

# From Frequency to Meaning: Vector Space Models of Semantics

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# Outline

- 1 Matrices
  - The Word – Context Matrix
  - The Pair – Pattern Matrix
- 2 Weighting the Elements
  - Positive PMI
- 3 Smoothing the Matrices
  - Truncated SVD
- 4 Efficient Comparisons
  - LSH
- 5 Applications
  - The Word – Context Matrix
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# The Word – Context Matrix

- Distributional hypothesis [2]
  - Words that occur in similar contexts tend to have similar meanings
  - “vague”, “obscure”
- The context is given by words, phrases, sentences, paragraphs, chapters, documents, or more exotic possibilities, such as sequences of characters or patterns [3]

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## The Word – Context Matrix (cont.)

- Example.

Whereof one cannot speak thereof one must be silent

Word	Co-occurents							
	whereof	one	cannot	speak	thereof	must	be	silent
whereof	0	1	0	0	0	0	0	0
one	1	0	1	0	1	1	0	0
cannot	0	1	0	1	0	0	0	0
speak	0	0	1	0	1	0	0	0
thereof	0	1	0	1	0	1	0	0
must	0	1	0	0	1	0	1	0
be	0	0	0	0	0	1	0	1
silent	0	0	0	0	0	0	1	0

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# The Pair – Pattern Matrix

- Extended distributional hypothesis [4]
  - Patterns that co-occur with similar pairs tend to have similar meanings
  - “X solves Y”, “Y is solved by X”
- Latent relation hypothesis [5]
  - Pairs of words that co-occur in similar patterns tend to have similar semantic relations
  - “committee:problem”, “congress:crisis”



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# The Pair – Pattern Matrix (cont.)

- Example.

Committee finds a solution to problem

Strike is solved by committee

Congress finds a solution to crisis

Congress solves crisis

Civil war is solved by committee

Pair	Pattern		
	X finds a solution to Y	Y is solved by X	X solves Y
committee:problem	1	0	0
strike:committee	0	1	0
congress:crisis	1	0	1
civil war:committee	0	1	0

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# Positive PMI

- An alternative to tf-idf which works well for both word-context matrices [6] and pair-pattern matrices [5]
- $F$  be a word-context frequency matrix with  $n_r$  rows and  $n_c$  columns
- $f_{ij}$  is the number of times that word  $w_i$  occurs in the context  $c_j$
- $X$  be the matrix that results when Positive PMI is applied to  $F$
- $x_{ij}$  in  $X$  is dened as follows

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## Positive PMI (cont.)

$$p_{ij} = \frac{f_{ij}}{\sum_{i=1}^{n_r} \sum_{j=1}^{n_c} f_{ij}}$$

$$p_{i*} = \frac{\sum_{j=1}^{n_c} f_{ij}}{\sum_{i=1}^{n_r} \sum_{j=1}^{n_c} f_{ij}}$$

$$p_{*j} = \frac{\sum_{i=1}^{n_r} f_{ij}}{\sum_{i=1}^{n_r} \sum_{j=1}^{n_c} f_{ij}}$$

$$\text{pmi}_{ij} = \log \left( \frac{p_{ij}}{p_{i*} p_{*j}} \right)$$

$$x_{ij} = \begin{cases} \text{pmi}_{ij} & \text{if } \text{pmi}_{ij} > 0 \\ 0 & \text{otherwise} \end{cases}$$

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# Truncated SVD

- An elegant way to improve similarity measurements can be applied to both documents [7] and words [8]
- SVD decomposes  $X$  into the product of three matrices  $U\Sigma V^T$
- $U$  and  $V$  are in column orthonormal form, and  $\Sigma$  is a diagonal matrix of singular values
- If  $X$  is of rank  $r$ , then  $\Sigma$  is also of rank  $r$ , let  $k < r$
- $\Sigma_k$ , the diagonal matrix formed from the top  $k$  singular values
- $U_k$  and  $V_k$ , the matrices produced by selecting the corresponding columns from  $U$  and  $V$
- $\tilde{X} = U_k \Sigma_k V_k^T$  is the matrix of rank  $k$  that best approximates the original matrix  $X$

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# LSH

- One general approach to LSH [9] is to “hash” items several times, in such a way that similar items are more likely to be hashed to the same bucket than dissimilar items are
- Definitions of LSH functions include the Min-wise independent function, such as Min-Hashing, that map vectors into short signatures or fingerprints
- After LSH, remained candidate pairs those pairs of signatures that we need to test for similarity

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# The Word – Context Matrix

- Open Source VSM System: Semantic Vectors [10]
  - Implementing the random projection approach to measuring word similarity
- Word similarity
  - Landauer and Dumais evaluated this approach with 80 multiple-choice synonym questions from the Test of English as a Foreign Language (TOEFL), achieving human-level performance [8]
- Word clustering
  - These algorithms are able to discover different senses of polysemous words, generating different clusters for each sense [6]

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## The Word – Context Matrix (cont.)

- Automatic thesaurus generation
  - Creating and maintaining such lexical resources is labour intensive, so it is natural to wonder whether the process can be automated to some degree [11]
- Context-sensitive spelling correction
  - These confusions cannot be detected by a simple dictionary-based spelling checker; they require context-sensitive spelling correction [12]
- Semantic role labeling
  - Word-context matrices can reliably predict the semantic frame to which an unknown lexical unit refers, with good levels of accuracy [13]

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# The Pair – Pattern Matrix

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  - Pattern frequencies are counted and then smoothed using SVD
- Relational similarity
  - Turney evaluated this approach to relational similarity with 374 multiple-choice analogy questions from the SAT college entrance test, achieving human-level performance [15]
- Relational clustering
  - The representative pairs to automatically generate multiple-choice analogy questions, in the style of SAT analogy questions [16]

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# The Pair – Pattern Matrix (cont.)

- Relational classification
  - Taint:poison is classified as strength (poisoning is stronger than tainting) and assess:review is classified as enablement (assessing is enabled by reviewing) [17]
- Relational search
  - A query for a relational search engine is “list all X such that X causes cancer”. In this example, the relation, cause, and one of the terms in the relation, cancer, are given by the user, and the task of the search engine is to find terms that satisfy the user’s query [18]
- Analogical mapping
  - With a pair-pattern matrix, we can solve proportional analogies by selecting the choice that maximizes relational similarity [5]

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