# Function Variation

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## 0. Function

$$f(x) = 2x^3 - 3x^2 - 2x + 3$$

## 1. Domain of a function ( $\mathbb{X}$ )

$$x \in \mathbb{R}$$

## 2. Zero of a function ( $x_0$ )

$$f(x) = 0$$
$$2x^3 - 3x^2 - 2x + 3 = 0$$
$$\vdots$$
$$(2x - 3)(x - 1)(x + 1) = 0$$
$$x_0 = -1 \quad \vee \quad x_0 = 1 \quad \vee \quad x_0 = \frac{3}{2}$$

## 3. Intersection of OY axis ( $y_0$ )

$$f(0) = y_0$$
$$f(0) = 2(0)^3 - 3(0)^2 - 2(0) + 3$$
$$y_0 = 3$$

## 4. Even or odd function

### 4.1 Even function

$$\forall_{x \in \mathbb{X}} \, f(x) \neq f(-x)$$
$$\text{Not even}$$

### 4.2 Odd function

$$\forall_{x \in \mathbb{X}} \, -f(x) \neq f(-x)$$
$$\text{Not odd}$$

## 5. Limits of a function

### 5.1 Limit in $-\infty$

$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} x^3 (2 - \frac{3}{x} - \frac{2}{x^2} + \frac{3}{x^3}) = -\infty$$

### 5.2 Limit in $\infty$

$$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} x^3 (2 - \frac{3}{x} - \frac{2}{x^2} + \frac{3}{x^3}) = \infty$$

## 6. Asymptotes of a functions

### 6.1 Vertical asymptote

Does not exist

### 6.2 Horizontal asymptote

Does not exist

### 6.3 Oblique asymptote

Does not exist

## 7. Monotonicity, maxima and minima of a function

### 7.1 Zero of a derivative

$$f'(x) = 6x^2 - 6x - 2$$
$$f'(x) = 0$$
$$6x^2 - 6x - 2 = 0$$
$$2(3x^2 - 3x - 1) = 0$$
$$x = \frac{1 - \sqrt{\frac{7}{3}}}{2} \approx -0.26 \quad \vee \quad x = \frac{1 + \sqrt{\frac{7}{3}}}{2} \approx 1.26$$

### 7.2 Sign of derivative

$$f'(x) > 0, \quad x \in (-\infty, \frac{1 - \sqrt{\frac{7}{3}}}{2}) \cup (\frac{1 + \sqrt{\frac{7}{3}}}{2}, +\infty)$$

$$f'(x) = 0, \quad x \in \{\frac{1 - \sqrt{\frac{7}{3}}}{2}, \frac{1 + \sqrt{\frac{7}{3}}}{2}\}$$

$$f'(x) < 0, \quad x \in (\frac{1 - \sqrt{\frac{7}{3}}}{2}, \frac{1 + \sqrt{\frac{7}{3}}}{2})$$

### 7.3 Monotonicity of a function

$$f \nearrow, \quad x \in (-\infty, \frac{1 - \sqrt{\frac{7}{3}}}{2}), \langle \frac{1 + \sqrt{\frac{7}{3}}}{2}, +\infty)$$

$$f \searrow, \quad x \in \langle \frac{1 - \sqrt{\frac{7}{3}}}{2}, \frac{1 + \sqrt{\frac{7}{3}}}{2} \rangle$$

### 7.4 Maxima and minima of a function

$$f''(x) = 12x - 6$$

$$f''(\frac{1 - \sqrt{\frac{7}{3}}}{2}) \approx -9.12 < 0 \Rightarrow \text{Maxima}$$

$$f''(\frac{1 + \sqrt{\frac{7}{3}}}{2}) \approx 9.12 > 0 \Rightarrow \text{Minima}$$

## 8. Concave or convex of a function

### 8.1 Zero of a second derivative

$$f''(x) = 12x - 6$$

$$f''(x) = 0$$
$$12x - 6 = 0$$
$$x = \frac{1}{2}$$

## 8.2 Sign of a second derivative

$$f''(x) < 0, \quad x \in (-\infty, \frac{1}{2})$$
$$f''(x) = 0, \quad x = \frac{1}{2}$$
$$f''(x) > 0, \quad x \in (\frac{1}{2}, -\infty)$$

## 8.3 Concave or convex of a function

$$x \in (-\infty, \frac{1}{2}) \Rightarrow \text{Concave function}$$
$$x \in (\frac{1}{2}, \infty) \Rightarrow \text{Convex function}$$

## 8.4 Inflection point

$$f''(\frac{1}{2}) = 0 \Rightarrow \text{Inflection point in } \frac{1}{2}$$

# 9. Table

| $x$ | $-\infty$ | $(-\infty, -1)$ | $-1$ | $(-1, \frac{1-\sqrt{\frac{7}{3}}}{2})$ | $\frac{1-\sqrt{\frac{7}{3}}}{2}$ | $(\frac{1-\sqrt{\frac{7}{3}}}{2}, 1)$ | $1$ | $(1, \frac{1}{2})$ | $\frac{1}{2}$ | $(\frac{1}{2}, \frac{1+\sqrt{\frac{7}{3}}}{2})$ | $\frac{1+\sqrt{\frac{7}{3}}}{2}$ | ( |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | $-\infty$ | $\uparrow\rightarrow$ | $0$ | $\uparrow\rightarrow$ | maxima | $\rightarrow\downarrow$ | $0$ | $\rightarrow\downarrow$ | inflection point | $\downarrow\rightarrow$ | minima | |
| $f'(x)$ | | $+$ | $+$ | $+$ | $0$ | $-$ | $-$ | $-$ | $-$ | $-$ | $0$ | |
| $f''(x)$ | | $-$ | $-$ | $-$ | $-$ | $-$ | $-$ | $-$ | $0$ | $+$ | $+$ | |

# 10. Plot