

"131, SECTOR 65, PHASE 2"

Cryptography

≈ 131

↳ Plaintext

→ Cypher Text → Plaintext

↑
Encryption

↑
Decryption

$$\begin{array}{r} 131 \\ +2 \\ \hline 353 \\ +1 \\ \hline \boxed{242} \end{array}$$

"Substitution Cypher"

Algo for performing encryption

X _____ X

Easy to break:

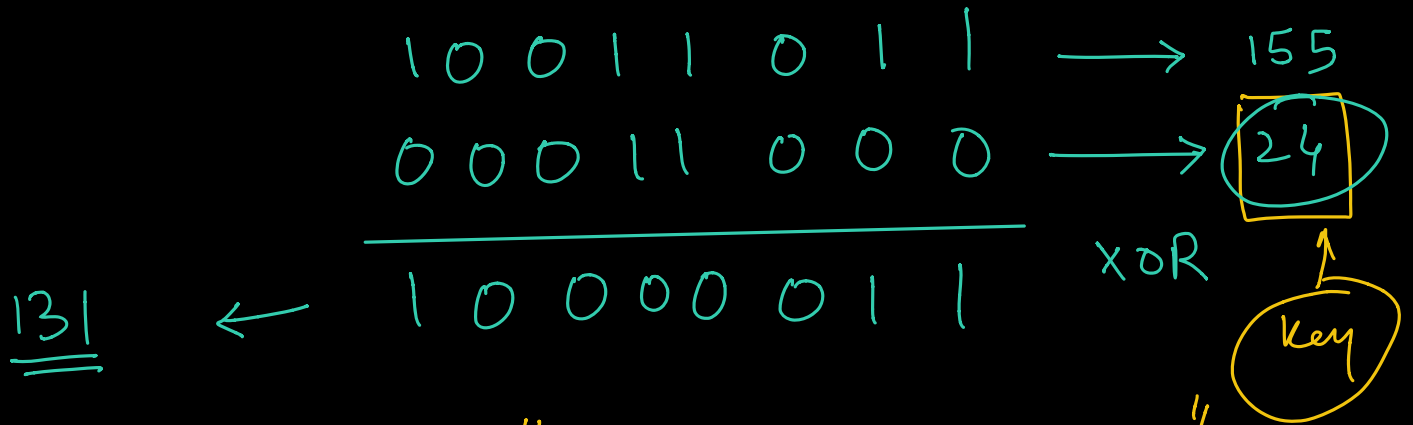
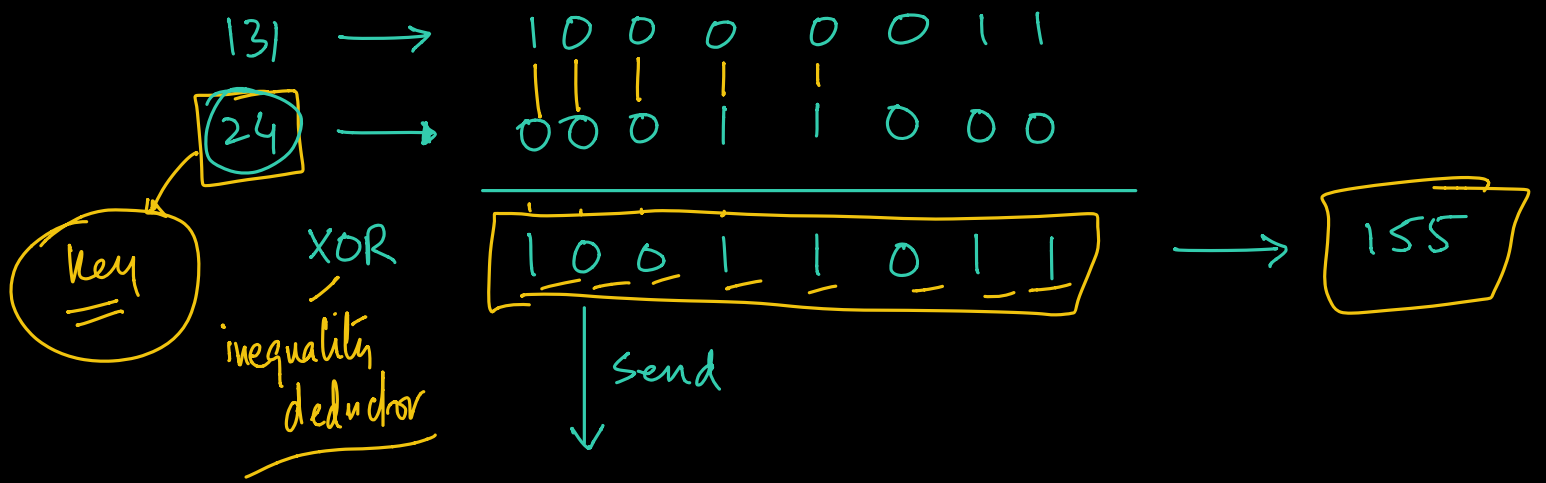
most commonly occurring letter: E

·	·	·	·	·
·	·	·	·	·
·	·	·	·	·

carefully ←

$$\begin{array}{c} \downarrow \downarrow \\ \text{e c t g w n n a} \\ \text{—} \end{array}$$

$$\begin{array}{c} e \\ \downarrow \\ g \end{array} \quad \boxed{+2}$$



"Shared key"

"Symmetric key"

"Private key"

8-bit key
keylength

crypto

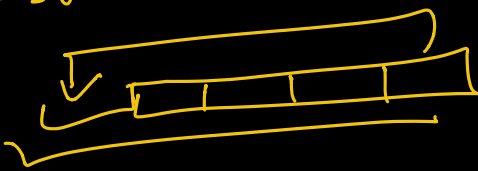
A
A XOR $\underbrace{K \text{ XOR } K}_0$

$\begin{array}{r} 1 \\ 0 \\ \hline 1 \end{array}$
 $\begin{array}{r} 0 \\ 0 \\ \hline 0 \end{array}$
 $\begin{array}{r} 00 \\ 00 \\ \hline 00 \end{array}$

$\begin{array}{r} 10 \\ 10 \\ \hline 00 \end{array}$
 $\begin{array}{r} 01 \\ 01 \\ \hline 00 \end{array}$

— Cryptanalysis

$\begin{array}{ccccccc} \text{---} & \text{---} & \text{---} & \text{---} & \text{---} & \text{---} & \text{---} \\ \text{---} & \text{---} & \text{---} & \text{---} & \text{---} & \text{---} & \text{---} \end{array}$
 8-bit



Public key Crypto:

message:

2

Bob's

$$p = 2$$

$$q = 7$$

} prime

$$n = p * q = 14$$

$$\phi = (p-1)(q-1) = (1)(6) = \underline{6}$$

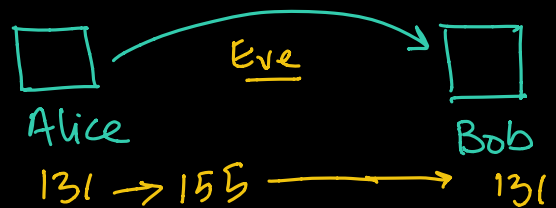
Need to find e and d

e need to be co-prime with r
"No shared factors"

$$e = \underline{5}$$

↳ public key

Publish: $(\overset{e}{5}, \overset{n}{14})$



6:	1, 2, 3, 6
x 2:	1, 2
x 3:	1, 3
x 4:	1, 2, 4
✓ 5:	<u>1, 5</u>

$$\approx \text{GCD}(e, r) = 1$$

Alice: $\boxed{2}$

$$\text{msg} \rightarrow 2 \xrightarrow{5 \rightarrow e} \text{mod } 14 \quad n \quad = \quad (6^5 \text{ mod } 14 = \underline{6})$$

$\boxed{4}$ → sends to Bob
 → enc

Bob:
 computes d such that:

$$e \cdot d \text{ mod } \textcircled{r} = 1$$

$$\begin{aligned} e &= 5 \\ r &= 6 \\ n &= 14 \end{aligned}$$

$$\boxed{d = 11}$$

5: $d \rightarrow$

$\textcircled{1}$	$\textcircled{2}$	3	4	$\textcircled{5}$...	$\textcircled{11}$
5	$\textcircled{10}$	15	20	25		55
$\textcircled{5}$	$\textcircled{4}$	3	2	<u>1</u>		<u>1</u>

To get message

$$\textcircled{\text{enc}}^{\textcircled{d}} \text{ mod } \textcircled{n}$$

$$\begin{aligned} \textcircled{4} \text{ mod } 14 &= \textcircled{2} \\ \textcircled{6} \text{ mod } 14 &= \underline{6} \end{aligned}$$

Reason:

$$\text{enc}^d \bmod n$$

$$\left(\text{msg}^{\cancel{e}} \right)^{\cancel{d}} \bmod n$$

$$\text{msg} \bmod n$$

(need the conditions to hold for this cancellation — Fermat's little theorem etc.)

This is the RSA algorithm: the backbone of all secure communication on the internet!!

Rivest
↑
Shamir
Adelman

"prime factorization"

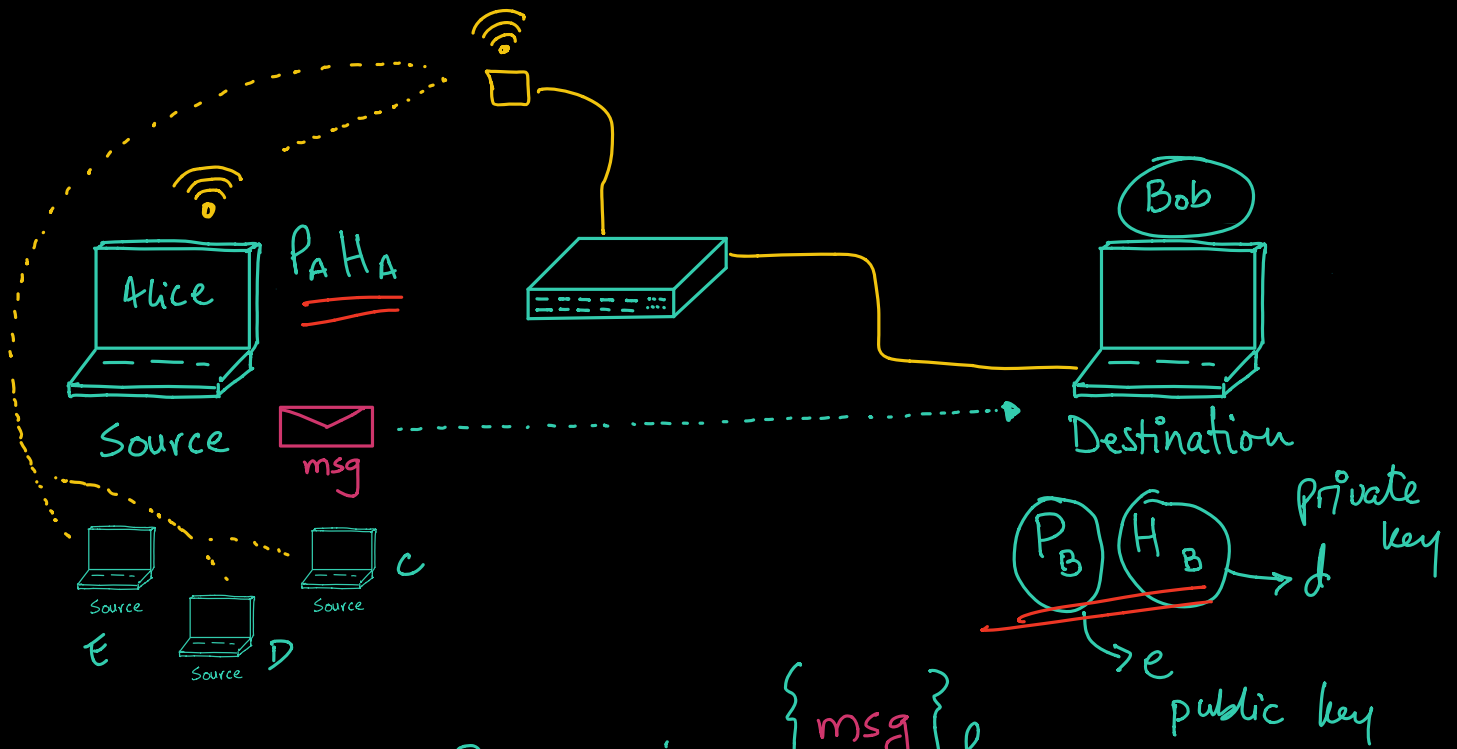
— Why is it hard to break?

- p and q are huge!
- easy to compute n from p and q
- ⊗ — very hard to go back.
- need p and q to compute d from e and n

$$\text{e} \cdot \text{d} \bmod r = 1$$
$$r = \underline{(p-1)(q-1)}$$

Demo!

Asymmetric key crypto



① A wants to send B a message

B decrypts

$$\{msg\}_{P_B}$$

$$\{\{msg\}_{P_B}\}_{H_B}$$

$\leadsto msg$

② ~~B~~^A wants to sign a message

Anyone wants to verify

$$\{msg\}_{H_B/A}$$

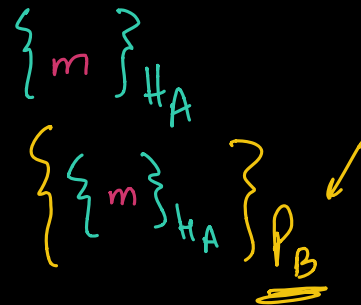
$$\{\{msg\}_{H_B/A}\}_{P_B/A}$$

$\leadsto msg$

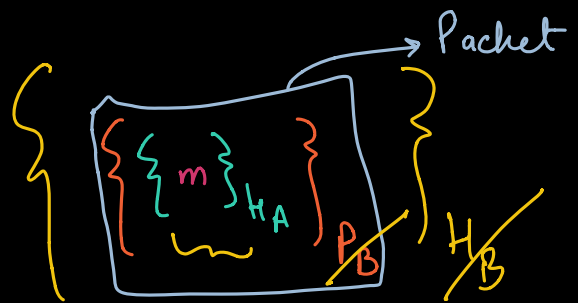
③ A wants to send B a message
But B needs verification that
message is from A

a) Sign message:

b) Encrypt for B:



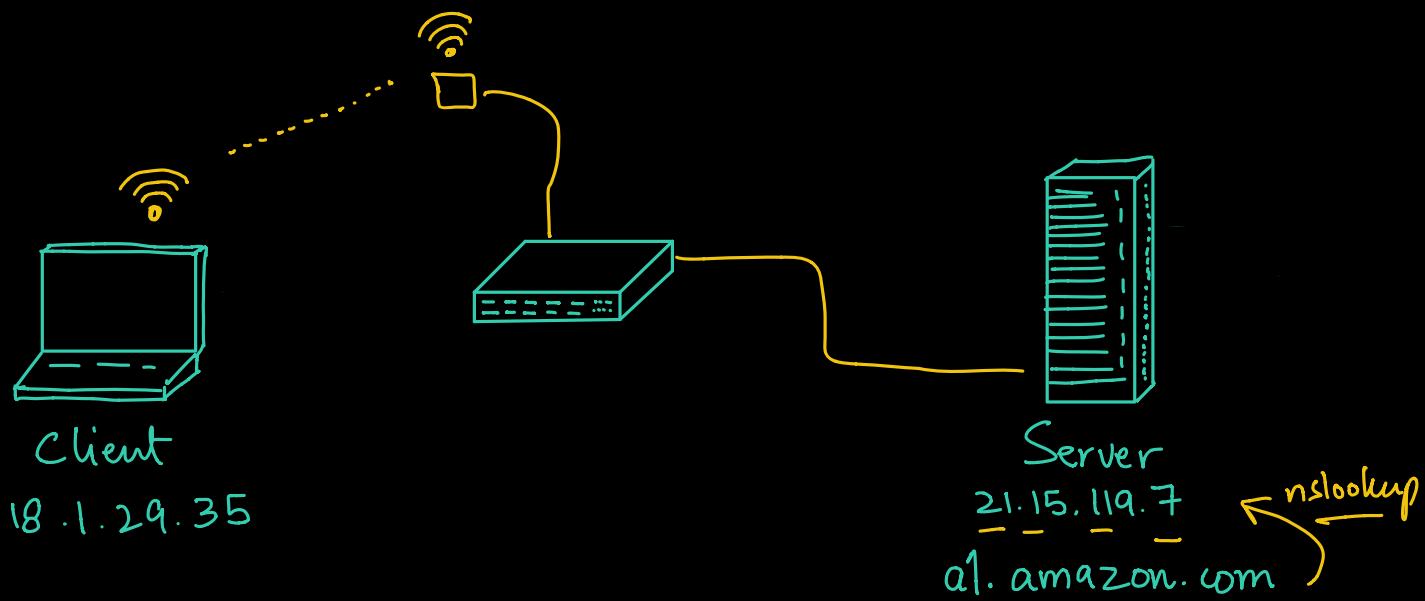
c) B opens the
package
Decryption



d) Verify the
signature



m

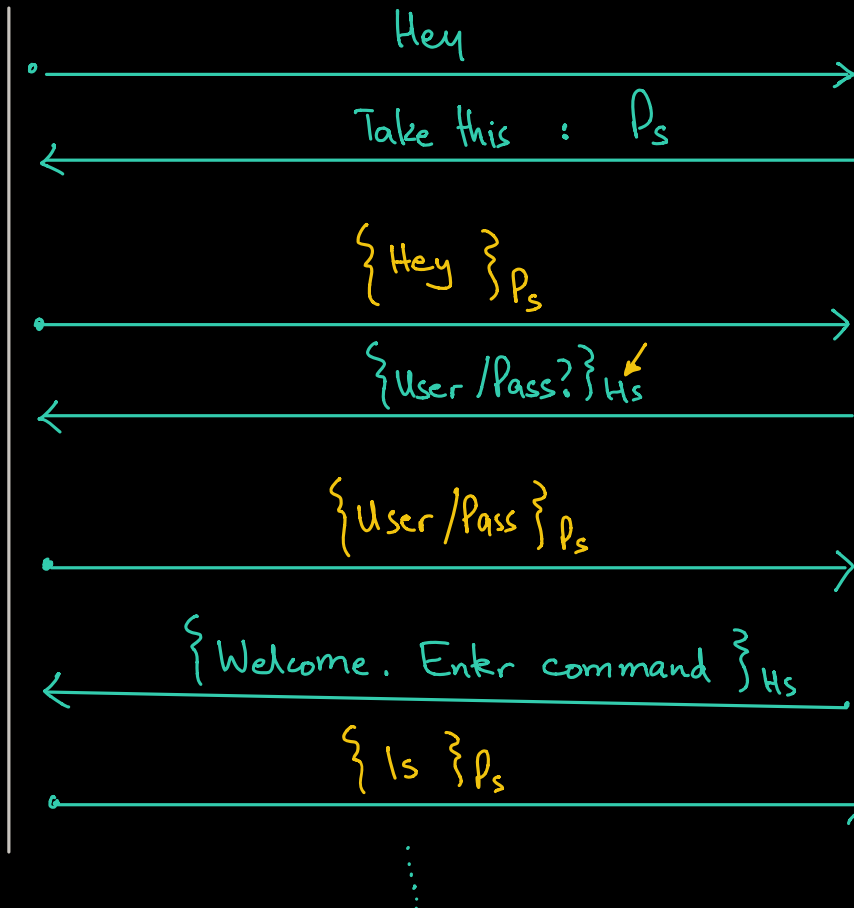


P_c, H_c

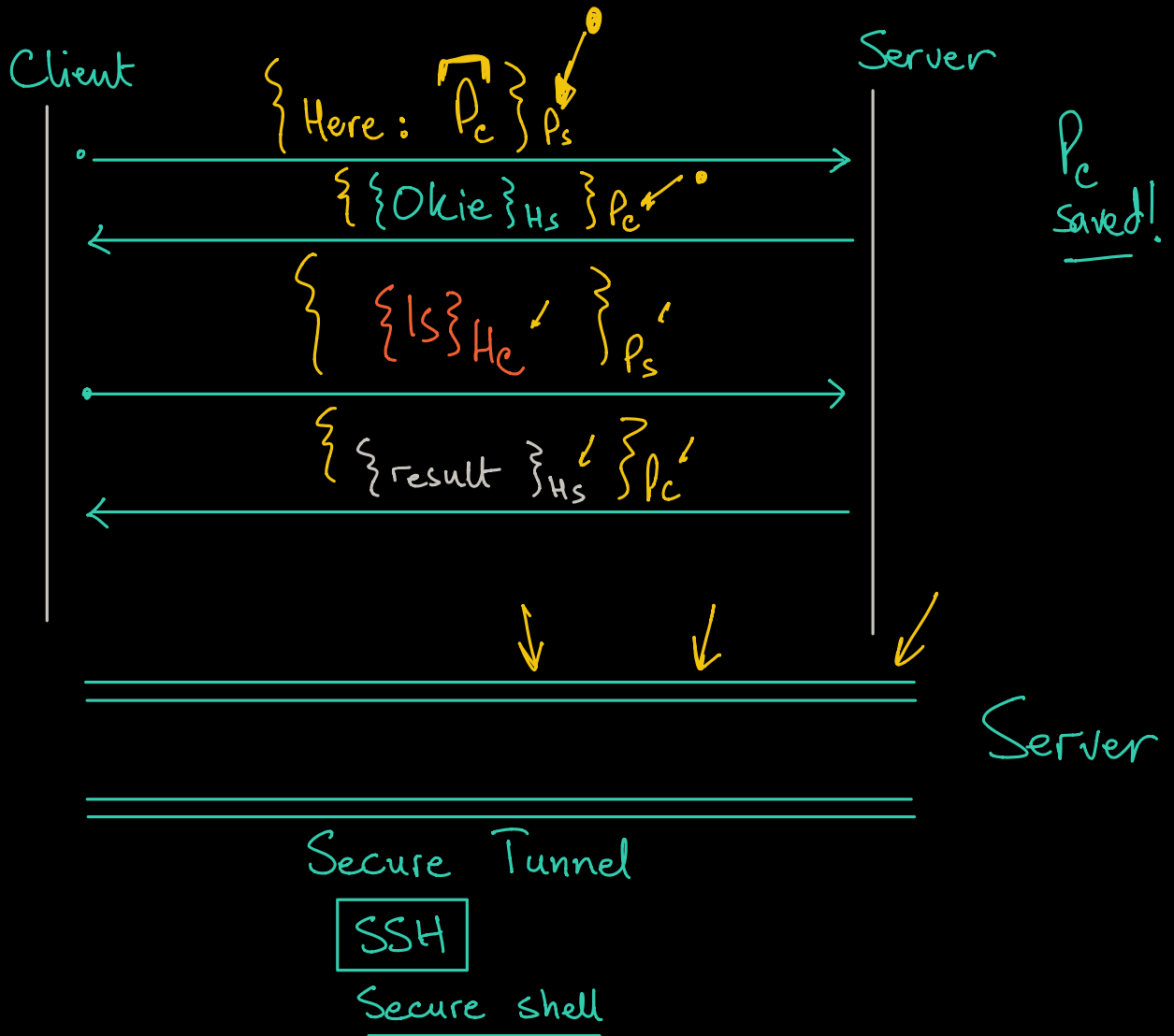
P_s, H_s

Client

Server



Verifying the client



→ Python notebooks

Symmetric ✓
Asymmetric ✓