Modeling Acoustic Pressure from Reflected Plane Waves in Time

Consider the propagation of a plane acoustic wave with frequency 500 Hz in one dimension through a medium characterized by a sound speed of 350 m/sec. The wave is reflected at a surface perpendicular to the direction of propagation with a reflection coefficient $R = -0.5e^{0.3i}$

Plot on the same diagram the real part of the incident, reflected, and total acoustic pressure at distances $x_1 = -4\,m$ *and* $x_2 = -7\,m$ from the origin of the axis where the reflection surface is located (a separate diagram for each distance), for a time span of three to six periods. Assume that at time 0, the incident wave has its maximum amplitude at the origin of the axis, where the reflection surface is located.

The time dependence is of the form: $e^{i\omega t}$

f = 500 Hz c = 350 m/s

Wavelength:

$$\lambda = \frac{c}{f} = \frac{350}{500} = 0.7\ \mathrm{m}$$

Time dependence of the wave is given as $e^{i\omega t}$

$\omega = 2\pi f = 2\pi \cdot 500 = 1000\pi$ rad/s = 3141.5926 rad/s

Reflection coefficient from the problem statement: $R = -0.5e^{0.3i}$

Wavenumber:

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.7} = 8.9759\ \mathrm{m}^{-1}$$

The ***acoustic pressure*** is written as:

$$P(x,t) = P_1(x)e^{i\omega t} = \left(A_1 e^{ikx} + A_2 e^{-ikx}\right)e^{i\omega t} = A_1 e^{i(kx+\omega t)} + A_2 e^{i(-kx+\omega t)}$$

The first term represents a wave traveling in the positive x-direction, the incident wave, and the second term a wave traveling in the negative x-direction, the reflected wave.

Given that the amplitude of the exponential function is 1, and the incident wave has amplitude 1, $A_1 = 1$.
Therefore, the reflection coefficient R expresses the fraction of the acoustic energy that is reflected.
So:

$$A_2 = A_1 \cdot R = R = -0.5e^{0.3i}$$

Final Pressure Expression:

$$p(x,t) = A_1 e^{i(kx+\omega t)} + A_2 e^{-i(kx-\omega t)} = e^{i(kx+\omega t)} - 0.5e^{0.3i}e^{-i(kx+\omega t)}$$

Using Euler's formula:

$$e^{i\theta} = \cos\theta + i\sin\theta$$

So:

$$p(x,t) = \cos(kx+\omega t) + i\sin(kx+\omega t) - 0.5[\cos(-kx-\omega t+0.3) + i\sin(-kx-\omega t+0.3)]$$

$$= \cos(kx+\omega t) - 0.5\cos(kx+\omega t+0.3) \ + i[\sin(kx+\omega t) - 0.5\sin(kx+\omega t+0.3)]$$

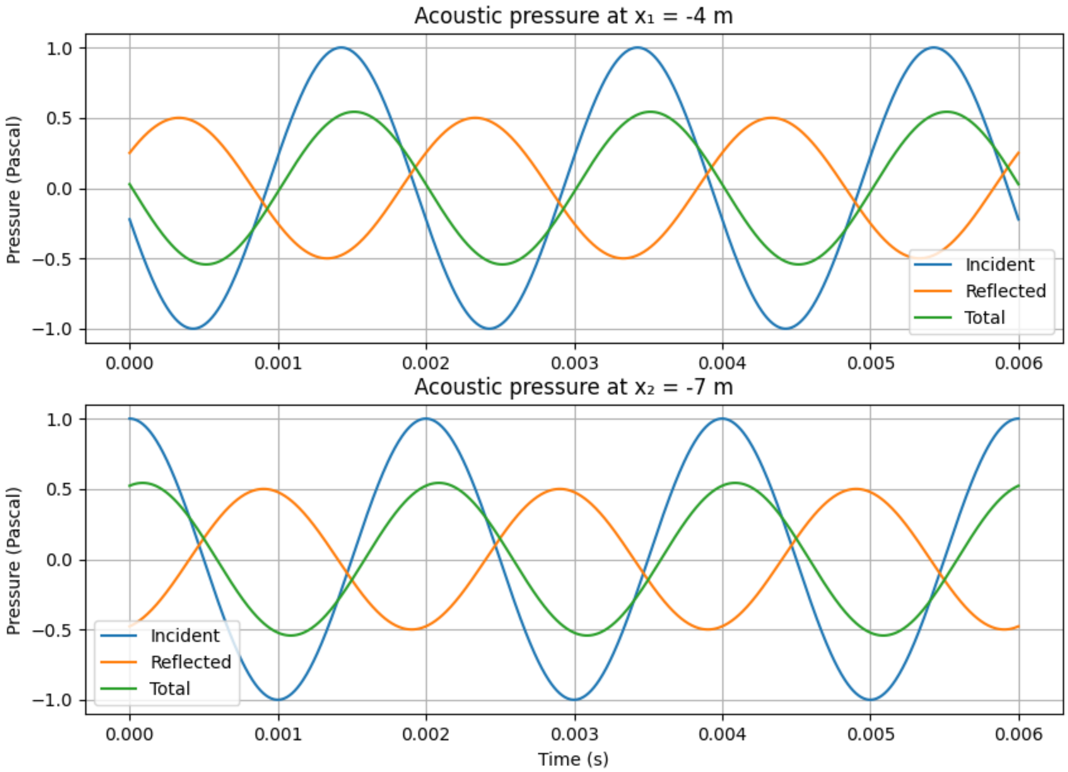
- For the **Incident Wave**, the equation is:

$$p_1(x,t) = e^{i(kx+\omega t)} = \cos(kx+\omega t) + i\sin(kx+\omega t)$$

- For the **Reflected Wave**, the equation is:

$$p_2(x,t) = Re^{i(-kx+\omega t)} = -0.5e^{0.3i}e^{i(-kx+\omega t)} = -0.5e^{i(-kx+\omega t+0.3)}$$

$$= -0.5\cos(-kx+\omega t+0.3) - i\cdot 0.5\sin(-kx+\omega t+0.3)$$


Acoustic pressure at x₁ = -4 m
Pressure (Pascal)
Incident
Reflected
Total
Acoustic pressure at x₂ = -7 m
Pressure (Pascal)
Incident
Reflected
Total
Time (s)