Solution (Chemical reaction in a reactor)

$$\frac{dC(t)}{dt} = \lambda C_{env} - \lambda C(t) - kC(t), \quad C(t=0) = C_0$$

We have a **PFR** (Plug Flow Reactor).

Non-dimensionalization

$\tilde{C} = \frac{C}{C_{env}}, \quad \tau = \frac{t}{T}$

But:

$T = \frac{1}{\lambda}$

So:

$\tau = \mu t \Rightarrow \tau = \lambda t$

Dimensionless parameter: $\beta = \frac{k}{\lambda}$

So:

$$\frac{d}{d\tau}\tilde{C}(\tau) = 1 - \tilde{C}(\tau) - \beta\tilde{C}(\tau)$$

$$\frac{d\tilde{C}(\tau)}{d\tau} = 1 - (1+\beta)\tilde{C}(\tau) \quad (1)$$

We have:

$$1 + \beta = \alpha$$

So the (1) is:

$$\frac{d\tilde{C}(\tau)}{d\tau} = 1 - \alpha\tilde{C}(\tau) \Leftrightarrow \frac{d\tilde{C}(\tau)}{1 - \alpha\tilde{C}(\tau)} = d\tau$$

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$$-\frac{1}{\alpha}\ln\left(1-\alpha\tilde{C}(\tau)\right)=\tau+c' \Rightarrow -\ln\left(1-\alpha\tilde{C}(\tau)\right)=\alpha\tau+\alpha c' \Rightarrow$$

$$1-\alpha\tilde{C}(\tau)=e^{-\alpha\tau}e^{-\alpha c'} \;\; \Rightarrow \;\; \alpha\tilde{C}(\tau)=1-e^{-\alpha\tau}e^{-\alpha c'} \;\; \Rightarrow$$

$$\tilde{C}(\tau)=\frac{1}{\alpha}-\frac{e^{-\alpha c'}}{\alpha}e^{-\alpha\tau}$$

For $\tau=0$:

$$\tilde{C}(0)=0 \Rightarrow \frac{1}{\alpha}-\frac{e^{-\alpha c'}}{\alpha}=0 \Rightarrow 1-e^{-\alpha c'}=0 \Rightarrow e^{-\alpha c'}=1 \Rightarrow e^{-\alpha c'}=e^{0} \Rightarrow \alpha c'=0 \Rightarrow c'=0$$

So:

$$\tilde{C}(\tau)=\frac{1}{1+\beta}-\frac{1}{1+\beta}e^{-(1+\beta)\tau} \;\; \Rightarrow$$

$$\tilde{C}(\tau)=\frac{1}{1+\beta}\left[1-e^{-(1+\beta)\tau}\right]$$