Soal untuk Tutorial 10 Aljali SI dan IF TA 2022/2023

1. . Let R^2 have the weighted Euclidean inner product $\langle \mathbf{u}, \mathbf{v} \rangle = 2u_1v_1 + 3u_2v_2$ and let $\mathbf{u} = (1, 1)$, $\mathbf{v} = (3, 2)$, $\mathbf{w} = (0, -1)$, and $k = 3$. Compute the stated quantities.
(a) $\langle \mathbf{u}, \mathbf{v} \rangle$
(b) $\langle k\mathbf{v}, \mathbf{w} \rangle$
(c) $\langle \mathbf{u} + \mathbf{v}, \mathbf{w} \rangle$
(d) $||\mathrm{v}||$
(e) $d(\mathbf{u}, \mathbf{v})$
(f) $||\ \mathbf{u} - k\mathbf{v}||$

2. find a matrix that generates the stated weighted inner product on R^2
a. $\langle \mathbf{u}, \mathbf{v} \rangle = 2u_1v_1 + 3u_2v_2$
b. $\langle \mathbf{u}, \mathbf{v} \rangle = 1/2\ u_1v + 5\ u_2v_2$

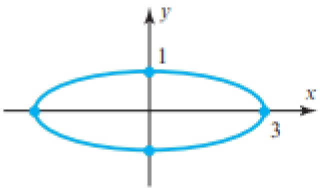
3. Compute the standard inner product on M_{22} of the given matrices.

$$U = \begin{bmatrix} 3 & -2 \\ 4 & 8 \end{bmatrix},\quad V = \begin{bmatrix} -1 & 3 \\ 1 & 1 \end{bmatrix}$$

4. Find $||U||$ and $d(U, V)$ relative to the standard inner product on M_{22}

$$U = \begin{bmatrix} 3 & -2 \\ 4 & 8 \end{bmatrix},\quad V = \begin{bmatrix} -1 & 3 \\ 1 & 1 \end{bmatrix}$$

5. find a weighted Euclidean inner product on R2 for which the “unit circle” is the ellipse shown in the accompanying figure



6. Find the cosine of the angle between the vectors with respect to the Euclidean inner product:

a. $\mathbf{u} = (-1, 5, 2)$, $\mathbf{v} = (2, 4, -9)$

b.

$$A = \begin{bmatrix} 2 & 6 \\ 1 & -3 \end{bmatrix},\quad B = \begin{bmatrix} 3 & 2 \\ 1 & 0 \end{bmatrix}$$

7. Show that the matrices are orthogonal with respect to the standard inner product on M_{22}

$$.\ U = \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix},\quad V = \begin{bmatrix} -3 & 0 \\ 0 & 2 \end{bmatrix}$$

8. If the vectors **u** = (1, 2) and **v** = (2, −4) are orthogonal with respect to the weighted Euclidean inner product <u, v> = $w_1u_1v_1 + w_2u_2v_2$, what must be true of the weights w_1 and w_2?

9. Determine whether the set of vectors is orthogonal and whether it is orthonormal with respect to the Euclidean inner product on R^2

(a) $(0, 1),\ (2, 0)$

(b) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right),\ \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

10. Show that the column vectors of A form an orthogonal basis for the column space of A with respect to the Euclidean inner product, and then find an orthonormal basis for that column space.

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 5 \\ -1 & 2 & 0 \end{bmatrix}$$