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A Variance-Ratio Test of Random Walks in Foreign Exchange Rates

CHRISTINA Y. LIU and JIA HE*

ABSTRACT

The separate variance-ratio tests under homoscedasticity and heteroscedasticity both provide evidence rejecting the random walk hypothesis, using five pairs of weekly nominal exchange rate series over the period from August 7, 1974 to March 29, 1989. The rejections cast doubt on the random walk hypothesis in exchange rates, which has received support in the existing literature. Furthermore, since the rejections are robust to heteroscedasticity, they suggest autocorelations of weekly increments in the nominal exchange rate series, which may be consistent with the exchange rate overshooting or undershooting phenomenon.

IN THE PAST DECADE, there has been a consensus that nominal exchange rates follow a random walk process. Since a "unit root" and "uncorrelated increments" are both required for a random walk process, the random walk is usually supported in the existing literature either because a "unit root" component is detected in the exchange rate series or because the increment in the exchange rate is found to be serially uncorrelated. For example, Meese and Singleton (1982) and Baillie and Bollerslev (1989) both find a "unit root" component in the exchange rate series, and the evidence provided in Giddy and Dufey (1975), Cornell and Dietrich (1978), Logue, Sweeney, and Willett (1978), and Hsieh (1988) all suggest that the exchange rate series contains uncorrelated increments.

In this paper, the random walk hypothesis for the nominal exchange rate series is reexamined, by applying a variance-ratio test developed in Lo and MacKinlay (1988) to five pairs of weekly nominal exchange rate series over the period from August 7, 1974 to March 29, 1989. While there are two implications of the random walk (unit root and uncorrelated increments), this paper focuses on the uncorrelated increments aspect. This is not only because there are some important departures from the random walk that unit root test cannot detect, but also because the autocorrelation aspect may yield

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interesting implications for alternative exchange rate models.[1] In particular, while an uncorrelated increment is consistent with the random walk model, the correlated increment suggests the possibility of either exchange rate overshooting (Dornbusch (1976)) or undershooting (Frenkel and Rodriguez (1982)).

The variance-ratio test is first implemented in this paper under the maintained hypothesis of homoscedasticity and then allows for possible heteroscedasticity in the null hypothesis.[2] Contrary to the suggestions in the literature, the random walk hypothesis is rejected for most of the foreign exchange rate series tested in this paper. To provide a comparison, the standard and heteroscedasticity-adjusted Box-Pierce Q tests are also performed. It is found that while the Box-Pierce tests suggest rejections of the random walk only due to heteroscedasticity, the variance-ratio tests suggest rejections due to autocorrelation. Therefore, the variance-ratio test results support the overshooting or undershooting hypotheses. The remainder of this paper is organized as follows: Section I describes the testing methodology; Section II presents the empirical results for the variance-ratio test and also provides a comparison with the Box-Pierce tests; Section III discusses the implications derived from these results; and Section IV summarizes and concludes the paper.

I. Testing Methodology

In testing the uncorrelatedness of increments in the exchange rate series, a variance-ratio test developed in Lo and MacKinlay (1988) is adopted. Similar to other variance-ratio tests, the test in Lo and MacKinlay exploits the fact that the variance of the increments in a random walk is linear in the sampling interval. That is, if a series follows a random walk process, the variance of its q-differences would be q times the variance of its first differences. Therefore, if we obtain $nq + 1$ nominal exchange rate observations S_0, S_1, $S_2, \ldots, S_{nq}$ at equally spaced intervals (q is any integer greater than one), the ratio of $1/q$ of the variance $S_t - S_{t-q}$ to the variance of $S_t - S_{t-1}$ would be equal to one.[3]

However, while the use of a point-estimate of the variance-ratio is not uncommon, the variance-ratio test-statistic (a Z-statistic) developed in Lo and

[1] This is because the random walk model is a proper subset of the unit root null hypothesis. Moreover, even though the variance-ratio test and unit root test are not direct competitors, it is shown in another paper by Lo and MacKinlay (1989) that the variance-ratio test is more reliable than the Dickey-Fuller test which is developed in Dickey and Fuller (1979) for detecting the unit root component and adopted in Meese and Singleton (1982) for examining the exchange rate series.

[2] In view of the increasing evidence that the exchange rate often possesses time-varying volatilities and deviations from normality, it is important to adopt a test which is robust to heteroscedasticity and nonnormality.

[3] While this variance-ratio would be exactly equal to one only under homoscedasticity, it still approaches one under the specification of the heteroscedasticity in Lo and MacKinlay (1988).

MacKinlay is unique for the following reasons.[4] First, after deriving an asymptotic distribution of the variance-ratio, the Z-statistic is developed by comparing the sample variance-ratio with the asymptotic variance of this variance-ratio, which hence provides an asymptotic standard normal test-statistic for the variance-ratio. Second, the refined Z^*-statistic, which is heteroscedasticity-consistent and able to use overlapping data, allows a more efficient and powerful test. Actually, it is shown in the Monte Carlo experiment performed in Lo and MacKinlay (1989) that under a heteroscedasticity random walk null, this variance-ratio test is more reliable than the Box-Pierce Q test, which is often adopted in the literature for detecting serial correlations. Moreover, the variance-ratio test is also shown to be as powerful as or more powerful than either the Box-Pierce or Dickey Fuller test against several interesting alternative hypotheses, including an AR(1), an ARIMA (1,1,1), and an ARIMA (1,1,0)

In testing the random walk in the exchange rate series in this paper, both the Z- and Z^*-statistics are calculated for various q's. By using 1-week as our base observation interval, Z- and Z^*-statistics are calculated for each q by comparing the variance of the base interval with that of 2-week, 4-week, 8-week, and 16-week observation intervals. That is, the variance-ratio, VR(q), for each interval q, and the variance of each variance-ratio, ϕ(q), will be calculated and used to generate the corresponding Z-statistics, Z(q), for each of the intervals q = 2, 4, 8, and 16. Similarly, the heteroscedasticity-consistent Z^*-statistics, Z^*(q), will also be calculated for each of the intervals q = 2, 4, 8, and 16. Note that since the Z- and Z^*-statistics are both asymptotic standard normal, the conventional critical value applies when they are adopted to test the random walk hypothesis. The formulas for the calculations are presented in the Appendix.

II. Empirical Results

To test the random walk in exchange rate series, we focus on the 763-week time span from August 7, 1974 to March 29, 1989 and examine five pairs of foreign currencies (Can.$/U.S.$, FF/U.S.$, DM/U.S.$, Yen/U.S.$, and Pound/U.S.$). All exchange rates are logs of the nominal rates on Wednesday, which are taken from the *International Financial Statistics* published by the International Montetary Fund. The data starts in 1974 due to the fact that most of the countries shifted into a flexible exchange rate regime around that time.

A. The Variance-Ratio Tests

By using 1-week as our base observation interval, the random walk hypothesis is tested by calculating the VR(q), ϕ(q), and the Z(q) for each of the cases q = 2, 4, 8, and 16. In addition, the heteroscedasticity-consistent variance-

[4] See, for example, Huizinga (1987), Fama and French (1988), and Cochrane (1988).

ratio test is also performed by calculating the VR(q), $\phi^*(q)$, and the $Z^*(q)$ for each of the cases $q = 2$, 4, 8, and 16. All these calculations are based on the formulas presented in the Appendix. The results from these calculations are presented in Tables I and II. While the variance-ratios VR(q) are reported in the main rows in each table, the Z- and Z^*-statistics are given in parentheses in Tables I and II, respectively.

It is shown in Table I that under the maintained hypothesis of homoscedasticity, there is evidence rejecting the random walk hypothesis for most of the currencies tested. For example, for the Japanese Yen, the Z-statistic associated with intervals q = 2, 4, 8, and 16 are 0.93, 2.47, 2.87, and 2.01, respectively. Compared with the conventional critical value (which is 1.96 for the five percent level), three out of these four Z's indicate that the variance-ratios are significantly different from one at the five percent level. The random walk hypothesis is therefore rejected for the Japanese Yen for three out four interval lengths examined. Based on a similar analysis, we can easily see that there are also rejections for the French franc, the DM, and the British Pound. It may be interesting to note at this point that the DM provides the strongest rejection among the currencies rejected (four rejections out of the four cases examined), while the British pound and the French franc provide only weak rejections (one rejection out of the four cases). Note that as

Table I

Estimates of Variance-Ratios VR(q) and Variance-Ratio Test Statistics $Z(q)$

Variance-ratio test of the random walk hypothesis for weekly nominal exchange rate series, for the sample period from August 1974 to March 1989. One-week is taken as a base observation interval. The variance ratio VR(q) is defined as $\sigma_c^2(q)/\sigma_a^2(q)$; where $\sigma_c^2(q)$ is an unbiased estimator of 1/q of the variance of the qth difference of the nominal exchange rate S_t, and $\sigma_a^2(q)$ is an unbiased estimator of the variance of the first difference of S_t. The variance ratios VR(q) are reported in the main rows. The homoscedasticity test statistic $Z(q)$, which tests the null hypothesis that VR(q) equals one, is given in the parenthesis immediately below each of the main row entries.

	Number q of base observations forming variance ratio			
Currency	2	4	8	16
	1.07	1.10	1.03	0.98
CAN$/US$	(1.90)	(1.62)	(0.37)	(−0.12)
	0.92	0.94	0.93	1.04
FF/US$	(−2.32)*	(−0.89)	(−0.61)	(0.24)
	1.08	1.20	1.22	1.36
DM/US$	(2.40)*	(3.23)*	(2.15)*	(2.29)*
	1.03	1.15	1.29	1.32
Yen/US$	(0.93)	(2.47)*	(2.87)*	(2.01)*
	1.02	1.04	1.17	1.33
Pound/US$	(0.64)	(0.66)	(1.66)	(2.13)*

*Variance ratios are statistically different from one at the five percent level of significance.

Table II

Estimates of Variance-Ratios VR(q) and Variance-Ratio Test Statistics $Z^*(q)$

Variance-ratio test of the random walk hypothesis for weekly nominal exchange rate series, for the sample period from August 1974 to March 1989. One-week is taken as a base observation interval. The variance ratio VR(q) is defined as $\sigma_c^2(q)/\sigma_a^2(q)$; where $\sigma_c^2(q)$ is an unbiased estimator of 1/q of the variance of the qth difference of the nominal exchange rate S_t, and $\sigma_a^2(q)$ is an unbiased estimator of the variance of the first difference of S_t. The variance ratios VR(q) are reported in the main rows. The heteroscedasticity-robust test statistic $Z^*(q)$, which tests the null hypothesis that VR(q) equals one, is given in the parenthesis immediately below each of the main row entries.

	Number q of base observations forming variance ratio			
Currency	2	4	8	16
	1.07	1.10	1.03	0.98
CAN$/US$	(1.44)	(1.16)	(0.43)	(−0.16)
	0.92	0.94	0.93	1.04
FF/US$	(−0.68)	(−0.43)	(−0.46)	(0.24)
	1.08	1.20	1.22	1.36
DM/US$	(2.16)*	(2.35)*	(2.57)*	(3.08)*
	1.03	1.15	1.29	1.32
Yen/US$	(0.59)	(2.23)*	(3.21)*	(2.60)*
	1.02	1.04	1.17	1.33
Pound/US$	(0.51)	(0.67)	(1.82)	(2.63)*

*Variance ratios are statistically different from one at the five percent level of significance.

shown in Lo and MacKinlay (1988), the variance-ratios associated with each q are not independent of each other. In fact, it is shown explicitly in Lo and MacKinlay that the variance-ratio (for each q) minus one is approximately $q - 1$ times the weighted sum of the first $q - 1$ autocorrelation coefficients. Under this scenario, the probability of rejection when one of the four statistics is large and three of them are small (as in the case of the pound) is not as high as when all four statistics are large (as in the case of the DM). Nevertheless, the Z-statistics in Table I present evidence rejecting the random walk hypothesis for most of the currencies tested, with the Canadian dollar being the only exception.

Further, since the results obtained from these $Z(q)$'s are under the maintained hypothesis of homoscedasticity, the rejections of the random walk may either be due to heteroscedasticity or to serial correlation. To investigate this issue, a heteroscedasticity-consistent variance-ratio test (the Z^*-test) is also implemented. The test results, presented in Table II, indicate that most of the rejections under homoscedasticity are robust to heteroscedasticity. This implies that the variance-ratio is different from one due to autocorrelation, rather than to heteroscedasticity. In other words, the random walk is rejected because of autocorrelations of weekly increments in the exchange rate series.

B. The Box-Pierce Q Tests

Note that while the above variance-ratio tests suggest that the rejection of the random walk is due to autocorrelation of weekly increments in exchange rates rather than to changes in their variances, they provide an interesting contrast with the findings in Hsieh (1988). By adopting a heteroscedasticity-adjusted Box-Pierce Q test, Hsieh (1988) finds that the rejection of the identical independently distributed (IID) hypothesis in the daily exchange rate series in his paper is primarily due to changes in variances. To explore further the discrepancy between these two sets of results, the standard and heteroscedasticity-adjusted Box-Pierce Q tests employed in Hsieh (1988) are also applied to the same data set used in this paper. To generate a test which is comparable with our previous variance-ratio tests, the Box-Pierce Q statistics of order 15 are estimated.[5]

Table III reports the estimates of each of the first 15 autocorrelation coefficients, as well as the standard and the adjusted Box-Pierce Q statistics. Based on the estimated standard Box-Pierce Q statistic, the null hypothesis of IID is not rejected for either the Canadian dollar or the French franc but is rejected for the DM, the Yen, and the British pound. In contrast, the heteroscedasticity-adjusted Box-Pierce Q test does not provide any rejection for any currency. Thus, similar to the findings in Hsieh (1988), the rejections of the random walk using the Box-Pierce test seem to be caused by heteroscedasticity, rather than by serial correlation.

However, it is also interesting to note that while the Box-Pierce and the variance-ratio test agree with one another under the assumption of homoscedasticity, they disagree when allowance is made for heteroscedasticity. Specifically, both the variance-ratio test and the Box-Pierce test reject the null of IID. The Box-Pierce results suggest that the IID is rejected due to heteroscedasticity, while the variance-ratio test suggests that autocorrelation is also present. The difference may be due to the particular autocorrelation function of exchange rate increments, or to the nature of the correction for heteroscedasticity used in these two tests.[6,7]

[5] Since the variance-ratio computed with an aggregation value q is approximately a linear combination of the first $q - 1$ autocorrelation coefficients, it is comparable with the Box-Pierce Q statistic of order $q - 1$, which is a linear combination of the first $q - 1$ *squared* autocorrelations. To present a contrasting result with our variance-ratio test of order 16 (the case of $q = 16$ presented in Tables I and II), the particular order of 15 is chosen for the Box-Pierce Q test.

[6] The basic difference between the Box-Pierce Q test and variance-ratio test is that the former adds up the squares of the autocorrelations while the latter adds up the autocorrelations themselves (with weights). Therefore, if the exchange rate increment exhibits small autocorrelations of the same sign for many lags, it is more likely that the Box-Pierce Q test does not reject the null while the variance-ratio test does. However, if the exchange rate autocorrelations are large and of varying signs, then the Box-Pierce Q test may be more powerful than the variance-ratio test. We are grateful to the referee for this insight.

[7] The heteroscedasticity-consistent estimators for the variance of autocorrelations are derived under different procedures for the Box-Pierce Q test and for the variance-ratio test. See Diebold (1986) and Lo and MacKinlay (1988) for more details.

Table III

Autocorrelation Coefficients and Standard and Heteroscedasticity-Adjusted Box-Pierce Q Statistics

Autocorrelation coefficients, standard and heteroscedasticity-adjusted Box-Pierce Q statistics for the first order difference of weekly nominal exchange rate series, for the sample period from August 1974 to March 1989 are reported in each columns. Heteroscedasticity-adjusted Box-Pierce Q statistic is described in Diebold (1986).

lag	Can$	FF	DM	Yen	Pound
1	0.07353	−0.08122	0.09163	0.03931	0.02629
2	−0.00925	0.06560	0.07808	0.07702	−0.01199
3	0.01125	0.00797	−0.00681	0.06123	0.04151
4	−0.04504	0.01582	0.00524	0.03710	0.07852
5	−0.01757	−0.02200	−0.00257	0.02979	0.05495
6	−0.04109	−0.06277	−0.08589	−0.04448	−0.03698
7	−0.03829	0.02406	0.01172	0.03861	−0.00042
8	0.00660	0.01266	0.06878	−0.04911	0.11803
9	−0.03561	0.04397	0.02615	−0.05099	−0.03417
10	0.00147	0.04628	0.06508	−0.04398	−0.00022
11	0.03490	0.05807	0.05823	0.05532	0.06204
12	0.02727	−0.01409	0.01040	0.06764	−0.01366
13	0.03176	0.02509	−0.02049	0.03516	−0.04832
14	0.02348	0.01242	0.00918	0.02271	0.01613
15	0.00947	0.01744	0.03620	0.00951	0.08924
Box-Pierce Q(15)	12.23509	19.09345	28.26979*	25.47353*	32.65846*
Adjusted Box-Pierce Q(15)	5.34453	7.36005	12.37362	10.95570	12.73092

*Significant at the five percent level.

C. The Variance-Ratio Test for Subperiods

To further check the robustness of the variance-ratio test results presented in Section II.A., the Z- and Z^*-statistics are also computed for two subperiods: from August 7, 1974 to October 10, 1979 and from October 17, 1979 to March 29, 1989. The reason to choose October 1979 as a cut-off point is because institutionally, the Fed changed its operating procedure in October, 1979, which is suspected to have caused some structural change. In addition, it has been reported by many studies that the relation between some of the economic variables seems to have had a reversal since the latter part of 1979 (Frenkel (1983) and Huizinga and Leiderman (1987)). It has therefore been common to examine the economic times series by splitting the data in reference to this date.

Before presenting our test results for these two subperiods, since both the Z- and Z^*-statistics are asymptotic standard normal, let us first note some finite sample properties of the variance-ratio test. Based on the Monte Carlo experiments performed in Lo and MacKinlay (1989), the empirical sizes of two-sided five percent variance-ratio test statistics are close to their nominal values for sample sizes greater than 32. This conclusion has been obtained in their work under the null hypothesis of random walks, with either homoscedastic or heteroscedastic disturbances. Since the sample size for each of our two subperiods is much greater than 32 (270 and 473, respectively), the adoption of the Z- and Z^*-statistics seems justifiable.[8]

The test results for these two subperiods are reported under homoscedasticity and heteroscedasticity in Tables IV and V, respectively. Again, the variance-ratios, VR(q) are reported in the main rows, and the Z- and Z^*-statistics are given in parentheses immediately below each of the main rows.

Compared with the test results from the overall sample period, it can be seen from Tables IV and V that the previous rejections seem to be robust. In particular, Table IV shows that under homoscedasticity, there is evidence leading to rejection for Can.$, DM, and British pound for the first subperiod, and rejection for the FF, DM, and Yen for the second subperiod. These rejections are also robust to heteroscedasticity, as indicated by the Z^*-statistics reported in Table V.

Overall, the variance-ratio tests performed in this paper provide evidence rejecting the random walk hypothesis for weekly nominal exchange rate series. These rejections are in general consistent under homoscedasticity or heteroscedasticity and are also robust for either the overall period or for our two subperiods. Even though the detailed reasons for these rejections are beyond the scope of this paper, some discussions of the implications derived from the rejections may still be helpful at this point.

III. Economic Implications

Note that since the rejections obtained from the variance-ratio test are robust to heteroscedasticity, they suggest autocorrelation of weekly increments in the exchange rate series. Further note that the existence of autocorrelation in financial assets does not necessarily imply any market inefficiency (Leroy (1973), Lucas (1978), and Levich (1979)). For the case of exchange rates, there are several alternative explanations for autocorrelation, including an exchange rate overshooting or undershooting phenomenon, risk aversion, and official interventions in foreign exchange markets.[9] In this

[8] Note that Monte Carlo simulations are limited by "specificity", the fact that simulations are performed for a specific data-generating process (DGP) and, consequently, the results of simulations apply for that DGP only. However, since the specific DGP's selected by Lo and MacKinlay (1989) are particularly relevant to empirical works in the stock prices and exchange rates (those DGP's include IID Gaussian returns, heteroscedastic Gaussian returns, an AR(1), and an integrated AR(1)), the application in this paper regarding the finite sample property derived in Lo and MacKinlay (1989) seems justifiable.

[9] The government intervention for the first subperiod in our sample is evidenced quantitatively in Taylor (1982). With government interventions, the increment in exchange rates may be positively or negatively correlated, depending on the objective of the intervention policy.

Table IV

Estimates of Variance-Ratios VR(q) and Variance-Ratio Test Statistics Z(q) for Subperiods Weekly: 1974:08 to 1979:10 and Weekly: 1979:10 to 1989:03

Variance-ratio test of the random walk hypothesis for weekly nominal exchange rate series. One-week is taken as a base observation interval. The variance ratio VR(q) is defined as $\sigma_c^2(q)/\sigma_a^2(q)$; where $\sigma_c^2(q)$ is an unbiased estimator of 1/q of the variance of the qth difference of the nominal exchange rate S_t and $\sigma_a^2(q)$ is an unbiased estimator of the variance of the first difference of S_t. The variance ratios VR(q) are reported in the main rows. The homoscedasticity test statistic Z(q), which tests the null hypothesis that VR(q) equals one, is given in the parenthesis immediately below each of the main row entries.

Time period	Number nq of base observations	Number q of base observations Aggregated to form variance ratio			
		2	4	8	16
A. CAN$/US$					
740807-791010	270	1.15	1.35	1.33	1.21
		(2.53)*	(3.34)*	(1.92)	(0.75)
791017-890329	473	1.02	1.01	0.95	0.89
		(0.50)	(0.20)	(−0.40)	(−0.55)
B. FF/US$					
740807-791010	270	0.70	0.59	0.47	0.43
		(−4.96)*	(−3.85)*	(−2.96)*	(−2.08)*
791010-890329	473	1.05	1.16	1.20	1.38
		(1.16)	(2.10)*	(1.64)	(1.96)*
C. DM/US$					
740807-791010	270	1.15	1.35	1.33	1.21
		(2.43)*	(2.38)*	(0.79)	(0.80)
791010-890329	473	1.06	1.16	1.20	1.35
		(1.31)	(2.07)*	(1.50)	(2.26)*
D. Yen/US$					
740807-791010	270	1.02	1.19	1.33	1.52
		(0.41)	(1.76)	(1.89)	(1.90)
791010-890329	473	1.03	1.14	1.27	1.24
		(0.63)	(1.75)	(2.03)*	(1.20)
E. Pound/US$					
740807-791010	270	1.09	1.21	1.52	1.77
		(1.14)	(1.98)	(3.00)*	(2.83)*
791010-890329	473	1.01	1.00	1.08	1.18
		(0.18)	(0.06)	(0.62)	(0.95)

*Variance ratios are statistically different from one at the five percent level of significance.

paper, since the adoption of the variance-ratio test also provides an easy way to distinguish between the overshooting and undershooting hypotheses, the possibility of overshooting or undershooting will be briefly examined below.

First, note that since the estimated variance-ratios, VR(q), associated with each q for most of the currencies are all greater than one, and since the variance-ratio minus one is approximately q − 1 times the weighted sum of the first q − 1 autocorrelation coefficients, the estimated variance-ratios

Table V

Estimates of Variance-Ratios VR(q) and Heteroscedasticity-Robust Variance-Ratio Test Statistics Z*(q) for Subperiods Weekly: 1974:08 to 1979:10 and Weekly: 1979:10 to 1989:03

Variance-ratio test of the random walk hypothesis for weekly nominal exchange rate series. One-week is taken as a base observation interval. The variance ratio VR(q) is defined as $\sigma_c^2(q)/\sigma_a^2(q)$; where $\sigma_c^2(q)$ is an unbiased estimator of 1/q of the variance of the qth difference of the nominal exchange rate S_t and $\sigma_a^2(q)$ is an unbiased estimator of the variance of the first difference of S_t. The variance ratios VR(q) are reported in the main rows. The heteroscedasticity-robust test statistic $Z^*(q)$, which tests the null hypothesis that VR(q) equals one, is given in the parenthesis immediately below eachof the main row entries.

Time period	Number nq of base observations	Number q of base observations Aggregated to form variance ratio			
		2	4	8	16
A. Can$/US$					
740807-791010	270	1.15	1.35	1.33	1.21
		(1.47)	(2.81)*	(2.23)*	(1.03)
791017-890329	473	1.02	1.01	0.95	0.89
		(0.41)	(0.21)	(−0.48)	(−0.76)
B. FF/US$					
740807-791010	270	0.96	1.07	1.02	1.10
		(−.45)	(0.61)	(0.15)	(0.53)
791010-890329	473	1.05	1.16	1.20	1.38
		(1.06)	(2.25)*	(2.08)*	(2.78)*
C. DM/US$					
740807-791010	270	1.15	1.35	1.33	1.21
		(2.07)*	(2.30)*	(0.94)	(1.03)
791010-890329	473	1.06	1.16	1.20	1.35
		(1.23)	(2.27)*	(1.86)	(2.50)*
D. Yen/US$					
740807-791010	270	1.02	1.19	1.33	1.52
		(0.29)	(1.60)	(1.84)	(2.19)*
791010-890329	473	1.03	1.14	1.27	1.24
		(0.41)	(1.63)	(2.41)*	(1.64)
E. Pound/US$					
740807-791010	270	1.09	1.21	1.52	1.77
		(1.11)	(1.97)*	(3.57)*	(4.07)*
791010-890329	473	1.01	1.00	1.08	1.18
		(0.15)	(0.07)	(0.72)	(1.23)

*Variance ratios are statistically different from one at the five percent level of significance.

obtained here suggest positive serial correlations, with the French franc as an exception.[10] Due to the evidence of these positive autocorrelations, the rejection of the random walk in this paper may be attributed to the exchange rate undershooting phenomenon, this provides an interesting contrast with the findings in Huizinga (1987).

By using a statistical technique designed to concentrate on the long-run behavior of time-series variables (i.e., the spectral analysis), Huizinga (1987) provides evidence of long-run negative serial correlations for the monthly real exchange rate. Interestingly, while he has found negative autocorrelations over the long horizon, he has also found positive autocorrelations for the first 4 years, which creates the "striking feature of the humped-shaped pattern for the serial correlation". It is explained in his paper that this humped-shaped pattern is evidence of positive serial correlations of the real exchange rate changes at low lags, which is more than fully offset by negative serial correlations at higher lags. In view of the evidence provided in his work and the close movement between nominal and real exchange rates (Mussa (1979, 1986)), it is not surprising that our results indicate positive weighted sum of serial correlations up to the first fifteen lags. While it would be interesting to determine if these positive autocorrelations in nominal exchange rates will also be offset by potential negative serial correlations at higher lags, we leave this question to future research.

IV. Conclusions

In this paper, the random walk hypothesis is tested by adopting a variance-ratio test developed in Lo and Mackinlay (1988), using five pairs of weekly nominal exchange rate series (the Can.$, FF, DM, Yen, Pound, all relative to the U.S.$) over the period from August 7, 1974 to March 29, 1989. It is found that contrary to previous suggestions in the literature, the random walk hypothesis is rejected at the five percent significance level. These rejections are robust to heteroscedasticity, as well as to the two sub-sample periods.

To provide comparisons, the standard and the heteroscedasticity-adjusted Box-Pierce Q tests are also applied to the same data set employed in this paper. It is found that while the variance-ratio tests reject the random walk hypothesis under either the assumption of homoscedasticity or heteroscedasticity, the Box-Pierce Q tests reject the random walk only under the null of homoscedasticity. Therefore, while the Box-Pierce Q tests suggest that those rejections may be caused by changes in variances, the variance-ratio tests suggest that they are due to autocorrelation. Lastly, a further examination of the estimated variance-ratios provides evidence showing positive serial correlations for most of the currencies examined. Due to these positive serial correlations, the rejection of the random walk in this paper may be attributed to the exchange rate undershooting phenomenon.

[10] Note that for the first subperiod, all the estimated variance-ratios for the French franc are less than one, which, different from other currencies, suggest exchange rate overshooting. Since it has been reported previously that the FF/U.S.$ sometimes behaves differently from other currencies over this particular period (for example, Taylor (1982) shows that the Central Bank in France was the only one which did not realize a loss form its intervention operations; in addition, Frenkel (1983) and Liu (1986) also show different behavior of the FF/U.S.$ from other foreign currencies), the underlying reasons for this particular result of the FF/U.S.$ may be closely tied to that of the other findings.

Appendix

This appendix presents the formulas for calculating the variance-ratio, the variance for the variance-ratio, and the variance-ratio test-statistics. The variance-ratio, VR(q):

$$VR(q) = \frac{\sigma_c^2(q)}{\sigma_a^2(q)}, \tag{1}$$

where σ_c^2(q) is an unbiased estimator of $1/q$ of the variance of the qth difference of the nominal exchange rate S_t, and $\sigma_a^2(q)$ is an unbiased estimator of the variance of the first difference of S_t. The formulas for calculating $\sigma_c^2(q)$ and $\sigma_a^2(q)$ are given below in equations (2) and (3):

$$\sigma_c^2(q) = \frac{1}{m} \sum_{t=q}^{nq} (S_t - S_{t-q} - q\hat{\mu})^2, \tag{2}$$

where $m = q(nq - q + 1)(1 - q/nq)$, and

$$\sigma_a^2(q) = \frac{1}{nq - 1} \sum_{t=1}^{nq} (S_t - S_{t-1} - \hat{\mu})^2, \tag{3}$$

where $\hat{\mu} = \frac{1}{nq}(S_{nq} - S_0)$. The asymptotic variance of the variance-ratio under homoscedasticity, $\phi(q)$:

$$\phi(q) = \frac{2(2q - 1)(q - 1)}{3q(nq)} \tag{4}$$

The standard normal test-statistic under homoscedasticity, Z(q), is then:

$$Z(q) = \frac{VR(q) - 1}{[\phi(q)]^{1/2}} \overset{a}{\sim} N(0,1) \tag{5}$$

Next, the heteroscedasticity-consistent asymptotic variance of the variance-ratio, ϕ^*(q):

$$\phi^*(q) = \sum_{j=1}^{q-1} \left[\frac{2(q-j)}{q}\right]^2 \hat{\delta}(j), \tag{6}$$

where

$$\hat{\delta}(j) = \frac{\sum_{t=j+1}^{nq} (S_t - S_{t-1} - \hat{\mu})^2 (S_{t-j} - S_{t-j-1} - \hat{\mu})^2}{\left[\sum_{k=1}^{nq} (S_t - S_{t-1} - \hat{\mu})^2\right]^2} \tag{7}$$

The heteroscedasticity-consistent standard normal test-statistic, Z*(q), is then given below:

$$Z^*(q) = \frac{VR(q) - 1}{[\phi^*(q)]^{1/2}} \overset{a}{\sim} N(0,1) \tag{8}$$

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