

24 Permutations of a 4 Part Chord Arpeggio

-by Richie Zellon

Before I go on to explain how I came up with the 24 permutations of a 4 part chord arpeggio as well as their application in the formation of new vocabulary, I'd like to share with you some relevant background information.

When I started my jazz studies in my mid 20's, I read somewhere that John Coltrane practiced out of a book called **Thesaurus of Scales and Melodic Patterns**. This book originally published in 1947, was written by Nicolas Slonimsky (1894 – 1995), a Russian-born American conductor, author, pianist, composer and lexicographer. It is said Coltrane and other seminal jazz musicians derived a wealth of ideas for their solos from it. So in my search for anything that might provide new resources as I began my study of improvisation, I went out and bought the book. Unlike the version currently available, it was an expensive heavy big blue book with a hard cover. I remember reading through the book off and on for several days, finally putting it away in frustration. Honestly, I didn't understand a word of the explanation provided. Neither could I make any practical sense of the musical content that filled its pages. I kept wondering what its relationship to jazz or anything that had to do with melody was? As the years went by I occasionally revisited the book hoping that my more advanced understanding of music theory would reveal some hidden musical treasure inside its pages. I finally put it away, forgot about it and did not open it back up until recently, over 30 years later. It was only then that my eyes were finally opened to its wealth of information!

While analysing the melodic construction of some Coltrane solos, it finally dawned on me how he developed lots of his vocabulary employing Slonimsky's system of permutation (I will share some actual proof of this later). Recently I've read that a multitude of other great musicians such as Freddie Hubbard, Frank Zappa and Allan Holdsworth (I always felt he was Coltrane on guitar) have also used it extensively. And I know you might be thinking...yeah that's all good for post-bop jazz but I just want to learn bebop. Well, remember as I stated earlier, this book was first published in 1946 which was practically the beginning of the bebop era. At the time there were no jazz books and if you read the biographies of Charlie Parker, Dizzy Gillespie, Thelonius Monk, and Miles Davis, you'll soon learn that they tried to get their hands on most of the Classical theory and harmony books of the day. It just so happens that **Thesaurus of Scales and Melodic Patterns** was the hot new book that every serious musician had to get their hands on back in 1946. So, keep this in mind next time you listen to Thelonius Monk, especially after you understand the system I am about to explain and hear its effect over a simple blues progression.

The Slonimsky System

Slonimsky defines a "permutation" as the distribution of notes of a given melodic pattern in different orders of succession. Said melodic pattern may consist of a varied amount of tones. In this application we will be initially using 2 tones.

Slonimsky defines 3 fundamental types of permutations which later form the basis for an additional set of sub-types:

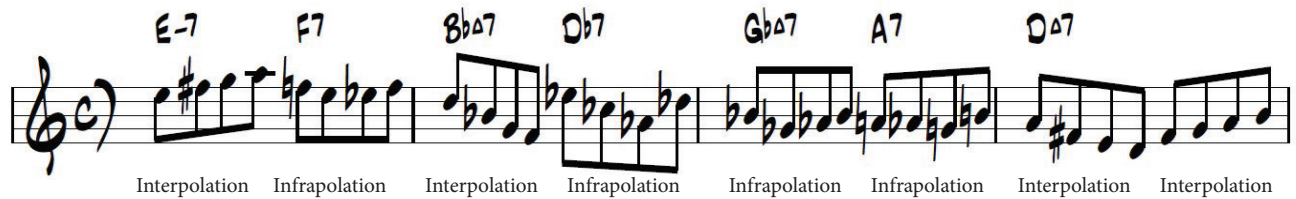
1. **Interpolation**- the insertion of one or more notes between the principal tones (the 1st and last notes).



2. **Infrapolation**- the insertion of one or more notes below a principal tone.



The 1st 4 bars of “Countdown” by John Coltrane happen to consist exclusively of Interpolation and Infrapolation as we can evidence from the following transcribed excerpt. Note that the principal tones are the 1st & last note in each 4 note grouping while the 2 middle notes are the permutation.



The remainder of the solo also makes extensive use of these permutations as do several other Coltrane solos recorded during his “Giant Steps” period. This was most likely inspired by the section on pentatonic scales within Slonimsky’s book.

3. **Ultrapolation**- the insertion of one or more notes above a principal tone.



Note that Infrapolation and Ultrapolation result in a shift of direction with the melodic line progressing in zigzags.

As previously mentioned, the remaining types of permutations Slonimsky defines, are a combination of Interpolation, Infrapolation and Ultrapolation:

4. **Inter-Infrapolation**- The insertion of a note in between and one below the principal tones.

5. **Inter-Ultrapolation**- The insertion of a note in between and one above the principal tones.

6. **Ultra-Infrapolation**- The insertion of a note above and one below the principal tones.

7. **Ultra-Interpolation**- The insertion of a note above and one in between the principal tones.

8. **Infra-Ultrapolation**- The insertion of a note below and one above the principal tones.

9. **Infra-Interpolation**- The insertion of a note below and 1 in between the principal tones.

Now that we have all the permutation types defined, let’s apply them to a 4 part 7th chord. In this case we are using a dominant 7, however the following rule applies to any 4 part chord: **There are 24 different permutations possible.** Within these 24 varieties, all conventional inversions of a 4 part chord (non inverted: 1,3,5,7/1st inv. 3,5,7,1/2nd inv. 5,7,1,3/3rd inv. 7,1,3,5) fall under the category of Interpolation. So if you’re ever bored from just playing the notes in the arpeggio descending or ascending in sequential order, the remaining 23 permutation types will provide an alternate resource to base your lines on!

You’ll notice that the examples that follow sometimes contain some unusually large leaps from one chord tone to the next. Use common sense and above all, your ear, when it comes time to apply them within the harmonic context of your choice.

24 Permutations of a 4 part 7th Chord using the Slonimsky System

Note: All permutations can be found between the first and last tone of each grouping.

by Richie Zellon

Interpolation (in between the principal tones)

C7

Infrapolation (below the principal tones)

Ultrapolation (above the principal tones)

Inter-Infrapolation

(1 in between & 1 below the principal tones)

Inter-Ultrapolation

(1 in between & 1 above the principal tones)

2

24 Permutations of a 4 part 7th Chord using the Slonimsky System

Ultra-Infrapolation

(1 above & 1 below the principal tones)

Ultra-Interpolation

(1 above & 1 in between the principal tones)

Diagram showing the first four permutations (17, 18, 19, 20) of a 4-part 7th chord using the Slonimsky System. The notation includes a treble clef, a key signature of one flat (Bb), and a 4-part chord structure. The permutations are displayed as a sequence of notes across four staves, with the first staff showing the principal tones and the subsequent staves showing the 1st and 2nd alterations (9, 10, 11, 8).

Infra-Ultrapolation

(1 below & 1 above the principal tones)

Infra-Interpolation

(1 below & 1 in between the principal tones)

Diagram showing the last four permutations (21, 22, 23, 24) of a 4-part 7th chord using the Slonimsky System. The notation includes a treble clef, a key signature of one flat (Bb), and a 4-part chord structure. The permutations are displayed as a sequence of notes across four staves, with the first staff showing the principal tones and the subsequent staves showing the 1st and 2nd alterations (9, 10, 11, 8).

Constructing A Solo Using The 4 Part Chords With Permutations

In the 3 examples that follow, I am going to show you how we can use the 4 part arpeggios as the foundation for a solo over a basic 12 bar blues progression.

In **Example 1**, we first lay down the foundation. Notice that even when using the arpeggios with permutations, we always observe the principle of smooth voice leading at the point of chord change. Upon hearing this first draft you will find that the use of quarter notes in combination with the occasional wide leaps aren't necessarily pleasing to the ear.

In **Example 2**, we take the first step in remedying the awkward effect the wide leaps have within the given harmonic progression. This is done by turning the 1's to 9's and the 5's to 13's. The new upper extensions are now idiomatically more compatible (at least from a jazz perspective) with the wide leaps which in turn make them stand out. You'll agree though, that it still sounds somewhat awkward.

In **Example 3**, we substitute the straight quarter notes with various syncopated rhythm cells and add a few chord tones to create more forward motion. The result is a diatonic solo exclusively consisting of chord tones and upper extensions (imagine the possibilities if you also incorporate chromatic approach notes). In spite of this we can already hear some shades of Monk in there, as well as the angular phrasing employed especially by post-bop saxophonists. Be forewarned though, that the intervals used to achieve this style require lots more technical effort on the guitar than on a horn.

Hope it won't take you 30 years like it took me, to unravel the power of this system as an alternate resource to derive improvisational vocabulary! Have Fun!

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EXAMPLE 1

Traditional 12 Bar Blues Progression

Using the arpeggio shapes from the Slonimsky system of permutation

Key of C **I7** (pat. 1)**IV7** (pat. 4)**I7**

5 3 1 b7 3 1 5 b7 3 1 b7 5

infrapolation infra-interpolation interpolation

IV7

b7 1 5 3 b7 3 1 5 3 5 1 b7

infra-interpolation infrapolation inter-infrapolation

I7 **V7** (pat. 5)

3 5 1 b7 5 b7 1 3 b7 5 3 1

inter-infrapolation ultra-infrapolation interpolation

IV7 **I7** **V7**

1 5 b7 3 5 b7 1 3 b7 1 3 5

ultrapolation interpolation infrapolation

EXAMPLE 2

Traditional 12 Bar Blues Progression w/ upper extensions
Using the arpeggio shapes from the Slonimsky system of permutation

Key of C **I7** (pat. 1)**IV7** (pat. 4)**I7**

13 3 9 b7 3 9 13 b7 3 9 b7 13

infrapolation infra-interpolation interpolation

IV7

b7 9 13 3 b7 3 9 13 3 13 9 b7

infra-interpolation infrapolation inter-infrapolation

I7 **V7** (pat. 5)

3 13 9 b7 13 b7 9 3 b7 13 3 9

inter-infrapolation ultra-infrapolation interpolation

IV7 **I7** **V7**

9 13 b7 3 5 b7 9 3 b7 9 3 13

ultrapotation interpolation infrapolation

EXAMPLE 3 **Traditional 12 Bar Blues Progression w/ upper extensions & added rhythms**
Using the arpeggio shapes from the Slonimsky system of permutation

(added notes are in parenthesis)

Key of C **I7** (pat. 1) **IV7** (pat. 4) **I7**

13 3 9 b7 3 9 (5) 13 b7 3 9 b7 13

infrapolation infra-interpolation interpolation

IV7

b7 9 13 (5) 3 b7 (3) 9 13 5 3 13 (3) 9 (5) b7

infra-interpolation * infra-interpolation (was originally infrapolation) inter-infrapolation

I7 **V7** (pat. 5)

3 13 9 b7 13 b7 9 3 b7 13 3 9 (b7)

inter-infrapolation ultra-infrapolation interpolation

IV7 **I7** **V7**

9 (5) 13 b7 (5) 3 5 b7 9 3 b7 9 3 13

ultrapolation interpolation infrapolation