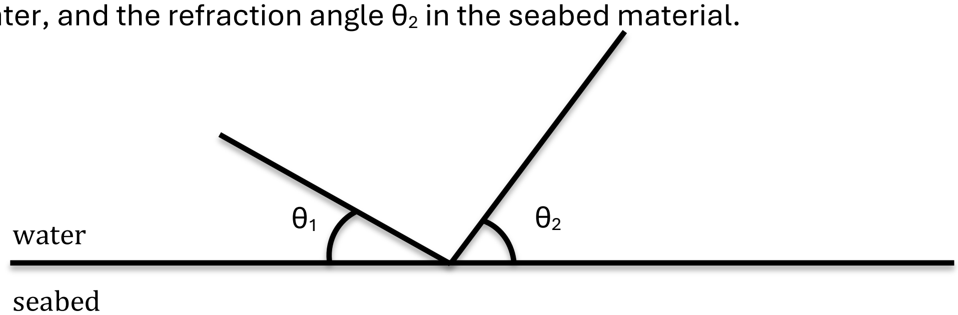
Analysis of Reflection Coefficients for Elastic and Fluid Seabed Models

Calculate the magnitude and phase of the reflection coefficient of plane acoustic waves at the interface between water and the seabed, which is assumed to consist of a semi-infinite elastic material, for incidence angles from 0° to 89° and plots their graph.

Given:
Speed of sound in water: $c_1 = 1500$ m/s
Density of water: $\rho_1 = 1000$ kg/m^3
Speed of longitudinal waves in the seabed: $c_{p_2} = 1800$ m/s
Density of the seabed: $\rho_2 = 1200$ kg/m^3
Speed of shear waves in the seabed: $c_{s_2} = 1000$ m/s
Frequency: f = 500 Hz

Next, assume that the seabed is a fluid material and plot the magnitude and phase of the transmission coefficient in this case.

To find the magnitude of the reflection coefficient R, we need the characteristic acoustic impedances for the water Z_1 and the seabed Z_2, the incident angle θ_1 in the water, and the refraction angle θ_2 in the seabed material.



$$Z_1 = \rho_1 \cdot c_1 = 1000 \cdot 1500 = 15 \cdot 10^5 \; kg/m^2s$$

For compressional waves:

$$Z_{p_2} = \rho_2 \cdot c_{p_2} = 1200 \cdot 1800 = 216 \cdot 10^4 \; kg/m^2s$$

For shear waves:

$$Z_{s_2} = \rho_2 \cdot c_{s_2} = 1200 \cdot 1000 = 12 \cdot 10^5 \; kg/m^2s$$

Snell's Law:

- For compressional waves: $\frac{sin\theta_1}{c_1} = \frac{sin\theta_2}{c_{p2}} \Rightarrow sin\theta_2 = sin\theta_1 \cdot \frac{c_{p2}}{c_1}$

- For shear waves: $\frac{sin\theta_1}{c_1} = \frac{sin\theta_2}{c_{s_2}} \Rightarrow sin\theta_2 = sin\theta_1 \cdot \frac{c_{s2}}{c_1}$

Substitute the incidence angles from 0° to 89° and calculate the refraction angles for the corresponding values. So:

$$cos\theta_2 = \sqrt{1 - \left(\frac{c_{p2}}{c_1}\right)^2 \cdot \sin^2\theta_1}$$ for compressional

$$cos\theta_2 = \sqrt{1 - \left(\frac{c_{s2}}{c_1}\right)^2 \cdot \sin^2\theta_1}$$ for shear

- The seabed is an elastic material:

The **reflection coefficient** for compressional waves is:

$$R_P = \frac{Z_{p_2} \cdot cos\theta_1 - Z_1 \cdot cos\theta_2}{Z_{p_2} \cdot cos\theta_1 - Z_1 \cdot cos\theta_2}$$

The **reflection coefficient** for shear waves is:

$$R_s = \frac{Z_{s_2} \cdot cos\theta_1 - Z_1 \cdot cos\theta_2}{Z_{s_2} \cdot cos\theta_1 - Z_1 \cdot cos\theta_2}$$

Everything is known, I have incidence angles from 0° to 89° and the corresponding reflection angles, so I can calculate the reflection coefficient for each type of wave

For compressional waves, we have: $c_{p_2} > c_1$

In this case, total reflection can occur, and we define the critical angle as:

$$\theta_{cr} = arcsin\left(\frac{c_1}{c_{p_2}}\right)$$

For compressional waves, we have: $c_{s_2} < c_1$

In this case, the total reflection is always a real number.

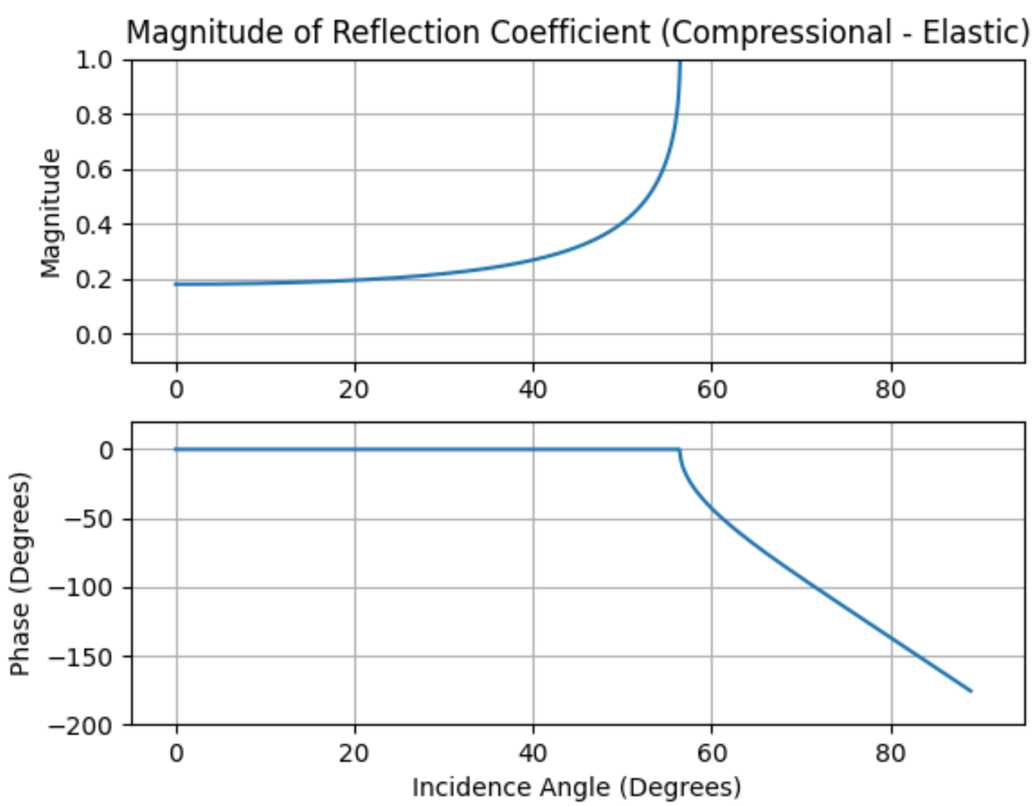
The magnitude of the reflection coefficient is $|R|$ and its phase is the angle of R in the complex plane.

For compressional waves:

The magnitude of the reflection coefficient remains almost constant for small incidence angles, but increases sharply as the angle approaches the critical angle (56.44°). This increase occurs because $c_{p_2} > c_1$

Correspondingly, the phase of reflection remains 0° for low incidence angles up to the critical angle, and becomes -180° for higher angles approaching 90°, i.e., we have total reflection.

Critical angle (degrees) for compressional waves: 56.44269023807929

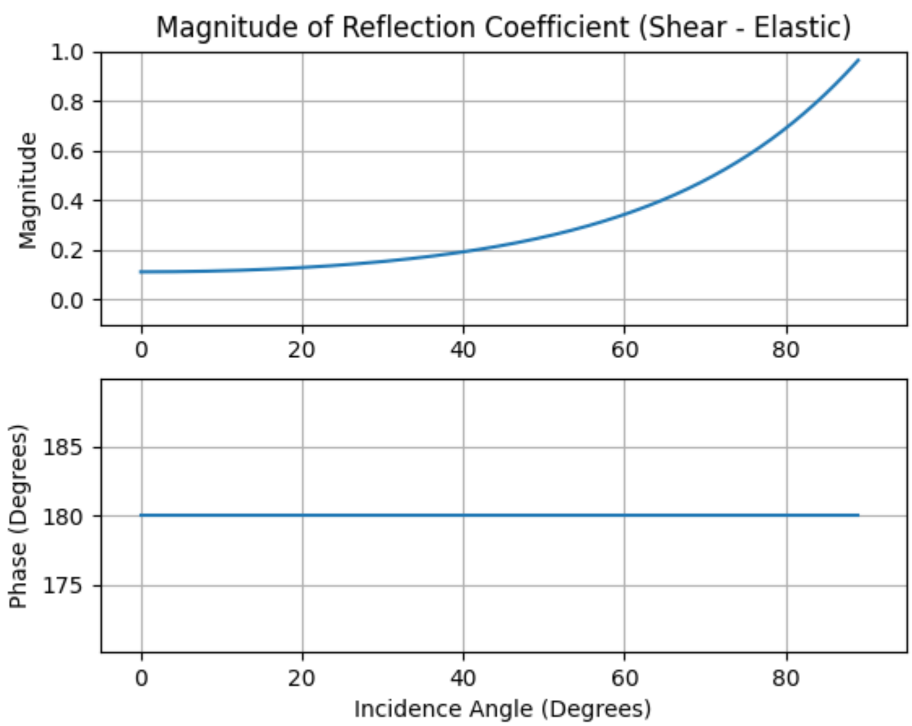


For shear waves:

The reflection coefficient in this case is always a real number.

The magnitude of the reflection coefficient increases at an accelerated rate, approaching 1 as the incidence angle approaches 90°.

The phase of the reflection coefficient remains constant and is independent of the incidence angle.



If the seabed were a **fluid material**, then shear waves cannot propagate.

The reflection coefficient is:

$$R = \frac{Z_2' - Z_1}{Z_2' + Z_1} \quad \text{with } Z_2' = C_{p_2} \cdot \rho_2 = Z_{p_2} = 216 \cdot 10^4 \, \mathrm{kg/m^2}s$$

Therefore,

$$R = \frac{216 \cdot 10^4 - 150 \cdot 10^4}{216 \cdot 10^4 + 150 \cdot 10^4} = \frac{66 \cdot 10^4}{366 \cdot 10^4} = \frac{11}{61} = 0.1803$$

for all incidence angles.

As with the compressional case, the reflection coefficient is a real number and is independent of the incidence angle.

Therefore, the magnitude of the reflection coefficient is

$$|R| = 0.1803$$

for all incidence angles, and its phase remains constant at 0°.