Function Variation

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0. Function

$$f(x) = 2x^3 - 3x^2 - 2x + 3$$

1. Domain of a function ($\mathbb{X}$)

$$x \in \mathbb{R}$$

2. Zero of a function (x_0)

$$\begin{gathered} f(x) = 0 \\ 2x^3 - 3x^2 - 2x + 3 = 0 \\ \vdots \\ (2x-3)(x-1)(x+1) = 0 \\ x_0 = -1 \quad \vee \quad x_0 = 1 \quad \vee \quad x_0 = \frac{3}{2} \end{gathered}$$

3. Intersection of OY axis (y_0)

$$\begin{gathered} f(0) = y_0 \\ f(0) = 2(0)^3 - 3(0)^2 - 2(0) + 3 \\ y_0 = 3 \end{gathered}$$

4. Even or odd function

4.1 Even function

$$\begin{gathered} \forall_{x \in \mathbb{X}} \, f(x) \neq f(-x) \\ \text{Not even} \end{gathered}$$

4.2 Odd function

$$\forall_{x \in \mathbb{X}} \ -f(x) \neq f(-x)$$
$$\text{Not odd}$$

5. Limits of a function

5.1 Limit in $-\infty$

$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} x^3(2 - \frac{3}{x} - \frac{2}{x^2} + \frac{3}{x^3}) = -\infty$$

5.2 Limit in ∞

$$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} x^3(2 - \frac{3}{x} - \frac{2}{x^2} + \frac{3}{x^3}) = \infty$$

6. Asymptotes of a functions

6.1 Vertical asymptote

$$\text{Does not exist}$$

6.2 Horizontal asymptote

$$\text{Does not exist}$$

6.3 Oblique asymptote

$$\text{Does not exist}$$

7. Monotonicity, maxima and minima of a function

7.1 Zero of a derivative

$$f'(x) = 6x^2 - 6x - 2$$
$$f'(x) = 0$$
$$6x^2 - 6x - 2 = 0$$
$$2(3x^2 - 3x - 1) = 0$$

$$x = \frac{1 - \sqrt{\frac{7}{3}}}{2} \approx -0.26 \quad \vee \quad x = \frac{1 + \sqrt{\frac{7}{3}}}{2} \approx 1.26$$

7.2 Sign of derivative

$$f'(x) > 0\,, \quad x \in (-\infty, \frac{1 - \sqrt{\frac{7}{3}}}{2}) \cup (\frac{1 + \sqrt{\frac{7}{3}}}{2}, +\infty)$$

$$f'(x) = 0\,, \quad x \in \{\frac{1 - \sqrt{\frac{7}{3}}}{2}, \frac{1 + \sqrt{\frac{7}{3}}}{2}\}$$

$$f'(x) < 0\,, \quad x \in (\frac{1 - \sqrt{\frac{7}{3}}}{2}, \frac{1 + \sqrt{\frac{7}{3}}}{2})$$

7.3 Monotonicity of a function

$$f \nearrow, \quad x \in (-\infty, \frac{1 - \sqrt{\frac{7}{3}}}{2}\rangle, \langle\frac{1 + \sqrt{\frac{7}{3}}}{2}, +\infty)$$

$$f \searrow, \quad x \in \langle\frac{1 - \sqrt{\frac{7}{3}}}{2}, \frac{1 + \sqrt{\frac{7}{3}}}{2}\rangle$$

7.4 Maxima and minima of a function

$$f''(x) = 12x - 6$$

$$f''(\frac{1 - \sqrt{\frac{7}{3}}}{2}) \approx -9.12 < 0 \Rightarrow \text{Maxima}$$

$$f''(\frac{1 + \sqrt{\frac{7}{3}}}{2}) \approx 9.12 > 0 \Rightarrow \text{Minima}$$

8. Concave or convex of a function

8.1 Zero of a second derivative

$$f''(x) = 12x - 6$$

$$f''(x) = 0$$
$$12x - 6 = 0$$
$$x = \frac{1}{2}$$

8.2 Sign of a second derivative

$$f''(x) < 0\,, \quad x \in (-\infty, \frac{1}{2})$$
$$f''(x) = 0\,, \quad x = \frac{1}{2}$$
$$f''(x) > 0\,, \quad x \in (\frac{1}{2}, -\infty)$$

8.3 Concave or convex of a function

$$x \in (-\infty, \frac{1}{2}) \Rightarrow \text{Concave function}$$
$$x \in (\frac{1}{2}, \infty) \Rightarrow \text{Convex function}$$

8.4 Inflection point

$$f''(\frac{1}{2}) = 0 \Rightarrow \text{Inflection point in } \frac{1}{2}$$

9. Table

x	$-\infty$	$(-\infty, -1)$	-1	$(-1, \frac{1-\sqrt{\frac{7}{3}}}{2})$	$\frac{1-\sqrt{\frac{7}{3}}}{2}$	$(\frac{1-\sqrt{\frac{7}{3}}}{2}, 1)$	1	$(1, \frac{1}{2})$
$f(x)$	$-\infty$	$\uparrow\rightarrow$	0	$\uparrow\rightarrow$	maxima	$\rightarrow\downarrow$	0	$\rightarrow\downarrow$
$f'(x)$		$+$	$+$	$+$	0	$-$	$-$	$-$
$f''(x)$		$-$	$-$	$-$	$-$	$-$	$-$	$-$

x	$\frac{1}{2}$	$(\frac{1}{2}, \frac{1+\sqrt{\frac{7}{3}}}{2})$	$\frac{1+\sqrt{\frac{7}{3}}}{2}$	$(\frac{1+\sqrt{\frac{7}{3}}}{2}, \frac{3}{2})$	$\frac{3}{2}$	$(\frac{3}{2}, +\infty)$	$+\infty$
$f(x)$	inflection point	$\downarrow\rightarrow$	minima	$\rightarrow\uparrow$	0	$\rightarrow\uparrow$	∞
$f'(x)$	$-$	$-$	0	$+$	$+$	$+$	
$f''(x)$	0	$+$	$+$	$+$	$+$	$+$	

10. Plot