Lecture 6 Relational Database Design

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Relational Database Design

- First Normal Form
- Pitfalls in Relational Database Design
- Functional Dependencies
- Decomposition
- Boyce-Codd Normal Form
- Third Normal Form
- Multi-valued Dependencies and Fourth Normal Form
- Overall Database Design Process

First Normal Form (第一范式)

- Domain is atomic if its elements are considered to be indivisible units
- A relational schema R is in 1NF if the domains of all attributes of R are atomic
- Non-atomic values complicate storage and encourage redundant storage of data
  - E.g. Set of accounts stored with each customer, and set of owners stored with each account
  - We assume all relations are in first normal form

First Normal Form (Contd.)

- Atomicity is actually a property of how the elements of the domain are used
  - E.g. Strings would normally be considered indivisible
    - Suppose that students are given roll numbers which are strings of the form *12307130220*
  - If the first two characters are extracted to find the year of enrollment, the domain of roll numbers is not atomic
  - Doing so is a bad idea: leads to encoding of information in application program rather than in the database

Relational Database Design

- RDB design is to find a "good" collection of schemas. A bad design may lead to
  - Repetition of Information
  - Inability to represent certain information
- Design Goals:
  - Avoid redundant data
  - Ensure that relationships among attributes are represented
  - Facilitate the checking of updates for violation of database integrity constraints

Example

- Consider the relation schema:

  Lending-schema = (branch-name, branch-city, assets, customer-name, loan-number, amount)

- Redundancy:

branch-name	*branch-city*	*assets*	*customer-name*	*loan-number*	*amount*
Downtown	Brooklyn	9000000	Jones	L-17	1000
Redwood	Palo Alto	2100000	Smith	L-23	2000
Perryridge	Horseneck	1700000	Hayes	L-15	1500
Downtown	Brooklyn	9000000	Jackson	L-14	1500

  - Data for *branch-name*, *branch-city*, assets are repeated for each loan that a branch makes
  - Wastes space
  - Complicates updating, introducing possibility of inconsistency of *assets* value

- Null values
  - Cannot store information about a branch if no loans exist
  - Can use null values, but they are difficult to handle.

Decomposition

- Decompose the relation schema *Lending-schema* into:

  Branch-schema = (branch-name, branch-city,assets)

  Loan-info-schema = (customer-name, loan-number, branch-name, amount)

- All attributes of an original schema *(R)* must appear in the decomposition (R_1, R_2):

  $$R = R_1 \cup R_2$$

- Lossless-join decomposition:
  For all possible relations *r* on schema *R*

  $$r = \Pi_{R1}(r) \bowtie \Pi_{R2}(r)$$

Example of Non Lossless-Join Decomposition

- Decomposition of $R = (A, B)$

  $R_1 = (A)$ $R_2 = (B)$

A	B
α	1
α	2
β	1

r

A
α
β

$\Pi_A(r)$

B
1
2

$\Pi_B(r)$

Π_A (r) $\bowtie$ Π_B (r)

A	B
α	1
α	2
β	1
β	2

Goal — Devise a Theory for the Following

- Decide whether a particular relation R is in "good" form
- In the case that a relation R is not in "good" form, decompose it into a set of relations $\{R_1, R_2, ..., R_n\}$ such that
  - each relation is in good form
  - the decomposition is a lossless-join decomposition
  - the decomposition is dependency-preservation
- Our theory is based on:
  - functional dependencies
  - Multi-valued dependencies

Functional Dependencies (函数依赖)

- Constraints on the set of legal relations
- Require that the value for a certain set of attributes determines uniquely the value for another set of attributes
- A functional dependency is a generalization of the notion of a *key*

Functional Dependencies (Cont.)

- Let R be a relation schema, $\alpha \subseteq R$ and $\beta \subseteq R$
- The functional dependency $\alpha \rightarrow \beta$ holds on R iff for **ANY** legal relations $r(R)$, whenever any two tuples t_1 and t_2 of r agree on the attributes α, they also agree on the attributes β. That is, $t_1[\alpha] = t_2[\alpha] \Rightarrow t_1[\beta] = t_2[\beta]$
- Example: Consider $r(A,B)$ with the following instance of r.

1	4
1	5
3	7

- On this instance, $A \rightarrow B$ does **NOT** hold, but $B \rightarrow A$ does hold.

Functional Dependencies (Cont.)

- K is a superkey for relation schema R iff $K \rightarrow R$
- K is a candidate key for R iff
  - $K \rightarrow R$, and
  - no $\alpha \subset K$, $\alpha \rightarrow R$
- FDs allow us to express constraints that cannot be expressed using superkeys. Consider the schema:

  Loan-info-schema = (customer-name, loan-number, branch-name, amount).

  We expect this set of FDs to hold:

  loan-number $\rightarrow$ *amount*
  loan-number $\rightarrow$ *branch-name*

  but would not expect the following to hold:

  loan-number $\rightarrow$ *customer-name*

Use of Functional Dependencies

- We use functional dependencies to:
  - test relations to see if they are legal under a given set of functional dependencies.
  - specify constraints on the set of legal relations
- Note: A specific instance of a relation schema may satisfy a functional dependency even if the functional dependency does not holds on all legal instances.
  - For example, a specific instance of *Loan-schema* may, by chance, satisfy

    loan-number $\rightarrow$ *customer-name.*

Functional Dependencies (Cont.)

- A functional dependency is **trivial** if it is satisfied by all instances of a relation
  - E.g.
    - *customer-name, loan-number* $\rightarrow$ *customer-name*
    - *customer-name* $\rightarrow$ *customer-name*
  - In general, $\alpha \rightarrow \beta$ is trivial if $\beta \subseteq \alpha$

- β is **fully dependent** on α, if there is no proper subset α' of α such that $\alpha' \rightarrow \beta$. *Otherwise,* β is **partially dependent** on α.

Closure of a Set of Functional Dependencies

- Given a set *F* of FDs, there are certain other FDs that are **logically implied** (逻辑蕴涵) by *F*
  - E.g. If $A \rightarrow B$ and $B \rightarrow C$, then we can infer that $A \rightarrow C$
  - The set of all FDs logically implied by *F* is the ***closure*** of *F*
- We denote the ***closure*** of *F* by F^+
- We can find all of F^+ by applying *Armstrong's Axioms*
  - if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$ **(reflexivity: 自反律)**
  - if $\alpha \rightarrow \beta$, then $\gamma\ \alpha \rightarrow \gamma\ \beta$ **(augmentation: 增广律)**
  - if $\alpha \rightarrow \beta$, and $\beta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$ **(transitivity: 传递律)**
- These rules are (正确且完备)
  - sound (generate only FDs that actually hold) and
  - complete (generate all FDs that hold)

Example

- $R = (A, B, C, G, H, I)$
  $F = \{ A \rightarrow B, A \rightarrow C, CG \rightarrow H, CG \rightarrow I, B \rightarrow H \}$
- some members of F^+
  - $A \rightarrow H$
    - by transitivity from $A \rightarrow B$ and $B \rightarrow H$
  - $AG \rightarrow I$
    - by augmenting $A \rightarrow C$ with G, to get $AG \rightarrow CG$ and then transitivity with $CG \rightarrow I$
  - $CG \rightarrow HI$
    - from $CG \rightarrow H$ and $CG \rightarrow I$: "union rule" can be inferred from
      - definition of functional dependencies, or
      - Augmentation of $CG \rightarrow I$ to infer $CG \rightarrow CGI$, augmentation of $CG \rightarrow H$ to infer $CGI \rightarrow HI$, and then transitivity

Procedure for Computing F^+

- To compute the closure of a set of FDs F:

$F^+ = F$

repeat

  for each FD f in F^+

    apply reflexivity and augmentation rules on f

    add the resulting FDs to F^+

  for each pair of FDs $f_1$and f_2 in F^+

    if f_1 and f_2 can be combined using transitivity

      then add the resulting FD to F^+

until F^+ does not change any further

NOTE: We will see an alternative procedure for this task later

Closure of Functional Dependencies (Cont.)

- We can further simplify manual computation of F^+ by using the following additional rules.
  - If $\alpha \rightarrow \beta$ holds and $\alpha \rightarrow \gamma$ holds, then $\alpha \rightarrow \beta\gamma$ holds **(union：合并规则)**
  - If $\alpha \rightarrow \beta\gamma$ holds, then $\alpha \rightarrow \beta$ holds and $\alpha \rightarrow \gamma$ holds **(decomposition：分解规则)**
  - If $\alpha \rightarrow \beta$ holds and $\gamma\beta \rightarrow \delta$ holds, then $\alpha\gamma \rightarrow \delta$ holds **(pseudotransitivity：伪传递规则)**

The above rules can be inferred from Armstrong's axioms

Closure of Attribute Sets

- Given a set of attributes α, define the *closure* of α under F (denoted by α^+) as the set of attributes that are functionally determined by α under F:

  $\alpha \rightarrow \beta$ **is in** $F^+ \Leftrightarrow \beta \subseteq \alpha^+$

- Algorithm to compute α^+ :

```
result := α;
while (changes to result) do
    for each β→γ in F do
      begin
        if β⊆result then result := result ∪ γ
      end
```

Example of Attribute Set Closure

- $R = (A, B, C, G, H, I)$
- $F = \{ A \rightarrow B, A \rightarrow C, CG \rightarrow H, CG \rightarrow I, B \rightarrow H \}$
- $(AG)^+$
  1. $result = AG$
  2. $result = ABCG$ ($A \rightarrow C$ and $A \rightarrow B$)
  3. $result = ABCGH$ ($CG \rightarrow H$ and $CG \subseteq AGBC$)
  4. $result = ABCGHI$ ($CG \rightarrow I$ and $CG \subseteq AGBCH$)
- Is AG a candidate key?
  1. Is AG a super key?
     1. Does $AG \rightarrow R$? == Is $(AG)^+ \supseteq R$
  2. Is any subset of AG a superkey?
     1. Does $A \rightarrow R$? == Is $(A)^+ \supseteq R$
     2. Does $G \rightarrow R$? == Is $(G)^+ \supseteq R$

Uses of Attribute Closure

There are several uses of the attribute closure:

- Testing for superkey
- Testing functional dependencies
  - To check if a functional dependency $\alpha \rightarrow \beta$ holds (or, in other words, is in F^+), just check if $\beta \subseteq \alpha^+$.
  - That is, we compute α^+ by using attribute closure, and then check if it contains β.
  - Is a simple and cheap test, and very useful
- Computing closure of F
  - For each $\gamma \subseteq R$, we find the closure γ^+, and for each $S \subseteq \gamma^+$, we output a functional dependency $\gamma \rightarrow S$.

Canonical Cover (最小覆盖)

- Sets of FDs may have redundant FDs that can be inferred from the others
  - Eg: $A \rightarrow C$ is redundant in: $\{A \rightarrow B, B \rightarrow C, A \rightarrow C\}$
  - Parts of a FD may be redundant
    - E.g. on RHS: $\{A \rightarrow B, B \rightarrow C, A \rightarrow CD\}$ can be simplified to $\{A \rightarrow B, B \rightarrow C, A \rightarrow D\}$
    - E.g. on LHS: $\{A \rightarrow B, B \rightarrow C, AC \rightarrow D\}$ can be simplified to $\{A \rightarrow B, B \rightarrow C, A \rightarrow D\}$
- Intuitively, **a canonical cover of F is a "minimal" set of FDs equivalent to F**, having no redundant FDs or redundant parts of FDs

Extraneous Attributes (额外属性)

- Consider a set F of FDs and the FD $\alpha \rightarrow \beta$ in F
  - Attribute A is extraneous in α if $A \in \alpha$ and F logically implies $(F - \{\alpha \rightarrow \beta\}) \cup \{(\alpha - A) \rightarrow \beta\}$.
  - Attribute A is extraneous in β if $A \in \beta$ and the set of FDs $(F - \{\alpha \rightarrow \beta\}) \cup \{\alpha \rightarrow (\beta - A)\}$ logically implies F.
- *Note:* implication in the opposite direction is trivial in each of the cases above
- Example: Given $F = \{A \rightarrow C, AB \rightarrow C\}$
  - B is extraneous in $AB \rightarrow C$ because $\{A \rightarrow C, AB \rightarrow C\}$ logically implies $A \rightarrow C$ (i.e. the result of dropping B from $AB \rightarrow C$).
- Example: Given $F = \{A \rightarrow C, AB \rightarrow CD\}$
  - C is extraneous in $AB \rightarrow CD$ since $AB \rightarrow CD$ can be inferred even after deleting C in $AB \rightarrow CD$

Testing if an Attribute is Extraneous

- Consider a set F of FDs and $\alpha \rightarrow \beta$ in F.
- To test if attribute $A \in \alpha$ is extraneous in α
  1. compute $(\{\alpha\} - A)^+$ using the dependencies in F
  2. check that $(\{\alpha\} - A)^+$ contains β; if it does, A is extraneous
- To test if attribute $A \in \beta$ is extraneous in β
  1. compute α^+ using only the dependencies in
     $$F' = (F - \{\alpha \rightarrow \beta\}) \cup \{\alpha \rightarrow (\beta - A)\},$$
  2. check that α^+ contains A; if it does, A is extraneous

Canonical Cover

- A *canonical cover* for F is a set of FDs F_c such that
  - F logically implies all dependencies in F_c, and
  - F_c logically implies all dependencies in F, and
  - No FD in F_c contains an extraneous attribute, and
  - Each left side of FD in F_c is unique.
- To compute a canonical cover for F :

repeat

Use the union rule to replace any dependencies in F

$\alpha_1 \rightarrow \beta_1$ and $\alpha_1 \rightarrow \beta_2$ with $\alpha_1 \rightarrow \beta_1\beta_2$

Find a FD $\alpha \rightarrow \beta$ with an extraneous attr. either in α or in β

If an extraneous attr. is found, delete it from $\alpha \rightarrow \beta$

until F does not change

Example of Computing a Canonical Cover

- $R = (A, B, C)$
  $F = \{A \rightarrow BC, B \rightarrow C, A \rightarrow B, AB \rightarrow C\}$
- Combine $A \rightarrow BC$ and $A \rightarrow B$ into $A \rightarrow BC$
  - Set is now $\{A \rightarrow BC, B \rightarrow C, AB \rightarrow C\}$
- A is extraneous in $AB \rightarrow C$
  - Check if the result of deleting A from $AB \rightarrow C$ is implied by the other dependencies
  - Set is now $\{A \rightarrow BC, B \rightarrow C\}$
- C is extraneous in $A \rightarrow BC$
  - Check if $A \rightarrow C$ is logically implied by $A \rightarrow B$ and the other dependencies
- The canonical cover is: $\{A \rightarrow B, B \rightarrow C\}$

Goals of Normalization

- Decide whether a particular relation R is in "good" form
- In the case that a relation R is not in "good" form, decompose it into a set of relations $\{R_1, R_2, ..., R_n\}$ such that
  - each relation is in good form
  - the decomposition is a lossless-join decomposition
- Our theory is based on:
  - functional dependencies
  - Multi-valued dependencies

Decomposition

- Decompose the relation schema *Lending-schema* into:

  Branch-schema = (branch-name, branch-city,assets)

  Loan-info-schema = (customer-name, loan-number, branch-name, amount)

- All attributes of an original schema (*R*) must appear in the decomposition (R_1, R_2):

  $$R = R_1 \cup R_2$$

- Lossless-join decomposition.
  For all possible relations *r* on schema *R*

  $$r = \Pi_{R1}(r) \bowtie \Pi_{R2}(r)$$

- A decomposition of R into R_1 and R_2 is lossless join iff at least one of the following dependencies is in F^+:
  - $R_1 \cap R_2 \rightarrow R_1$
  - $R_1 \cap R_2 \rightarrow R_2$

Normalization Using Functional Dependencies

- When we decompose a relation schema R with a set of FDs F into $R_1, R_2, .., R_n$ we want
  - Lossless-join decomposition: Otherwise decomposition would result in information loss.
  - No redundancy: The relations R_i preferably should be in either BCNF or 3NF.
  - Dependency preservation: Let F_i be the subset of dependencies F^+ that include only attributes in R_i.
    - $(F_1 \cup F_2 \cup \ldots \cup F_n)^+ = F^+$
    - Otherwise, checking updates for violation of FDs may require computing joins, which is expensive.

Example

- $R = (A, B, C)$
  $F = \{A \rightarrow B, B \rightarrow C\}$
  - Can be decomposed in two different ways
- $R_1 = (A, B), \quad R_2 = (B, C)$
  - Lossless-join decomposition: $R_1 \cap R_2 = \{B\}$ and $B \rightarrow BC$
  - Dependency preserving
- $R_1 = (A, B), \quad R_2 = (A, C)$
  - Lossless-join decomposition: $R_1 \cap R_2 = \{A\}$ and $A \rightarrow AB$
  - Not dependency preserving
    (cannot check $B \rightarrow C$ without computing $R_1 \bowtie R_2$)

Testing for Dependency Preservation

- To check if $\alpha \rightarrow \beta$ is preserved in a decomposition of R into $R_1, R_2, \ldots, R_n$ we apply the following simplified test
  - *result* = α
    while (changes to *result*) do
      for each R_i in the decomposition
        $t = (result \cap R_i)^+ \cap R_i$ (Note: here the closure is under the FD set F)
        $result = result \cup t$
  - If *result* contains all attributes in β, then the functional dependency $\alpha \rightarrow \beta$ is preserved.
- We apply the test on all dependencies in F to check if a decomposition is dependency preserving
- This procedure takes polynomial time, instead of the exponential time required to compute F^+ and $(F_1 \cup F_2 \cup \ldots \cup F_n)^+$

Boyce-Codd Normal Form

A relation schema R is in BCNF with respect to a set F of functional dependencies if for all functional dependencies in F^+ of the form $\alpha \rightarrow \beta$, where $\alpha \subseteq R$ and $\beta \subseteq R$, at least one of the following holds:

- $\alpha \rightarrow \beta$ is trivial (i.e., $\beta \subseteq \alpha$)
- α is a superkey for R

Example

- $R = (A, B, C)$
  $F = \{A \rightarrow B, B \rightarrow C\}$
  Key = $\{A\}$
- R is not in BCNF
- Decomposition $R_1 = (A, B)$, $R_2 = (B, C)$
  - R_1 and R_2 in BCNF
  - Lossless-join decomposition
  - Dependency preserving

Testing for BCNF

- To check if a non-trivial dependency $\alpha \rightarrow \beta$ causes a violation of BCNF
  1. compute α^+ (the attribute closure of α), and
  2. verify that it includes all attributes of *R*, that is, it is a superkey of *R*.
- Simplified test: To check if a relation schema *R* is in BCNF, **it suffices to check only the FDs *F*** for violation of BCNF, rather than checking all dependencies in F^+.
  - If none of the dependencies in *F* causes a violation of BCNF, then none of the dependencies in F^+ will cause a violation of BCNF either

Testing for BCNF (Cont.)

- However, using only F is incorrect when testing a relation in a decomposition of R
  - E.g. Consider R (A, B, C, D), with F = {$A \rightarrow B$, $B \rightarrow C$}
    - Decompose R into $R_1(A,B)$ and $R_2(A,C,D)$
    - Neither of the dependencies in F contain only attributes from (A,C,D) so we might be mislead into thinking R_2 satisfies BCNF.
    - In fact, dependency $A \rightarrow C$ in F^+ shows R_2 is not in BCNF.

BCNF Decomposition Algorithm

result := {*R*};
done := false;
compute F^+;
while (not *done*) **do**
  if (there is a schema R_i in *result* that is not in BCNF)
    then begin
      let $\alpha \rightarrow \beta$ be a nontrivial FD that holds on R_i
        such that $\alpha \rightarrow R_i$ is not in F^+,
        and $\alpha \cap \beta = \varnothing$;
        result := (*result* – R_i) $\cup$ ($R_i - \beta$) $\cup$ (α, β);
      end
    else *done* := **true**;

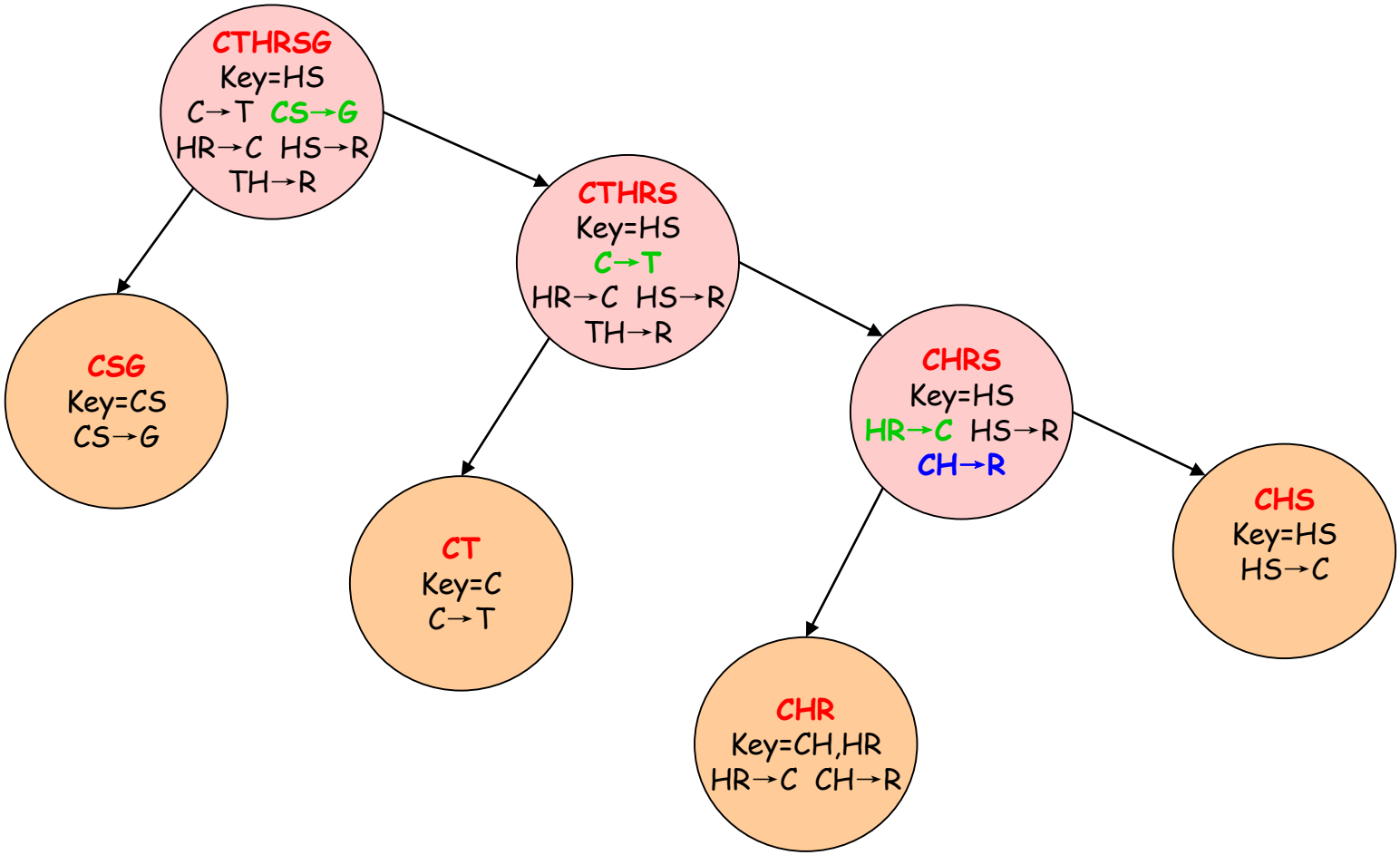
Note: each R_i is in BCNF, and decomposition is lossless-join.

Example

Let us consider the relation scheme ***CTHRSG***, where C=course, T=teacher, H=hour, R=room, S=student, and G=grade. The functional dependencies ***F*** we assume are:

- ***C→T*** each course has one teacher
- ***HR→C*** only one course can meet in a room at one time
- ***HT→R*** a teacher can be in only one room at one time
- ***CS→G*** each student has one grade in each course
- ***HS→R*** a student can be in only one room at one time

Decomposition Tree



Example of BCNF Decomposition

- *R = (branch-name, branch-city, assets,*
  customer-name, loan-number, amount)
  F = {branch-name → assets branch-city,
  loan-number → amount branch-name}
  Key = *{loan-number, customer-name}*
- Decomposition
  - R_1 = *(branch-name, branch-city, assets)*
  - R_2 = *(branch-name, customer-name, loan-number, amount)*
  - R_3 = *(branch-name, loan-number, amount)*
  - R_4 = *(customer-name, loan-number)*
- Final decomposition
  R_1, R_3, R_4

Testing Decomposition for BCNF

- To check if a relation R_i in a decomposition of R is in BCNF,
  - Either test R_i for BCNF w.r.t. the restriction of F to R_i (that is, all FDs in F^+ that contain only attributes from R_i)
  - or use the original set of dependencies F that hold on R, but with the following test:
    - for every set of attributes $\alpha \subseteq R_i$, check that α^+ either includes no attri. of $R_i - \alpha$, or includes all attributes of R_i.
    - If the condition is violated by some α, then the FD
      $\alpha \rightarrow (\alpha^+ - \alpha) \cap R_i$
      can be shown to hold on R_i, and R_i violates BCNF.
    - We use above dependency to decompose R_i

BCNF and Dependency Preservation

It is not always possible to get a BCNF decomposition that is dependency preserving

- $R = (J, K, L)$
  $F = \{JK \rightarrow L, L \rightarrow K\}$
  Two candidate keys = JK and JL
- R is not in BCNF
- Any decomposition of R will fail to preserve

$$JK \rightarrow L$$

Third Normal Form: Motivation

- There are some situations where
  - BCNF is not dependency preserving, and
  - efficient checking for FD violation on updates is important
- Solution: define a weaker normal form, called Third Normal Form.
  - Allows some redundancy
  - But FDs can be checked on individual relations without computing a join
  - There is always a lossless-join, dependency-preserving decomposition into 3NF

Third Normal Form

- A relation schema R is in 3NF if for all:

  $$\alpha \rightarrow \beta \text{ in } F^+$$

  at least one of the following holds:
  - $\alpha \rightarrow \beta$ is trivial (i.e., $\beta \in \alpha$)
  - α is a superkey for R
  - Each attr. A in $\beta - \alpha$ is contained in a candidate key for R.

    (*NOTE: each attribute may be in a different candidate key*)
- If a relation is in BCNF it is in 3NF (since in BCNF one of the first two conditions above must hold)
- Third condition is a minimal relaxation of BCNF to ensure dependency preservation

3NF (Cont.)

- Example
  - $R = (J, K, L)$
    $F = \{JK \rightarrow L, L \rightarrow K\}$
  - Two candidate keys: JK and JL
  - R is in 3NF

    $JK \rightarrow L$ JK is a superkey
    $L \rightarrow K$ K is contained in a candidate key
  - BCNF decomposition has (JL) and (LK)
    - Testing for $JK \rightarrow L$ requires a join
- There is some redundancy in this schema
- Equivalent to example in the book:

  Banker-schema = (branch-name, customer-name, banker-name)

  banker-name $\rightarrow$ branch name

  branch name customer-name $\rightarrow$ banker-name

Testing for 3NF

- Optimization: Need to check only FDs in F
- Use attribute closure to check for each dependency $\alpha \rightarrow \beta$, if α is a superkey.
- If α is not a superkey, we have to verify if each attr. in β is contained in a candidate key of R
  - this test is rather more expensive, since it involve finding candidate keys
  - testing for 3NF has been shown to be NP-hard
  - Interestingly, decomposition into 3NF can be done in polynomial time

3NF Decomposition Algorithm

Let F_c be a canonical cover for F;
$i := 0$;
for each FD $\alpha \rightarrow \beta$ in F_c **do**
 if none of the schemas R_j, $1 \le j \le i$ contains α, β
 then begin
 $i := i + 1$;
 $R_i := \alpha \beta$
 end
if none of the schemas R_j, $1 \le j \le i$ contains a candidate key for R
 then begin
 $i := i + 1$;
 R_i := any candidate key for R;
 end
return $(R_1, R_2, \ldots, R_i)$

3NF Decomposition Algorithm (Cont.)

- The algorithm ensures:
  - each relation schema R_i is in 3NF
  - decomposition is dependency preserving and lossless-join

Example

- Relation schema:

  Banker-info-schema = (branch-name, customer-name, banker-name, office-number)

- The FDs for this relation schema are:

  banker-name → branch-name office-number
  customer-name branch-name → banker-name

- The key is:

  {customer-name, branch-name}

Applying 3NF to *Banker-info-schema*

- The **for** loop in the algorithm causes us to include the following schemas in our decomposition:

  Banker-office-schema = (banker-name, branch-name, office-number)
  Banker-schema = (customer-name, branch-name, banker-name)

- Since *Banker-schema* contains a candidate key for *Banker-info-schema*, we are done with the decomposition process

Comparison of BCNF and 3NF

- It is always possible to decompose a relation into relations in 3NF and
  - the decomposition is lossless
  - the dependencies are preserved
- It is always possible to decompose a relation into relations in BCNF and
  - the decomposition is lossless
  - it may not be possible to preserve dependencies.

Comparison of BCNF and 3NF (Cont.)

- Example of problems due to redundancy in 3NF
  - $R = (J, K, L)$
    $F = \{JK \rightarrow L, L \rightarrow K\}$

J	L	K
j_1	l_1	k_1
j_2	l_1	k_1
j_3	l_1	k_1
null	l_2	k_2

A schema that is in 3NF but not in BCNF has the problems of

- repetition of information (e.g., the relationship l_1, k_1)
- need to use null values (e.g., to represent the relationship l_2, k_2 where there is no corresponding value for J).

Design Goals

- Goal for a relational database design is:
  - BCNF
  - Lossless join
  - Dependency preservation
- If we cannot achieve this, we accept one of
  - Lack of dependency preservation
  - Redundancy due to use of 3NF

Design Goals (Cont.)

- Interestingly, SQL does not provide a direct way of specifying FDs other than superkeys.

  Can specify FDs using assertions, but they are expensive to test

- Even if we had a dependency preserving decomposition, if we use SQL we can only test efficiently these FDs whose left side is key
  - i.e., using SQL we would not be able to efficiently test a FD whose left hand side is not a key.

Testing for FDs Across Relations

- If decomposition is not dependency preserving, we can have an extra **materialized view** for each dependency $\alpha \rightarrow \beta$ in F_c that is not preserved in the decomposition
- The materialized view is defined as a projection on $\alpha\ \beta$ of the join of the relations in the decomposition
- Many newer database systems support materialized views and database system maintains the view when the relations are updated.
  - No extra coding effort for programmer

Testing for FDs Across Relations (Cont.)

- The functional dependency $\alpha \rightarrow \beta$ is expressed by declaring α as a candidate key on the materialized view
- Checking for candidate key cheaper than checking $\alpha \rightarrow \beta$. DBMS can do candidate key check unique(α) efficiently
- BUT:
  - Space overhead: for storing the materialized view
  - Time overhead: Need to keep materialized view up to date when relations are updated
  - Database system may not support key declarations on materialized views

Multivalued Dependencies

- There are database schemas in BCNF that do not seem to be sufficiently normalized
- Consider a database

  classes(course, teacher, book)

  such that $(c,t,b) \in classes$ means that *t* is qualified to teach *c*, and *b* is a required textbook for *c*
- The database is supposed to list for each course the set of teachers any one of which can be the course's instructor, and the set of books, all of which are required for the course

course	teacher	book
database	Avi	DB Concepts
database	Avi	Ullman
database	Hank	DB Concepts
database	Hank	Ullman
database	Sudarshan	DB Concepts
database	Sudarshan	Ullman
operating systems	Avi	OS Concepts
operating systems	Avi	Shaw
operating systems	Jim	OS Concepts
operating systems	Jim	Shaw

classes

- There are no non-trivial functional dependencies and therefore the relation is in BCNF
- Insertion anomalies – i.e., if Sara is a new teacher that can teach database, two tuples need to be inserted (database, Sara, DB Concepts)
  (database, Sara, Ullman)

Multivalued Dependencies (Cont.)

- Therefore, it is better to decompose *classes* into:

course	*teacher*
database	Avi
database	Hank
database	Sudarshan
operating systems	Avi
operating systems	Jim

teaches

course	*book*
database	DB Concepts
database	Ullman
operating systems	OS Concepts
operating systems	Shaw

text

We shall see that these two relations are in Fourth Normal Form (4NF)

Multivalued Dependencies (MVDs)

- Let R be a relation schema and let $\alpha \subseteq R$ and $\beta \subseteq R$. The *multivalued dependency*

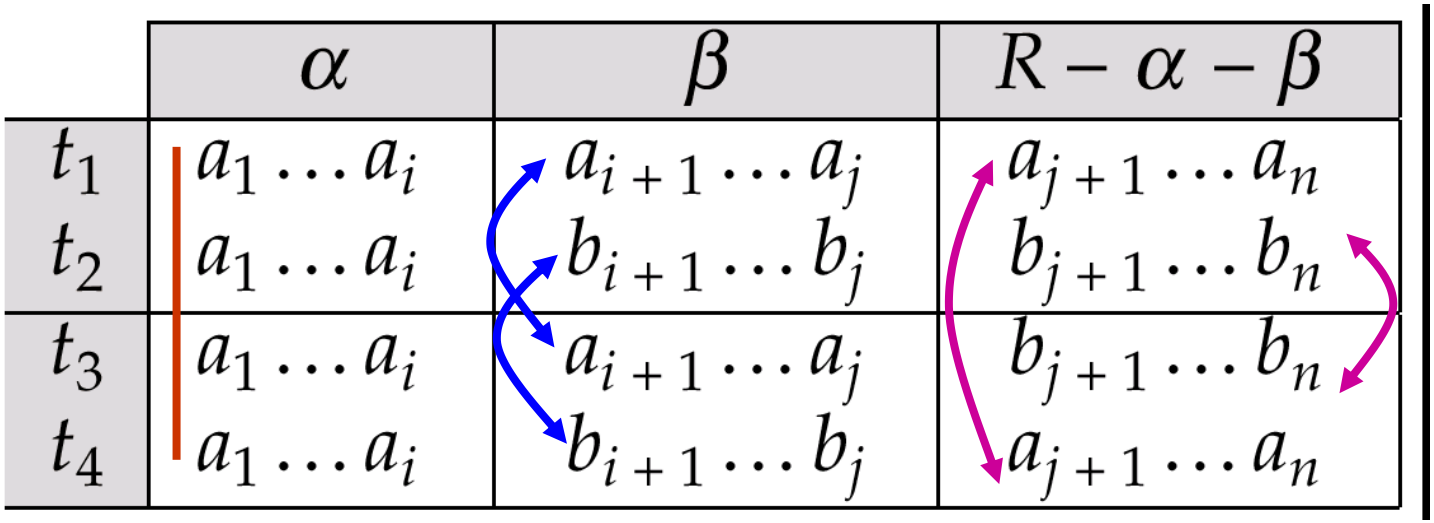
$$\alpha \twoheadrightarrow \beta$$

holds on R if in any legal relation $r(R)$, for all pairs for tuples t_1 and t_2 in r such that $t_1[\alpha] = t_2[\alpha]$, there exist tuples t_3 and t_4 in r such that:

$$t_1[\alpha] = t_2[\alpha] = t_3[\alpha] = t_4[\alpha]$$
$$t_3[\beta] = t_1[\beta]$$
$$t_3[R - \beta] = t_2[R - \beta]$$
$$t_4[\beta] = t_2[\beta]$$
$$t_4[R - \beta] = t_1[R - \beta]$$

MVD (Cont.)

- Tabular representation of $\alpha \twoheadrightarrow \beta$



- In the above example, we have course $\twoheadrightarrow$ teacher and course $\twoheadrightarrow$ book

MVD (Cont.)

- Properties of MVD
  - Symmetry: if X->->Y then X->-> Z, here Z=U-X-Y
  - Transitivity: if X->->Y, Y->->Z, then X->->Z-Y
  - If X->->Y, X->->Z, then X->->YZ
  - If X->->Y, X->->Z, then X->->Y ∩ Z
  - If X->->Y, X->->Z, then X->->Y-Z, X->->Z-Y
  - ……

Example

- Let R be a relation schema with a set of attributes that are partitioned into 3 nonempty subsets.

$$Y, Z, W$$

- We say that $Y \twoheadrightarrow Z$ (Y multi-determines Z) iff for all possible relations $r(R)$

$$< y, z_1, w_1 > \in r \text{ and } < y, z_2, w_2 > \in r \text{ then}$$

$$< y, z_1, w_2 > \in r \text{ and } < y, z_2, w_1 > \in r$$

- Note that since the behavior of Z and W are identical it follows that $Y \twoheadrightarrow Z$ if $Y \twoheadrightarrow W$

Example (Cont.)

- In our example:

$$course \rightarrow\!\!\!\!\rightarrow teacher$$
$$course \rightarrow\!\!\!\!\rightarrow book$$

- The above formal definition is supposed to formalize the notion that given a particular value of Y (*course*) it has associated with it a set of values of Z (*teacher*) and a set of values of W (*book*), and these two sets are in some sense independent of each other
- Note:
  - If $Y \rightarrow Z$ then $Y \rightarrow\!\!\!\!\rightarrow Z$
  - Indeed we have (in above notation) $Z_1 = Z_2$ The claim follows.

Use of Multivalued Dependencies

- We use MVDs in two ways:
  1. To test relations to determine whether they are legal under a given set of FDs and MVDs
  2. To specify constraints on the set of legal relations. We shall concern ourselves with relations that satisfy a given set of FDs and MVDs.
- If a relation *r* fails to satisfy a given MVD, we can construct a relations *r′* that does satisfy the MVD by adding tuples to *r*

Theory of MVDs

- From the definition of multivalued dependency, we can derive the following rule:
  - If $\alpha \rightarrow \beta$, then $\alpha \twoheadrightarrow \beta$; That is, every FD is also a MVD
- The **closure** D^+ of D is the set of all FDs and MVDs logically implied by D.
  - We can compute D^+ from D, using the formal definitions of FDs and MVDs.
  - We can manage with such reasoning for very simple MVDs, which seem to be common in practice
  - For complex MVDs, it is better to reason about sets of dependencies using a system of inference rules (see Appendix C).

Fourth Normal Form

- A relation schema R is in 4NF w.r.t. a set D of FDs and MVDs if for all MVDs in D^+ of the form $\alpha \rightarrow\rightarrow \beta$, where $\alpha \subseteq R$ and $\beta \subseteq R$, at least one of the following hold:
  - $\alpha \rightarrow\rightarrow \beta$ is trivial (i.e., $\beta \subseteq \alpha$ or $\alpha \cup \beta = R$)
  - α is a superkey for schema R
- If a relation is in 4NF it is in BCNF

Restriction of Multivalued Dependencies

- The restriction of D to R_i is the set D_i consisting of
  - All FDs in D^+ that include only attributes of R_i
  - All MVDs of the form

    $\alpha \rightarrow\rightarrow (\beta \cap R_i)$

    where $\alpha \subseteq R_i$ and $\alpha \rightarrow\rightarrow \beta$ is in D^+

4NF Decomposition Algorithm

result: = {*R*};
done := false;
compute D^+;
Let D_i denote the restriction of D^+ to R_i

while (**not** *done*)
  if (there is a schema $\mathbf{R}_i$ in *result* that is not in 4NF) **then**
    begin

    let $\alpha \rightarrow\rightarrow \beta$ be a nontrivial MVD that holds
      on R_i such that $\alpha \rightarrow R_i$ is not in D_i, and $\alpha \cap \beta = \phi$;
    result := (*result* - R_i) $\cup$ (R_i - β) $\cup$ (α, β);
    end
  else *done*:= true;

Note: each R_i is in 4NF, and decomposition is lossless-join

Example

- $R = (A, B, C, G, H, I)$

  $F = \{ A \twoheadrightarrow B$

  $B \twoheadrightarrow HI$

  $CG \twoheadrightarrow H \}$

- R is not in 4NF since $A \twoheadrightarrow B$ and A is not a superkey for R
- Decomposition

  a) $R_1 = (A, B)$ (R_1 is in 4NF)

  b) $R_2 = (A, C, G, H, I)$ (R_2 is not in 4NF)

  c) $R_3 = (C, G, H)$ (R_3 is in 4NF)

  d) $R_4 = (A, C, G, I)$ (R_4 is not in 4NF)

- Since $A \twoheadrightarrow B$ and $B \twoheadrightarrow HI$, $A \twoheadrightarrow HI$, $A \twoheadrightarrow I$

  e) $R_5 = (A, I)$ (R_5 is in 4NF)

  f) $R_6 = (A, C, G)$ (R_6 is in 4NF)

Further Normal Forms

- **Join dependencies** generalize MVDs
  - lead to **project-join normal form** (PJNF) (also called **fifth normal form**)
- A class of even more general constraints, leads to a normal form called **domain-key normal form**
- Problem with these generalized constraints: are hard to reason with, and no set of sound and complete set of inference rules exists
- Hence rarely used

Overall Database Design Process

- We have assumed schema *R* is given
  - *R* could have been generated when converting E-R diagram to a set of tables.
  - *R* could have been a single relation containing *all* attributes that are of interest (called **universal relation**).
  - Normalization breaks *R* into smaller relations
  - *R* could have been the result of some ad hoc design of relations, which we then test/convert to normal form

ER Model and Normalization

- When an E-R diagram is carefully designed, identifying all entities correctly, the tables generated from the E-R diagram should not need further normalization
- However, in a real (imperfect) design there can be FDs from non-key attributes of an entity to other attributes of the entity
- E.g. *employee* entity with attributes *department-number* and *department-address*, and an FD

  department-number → *department-address*
  - Good design would have made department an entity
- FDs from non-key attributes of a relationship set possible, but rare --- most relationships are binary

Universal Relation Approach

- **Dangling tuples** – Tuples that "disappear" in computing a join.
  - Let r_1 (R_1), r_2 (R_2), …., r_n (R_n) be a set of relations
  - A tuple r of the relation r_i is a dangling tuple if r is not in the relation:

$$\Pi_{Ri} (r_1 \bowtie r_2 \bowtie \ldots \bowtie r_n)$$

- The relation $r_1 \bowtie r_2 \bowtie \ldots \bowtie r_n$ is called a *universal relation* since it involves all the attributes in the "universe" defined by $R_1 \cup R_2 \cup \ldots \cup R_n$
- If dangling tuples are allowed in the database, instead of decomposing a universal relation, we may prefer to synthesize a collection of normal form schemas from a given set of attributes.

Universal Relation Approach

- Dangling tuples may occur in practical database applications
- They represent incomplete information
- E.g. may want to break up information about loans into:

  (branch-name, loan-number)

  (loan-number, amount)

  (loan-number, customer-name)
- Universal relation would require null values, and have dangling tuples

Universal Relation Approach (Contd.)

- A particular decomposition defines a restricted form of incomplete information that is acceptable in our database.
  - Above decomposition requires at least one of customer-name, branch-name or amount in order to enter a loan number without using null values
  - Rules out storing of customer-name, amount without an appropriate loan-number (since it is a key, it can't be null either!)
- Universal relation requires to impose the **unique role assumption, i.e., all attribute names are unique**
- Reuse of attribute names is natural in SQL since relation names can be prefixed to disambiguate names

Denormalization for Performance

- May want to use non-normalized schema for performance
- E.g. displaying *customer-name* along with *account-number* and *balance* requires join of *account* with *depositor*
- Alternative 1: Use denormalized relation containing attributes of *account* as well as *depositor* with all above attributes
  - faster lookup
  - Extra space and extra execution time for updates
  - extra coding work for programmer and possibility of error in extra code
- Alternative 2: use a materialized view defined as

  account ⋈ depositor
  - Benefits and drawbacks same as above, except no extra coding work for programmer and avoids possible errors

Other Design Issues

- Some aspects of database design are not caught by normalization
- Examples of bad database design, to be avoided:

  Instead of *earnings(company-id, year, amount)*, use

  - *earnings-2000*, *earnings-2001*, *earnings-2002*, etc., all on the schema (*company-id*, *earnings*).
    - Above are in BCNF, but make querying across years difficult and needs new table each year
  - *company-year*(*company-id*, *earnings-2000*, *earnings-2001*, *earnings-2002*)
    - Also in BCNF, but also makes querying across years difficult and requires new attribute each year.
    - Is an example of a **crosstab**, where values for one attribute become column names
    - Used in spreadsheets, and in data analysis tools

Assignments

- Practice exercises
  - 7.1, 7.11
- Exercises
  - 7.23, 7.25, 7.27