Ausplots

In this vignette, we explore show how to fit our point process model to the TERN Ausplots dataset. We begin by installing the `ppjsdm` package which implements the saturated pairwise interaction Gibbs point process. We also load the `rgdal` library for its coordinate systems implementation.

```r
need_ppjsdm <- !("ppjsdm" %in% installed.packages()[, "Package"])
if (need_ppjsdm) {
    library(devtools)
    devtools::install_github("iflint1/ppjsdm", upgrade = "never", quiet = TRUE)
}

suppressMessages(library(ppjsdm))
suppressMessages(library(rgdal))
```

Dataset load

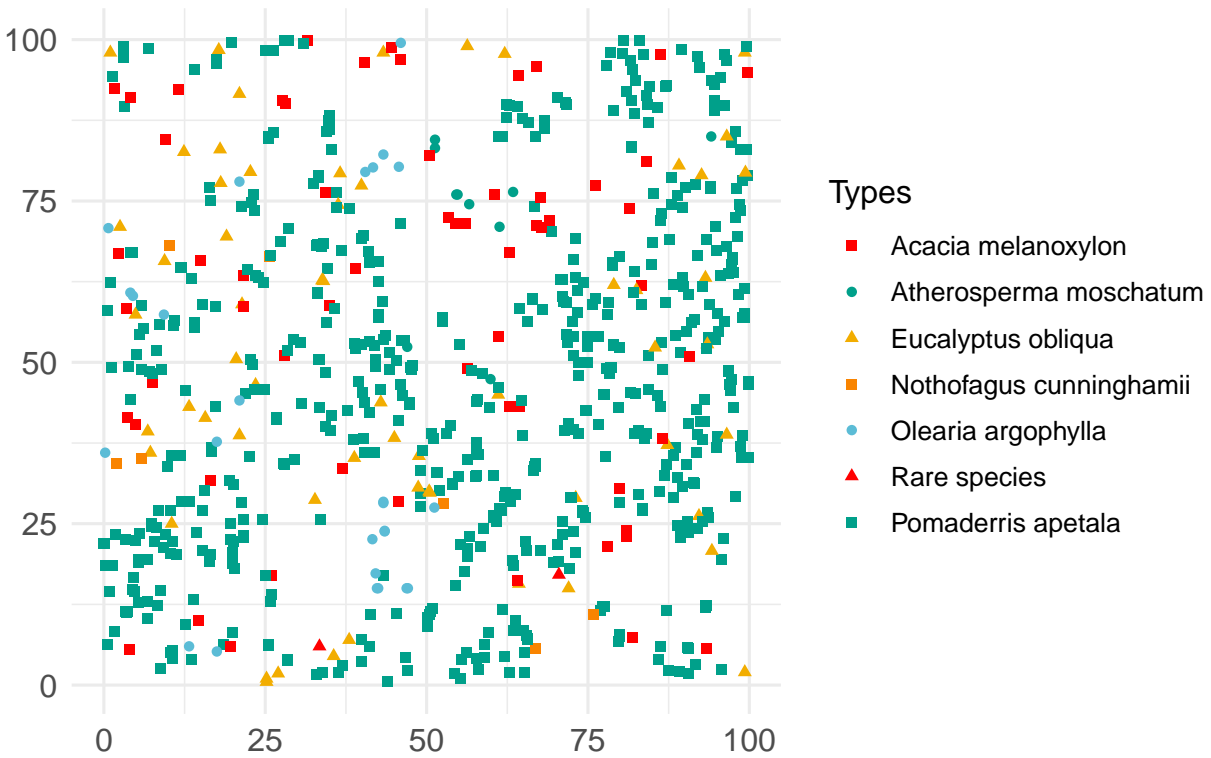The dataset is loaded by a single call to an included utility function, see the code block below.

```r
source("R/ausplots.R")
load(ausplots())
```

The loaded data is in the following form. First, we include the plot by plot data in the `configurations` and `windows` objects. These are lists of `ppjsdm` configurations and windows, respectively. Each entry corresponds to a forest plot, with names shown below.

```r
print(names(configurations))
#>  [1] "Frankland"     "Collins"      "Clare"         "Sutton"
#>  [5] "Giants"        "Warren"       "Dawson"        "Carey"
#>  [9] "Dombakup"      "Bird"         "Flowerdale"    "Dip"
#> [13] "Weld"          "ZigZag"       "NorthStyx"     "Newline"
#> [17] "WaratahMix"    "WogWay"       "Goodenia"      "Candelo"
#> [21] "ANU101"        "Ada Tree"     "Weeaproinah"   "ANU363"
#> [25] "ANU589"        "Turtons"      "Lardner"       "HardyCreek"
#> [29] "MinesRd"       "Tinebank"     "Lorne"         "BirdTree"
#> [33] "A-Tree"        "BlackBull"    "Bruxner"       "Osullivans"
#> [37] "Herberton"     "Lamb Range"   "Baldy"         "Koombooloomba"
#> [41] "BlackRiver"    "BondTier"     "Supersite"     "Mackenzie"
#> [45] "MtMaurice"     "BenRidge"     "MtField"       "Caveside"
```
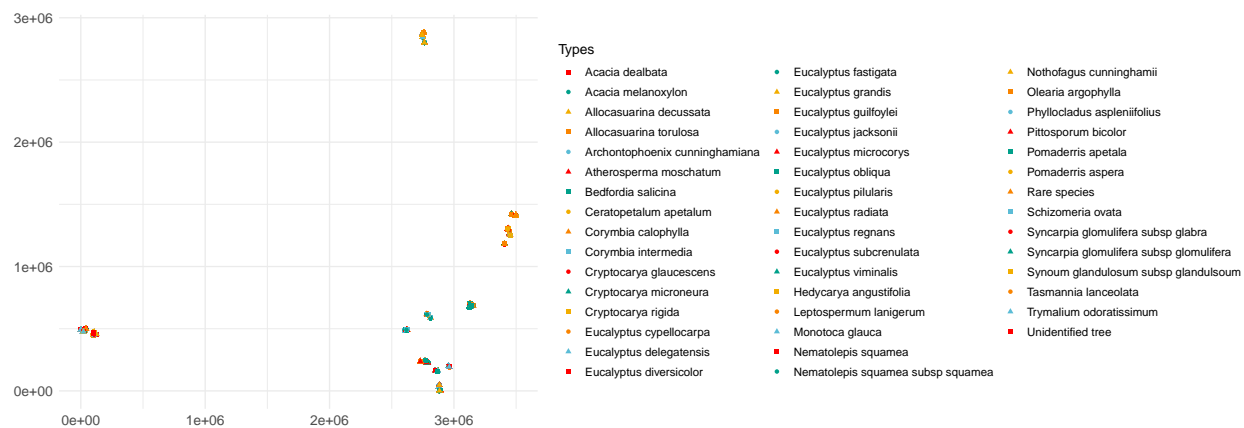
The objects can be accessed as follows.

```
plot(configurations$Dip)
```



```
print(windows$Dip)
#> $x_range
#> [1]    0 100
#>
#> $y_range
#> [1]    0 100
#>
#> attr(,"class")
#> [1] "Rectangle_window"
```

We also include the full configuration and corresponding window in the respective `configuration` and `window` objects.

```
plot(configuration)
```

Types

Acacia dealbata	Eucalyptus fastigata	Nothofagus cunninghamii
Acacia melanoxylon	Eucalyptus grandis	Olearia argophylla
Allocasuarina decussata	Eucalyptus guilfoylei	Phyllocladus aspleniifolius
Allocasuarina torulosa	Eucalyptus jacksonii	Pittosporum bicolor
Archontophoenix cunninghamiana	Eucalyptus microcorys	Pomaderris apetala
Atherosperma moschatum	Eucalyptus obliqua	Pomaderris aspera
Bedfordia salicina	Eucalyptus pilularis	Rare species
Ceratopetalum apetalum	Eucalyptus radiata	Schizomeria ovata
Corymbia calophylla	Eucalyptus regnans	Syncarpia glomulifera subsp glabra
Corymbia intermedia	Eucalyptus subcrenulata	Syncarpia glomulifera subsp glomulifera
Cryptocarya glaucescens	Eucalyptus viminalis	Synoum glandulosum subsp glandulsoum
Cryptocarya microneura	Hedycarya angustifolia	Tasmannia lanceolata
Cryptocarya rigida	Leptospermum lanigerum	Trymalium odoratissimum
Eucalyptus cypellocarpa	Monotoca glauca	Unidentified tree
Eucalyptus delegatensis	Nematolepis squamea	
Eucalyptus diversicolor	Nematolepis squamea subsp squamea	

```
str(window)
#> List of 2
#>  $ x_ranges:List of 48
#>   ..$ : num [1:2] 107165 107265
#>   ..$ : num [1:2] 31550 31650
#>   ..$ : num [1:2] 100859 100959
#>   ..$ : num [1:2] 41301 41401
#>   ..$ : num [1:2] 122043 122143
#>   ..$ : num [1:2] 16550 16650
#>   ..$ : num [1:2] 98717 98817
#>   ..$ : num [1:2] 0 100
#>   ..$ : num [1:2] 20778 20878
#>   ..$ : num [1:2] 2882620 2882720
#>   ..$ : num [1:2] 2791146 2791246
#>   ..$ : num [1:2] 2777590 2777690
#>   ..$ : num [1:2] 2890793 2890893
#>   ..$ : num [1:2] 2891016 2891116
#>   ..$ : num [1:2] 2879678 2879778
#>   ..$ : num [1:2] 3129024 3129124
#>   ..$ : num [1:2] 3123531 3123631
#>   ..$ : num [1:2] 3123651 3123751
#>   ..$ : num [1:2] 3153811 3153911
#>   ..$ : num [1:2] 3142958 3143058
#>   ..$ : num [1:2] 2780598 2780698
#>   ..$ : num [1:2] 2811941 2812041
#>   ..$ : num [1:2] 2604852 2604952
#>   ..$ : num [1:2] 2790421 2790521
```

```
#>    ..$ : num [1:2] 2791483 2791583
#>    ..$ : num [1:2] 2624631 2624731
#>    ..$ : num [1:2] 2617921 2618021
#>    ..$ : num [1:2] 2783294 2783394
#>    ..$ : num [1:2] 3439025 3439125
#>    ..$ : num [1:2] 3438409 3438509
#>    ..$ : num [1:2] 3445158 3445258
#>    ..$ : num [1:2] 3450652 3450752
#>    ..$ : num [1:2] 3432031 3432131
#>    ..$ : num [1:2] 3463563 3463663
#>    ..$ : num [1:2] 3498325 3498425
#>    ..$ : num [1:2] 3406828 3406928
#>    ..$ : num [1:2] 2748620 2748720
#>    ..$ : num [1:2] 2758576 2758676
#>    ..$ : num [1:2] 2743571 2743671
#>    ..$ : num [1:2] 2761661 2761761
#>    ..$ : num [1:2] 2767227 2767327
#>    ..$ : num [1:2] 2730033 2730133
#>    ..$ : num [1:2] 2883450 2883550
#>    ..$ : num [1:2] 2849880 2849980
#>    ..$ : num [1:2] 2956721 2956821
#>    ..$ : num [1:2] 2962046 2962146
#>    ..$ : num [1:2] 2882923 2883023
#>    ..$ : num [1:2] 2870138 2870238
#>  $ y_ranges:List of 48
#>    ..$ : num [1:2] 471578 471678
#>    ..$ : num [1:2] 486985 487085
#>    ..$ : num [1:2] 447526 447626
#>    ..$ : num [1:2] 497895 497995
#>    ..$ : num [1:2] 456521 456621
#>    ..$ : num [1:2] 476941 477041
#>    ..$ : num [1:2] 465526 465626
#>    ..$ : num [1:2] 491927 492027
#>    ..$ : num [1:2] 474351 474451
#>    ..$ : num [1:2] 1347 1447
#>    ..$ : num [1:2] 227413 227513
#>    ..$ : num [1:2] 228326 228426
#>    ..$ : num [1:2] 6710 6810
#>    ..$ : num [1:2] 0 100
#>    ..$ : num [1:2] 32199 32299
#>    ..$ : num [1:2] 701232 701332
#>    ..$ : num [1:2] 673185 673285
#>    ..$ : num [1:2] 674915 675015
#>    ..$ : num [1:2] 684507 684607
#>    ..$ : num [1:2] 689317 689417
#>    ..$ : num [1:2] 617358 617458
#>    ..$ : num [1:2] 587373 587473
#>    ..$ : num [1:2] 489015 489115
#>    ..$ : num [1:2] 610632 610732
#>    ..$ : num [1:2] 611099 611199
#>    ..$ : num [1:2] 490186 490286
#>    ..$ : num [1:2] 491115 491215
#>    ..$ : num [1:2] 613134 613234
```

```
#>    ..$ : num [1:2] 1298191 1298291
#>    ..$ : num [1:2] 1306328 1306428
#>    ..$ : num [1:2] 1263211 1263311
#>    ..$ : num [1:2] 1252380 1252480
#>    ..$ : num [1:2] 1302835 1302935
#>    ..$ : num [1:2] 1422023 1422123
#>    ..$ : num [1:2] 1411009 1411109
#>    ..$ : num [1:2] 1181245 1181345
#>    ..$ : num [1:2] 2845934 2846034
#>    ..$ : num [1:2] 2879783 2879883
#>    ..$ : num [1:2] 2862077 2862177
#>    ..$ : num [1:2] 2798972 2799072
#>    ..$ : num [1:2] 242735 242835
#>    ..$ : num [1:2] 236421 236521
#>    ..$ : num [1:2] 627 727
#>    ..$ : num [1:2] 163197 163297
#>    ..$ : num [1:2] 198694 198794
#>    ..$ : num [1:2] 192436 192536
#>    ..$ : num [1:2] 46485 46585
#>    ..$ : num [1:2] 158910 159010
#>   - attr(*, "class")= chr "Rectangle_window_union"
```

Example fit on full dataset

```r
load(ausplots(to_consider = 5)) # Consider only 5 plots so that fit is not excessively slow
ntypes <- nlevels(configuration$types)
tm <- Sys.time()
fit_complete <- gibbsm(configuration,
                       window = window,
                       model = "square_exponential", # Short-range interaction
                       medium_range_model = "tanh", # Medium-range interaction
                       short_range = matrix(10, ntypes, ntypes), # Matrix of short-range interaction di
                       medium_range = matrix(200, ntypes, ntypes), # Matrix of medium-range interaction
                       long_range = matrix(1000, ntypes, ntypes), # Matrix of long-range interaction di
                       saturation = 10, # Saturation parameter
                       fitting_package = "glmnet", # Fitting package
                       which_lambda = "smallest", # Which lambda to choose if regularization
                       min_dummy = 1e5, # Force at least 1e5 dummy points per species
                       max_dummy = 1e5, # Force at maximum 1e5 dummy points per species
                       dummy_distribution = "binomial") # Distribution of dummy points
print(Sys.time() - tm)
#> Time difference of 23.66021 secs
print(fit_complete$coefficients$alpha)
#>                         Allocasuarina decussata Eucalyptus diversicolor
#> Allocasuarina decussata            0.22412375             -0.017794693
#> Eucalyptus diversicolor           -0.01779469              0.011321723
#> Eucalyptus guilfoylei              0.06875874             -0.006676091
#> Eucalyptus jacksonii              -0.03300270             -0.157695880
#> Rare species                      -0.07829415             -0.009741903
#>                         Eucalyptus guilfoylei Eucalyptus jacksonii Rare species
#> Allocasuarina decussata            0.068758741          -0.03300270 -0.078294153
#> Eucalyptus diversicolor           -0.006676091          -0.15769588 -0.009741903
```
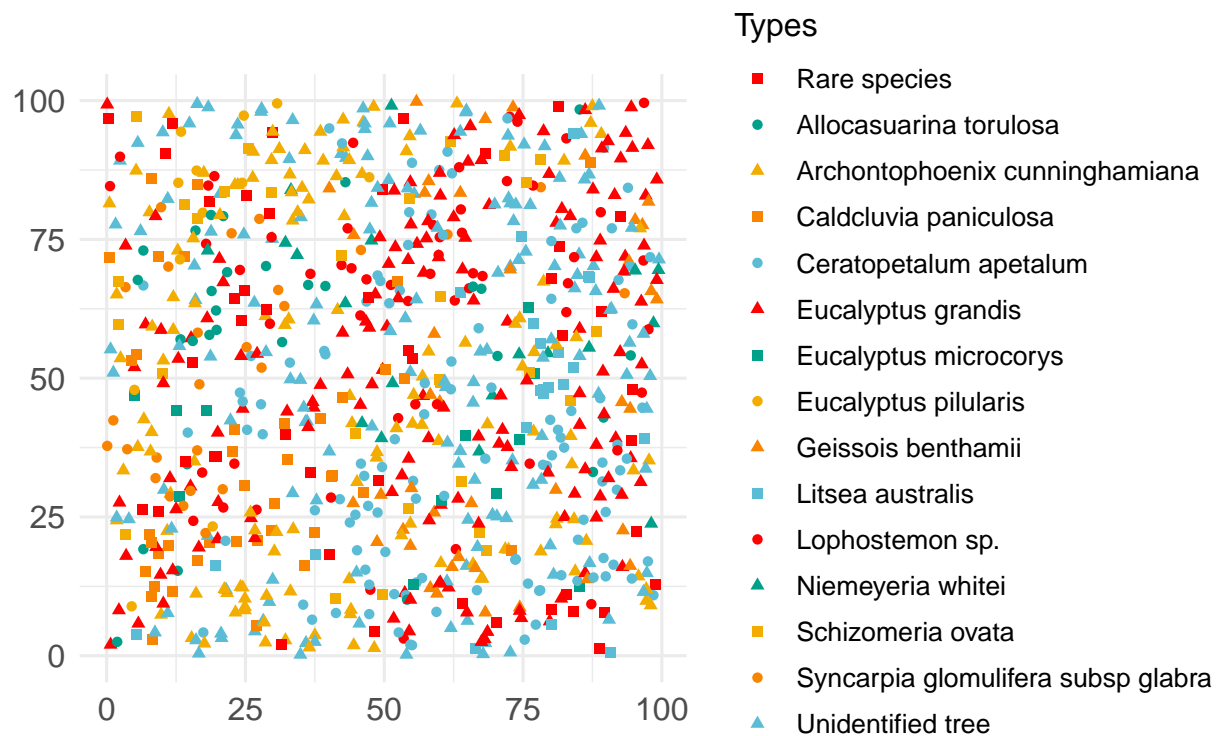
More in-depth exploration of a single plot

We explore here a single plot which has the second most number of species and third most number of individuals.

```
load(ausplots(plot_minimum_abundance = 10, overall_minimum_abundance = 1))
Bruxner <- configurations$Bruxner
Bruxner_window <- windows$Bruxner
plot(Bruxner)
```


And let's fit the model.

```
ntypes <- nlevels(Bruxner$types)
tm <- Sys.time()
fit <- gibbsm(Bruxner,
              window = Bruxner_window,
              model = "square_exponential", # Short-range interaction
              medium_range_model = "tanh", # Medium-range interaction
              short_range = matrix(5, ntypes, ntypes), # Matrix of short-range interaction distances
              medium_range = matrix(20, ntypes, ntypes), # Matrix of medium-range interaction distances
              long_range = matrix(40, ntypes, ntypes), # Matrix of long-range interaction distances
              saturation = 10, # Saturation parameter
```

```r
                fitting_package = "glmnet", # Fitting package
                min_dummy = 1e4, # Force at least 1e5 dummy points per species
                max_dummy = 1e4, # Force at maximum 1e5 dummy points per species
                dummy_distribution = "stratified") # Distribution of dummy points
print(Sys.time() - tm)
#> Time difference of 21.70018 secs
summ <- summary(fit)
print(summ)
#>           coefficients         se        CI95_lo      CI95_hi Ztest
#> beta0_1    -8.572313e+00 5.60850291 -1.956478e+01  2.42015105
#> beta0_2    -1.011139e+01 5.63502212 -2.115583e+01  0.93305376
#> beta0_3    -5.278210e+00 3.37618397 -1.189541e+01  1.33898863
#> beta0_4    -3.976493e+00 2.11604215 -8.123859e+00  0.17087356
#> beta0_5    -9.143888e+00 6.61353949 -2.210619e+01  3.81841133
#> beta0_6    -6.206804e-01 2.37608116 -5.277714e+00  4.03635311
#> beta0_7    -1.049393e+01 4.91034519 -2.011803e+01 -0.86982894     *
#> beta0_8    -4.905219e+00 6.12661678 -1.691317e+01  7.10272973
#> beta0_9    -9.214283e+00 3.76862375 -1.660065e+01 -1.82791576     *
#> beta0_10   -1.159017e+01 4.84082351 -2.107801e+01 -2.10232656     *
#> beta0_11   -7.598318e+00 2.61957862 -1.273260e+01 -2.46403777    **
#> beta0_12   -8.039097e+00 4.66236245 -1.717716e+01  1.09896566
#> beta0_13   -9.525965e+00 6.15545793 -2.159044e+01  2.53851095
#> beta0_14   -7.308584e+00 5.45665650 -1.800343e+01  3.38626601
#> beta0_15   -3.918843e+00 3.49476201 -1.076845e+01  2.93076434
#> alpha_1_1  -9.436439e-02 0.29334110 -6.693024e-01  0.48057359
#> alpha_1_2   6.874193e-02 0.26977572 -4.600088e-01  0.59749262
#> alpha_1_3   1.051674e-02 0.21457967 -4.100517e-01  0.43108517
#> alpha_1_4  -2.364734e-01 0.31223948 -8.484515e-01  0.37550475
#> alpha_1_5  -2.935726e-02 0.11555908 -2.558489e-01  0.19713438
#> alpha_1_6  -1.704416e-01 0.11009827 -3.862302e-01  0.04534710
#> alpha_1_7   5.752740e-01 0.57164408 -5.451278e-01  1.69567579
#> alpha_1_8   5.355308e-01 0.60030931 -6.410538e-01  1.71211546
#> alpha_1_9   8.414219e-03 0.24181505 -4.655346e-01  0.48236300
#> alpha_1_10  6.122957e-02 0.25411988 -4.368362e-01  0.55929539
#> alpha_1_11 -7.040205e-02 0.19135276 -4.454466e-01  0.30464248
#> alpha_1_12 -4.419005e-01 0.52011613 -1.461309e+00  0.57750838
#> alpha_1_13  3.975007e-02 0.31856337 -5.846227e-01  0.66412281
#> alpha_1_14  4.365841e-02 0.23290903 -4.128349e-01  0.50015171
#> alpha_1_15  1.555760e-02 0.11742604 -2.145932e-01  0.24570841
#> alpha_2_2  -5.558255e-01 0.47501604 -1.486840e+00  0.37518881
#> alpha_2_3   9.783605e-02 0.18033618 -2.556164e-01  0.45128847
#> alpha_2_4   2.562088e-02 0.27114655 -5.058166e-01  0.55705835
#> alpha_2_5  -3.216732e-01 0.28942573 -8.889372e-01  0.24559075
#> alpha_2_6   3.264728e-01 0.15868381  1.545828e-02  0.63748738     *
#> alpha_2_7  -4.551175e-01 0.92768517 -2.273347e+00  1.36311203
#> alpha_2_8   9.041109e-01 0.89570797 -8.514444e-01  2.65966631
#> alpha_2_9   1.859257e-01 0.46551494 -7.264668e-01  1.09831826
#> alpha_2_10 -3.895386e-01 0.55382955 -1.475025e+00  0.69594735
#> alpha_2_11  2.130961e-01 0.28853720 -3.524264e-01  0.77861865
#> alpha_2_12  1.207746e-01 0.53457360 -9.269704e-01  1.16851964
#> alpha_2_13 -1.780746e-02 0.46021298 -9.198083e-01  0.88419341
#> alpha_2_14  1.499522e-01 0.39220509 -6.187557e-01  0.91866003
#> alpha_2_15 -1.388818e-01 0.18569286 -5.028331e-01  0.22506947
```

```
#> alpha_3_3     7.468924e-04 0.07984846 -1.557532e-01  0.15724699
#> alpha_3_4    -4.689609e-02 0.10054856 -2.439676e-01  0.15017546
#> alpha_3_5     3.752703e-03 0.06114647 -1.160922e-01  0.12359758
#> alpha_3_6    -1.916781e-01 0.06347700 -3.160908e-01 -0.06726550   **
#> alpha_3_7    -1.647562e-01 0.30033898 -7.534098e-01  0.42389734
#> alpha_3_8    -2.113622e-01 0.28428867 -7.685577e-01  0.34583338
#> alpha_3_9     2.390065e-03 0.13969548 -2.714080e-01  0.27618817
#> alpha_3_10   -1.328460e-02 0.13134732 -2.707206e-01  0.24415141
#> alpha_3_11   -1.811716e-01 0.11135736 -3.994280e-01  0.03708486
#> alpha_3_12    1.857497e-01 0.20160746 -2.093936e-01  0.58089308
#> alpha_3_13    1.044913e-01 0.16217653 -2.133688e-01  0.42235150
#> alpha_3_14    1.866098e-02 0.16941600 -3.133883e-01  0.35071024
#> alpha_3_15   -3.655958e-02 0.06373835 -1.614844e-01  0.08836529
#> alpha_4_4    -3.398202e-01 0.25503426 -8.396782e-01  0.16003774
#> alpha_4_5    -2.236493e-01 0.14737804 -5.125050e-01  0.06520633
#> alpha_4_6     2.125645e-01 0.10816603  5.629556e-04  0.42456602   *
#> alpha_4_7    -4.404213e-01 0.65073948 -1.715847e+00  0.83500466
#> alpha_4_8     5.667659e-01 0.44092822 -2.974376e-01  1.43096929
#> alpha_4_9    -4.919119e-02 0.31928661 -6.749815e-01  0.57659908
#> alpha_4_10   -1.061774e-02 0.47955019 -9.505188e-01  0.92928335
#> alpha_4_11    1.608577e-01 0.23781389 -3.052489e-01  0.62696437
#> alpha_4_12   -6.663662e-01 0.74150710 -2.119693e+00  0.78696097
#> alpha_4_13    8.333981e-02 0.32915704 -5.617961e-01  0.72847575
#> alpha_4_14   -2.623403e-01 0.26370794 -7.791984e-01  0.25451776
#> alpha_4_15   -1.507864e-01 0.11454640 -3.752932e-01  0.07372044
#> alpha_5_5     5.069051e-02 0.07190027 -9.023144e-02  0.19161246
#> alpha_5_6     7.754164e-02 0.05039628 -2.123325e-02  0.17631654
#> alpha_5_7     1.077878e-01 0.24906694 -3.803744e-01  0.59595001
#> alpha_5_8    -3.821312e-01 0.89566088 -2.137594e+00  1.37333188
#> alpha_5_9     8.755584e-02 0.10272305 -1.137776e-01  0.28888932
#> alpha_5_10   -2.152548e-01 0.15284729 -5.148300e-01  0.08432034
#> alpha_5_11   -1.268472e-03 0.10552055 -2.080849e-01  0.20554800
#> alpha_5_12   -5.417548e-01 0.29732052 -1.124492e+00  0.04098266
#> alpha_5_13   -1.771400e-02 0.16956396 -3.500533e-01  0.31462525
#> alpha_5_14   -1.372883e-01 0.28186392 -6.897314e-01  0.41515487
#> alpha_5_15   -1.904298e-01 0.06977882 -3.271937e-01 -0.05366579   **
#> alpha_6_6    -5.550041e-02 0.07541750 -2.033160e-01  0.09231517
#> alpha_6_7     4.303055e-01 0.27097143 -1.007887e-01  0.96139976
#> alpha_6_8    -2.425137e-01 0.40073595 -1.027942e+00  0.54291431
#> alpha_6_9     7.339394e-02 0.10695490 -1.362338e-01  0.28302170
#> alpha_6_10    6.892043e-02 0.13236856 -1.905172e-01  0.32835804
#> alpha_6_11   -4.606155e-02 0.07738033 -1.977242e-01  0.10560112
#> alpha_6_12   -4.076560e-02 0.21452094 -4.612189e-01  0.37968772
#> alpha_6_13   -1.977828e-01 0.15361117 -4.988552e-01  0.10328957
#> alpha_6_14    2.926585e-01 0.19683642 -9.313375e-02  0.67845083
#> alpha_6_15   -4.071684e-02 0.06032642 -1.589545e-01  0.07752078
#> alpha_7_7    -5.808706e+00 1.78090922 -9.299224e+00 -2.31818812   **
#> alpha_7_8     1.902746e+00 1.20748487 -4.638813e-01  4.26937238
#> alpha_7_9    -2.874211e-01 0.51235747 -1.291623e+00  0.71678109
#> alpha_7_10    5.939663e-01 0.51084806 -4.072775e-01  1.59521011
#> alpha_7_11   -5.467864e-01 0.60353588 -1.729695e+00  0.63612220
#> alpha_7_12   -5.838990e-01 1.24969348 -3.033253e+00  1.86545521
#> alpha_7_13   -9.472964e-01 1.28339254 -3.462700e+00  1.56810672
```

```
#> alpha_7_14   -8.099983e-02 0.59958676 -1.256168e+00  1.09416863
#> alpha_7_15   -5.720071e-01 0.29543315 -1.151045e+00  0.00703127
#> alpha_8_8    -2.310647e+00 1.17157074 -4.606884e+00 -0.01441097    *
#> alpha_8_9    -2.847111e-01 1.34986361 -2.930395e+00  2.36097301
#> alpha_8_10    6.704532e-01 0.76684612 -8.325376e-01  2.17344399
#> alpha_8_11    3.081658e-01 0.48624850 -6.448638e-01  1.26119534
#> alpha_8_12   -6.751277e-01 1.42860042 -3.475133e+00  2.12487770
#> alpha_8_13    4.580234e-01 0.70593418 -9.255821e-01  1.84162901
#> alpha_8_14    1.208779e-01 0.53598608 -9.296355e-01  1.17139130
#> alpha_8_15    2.852003e-01 0.27797214 -2.596150e-01  0.83001573
#> alpha_9_9    -3.734926e-02 0.26344006 -5.536823e-01  0.47898376
#> alpha_9_10   -4.971358e-01 0.47507264 -1.428261e+00  0.43398945
#> alpha_9_11    5.596648e-03 0.24060179 -4.659742e-01  0.47716750
#> alpha_9_12    6.014135e-02 0.39075563 -7.057256e-01  0.82600831
#> alpha_9_13    4.183365e-02 0.43870263 -8.180077e-01  0.90167500
#> alpha_9_14   -9.064435e-01 0.64000423 -2.160829e+00  0.34794175
#> alpha_9_15   -6.821440e-02 0.11455969 -2.927473e-01  0.15631846
#> alpha_10_10   2.308658e-02 0.22402930 -4.160028e-01  0.46217593
#> alpha_10_11  -6.554927e-02 0.24554507 -5.468088e-01  0.41571024
#> alpha_10_12   1.075382e-01 0.60894020 -1.085963e+00  1.30103903
#> alpha_10_13   1.530879e-01 0.40630425 -6.432538e-01  0.94942958
#> alpha_10_14  -1.388868e-01 0.91182016 -1.926021e+00  1.64824793
#> alpha_10_15   6.954155e-02 0.11867746 -1.630620e-01  0.30214511
#> alpha_11_11  -1.708340e-01 0.18974793 -5.427331e-01  0.20106509
#> alpha_11_12  -1.209194e-01 0.34632274 -7.996995e-01  0.55786065
#> alpha_11_13   2.051978e-01 0.20727034 -2.010446e-01  0.61144019
#> alpha_11_14   6.034082e-01 0.28305801  4.862472e-02  1.15819174    *
#> alpha_11_15  -1.604833e-01 0.09531759 -3.473023e-01  0.02633576
#> alpha_12_12  -1.281819e+00 1.11457558 -3.466347e+00  0.90270923
#> alpha_12_13   6.243618e-01 0.57908237 -5.106188e-01  1.75934237
#> alpha_12_14   1.777764e-01 0.61562509 -1.028827e+00  1.38437939
#> alpha_12_15  -2.518254e-01 0.22580067 -6.943866e-01  0.19073580
#> alpha_13_13  -6.302059e-01 0.64649431 -1.897311e+00  0.63689961
#> alpha_13_14  -6.235840e-02 0.61136524 -1.260612e+00  1.13589545
#> alpha_13_15   4.807311e-02 0.18782725 -3.200615e-01  0.41620777
#> alpha_14_14  -4.634982e-01 0.38062294 -1.209505e+00  0.28250909
#> alpha_14_15  -3.932984e-03 0.17747856 -3.517846e-01  0.34391860
#> alpha_15_15  -1.239916e-01 0.08984055 -3.000758e-01  0.05209263
#> gamma_1_1    -2.996597e-01 0.56363434 -1.404363e+00  0.80504337
#> gamma_1_2     6.296289e-03 0.15862967 -3.046121e-01  0.31720472
#> gamma_1_3     3.116316e-01 0.42125097 -5.140052e-01  1.13726829
#> gamma_1_4    -6.192550e-02 0.10363496 -2.650463e-01  0.14119530
#> gamma_1_5    -3.997557e-02 0.17932239 -3.914410e-01  0.31148986
#> gamma_1_6    -1.155231e-01 0.16098017 -4.310384e-01  0.19999224
#> gamma_1_7     8.221621e-02 0.26320381 -4.336538e-01  0.59808620
#> gamma_1_8    -1.741335e-01 0.25123486 -6.665447e-01  0.31827782
#> gamma_1_9     1.296378e-01 0.20128938 -2.648821e-01  0.52415778
#> gamma_1_10   -4.546046e-02 0.13878939 -3.174827e-01  0.22656174
#> gamma_1_11   -8.081449e-02 0.15024539 -3.752900e-01  0.21366106
#> gamma_1_12   -1.567440e-01 0.18703052 -5.233171e-01  0.20982912
#> gamma_1_13    7.335734e-02 0.24075356 -3.985110e-01  0.54522564
#> gamma_1_14   -4.446449e-03 0.15808569 -3.142887e-01  0.30539582
#> gamma_1_15    2.386713e-01 0.23658844 -2.250335e-01  0.70237613
```

```
#> gamma_2_2    -3.600605e-01 0.28831924 -9.251558e-01  0.20503480
#> gamma_2_3     3.161401e-02 0.16001424 -2.820081e-01  0.34523616
#> gamma_2_4     8.966073e-04 0.12129363 -2.368345e-01  0.23862776
#> gamma_2_5    -9.323325e-03 0.12178719 -2.480218e-01  0.22937519
#> gamma_2_6     2.411476e-02 0.12419087 -2.192949e-01  0.26752439
#> gamma_2_7     1.744744e-01 0.25526333 -3.258325e-01  0.67478137
#> gamma_2_8     3.395589e-01 0.39255359 -4.298320e-01  1.10894981
#> gamma_2_9    -3.031392e-02 0.16621466 -3.560887e-01  0.29546084
#> gamma_2_10   -3.810438e-01 0.23633955 -8.442608e-01  0.08217320
#> gamma_2_11    1.321920e-01 0.17514462 -2.110851e-01  0.47546919
#> gamma_2_12    1.946769e-01 0.24739634 -2.902110e-01  0.67956483
#> gamma_2_13   -4.129657e-03 0.18421182 -3.651782e-01  0.35691888
#> gamma_2_14    7.703537e-02 0.23241004 -3.784799e-01  0.53255068
#> gamma_2_15    7.440190e-02 0.11484440 -1.506890e-01  0.29949279
#> gamma_3_3    -8.346556e-02 0.34775507 -7.650530e-01  0.59812186
#> gamma_3_4    -9.450373e-02 0.09373359 -2.782182e-01  0.08921072
#> gamma_3_5     4.742935e-02 0.19766245 -3.399819e-01  0.43484064
#> gamma_3_6    -3.444620e-02 0.18626784 -3.995245e-01  0.33063206
#> gamma_3_7    -2.030289e-01 0.12887309 -4.556155e-01  0.04955772
#> gamma_3_8    -1.600616e-01 0.12765224 -4.102554e-01  0.09013222
#> gamma_3_9     6.425304e-02 0.12033686 -1.716029e-01  0.30010896
#> gamma_3_10    1.352581e-01 0.10745070 -7.534136e-02  0.34585766
#> gamma_3_11   -7.510739e-02 0.10942623 -2.895789e-01  0.13936408
#> gamma_3_12   -1.748925e-01 0.15267063 -4.741215e-01  0.12433640
#> gamma_3_13   -3.646624e-02 0.14758436 -3.257263e-01  0.25279380
#> gamma_3_14    2.186390e-01 0.24096996 -2.536534e-01  0.69093147
#> gamma_3_15    5.239273e-02 0.32878251 -5.920092e-01  0.69679462
#> gamma_4_4     1.011918e-01 0.11870648 -1.314687e-01  0.33385222
#> gamma_4_5    -1.451361e-02 0.07336810 -1.583124e-01  0.12928522
#> gamma_4_6    -5.969541e-02 0.06940022 -1.957173e-01  0.07632651
#> gamma_4_7     5.152251e-01 0.16129624  1.990903e-01  0.83135991    **
#> gamma_4_8    -1.249475e-01 0.13346962 -3.865432e-01  0.13664812
#> gamma_4_9    -6.156803e-02 0.10560843 -2.685567e-01  0.14542069
#> gamma_4_10   -1.098820e-01 0.11153769 -3.284918e-01  0.10872787
#> gamma_4_11   -8.188266e-02 0.10471907 -2.871283e-01  0.12336294
#> gamma_4_12    7.882878e-02 0.15127224 -2.176594e-01  0.37531692
#> gamma_4_13    1.233036e-01 0.18804456 -2.452569e-01  0.49186418
#> gamma_4_14    5.742657e-02 0.13119139 -1.997038e-01  0.31455697
#> gamma_4_15    4.793424e-02 0.06769426 -8.474407e-02  0.18061256
#> gamma_5_5     6.450380e-01 0.70683075 -7.403249e-01  2.03040077
#> gamma_5_6    -1.817057e-01 0.15367197 -4.828973e-01  0.11948581
#> gamma_5_7     1.092784e-01 0.09910465 -8.496311e-02  0.30351998
#> gamma_5_8     1.492017e-02 0.16062502 -2.998991e-01  0.32973943
#> gamma_5_9     6.367975e-02 0.15072700 -2.317397e-01  0.35909924
#> gamma_5_10   -8.310234e-02 0.10311030 -2.851948e-01  0.11899013
#> gamma_5_11    6.704443e-02 0.11796889 -1.641704e-01  0.29825921
#> gamma_5_12    5.381685e-02 0.15012691 -2.404265e-01  0.34806019
#> gamma_5_13    1.112957e-01 0.18337880 -2.481201e-01  0.47071155
#> gamma_5_14    4.816857e-02 0.11153775 -1.704414e-01  0.26677854
#> gamma_5_15   -1.433647e-01 0.15707506 -4.512262e-01  0.16449676
#> gamma_6_6    -1.450508e-01 0.21784156 -5.720124e-01  0.28191079
#> gamma_6_7     7.515534e-02 0.12355289 -1.670039e-01  0.31731455
#> gamma_6_8     3.507727e-02 0.12623554 -2.123398e-01  0.28249438
```

```
#> gamma_6_9     6.101380e-02 0.15469743 -2.421876e-01  0.36421519
#> gamma_6_10    1.086436e-01 0.10636469 -9.982734e-02  0.31711458
#> gamma_6_11   -2.277038e-02 0.10662465 -2.317509e-01  0.18621010
#> gamma_6_12    1.537057e-01 0.12232947 -8.605563e-02  0.39346708
#> gamma_6_13   -9.598774e-02 0.14285828 -3.759848e-01  0.18400933
#> gamma_6_14    5.675339e-02 0.11629541 -1.711814e-01  0.28468820
#> gamma_6_15   -3.549532e-02 0.23774711 -5.014711e-01  0.43048044
#> gamma_7_7    -3.364001e-01 0.50364161 -1.323520e+00  0.65071932
#> gamma_7_8    -9.022205e-02 0.43206422 -9.370524e-01  0.75660826
#> gamma_7_9    -1.243546e-01 0.21559803 -5.469190e-01  0.29820978
#> gamma_7_10    3.783407e-03 0.24593835 -4.782469e-01  0.48581371
#> gamma_7_11   -1.287614e-03 0.18485977 -3.636061e-01  0.36103088
#> gamma_7_12    5.326955e-01 0.31620332 -8.705166e-02  1.15244258
#> gamma_7_13   -2.711137e-02 0.28785167 -5.912903e-01  0.53706753
#> gamma_7_14   -4.819917e-01 0.37924628 -1.225301e+00  0.26131730
#> gamma_7_15   -5.770691e-02 0.10426142 -2.620555e-01  0.14664171
#> gamma_8_8    -2.966371e-01 0.65752840 -1.585369e+00  0.99209492
#> gamma_8_9    -4.417835e-02 0.30479812 -6.415717e-01  0.55321498
#> gamma_8_10   -3.318545e-01 0.26092818 -8.432643e-01  0.17955535
#> gamma_8_11    4.196803e-02 0.19293085 -3.361695e-01  0.42010556
#> gamma_8_12    9.614438e-02 0.32076644 -5.325463e-01  0.72483504
#> gamma_8_13   -2.083417e-01 0.46448720 -1.118720e+00  0.70203650
#> gamma_8_14    3.746527e-01 0.28747768 -1.887932e-01  0.93809863
#> gamma_8_15   -1.539061e-02 0.13088517 -2.719208e-01  0.24113961
#> gamma_9_9    -9.422115e-02 0.23273958 -5.503823e-01  0.36194004
#> gamma_9_10    3.916448e-01 0.25555077 -1.092255e-01  0.89251510
#> gamma_9_11    3.292883e-02 0.15162312 -2.642470e-01  0.33010468
#> gamma_9_12   -3.130620e-01 0.26647494 -8.353433e-01  0.20921932
#> gamma_9_13   -2.449407e-01 0.22841578 -6.926274e-01  0.20274596
#> gamma_9_14   -1.194107e-01 0.19649968 -5.045430e-01  0.26572156
#> gamma_9_15    1.310447e-01 0.13259421 -1.288352e-01  0.39092462
#> gamma_10_10  -1.159583e-01 0.17976744 -4.682960e-01  0.23637945
#> gamma_10_11   1.357327e-02 0.13682125 -2.545915e-01  0.28173799
#> gamma_10_12   3.885007e-02 0.20695256 -3.667695e-01  0.44446964
#> gamma_10_13  -1.032359e-01 0.17145526 -4.392820e-01  0.23281023
#> gamma_10_14   1.422180e-01 0.20783190 -2.651251e-01  0.54956100
#> gamma_10_15   5.006730e-02 0.10894250 -1.634561e-01  0.26359068
#> gamma_11_11   6.865860e-02 0.22419746 -3.707604e-01  0.50807756
#> gamma_11_12   1.821288e-01 0.25141378 -3.106332e-01  0.67489072
#> gamma_11_13   1.847389e-01 0.31427844 -4.312355e-01  0.80071334
#> gamma_11_14  -8.195300e-02 0.16933646 -4.138464e-01  0.24994036
#> gamma_11_15   6.087269e-02 0.15272807 -2.384688e-01  0.36021421
#> gamma_12_12  -9.161921e-02 0.47683564 -1.026200e+00  0.84296146
#> gamma_12_13   5.147053e-01 0.31719487 -1.069852e-01  1.13639587
#> gamma_12_14  -3.157237e-01 0.27109904 -8.470681e-01  0.21562065
#> gamma_12_15  -8.945589e-02 0.12767461 -3.396935e-01  0.16078175
#> gamma_13_13  -2.054746e-01 0.54239544 -1.268550e+00  0.85760090
#> gamma_13_14   4.162329e-03 0.23597353 -4.583373e-01  0.46666194
#> gamma_13_15   7.459013e-02 0.17127066 -2.610942e-01  0.41027446
#> gamma_14_14  -1.522956e-01 0.26016617 -6.622119e-01  0.35762071
#> gamma_14_15  -1.316585e-01 0.11205285 -3.512780e-01  0.08796110
#> gamma_15_15  -1.072822e-01 0.52525201 -1.136757e+00  0.92219285
#>                       Pval        Zval se_numerical_proportion
```

```
#> beta0_1     0.126401008 -1.528449356           0.001711542
#> beta0_2     0.072752094 -1.794382779           0.002617281
#> beta0_3     0.117966593 -1.563365742           0.005158860
#> beta0_4     0.060215484 -1.879212493           0.005557154
#> beta0_5     0.166787170 -1.382601236           0.004478648
#> beta0_6     0.793922705 -0.261220195           0.005833772
#> beta0_7     0.032589363 -2.137106100           0.005878321
#> beta0_8     0.423339704 -0.800640661           0.004226274
#> beta0_9     0.014485241 -2.444999341           0.003186653
#> beta0_10    0.016654162 -2.394255084           0.002866091
#> beta0_11    0.003724636 -2.900587697           0.003781300
#> beta0_12    0.084662010 -1.724253941           0.003662024
#> beta0_13    0.121727312 -1.547563969           0.002319187
#> beta0_14    0.180444157 -1.339388728           0.002761004
#> beta0_15    0.262139890 -1.121347696           0.005190619
#> alpha_1_1   0.747688860 -0.321688277           0.006072114
#> alpha_1_2   0.798868751  0.254811425           0.007451957
#> alpha_1_3   0.960910633  0.049010871           0.003260125
#> alpha_1_4   0.448842480 -0.757346198           0.003891151
#> alpha_1_5   0.799460445 -0.254045450           0.005971854
#> alpha_1_6   0.121601686 -1.548085611           0.006313722
#> alpha_1_7   0.314247282  1.006349905           0.005237178
#> alpha_1_8   0.372343878  0.892091523           0.003976795
#> alpha_1_9   0.972242335  0.034796095           0.004742083
#> alpha_1_10  0.809595741  0.240947585           0.005455600
#> alpha_1_11  0.712934670 -0.367917608           0.004746843
#> alpha_1_12  0.395536986 -0.849618926           0.003861638
#> alpha_1_13  0.900698390  0.124779156           0.004392019
#> alpha_1_14  0.851309139  0.187448321           0.005986721
#> alpha_1_15  0.894597924  0.132488493           0.005988700
#> alpha_2_2   0.241952897 -1.170119464           0.006164206
#> alpha_2_3   0.587460088  0.542520357           0.005586556
#> alpha_2_4   0.924719211  0.094490900           0.005445160
#> alpha_2_5   0.266388043 -1.111418987           0.004236471
#> alpha_2_6   0.039649724  2.057379622           0.007764365
#> alpha_2_7   0.623713090 -0.490594765           0.005429582
#> alpha_2_8   0.312791771  1.009381367           0.003837778
#> alpha_2_9   0.689599950  0.399398024           0.004009410
#> alpha_2_10  0.481834718 -0.703354719           0.004136708
#> alpha_2_11  0.460186681  0.738539488           0.004806850
#> alpha_2_12  0.821258161  0.225927048           0.005312672
#> alpha_2_13  0.969134400 -0.038693946           0.003961491
#> alpha_2_14  0.702215847  0.382331022           0.005354777
#> alpha_2_15  0.454513485 -0.747911592           0.005073501
#> alpha_3_3   0.992536798  0.009353873           0.008349537
#> alpha_3_4   0.640927461 -0.466402446           0.006059293
#> alpha_3_5   0.951062659  0.061372367           0.006880009
#> alpha_3_6   0.002530694 -3.019647048           0.006534901
#> alpha_3_7   0.583302198 -0.548567640           0.006602223
#> alpha_3_8   0.457192763 -0.743477295           0.006164919
#> alpha_3_9   0.986349571  0.017109111           0.005278199
#> alpha_3_10  0.919438503 -0.101141036           0.006149348
#> alpha_3_11  0.103750198 -1.626938322           0.005454938
```

```
#> alpha_3_12   0.356871137   0.921343461          0.005903160
#> alpha_3_13   0.519376874   0.644306233          0.005812117
#> alpha_3_14   0.912291335   0.110148840          0.005335535
#> alpha_3_15   0.566246353  -0.573588416          0.006216512
#> alpha_4_4    0.182712586  -1.332449316          0.008043838
#> alpha_4_5    0.129135101  -1.517521374          0.005470282
#> alpha_4_6    0.049394734   1.965168535          0.007228304
#> alpha_4_7    0.498532084  -0.676801220          0.005442286
#> alpha_4_8    0.198655132   1.285392581          0.007610854
#> alpha_4_9    0.877557752  -0.154065929          0.004221212
#> alpha_4_10   0.982335448  -0.022141042          0.006745471
#> alpha_4_11   0.498785637   0.676401702          0.005026856
#> alpha_4_12   0.368831298  -0.898664675          0.004878670
#> alpha_4_13   0.800120126   0.253191641          0.004755000
#> alpha_4_14   0.319826823  -0.994813820          0.006187356
#> alpha_4_15   0.188047132  -1.316378212          0.007060450
#> alpha_5_5    0.480803179   0.705011339          0.008071088
#> alpha_5_6    0.123892649   1.538638195          0.007961546
#> alpha_5_7    0.665184524   0.432766347          0.006604528
#> alpha_5_8    0.669636323  -0.426647187          0.004222099
#> alpha_5_9    0.394020639   0.852348601          0.005999343
#> alpha_5_10   0.159042258  -1.408299991          0.006463333
#> alpha_5_11   0.990408786  -0.012021094          0.005796103
#> alpha_5_12   0.068436177  -1.822123975          0.005037194
#> alpha_5_13   0.916797997  -0.104467956          0.005733343
#> alpha_5_14   0.626206706  -0.487072850          0.005063857
#> alpha_5_15   0.006351742  -2.729048211          0.006665119
#> alpha_6_6    0.461786102  -0.735908963          0.007741665
#> alpha_6_7    0.112283938   1.588010620          0.006852737
#> alpha_6_8    0.545065460  -0.605170883          0.005498325
#> alpha_6_9    0.492578221   0.686213921          0.006370259
#> alpha_6_10   0.602596167   0.520670731          0.006793455
#> alpha_6_11   0.551668563  -0.595261685          0.007274567
#> alpha_6_12   0.849284945  -0.190030864          0.005759377
#> alpha_6_13   0.197900999  -1.287554751          0.005819428
#> alpha_6_14   0.137064751   1.486810939          0.005447244
#> alpha_6_15   0.499712576  -0.674942062          0.006733326
#> alpha_7_7    0.001107650  -3.261651959          0.007496128
#> alpha_7_8    0.115073645   1.575792443          0.005457090
#> alpha_7_9    0.574812775  -0.560977648          0.005168506
#> alpha_7_10   0.244948637   1.162706418          0.006064892
#> alpha_7_11   0.364950865  -0.905971636          0.004678264
#> alpha_7_12   0.640332627  -0.467233781          0.005484035
#> alpha_7_13   0.460442131  -0.738119013          0.003446385
#> alpha_7_14   0.892538539  -0.135092754          0.006599493
#> alpha_7_15   0.052847604  -1.936164118          0.005478060
#> alpha_8_8    0.048579416  -1.972264538          0.005178892
#> alpha_8_9    0.832950952  -0.210918392          0.002782891
#> alpha_8_10   0.381955145   0.874299549          0.006086939
#> alpha_8_11   0.526236196   0.633761938          0.005269070
#> alpha_8_12   0.636513010  -0.472579782          0.004971058
#> alpha_8_13   0.516455443   0.648818895          0.004837584
#> alpha_8_14   0.821571421   0.225524303          0.005064251
```

```
#> alpha_8_15   0.304890007   1.026003346        0.005659471
#> alpha_9_9    0.887257580  -0.141775193        0.005720042
#> alpha_9_10   0.295357154  -1.046441688        0.004938970
#> alpha_9_11   0.981442049   0.023261040        0.004789085
#> alpha_9_12   0.877680391   0.153910391        0.005696146
#> alpha_9_13   0.924030759   0.095357643        0.003814970
#> alpha_9_14   0.156685172  -1.416308595        0.004095905
#> alpha_9_15   0.551543648  -0.595448599        0.005988768
#> alpha_10_10  0.917922012   0.103051608        0.008430751
#> alpha_10_11  0.789504495  -0.266954108        0.004933617
#> alpha_10_12  0.859823442   0.176598921        0.005377933
#> alpha_10_13  0.706336075   0.376781385        0.004541701
#> alpha_10_14  0.878936017  -0.152318142        0.004906676
#> alpha_10_15  0.557894985   0.585971028        0.006572904
#> alpha_11_11  0.367949506  -0.900320893        0.005438980
#> alpha_11_12  0.726974833  -0.349152489        0.004696854
#> alpha_11_13  0.322173770   0.990000714        0.005886903
#> alpha_11_14  0.033027605   2.131747557        0.005234892
#> alpha_11_15  0.092245623  -1.683669170        0.005553761
#> alpha_12_12  0.250122864  -1.150051005        0.003819417
#> alpha_12_13  0.280948231   1.078191665        0.004268351
#> alpha_12_14  0.772754514   0.288773760        0.005067540
#> alpha_12_15  0.264741057  -1.115255247        0.005046260
#> alpha_13_13  0.329656950  -0.974805095        0.004074124
#> alpha_13_14  0.918757787  -0.101998600        0.003748735
#> alpha_13_15  0.797994680   0.255943221        0.004923473
#> alpha_14_14  0.223324398  -1.217735773        0.004991197
#> alpha_14_15  0.982320060  -0.022160333        0.005793393
#> alpha_15_15  0.167546727  -1.380129659        0.006705790
#> gamma_1_1    0.594964187  -0.531656122        0.001764391
#> gamma_1_2    0.968338881   0.039691747        0.003720663
#> gamma_1_3    0.459435628   0.739776487        0.001566337
#> gamma_1_4    0.550150345  -0.597534852        0.004018794
#> gamma_1_5    0.823593351  -0.222925678        0.003069025
#> gamma_1_6    0.472989670  -0.717623165        0.003070082
#> gamma_1_7    0.754761554   0.312367096        0.003248041
#> gamma_1_8    0.488240346  -0.693110281        0.003072784
#> gamma_1_9    0.519551358   0.644037129        0.003201475
#> gamma_1_10   0.743251926  -0.327550008        0.004251382
#> gamma_1_11   0.590657599  -0.537883328        0.003165678
#> gamma_1_12   0.401993510  -0.838066227        0.003430735
#> gamma_1_13   0.760595499   0.304698885        0.003058283
#> gamma_1_14   0.977560997  -0.028126828        0.003609597
#> gamma_1_15   0.313068761   1.008803753        0.001812729
#> gamma_2_2    0.211728773  -1.248825862        0.003100587
#> gamma_2_3    0.843381512   0.197569995        0.002787192
#> gamma_2_4    0.994102060   0.007392039        0.003893899
#> gamma_2_5    0.938978172  -0.076554229        0.003689647
#> gamma_2_6    0.846038869   0.194174985        0.003793636
#> gamma_2_7    0.494286151   0.683507598        0.003547581
#> gamma_2_8    0.387038726   0.865000115        0.003148519
#> gamma_2_9    0.855285992  -0.182378125        0.003041322
#> gamma_2_10   0.106902606  -1.612272725        0.003080615
```

```
#> gamma_2_11   0.450393373   0.754759376                0.004014067
#> gamma_2_12   0.431338687   0.786902938                0.003204049
#> gamma_2_13   0.982114539  -0.022417979                0.003196896
#> gamma_2_14   0.740294657   0.331463168                0.002894308
#> gamma_2_15   0.517082226   0.647849608                0.003418918
#> gamma_3_3    0.810320574  -0.240012490                0.005286306
#> gamma_3_4    0.313350621  -1.008216329                0.003844242
#> gamma_3_5    0.810368046   0.239951254                0.004222768
#> gamma_3_6    0.853285261  -0.184928316                0.004404235
#> gamma_3_7    0.115160148  -1.575417326                0.004704440
#> gamma_3_8    0.209882838  -1.253887668                0.004787215
#> gamma_3_9    0.593380889   0.533943118                0.003238925
#> gamma_3_10   0.208105250   1.258792610                0.003611441
#> gamma_3_11   0.492476868  -0.686374679                0.004101316
#> gamma_3_12   0.251979488  -1.145554608                0.003512964
#> gamma_3_13   0.804840587  -0.247087397                0.003070835
#> gamma_3_14   0.364232875   0.907328940                0.002020144
#> gamma_3_15   0.873390165   0.159353764                0.005010148
#> gamma_4_4    0.393962333   0.852453688                0.005292816
#> gamma_4_5    0.843186610  -0.197819089                0.004314955
#> gamma_4_6    0.389699874  -0.860161772                0.004733868
#> gamma_4_7    0.001401809   3.194278415                0.004045583
#> gamma_4_8    0.349196185  -0.936149567                0.004954183
#> gamma_4_9    0.559904048  -0.582984035                0.003578575
#> gamma_4_10   0.324547623  -0.985155647                0.003615220
#> gamma_4_11   0.434257502  -0.781926956                0.004112890
#> gamma_4_12   0.602293332   0.521105425                0.003988483
#> gamma_4_13   0.512007554   0.655714940                0.002898287
#> gamma_4_14   0.661581112   0.437731252                0.004278166
#> gamma_4_15   0.478883766   0.708099023                0.004352475
#> gamma_5_5    0.361464709   0.912577658                0.004656902
#> gamma_5_6    0.237036757  -1.182425886                0.005667047
#> gamma_5_7    0.270176161   1.102656967                0.004338408
#> gamma_5_8    0.925992368   0.092888216                0.004088704
#> gamma_5_9    0.672671768   0.422484012                0.003650899
#> gamma_5_10   0.420268366  -0.805955747                0.003959638
#> gamma_5_11   0.569815693   0.568322958                0.003322285
#> gamma_5_12   0.719987337   0.358475718                0.003056547
#> gamma_5_13   0.543905975   0.606917031                0.002407769
#> gamma_5_14   0.665843961   0.431858910                0.003891474
#> gamma_5_15   0.361392645  -0.912714635                0.005039221
#> gamma_6_6    0.505503995  -0.665854673                0.005765262
#> gamma_6_7    0.542998645   0.608284732                0.003915020
#> gamma_6_8    0.781110940   0.277871581                0.004554181
#> gamma_6_9    0.693280313   0.394407357                0.003181269
#> gamma_6_10   0.307052814   1.021425658                0.004103759
#> gamma_6_11   0.830892985  -0.213556423                0.004360827
#> gamma_6_12   0.208938464   1.256489738                0.004042431
#> gamma_6_13   0.501641715  -0.671908867                0.003524949
#> gamma_6_14   0.625542359   0.488010563                0.003842639
#> gamma_6_15   0.881317987  -0.149298642                0.005836910
#> gamma_7_7    0.504174778  -0.667935480                0.004557112
#> gamma_7_8    0.834591653  -0.208816291                0.004156609
```

```
#> gamma_7_9    0.564081950 -0.576789077            0.003555972
#> gamma_7_10   0.987726179  0.015383560            0.003788324
#> gamma_7_11   0.994442494 -0.006965358            0.003661240
#> gamma_7_12   0.092053959  1.684661187            0.003525673
#> gamma_7_13   0.924962024 -0.094185221            0.002995428
#> gamma_7_14   0.203757046 -1.270920181            0.002882694
#> gamma_7_15   0.579932788 -0.553482898            0.005038848
#> gamma_8_8    0.651888984 -0.451139536            0.003508227
#> gamma_8_9    0.884755886 -0.144942988            0.002801680
#> gamma_8_10   0.203435970 -1.271823126            0.006531205
#> gamma_8_11   0.827796207  0.217528875            0.003445787
#> gamma_8_12   0.764380612  0.299733279            0.003556634
#> gamma_8_13   0.653762584 -0.448541303            0.002467977
#> gamma_8_14   0.192492449  1.303241141            0.003970979
#> gamma_8_15   0.906393605 -0.117588642            0.003772638
#> gamma_9_9    0.685598722 -0.404835106            0.003153426
#> gamma_9_10   0.125386339  1.532551743            0.003398967
#> gamma_9_11   0.828071552  0.217175533            0.003415901
#> gamma_9_12   0.240063921 -1.174827037            0.002840561
#> gamma_9_13   0.283564661 -1.072345982            0.002728219
#> gamma_9_14   0.543393613 -0.607689221            0.002899909
#> gamma_9_15   0.322998833  0.988314119            0.003353265
#> gamma_10_10 0.518897414 -0.645045942            0.004285504
#> gamma_10_11 0.920975988  0.099204391            0.004021225
#> gamma_10_12 0.851092609  0.187724518            0.003511297
#> gamma_10_13 0.547097157 -0.602115651            0.003763362
#> gamma_10_14 0.493790044  0.684293194            0.003384572
#> gamma_10_15 0.645821005  0.459575421            0.003738558
#> gamma_11_11 0.759420655  0.306241660            0.003602731
#> gamma_11_12 0.468808913  0.724418390            0.002852170
#> gamma_11_13 0.556653607  0.587819272            0.002557872
#> gamma_11_14 0.628410359 -0.483965493            0.003195236
#> gamma_11_15 0.690210741  0.398569090            0.003485597
#> gamma_12_12 0.847632523 -0.192140025            0.002740570
#> gamma_12_13 0.104658121  1.622678634            0.003192457
#> gamma_12_14 0.244178312 -1.164606498            0.003029746
#> gamma_12_15 0.483518180 -0.700655266            0.003643999
#> gamma_13_13 0.704815539 -0.378828073            0.002027282
#> gamma_13_14 0.985926873  0.017638964            0.002791689
#> gamma_13_15 0.663192071  0.435510291            0.002628797
#> gamma_14_14 0.558293425 -0.585378228            0.003508762
#> gamma_14_15 0.240007737 -1.174967454            0.003667130
#> gamma_15_15 0.838158943 -0.204248973            0.004792967
```
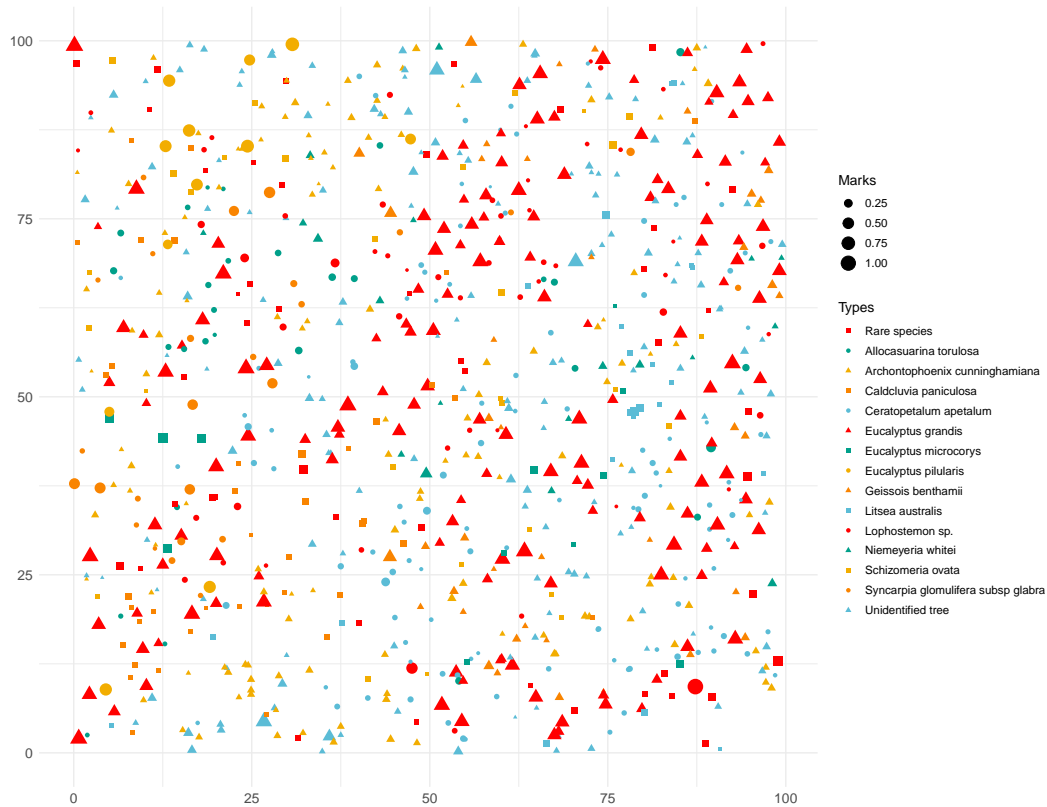
The model is also able to account for the trees' diameters, see code below. In this case, the distances matrices are measured as multiples of the trees' dbhs. We do not model any medium-range interactions to also illustrate how that would work.

```
load(ausplots(use_marks = TRUE, plot_minimum_abundance = 10, overall_minimum_abundance = 1))
Bruxner <- configurations$Bruxner
Bruxner_window <- windows$Bruxner
plot(Bruxner)
```

```r
ntypes <- nlevels(Bruxner$types)
tm <- Sys.time()
fit <- gibbsm(Bruxner,
              window = Bruxner_window,
              model = "square_exponential", # Short-range interaction
              short_range = matrix(50, ntypes, ntypes), # Matrix of short-range interaction ratios
              saturation = 10, # Saturation parameter
              fitting_package = "glmnet", # Fitting package
              min_dummy = 1e4, # Force at least 1e5 dummy points per species
              max_dummy = 1e4, # Force at maximum 1e5 dummy points per species
              dummy_distribution = "stratified") # Distribution of dummy points
print(Sys.time() - tm)
#> Time difference of 7.901089 secs
summ <- summary(fit)
print(summ)
#>          coefficients        se      CI95_lo      CI95_hi Ztest
#> beta0_1    0.157136228 0.92272133 -1.651364346  1.965636802
#> beta0_2   -2.665935639 1.29059337 -5.195452155 -0.136419123     *
#> beta0_3    0.862245936 0.40563799  0.067210094  1.657281779     *
#> beta0_4    0.147714991 0.62907072 -1.085240960  1.380670942
#> beta0_5   -0.151265280 0.43284225 -0.999620500  0.697089940
#> beta0_6   -5.070307863 0.90851344 -6.850961480 -3.289654246   ***
#> beta0_7   -2.510552428 2.72619204 -7.853790644  2.832685787
#> beta0_8   -4.732996302 3.46863483 -11.531395636  2.065403032
#> beta0_9   -2.679426982 0.77475658 -4.197921984 -1.160931979   ***
#> beta0_10  -1.476182872 1.10811023 -3.648039021  0.695673278
#> beta0_11  -0.986378478 0.85513259 -2.662407550  0.689650595
```

```
#> beta0_12   -2.141432750 1.31200020  -4.712905891  0.430040390
#> beta0_13   -1.970434943 2.36603223  -6.607772895  2.666903008
#> beta0_14   -5.155353652 1.87796175  -8.836091042 -1.474616262    **
#> beta0_15    0.024252321 0.46622540  -0.889532680  0.938037322
#> alpha_1_1  -0.185555795 0.13767744  -0.455398610  0.084287019
#> alpha_1_2  -0.047599287 0.10352409  -0.250502779  0.155304204
#> alpha_1_3  -0.212972999 0.05480726  -0.320393251 -0.105552747   ***
#> alpha_1_4  -0.129061306 0.08266817  -0.291087943  0.032965331
#> alpha_1_5  -0.004290146 0.06752582  -0.136638321  0.128058029
#> alpha_1_6  -0.102320644 0.06012706  -0.220167514  0.015526227
#> alpha_1_7  -0.121536660 0.28110683  -0.672495928  0.429422607
#> alpha_1_8   0.068692041 0.15255344  -0.230307212  0.367691294
#> alpha_1_9   0.069865133 0.10483800  -0.135613567  0.275343834
#> alpha_1_10  0.163422388 0.13379228  -0.098805653  0.425650429
#> alpha_1_11 -0.039596356 0.09155601  -0.219042841  0.139850129
#> alpha_1_12 -0.131152910 0.14502364  -0.415394021  0.153088201
#> alpha_1_13 -0.321656021 0.20045958  -0.714549570  0.071237529
#> alpha_1_14  0.139899867 0.13544700  -0.125571366  0.405371100
#> alpha_1_15 -0.044127705 0.05570128  -0.153300215  0.065044805
#> alpha_2_2   0.085404262 0.10300805  -0.116487797  0.287296320
#> alpha_2_3  -0.082565710 0.06531942  -0.210589418  0.045457998
#> alpha_2_4  -0.238184127 0.09942214  -0.433047941 -0.043320314    *
#> alpha_2_5  -0.085802485 0.08341034  -0.249283749  0.077678779
#> alpha_2_6   0.020983515 0.08359975  -0.142868976  0.184836006
#> alpha_2_7  -0.133485380 0.25469961  -0.632687447  0.365716687
#> alpha_2_8   0.087924740 0.20712464  -0.318032088  0.493881567
#> alpha_2_9  -0.092563640 0.18186803  -0.449018436  0.263891156
#> alpha_2_10  0.099823517 0.12537504  -0.145907046  0.345554080
#> alpha_2_11 -0.057229699 0.10668899  -0.266336276  0.151876879
#> alpha_2_12  0.053879394 0.15390064  -0.247760317  0.355519105
#> alpha_2_13 -0.029480279 0.23371700  -0.487557191  0.428596633
#> alpha_2_14  0.101366872 0.12346649  -0.140623008  0.343356752
#> alpha_2_15 -0.118339955 0.06642186  -0.248524408  0.011844499
#> alpha_3_3  -0.041359117 0.03305918  -0.106153917  0.023435682
#> alpha_3_4  -0.167408845 0.03817572  -0.242231887 -0.092585804   ***
#> alpha_3_5  -0.076833506 0.03415922  -0.143784352 -0.009882661    *
#> alpha_3_6  -0.093184297 0.03188510  -0.155677937 -0.030690657    **
#> alpha_3_7  -0.016563249 0.12221040  -0.256091236  0.222964738
#> alpha_3_8   0.038354655 0.09813263  -0.153981768  0.230691079
#> alpha_3_9  -0.030016990 0.05210163  -0.132134304  0.072100323
#> alpha_3_10 -0.029605132 0.06530410  -0.157598818  0.098388554
#> alpha_3_11 -0.160401897 0.05793005  -0.273942715 -0.046861079    **
#> alpha_3_12  0.021207364 0.06480471  -0.105807540  0.148222268
#> alpha_3_13 -0.091128137 0.08532900  -0.258369909  0.076113634
#> alpha_3_14 -0.064008530 0.07238638  -0.205883231  0.077866171
#> alpha_3_15 -0.079303888 0.02646284  -0.131170109 -0.027437667    **
#> alpha_4_4   0.088127276 0.04886427  -0.007644927  0.183899478
#> alpha_4_5  -0.047792768 0.04060971  -0.127386340  0.031800805
#> alpha_4_6  -0.027202890 0.03519722  -0.096188170  0.041782390
#> alpha_4_7  -0.126546126 0.11938654  -0.360539444  0.107447192
#> alpha_4_8  -0.009100681 0.10346108  -0.211880664  0.193679302
#> alpha_4_9  -0.233376096 0.11806082  -0.464771057 -0.001981135    *
#> alpha_4_10 -0.216735813 0.21505388  -0.638233674  0.204762049
```

```
#> alpha_4_11   -0.070968241 0.06934747  -0.206886784  0.064950302
#> alpha_4_12   -0.019688361 0.10313177  -0.221822912  0.182446191
#> alpha_4_13   -0.097131870 0.14030693  -0.372128407  0.177864666
#> alpha_4_14   -0.036836901 0.06919386  -0.172454377  0.098780575
#> alpha_4_15   -0.136592479 0.04106779  -0.217083859 -0.056101099   ***
#> alpha_5_5    -0.067773440 0.04780591  -0.161471301  0.025924422
#> alpha_5_6     0.024775033 0.04042133  -0.054449320  0.103999386
#> alpha_5_7     0.154634049 0.19142066  -0.220543558  0.529811656
#> alpha_5_8    -0.169863955 0.11159741  -0.388590858  0.048862948
#> alpha_5_9    -0.037852606 0.05631879  -0.148235404  0.072530193
#> alpha_5_10   -0.256450268 0.11477287  -0.481400957 -0.031499579    *
#> alpha_5_11    0.044440407 0.06470374  -0.082376603  0.171257416
#> alpha_5_12   -0.195429664 0.09178549  -0.375325927 -0.015533401    *
#> alpha_5_13   -0.194133498 0.12797008  -0.444950253  0.056683257
#> alpha_5_14   -0.233955644 0.12111469  -0.471336069  0.003424782
#> alpha_5_15   -0.121051580 0.03583183  -0.191280679 -0.050822481   ***
#> alpha_6_6     0.245826387 0.11538995   0.019666247  0.471986528    *
#> alpha_6_7    -0.087667982 0.14629743  -0.374405672  0.199069708
#> alpha_6_8    -0.018325407 0.18049750  -0.372093999  0.335443184
#> alpha_6_9     0.051251026 0.06245610  -0.071160687  0.173662738
#> alpha_6_10    0.012817439 0.07923648  -0.142483209  0.168118087
#> alpha_6_11   -0.056016766 0.06695647  -0.187249035  0.075215503
#> alpha_6_12   -0.056548594 0.07885644  -0.211104370  0.098007183
#> alpha_6_13   -0.020446996 0.13227577  -0.279702737  0.238808745
#> alpha_6_14    0.093764356 0.11185039  -0.125458385  0.312987097
#> alpha_6_15   -0.013633173 0.04608449  -0.103957115  0.076690769
#> alpha_7_7    -0.933975195 1.58088660  -4.032455999  2.164505610
#> alpha_7_8    -0.613848805 0.59291932  -1.775949320  0.548251710
#> alpha_7_9    -0.068290156 0.20049489  -0.461252912  0.324672601
#> alpha_7_10    0.186200396 0.27597934  -0.354709167  0.727109960
#> alpha_7_11   -0.444733656 0.33378199  -1.098934332  0.209467019
#> alpha_7_12    0.163656344 0.26630750  -0.358296768  0.685609456
#> alpha_7_13    0.520219122 0.38611406  -0.236550522  1.276988765
#> alpha_7_14    0.697233793 0.61803044  -0.514083620  1.908551205
#> alpha_7_15   -0.034801459 0.14309790  -0.315268192  0.245665274
#> alpha_8_8    -0.376405841 0.65045416  -1.651272562  0.898460879
#> alpha_8_9    -0.189018604 0.15992470  -0.502465258  0.124428050
#> alpha_8_10   -0.369073061 0.26321981  -0.884974416  0.146828294
#> alpha_8_11   -0.052102180 0.13268528  -0.312160559  0.207956199
#> alpha_8_12   -0.059947249 0.18565769  -0.423829628  0.303935129
#> alpha_8_13    0.296196443 0.33357624  -0.357600976  0.949993862
#> alpha_8_14    0.185370023 0.31879027  -0.439447428  0.810187473
#> alpha_8_15    0.037040075 0.10208485  -0.163042545  0.237122695
#> alpha_9_9     0.002941054 0.10190421  -0.196787520  0.202669627
#> alpha_9_10   -0.699600697 0.24486552  -1.179528306 -0.219673087   **
#> alpha_9_11   -0.015953205 0.09651140  -0.205112069  0.173205660
#> alpha_9_12    0.131788108 0.12318031  -0.109640859  0.373217074
#> alpha_9_13    0.178908369 0.16298819  -0.140542607  0.498359345
#> alpha_9_14   -0.220382795 0.22732388  -0.665929409  0.225163819
#> alpha_9_15   -0.049498592 0.05986475  -0.166831350  0.067834166
#> alpha_10_10   0.024141508 0.11957544  -0.210222051  0.258505066
#> alpha_10_11   0.038359418 0.11516479  -0.187359418  0.264078253
#> alpha_10_12   0.003746273 0.13170748  -0.254395644  0.261888191
```

```
#> alpha_10_13  0.104436534 0.21064990  -0.308429677  0.517302745
#> alpha_10_14 -0.410534242 0.27293840  -0.945483668  0.124415184
#> alpha_10_15 -0.010756329 0.06511745  -0.138384181  0.116871523
#> alpha_11_11 -0.104170251 0.10392323  -0.307856034  0.099515531
#> alpha_11_12  0.091039340 0.11547303  -0.135283637  0.317362317
#> alpha_11_13  0.197094439 0.14160748  -0.080451122  0.474639999
#> alpha_11_14  0.211126636 0.15155067  -0.085907221  0.508160492
#> alpha_11_15 -0.107406867 0.04933271  -0.204097204 -0.010716529     *
#> alpha_12_12  0.241256377 0.20630162  -0.163087366  0.645600120
#> alpha_12_13 -0.109361361 0.22110574  -0.542720656  0.323997935
#> alpha_12_14 -0.171447204 0.18809136  -0.540099491  0.197205084
#> alpha_12_15 -0.098848571 0.07011838  -0.236278071  0.038580929
#> alpha_13_13 -0.635629490 0.54115792  -1.696279528  0.425020548
#> alpha_13_14 -0.334154255 0.27548565  -0.874096198  0.205787688
#> alpha_13_15 -0.055891056 0.10774500  -0.267067377  0.155285266
#> alpha_14_14 -0.104176404 0.22154951  -0.538405470  0.330052662
#> alpha_14_15 -0.077921113 0.08813787  -0.250668164  0.094825939
#> alpha_15_15 -0.062647079 0.03918641  -0.139451030  0.014156872
#>                     Pval        Zval se_numerical_proportion
#> beta0_1     8.647770e-01  0.17029652              0.18608016
#> beta0_2     3.885997e-02 -2.06566662              0.08185909
#> beta0_3     3.353209e-02  2.12565383              0.27814890
#> beta0_4     8.143526e-01  0.23481460              0.23280487
#> beta0_5     7.267367e-01 -0.34946977              0.24061727
#> beta0_6     2.392996e-08 -5.58088373              0.07489315
#> beta0_7     3.571022e-01 -0.92090080              0.05482403
#> beta0_8     1.724063e-01 -1.36451271              0.04885190
#> beta0_9     5.433710e-04 -3.45841137              0.11821619
#> beta0_10    1.828068e-01 -1.33216247              0.17313382
#> beta0_11    2.487134e-01 -1.15348017              0.11183835
#> beta0_12    1.026396e-01 -1.63218935              0.08335727
#> beta0_13    4.049567e-01 -0.83280140              0.07409845
#> beta0_14    6.047667e-03 -2.74518566              0.05642211
#> beta0_15    9.585140e-01  0.05201845              0.22683002
#> alpha_1_1   1.777364e-01 -1.34775749              0.14682044
#> alpha_1_2   6.456673e-01 -0.45978947              0.12368718
#> alpha_1_3   1.019708e-04 -3.88585392              0.19435334
#> alpha_1_4   1.184773e-01 -1.56119708              0.15101283
#> alpha_1_5   9.493418e-01 -0.06353342              0.15786321
#> alpha_1_6   8.880405e-02 -1.70174037              0.12986675
#> alpha_1_7   6.654867e-01 -0.43235043              0.07182476
#> alpha_1_8   6.525072e-01  0.45028182              0.14719884
#> alpha_1_9   5.051488e-01  0.66641041              0.11358468
#> alpha_1_10  2.219106e-01  1.22146355              0.12275975
#> alpha_1_11  6.653909e-01 -0.43248231              0.12453286
#> alpha_1_12  3.658070e-01 -0.90435539              0.10881914
#> alpha_1_13  1.085834e-01 -1.60459294              0.12162118
#> alpha_1_14  3.016622e-01  1.03287538              0.09472011
#> alpha_1_15  4.282321e-01 -0.79222061              0.17782649
#> alpha_2_2   4.070462e-01  0.82910283              0.12720721
#> alpha_2_3   2.062192e-01 -1.26403008              0.14667811
#> alpha_2_4   1.658934e-02 -2.39568499              0.13546281
#> alpha_2_5   3.036304e-01 -1.02867923              0.12263250
```

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#> alpha_2_6    8.018143e-01   0.25099974           0.07339592
#> alpha_2_7    6.002163e-01  -0.52408945           0.07356391
#> alpha_2_8    6.712000e-01   0.42450160           0.05744384
#> alpha_2_9    6.107799e-01  -0.50896047           0.07847583
#> alpha_2_10   4.259162e-01   0.79619929           0.11066291
#> alpha_2_11   5.916710e-01  -0.53641617           0.08703512
#> alpha_2_12   7.262696e-01   0.35009207           0.09099939
#> alpha_2_13   8.996238e-01  -0.12613664           0.07867154
#> alpha_2_14   4.116422e-01   0.82100713           0.07573087
#> alpha_2_15   7.480768e-02  -1.78164169           0.13525651
#> alpha_3_3    2.109115e-01  -1.25106306           0.30304897
#> alpha_3_4    1.158700e-05  -4.38521745           0.23286645
#> alpha_3_5    2.449497e-02  -2.24927563           0.18423046
#> alpha_3_6    3.472300e-03  -2.92250326           0.17245458
#> alpha_3_7    8.921924e-01  -0.13553060           0.11970326
#> alpha_3_8    6.959118e-01   0.39084507           0.14737336
#> alpha_3_9    5.645315e-01  -0.57612385           0.14921901
#> alpha_3_10   6.503020e-01  -0.45334262           0.17543052
#> alpha_3_11   5.624777e-03  -2.76888917           0.13879179
#> alpha_3_12   7.434786e-01   0.32725033           0.14577145
#> alpha_3_13   2.855376e-01  -1.06796206           0.14573269
#> alpha_3_14   3.765548e-01  -0.88426205           0.14574231
#> alpha_3_15   2.728284e-03  -2.99680143           0.25330512
#> alpha_4_4    7.130789e-02   1.80351168           0.26166250
#> alpha_4_5    2.392433e-01  -1.17688025           0.21264233
#> alpha_4_6    4.395990e-01  -0.77287046           0.13026238
#> alpha_4_7    2.891583e-01  -1.05996979           0.11577063
#> alpha_4_8    9.299066e-01  -0.08796237           0.11299087
#> alpha_4_9    4.807051e-02  -1.97674461           0.10852599
#> alpha_4_10   3.135404e-01  -1.00782098           0.11521622
#> alpha_4_11   3.061321e-01  -1.02337175           0.13728569
#> alpha_4_12   8.486001e-01  -0.19090491           0.09123360
#> alpha_4_13   4.887607e-01  -0.69228133           0.11495885
#> alpha_4_14   5.944681e-01  -0.53237239           0.13010725
#> alpha_4_15   8.809404e-04  -3.32602497           0.21400986
#> alpha_5_5    1.562845e-01  -1.41767911           0.19928534
#> alpha_5_6    5.399294e-01   0.61291976           0.16753688
#> alpha_5_7    4.191924e-01   0.80782318           0.06952670
#> alpha_5_8    1.279806e-01  -1.52211378           0.12105574
#> alpha_5_9    5.015116e-01  -0.67211327           0.15060960
#> alpha_5_10   2.545575e-02  -2.23441542           0.13224177
#> alpha_5_11   4.921905e-01   0.68682897           0.12009015
#> alpha_5_12   3.323773e-02  -2.12919989           0.12853095
#> alpha_5_13   1.292610e-01  -1.51702252           0.10296340
#> alpha_5_14   5.339817e-02  -1.93168680           0.09168301
#> alpha_5_15   7.292877e-04  -3.37832525           0.17853645
#> alpha_6_6    3.313886e-02   2.13039691           0.07281293
#> alpha_6_7    5.490096e-01  -0.59924486           0.05676140
#> alpha_6_8    9.191320e-01  -0.10152721           0.05818043
#> alpha_6_9    4.118783e-01   0.82059275           0.09994003
#> alpha_6_10   8.714934e-01   0.16176184           0.09652831
#> alpha_6_11   4.028092e-01  -0.83661469           0.08799619
#> alpha_6_12   4.733074e-01  -0.71710815           0.07357682
```

```
#> alpha_6_13  8.771536e-01 -0.15457855         0.06492921
#> alpha_6_14  4.018614e-01  0.83830154         0.05989060
#> alpha_6_15  7.673599e-01 -0.29582996         0.16223481
#> alpha_7_7   5.546598e-01 -0.59079202         0.03143524
#> alpha_7_8   3.005293e-01 -1.03529904         0.04455315
#> alpha_7_9   7.333987e-01 -0.34060797         0.08602599
#> alpha_7_10  4.998730e-01  0.67468963         0.06441851
#> alpha_7_11  1.827263e-01 -1.33240760         0.06529219
#> alpha_7_12  5.388592e-01  0.61453899         0.06752867
#> alpha_7_13  1.778773e-01  1.34731982         0.06897916
#> alpha_7_14  2.592547e-01  1.12815444         0.03703433
#> alpha_7_15  8.078502e-01 -0.24320035         0.09833597
#> alpha_8_8   5.628040e-01 -0.57868158         0.04863551
#> alpha_8_9   2.372364e-01 -1.18192251         0.08311883
#> alpha_8_10  1.608711e-01 -1.40214772         0.07570109
#> alpha_8_11  6.945596e-01 -0.39267489         0.10273293
#> alpha_8_12  7.467776e-01 -0.32289129         0.07104881
#> alpha_8_13  3.745718e-01  0.88794226         0.06295955
#> alpha_8_14  5.609173e-01  0.58147955         0.04546399
#> alpha_8_15  7.167273e-01  0.36283618         0.14917886
#> alpha_9_9   9.769755e-01  0.02886096         0.12349144
#> alpha_9_10  4.275564e-03 -2.85708124         0.09408476
#> alpha_9_11  8.687089e-01 -0.16529866         0.10158831
#> alpha_9_12  2.846735e-01  1.06987968         0.10491989
#> alpha_9_13  2.723456e-01  1.09767691         0.11736371
#> alpha_9_14  3.323127e-01 -0.96946611         0.06605941
#> alpha_9_15  4.083276e-01 -0.82684034         0.13761318
#> alpha_10_10 8.400000e-01  0.20189353         0.15921305
#> alpha_10_11 7.390717e-01  0.33308287         0.10847115
#> alpha_10_12 9.773081e-01  0.02844389         0.10274453
#> alpha_10_13 6.200479e-01  0.49578251         0.08766147
#> alpha_10_14 1.325485e-01 -1.50412785         0.06667602
#> alpha_10_15 8.687995e-01 -0.16518352         0.15425654
#> alpha_11_11 3.161616e-01 -1.00237699         0.10459023
#> alpha_11_12 4.304607e-01  0.78840350         0.09210261
#> alpha_11_13 1.639720e-01  1.39183635         0.10592574
#> alpha_11_14 1.635868e-01  1.39310921         0.07086845
#> alpha_11_15 2.946612e-02 -2.17719366         0.14161068
#> alpha_12_12 2.422283e-01  1.16943521         0.09169449
#> alpha_12_13 6.208746e-01 -0.49461112         0.08886403
#> alpha_12_14 3.620266e-01 -0.91151027         0.06863998
#> alpha_12_15 1.586170e-01 -1.40973836         0.14004279
#> alpha_13_13 2.401656e-01 -1.17457301         0.07340572
#> alpha_13_14 2.251435e-01 -1.21296431         0.06656048
#> alpha_13_15 6.039459e-01 -0.51873456         0.11556974
#> alpha_14_14 6.381998e-01 -0.47021726         0.06427866
#> alpha_14_15 3.766519e-01 -0.88408209         0.11823706
#> alpha_15_15 1.098886e-01 -1.59869405         0.23995342
```