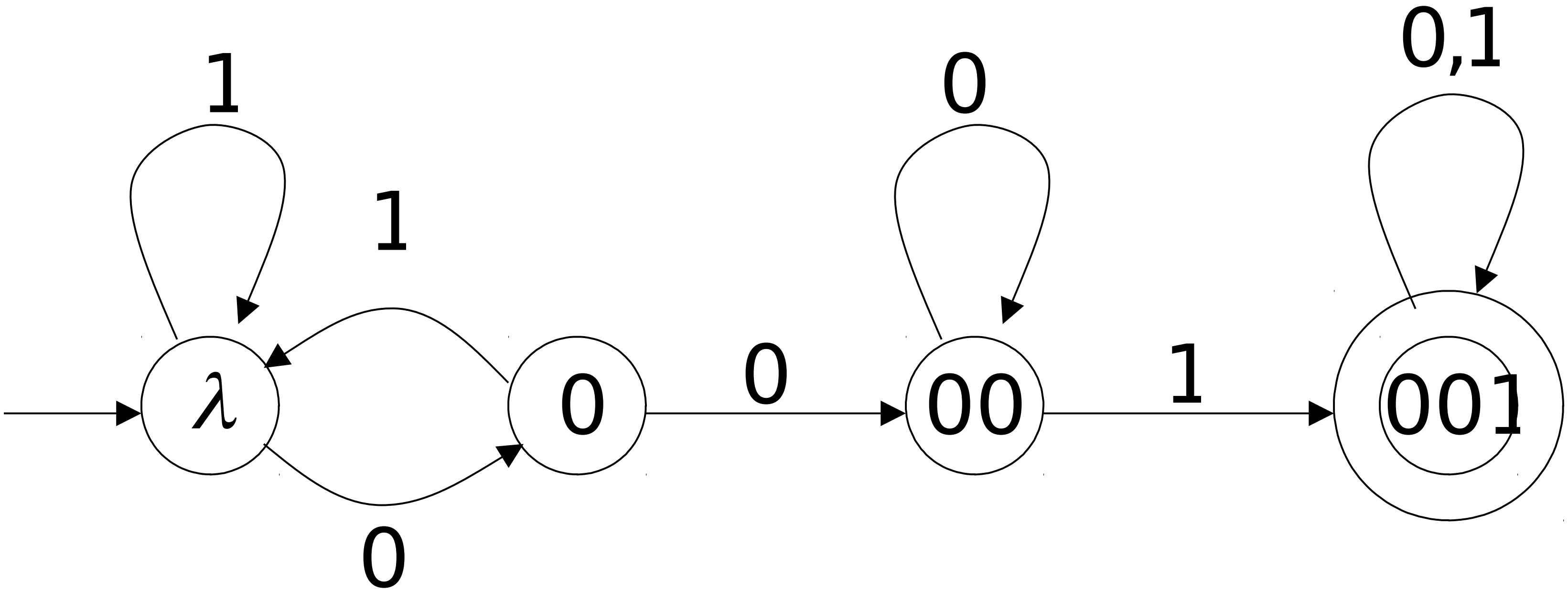
Infinite Languages

- *Example:* $\Sigma = \{a,b\}$

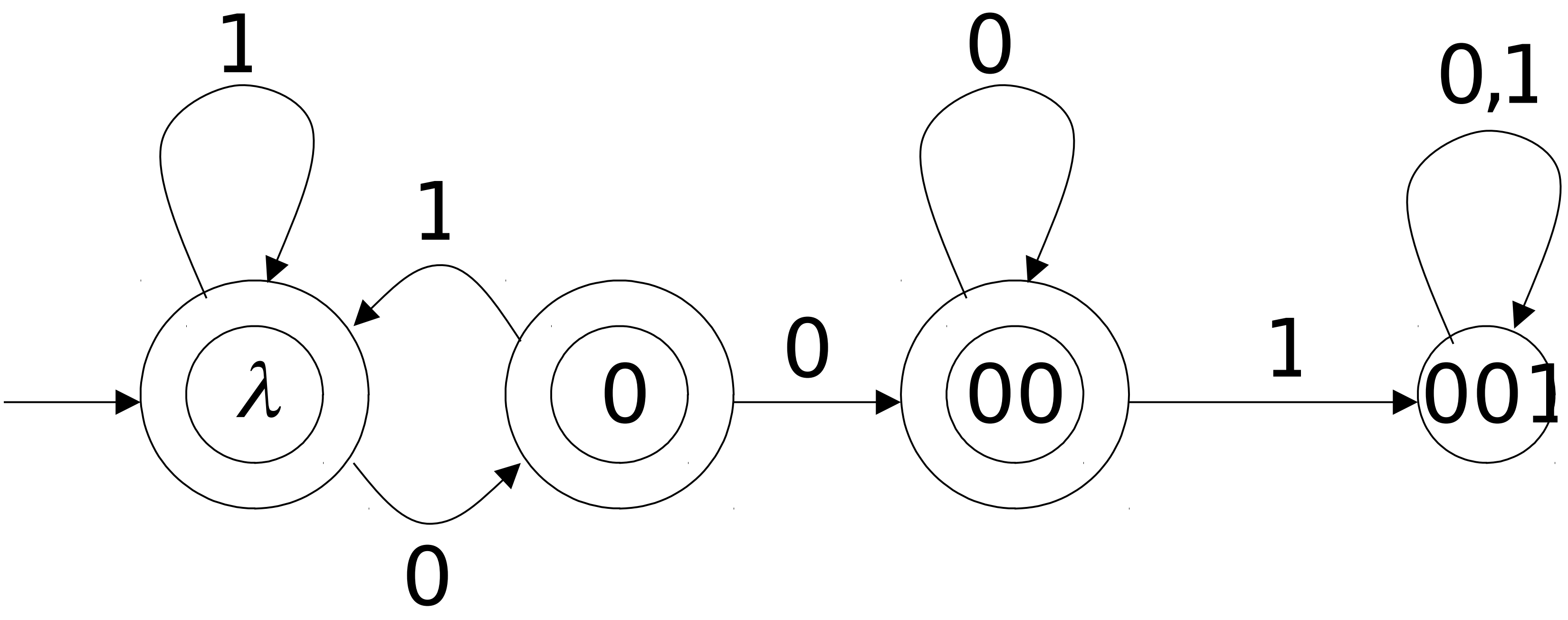
  $L = \{s : |s| \text{ is odd}\}$

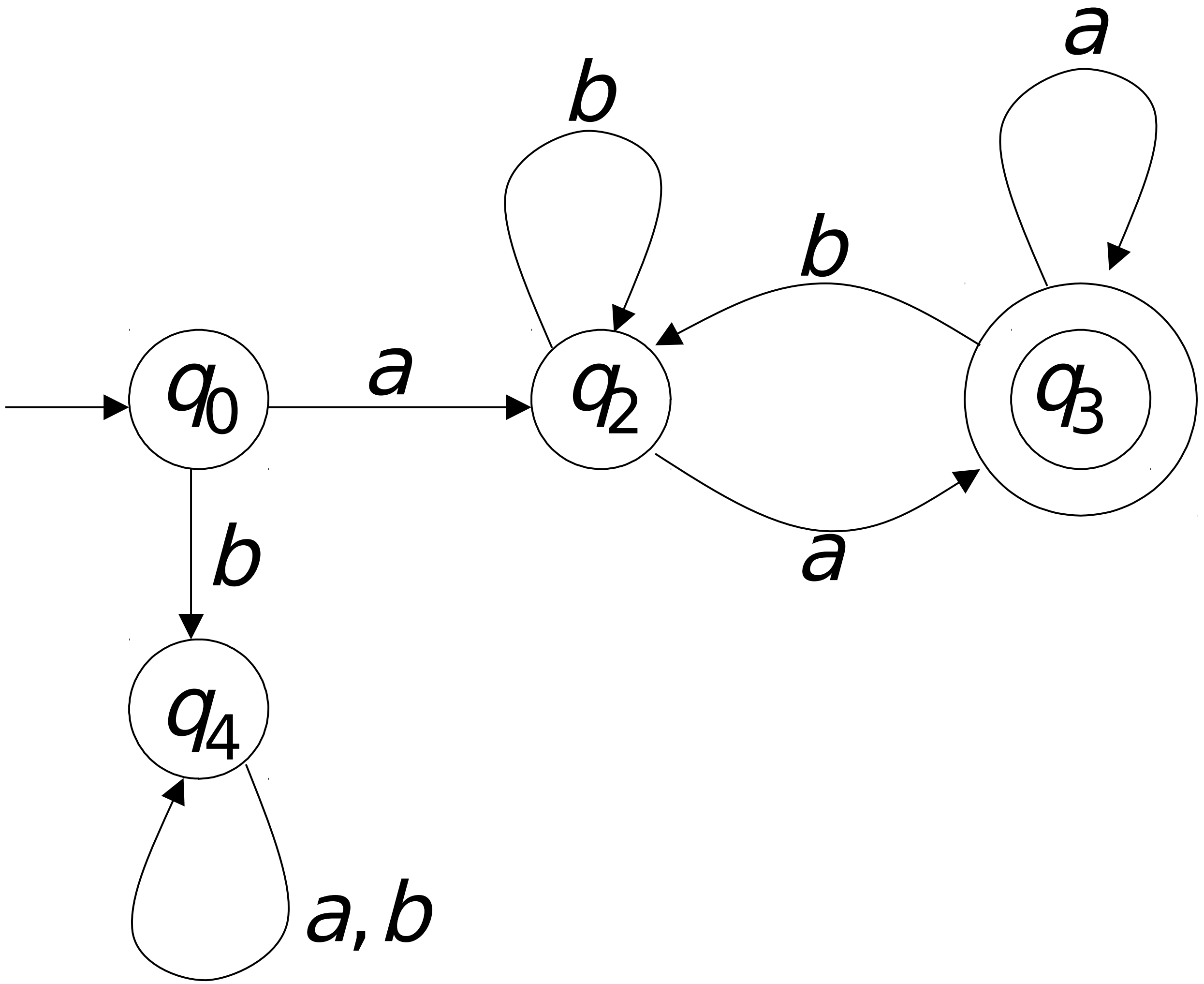


$L(M) =$ { all binary strings containing substring 001 }



$L(M) =$ { all binary strings without substring 001 }





a
b
b
a
a
b
a, b
q_0
q_2
q_3
q_4

$$L(M) = \{ awa : w \in \{a, b\}^* \}$$