Foundation of Cryptography (0368-4162-01), Lecture 7

Encryption Schemes

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Section 1

Definitions

Correctness

Definition 1 (encryption scheme)

A trippet of PPT's $(G, \mathrm{E}, \mathrm{D})$ such that

1. $G(1^n)$ outputs a key $(e, d) \in \{0,1\}^* \times \{0,1\}^*$
2. $\mathrm{E}(e, m)$ outputs a string in $c \in \{0,1\}^*$
3. $\mathrm{D}(d, c)$ outputs $m \in \{0,1\}^*$

Correctness: $\mathrm{D}(d, \mathrm{E}(e, m)) = m$, for any $(e, d) \in \mathrm{Supp}(G(1^n))$ and $m \in \{0,1\}^*$

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- e – encryption key, d – decryption key
- m – plaintext, $c = \mathrm{E}(e, m)$ – ciphertext
- $E_e(m) \equiv E(e, m)$ and $D_d(c) \equiv D(d, c)$,

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- e – encryption key, d – decryption key
- m – plaintext, $c = \mathrm{E}(e, m)$ – ciphertext
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- public/private key

Security

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  - Other concerns, e.g., multiple encryptions, active adversary

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2. Formulate via the simulation paradigm
3. Cannot hide the message length

Semantic security – private-key model

Definition 2 (Semantic Security – private-key model)

An encryption scheme $(G, \mathsf{E}, \mathsf{D})$ is semantically secure in the private-key model, if for any PPT A, $\exists$ PPT A$'$ s.t. $\forall$ poly-bounded dist. ensemble $\mathcal{M} = \{\mathcal{M}_n\}_{n\in\mathbb{N}}$ and poly-bounded functions $h, f \colon \{0,1\}^* \mapsto \{0,1\}^*$

$$\Big|\Pr_{m\leftarrow\mathcal{M}_n, e\leftarrow G(1^n)_1}[\mathsf{A}(1^n, 1^{|m|}, h(1^n, m), E_e(m)) = f(1^n, m)] - \Pr_{m\leftarrow\mathcal{M}_n}[\mathsf{A}'(1^n, 1^{|m|}, h(1^n, m)) = f(1^n, m)]\Big| = \operatorname{neg}(n)$$

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- Reflection to ZK
- public-key variant – A gets e

Indistinguishablity of encryptions

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- Less intuitive than semantic security, but easier to work with

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An encryption scheme $(G, \mathsf{E}, \mathsf{D})$ has indistinguishable encryptions in the private-key model, if for any $p, \ell \in \text{poly}$, $\{x_n, y_n \in \{0,1\}^{\ell(n)}\}_{n\in\mathbb{N}}$, $\{z_n \in \{0,1\}^{p(n)}\}_{n\in\mathbb{N}}$ and poly-time B,

$$\left|\Pr_{e\leftarrow G(1^n)_1}[\mathsf{B}(z_n, E_e(x_n)) = 1] - \Pr_{e\leftarrow G(1^n)_1}[\mathsf{B}(z_n, E_e(y_n)) = 1]\right| = \text{neg}(n)$$

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- Non-uniform definition
- Public-key variant

Equivalence of definitions

Theorem 4

An encryption scheme $(G, \mathrm{E}, \mathrm{D})$ *is semantically secure iff is has indistinguishable encryptions.*

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We prove the private key case

Indistinguishablity $\Longrightarrow$ Semantic Security

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Fix $\mathcal{M}$, A, f and h, be as in Definition 2.

Indistinguishablity $\Longrightarrow$ Semantic Security

Fix $\mathcal{M}$, A, f and h, be as in Definition 2. We construct A′ as

Algorithm 5 (A′)

Input: 1^n, $1^{|m|}$ and $h(m)$

1. $e \leftarrow G(1^n)_1$
2. $c = E_e(1^{|m|})$
3. Output $\mathrm{A}(1^n, 1^{|m|}, h(m), c)$

Indistinguishablity $\implies$ Semantic Security

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Claim 6

A′ is a good simulator for A (according to Definition 2)

Proving Claim 6

Assume exists infinite $\mathcal{I} \subseteq \mathbb{N}$ and $p \in$ poly s.t. for any $n \in \mathcal{I}$:

$$\Big|\Pr_{m \leftarrow \mathcal{M}_n, e \leftarrow G(1^n)_1}[\mathsf{A}(1^n, 1^{|m|}, h(1^n, m), E_e(m)) = f(1^n, m)] \quad (1)$$
$$-\Pr_{m \leftarrow \mathcal{M}_n}[\mathsf{A}'(1^n, 1^{|m|}, h(1^n, m)) = f(1^n, m)]\Big| > 1/p(n)$$

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Fix $n \in \mathcal{I}$ and let $x_n \in \mathrm{Supp}(\mathcal{M}_n)$ be a value that maximize Equation (1).

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Assume exits algorithm B that contradicts the indistinguishability of the scheme with respect to $\{(x_n, y_n = 1^{|x_n|})\}_{n \in \mathbb{N}}$ and $\{z_n = (1^n, 1^{|x_n|}, h(1^n, x_n), f(1^n, x_n))\}_{n \in \mathbb{N}}$.

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Algorithm 7 (B)

Input: $z_n = (1^n, 1^{|x_n|}, h(1^n, x_n), f(1^n, x_n)), c$
Output 1 iff $\mathsf{A}(1^n, 1^{|x_n|}, h(x + n), c) = f(1^n, x_n)$

Semantic Security $\implies$ Indistinguishablity

Assume $\exists$ B, $\{x_n, y_n \in \{0,1\}^{\ell(n)}\}_{n\in\mathbb{N}}$ and a $\{z_n\}_{n\in\mathbb{N}}$, such that (wlog) for infinitely many n's:

$$\Pr_{e\leftarrow G(1^n)_1}[\mathsf{B}(z_n, E_e(x_n)) = 1] - \Pr_{e\leftarrow G(1^n)_1}[\mathsf{B}(z_n, E_e(y_n)) = 1] \geq \frac{1}{p(n)}$$

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- Let $\mathcal{M}_n$ be x_n wp $\frac{1}{2}$ and y_n otherwise.
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$$\Pr_{m\leftarrow\mathcal{M}_n, e\leftarrow G(1^n)_1}[\mathrm{A}(1^n, 1^{|m|}, h(1^n, m), E_e(m)) = f(1^n, m)] \geq \frac{1}{2} + \frac{1}{p(n)}$$

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For *any* A′

$$\Pr_{m\leftarrow\mathcal{M}_n, e\leftarrow G(1^n)_1}[\mathrm{A}(1^n, 1^{|m|}, h(1^n, m), E_e(m)) = f(1^n, m)] \leq \frac{1}{2}$$

Security Under Multiple Encryptions

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An encryption scheme $(G, \mathrm{E}, \mathrm{D})$ has indistinguishable encryptions for multiple messages in the private-key model, if for any $p, \ell, t \in \text{poly}$,
$\{x_{n,1}, \ldots x_{n,t(n)}, y_{n,1}, \ldots, y_{n,t(n)} \in \{0,1\}^{\ell(n)}\}_{n \in \mathbb{N}}$,
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Extensions:

- Different length messages

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Extensions:

- Different length messages
- Semantic security version
- Public-key definition

Multiple Encryption in the Public-Key Model

Theorem 9

A public-key encryption scheme has indistinguishable encryptions for multiple messages, iff it has indistinguishable encryptions for a single message.

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Proof: Assume $(G, \mathrm{E}, \mathrm{D})$ is public-key secure for a single message and not for multiple messages with respect to B, $\{x_{1,t(n)}, \ldots x_{n,t(n)}, y_{n,1}, \ldots, y_{n,t(n)} \in \{0,1\}^{\ell(n)}\}_{n \in \mathbb{N}}$, $\{z_n \in \{0,1\}^{p(n)}\}_{n \in \mathbb{N}}$.

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Theorem 9

A public-key encryption scheme has indistinguishable encryptions for multiple messages, iff it has indistinguishable encryptions for a single message.

Proof: Assume $(G, \mathrm{E}, \mathrm{D})$ is public-key secure for a single message and not for multiple messages with respect to B, $\{x_{1,t(n)}, \ldots x_{n,t(n)}, y_{n,1}, \ldots, y_{n,t(n)} \in \{0,1\}^{\ell(n)}\}_{n \in \mathbb{N}}$, $\{z_n \in \{0,1\}^{p(n)}\}_{n \in \mathbb{N}}$.
It follows that for some function $i(n) \in [t(n)]$

$$\Big|\Pr[\mathrm{B}(1^n, e, E_e(x_{n,1}), \ldots, E_e(x_{n,i-1}), E_e(y_{n,i}) \ldots, E_e(y_{n,t(n)})) = 1]$$
$$-\Pr[\mathrm{B}(1^n, e, E_e(x_{n,1}), \ldots, E_e(x_{n,i}), E_e(y_{n,i+1}) \ldots, E_e(y_{n,t(n)})) = 1]\Big|$$
$$> \mathrm{neg}(n)$$

where in both cases $e \leftarrow G(1^n)_1$

Algorithm 10 (B′)

Input: 1^n, $z_n = (i(n), x_{1,t(n)}, \ldots x_{n,t(n)}, y_{n,1}, \ldots, y_{n,t(n)}, e\ , c$
Return $\mathrm{B}(c, E_e(x_{n,1}), \ldots, E_e(x_{n,i-1}), c, E_e(y_{n,i+1}) \ldots, E_e(y_{n,t(n)}))$

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B′ is critically using the public key

Multiple Encryption in the Private-Key Model

Fact 11

Assuming (non uniform) OWFs exists, there exists an encryption scheme that has private-key indistinguishable encryptions for a single messages, but not *for multiple messages*

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Construction 12

- $G(1^n)$ outputs $e \leftarrow \{0,1\}^n$,
- $\mathrm{E}_e(m)$ outputs $g^{|m|}(e) \oplus m$
- $\mathrm{D}_e(c)$ outputs $g^{|c|}(e) \oplus c$

Claim 13

$(G, \mathrm{E}, \mathrm{D})$ has private-key indistinguishable encryptions for a single message

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$$\left|\Pr[\mathrm{B}(z_n, g^{|x_n|}(U_n) \oplus x_n) = 1] - \Pr[\mathrm{B}(z_n, U_{|x_n|} \oplus x_n) = 1]\right| > \mathrm{neg}(n) \tag{2}$$

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Proof: Take $x_{n,1} = x_{n,2}$, $y_{n,1} \neq y_{n,2}$ and $\mathrm{D}(c_1, c_2)$ outputs 1 iff $c_1 = c_2$

Section 2

Constructions

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- $\mathrm{E}_e(m)$: choose $r \leftarrow \{0,1\}^n$ and output $(r, e(r) \oplus m)$
- $\mathrm{D}_e(r, c)$: output $e(r) \oplus c$

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- We believe that public-key encryptions are of different complexity than private-key ones

Section 3

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- In the public-key settings, the adversary is also given the public key
- We focus on indistinguishability, but each of the above definitions has an equivalent semantic security variant.

CPA Security

Let $(G, \mathsf{E}, \mathsf{D})$ be an an encryption scheme. For a pair of alg. $\mathsf{A} = (\mathsf{A}_1, \mathsf{A}_2)$, $n \in \mathbb{N}$, $z \in \{0,1\}^*$ and $b \in \{0,1\}$, we let:

Experiment 19 ($\mathsf{Exp}^{\mathsf{CPA}}_{\mathsf{A},n,z_n}(b)$)

1. $(e, d) \leftarrow G(1^n)$
2. $(x_0, x_1, s) \leftarrow \mathsf{A}_1^{E_e(\cdot)}(1^n, z)$
3. $c \leftarrow \mathsf{E}_e(x_b)$
4. Output $\mathsf{A}_2^{E_e(\cdot)}(1^n, s, c)$

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Definition 20 (private key CPA)

$(G, \mathsf{E}, \mathsf{D})$ has indistinguishable encryptions in the private-key model under CPA attack, if $\forall$ PPT $\mathsf{A}_1, \mathsf{A}_2$, and poly-bounded $\{z_n\}_{n \in \mathbb{N}}$:

$$|\Pr[\mathrm{Exp}^{\mathrm{CPA}}_{\mathsf{A},n,z_n}(0) = 1] - \Pr[\mathrm{Exp}^{\mathrm{CPA}}_{\mathsf{A},n,z_n}(1) = 1]| = \mathrm{neg}(n)$$

- The scheme from Construction 15 has indistinguishable encryptions in the private-key model (for short, private-key CPA secure)

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- The scheme from Construction 17 has indistinguishable encryptions in the public-key model (for short, public-key CPA secure)
- In both cases, definitions are *not* equivalent

CCA Security

Experiment 21 ($\mathrm{Exp}^{\mathrm{CCA1}}_{\mathrm{A},n,z_n}(b)$)

1. $(e, d) \leftarrow G(1^n)$
2. $(x_0, x_1, s) \leftarrow \mathrm{A}_1^{E_e(\cdot), D_d(\cdot)}(1^n, z)$
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Experiment 22 ($\mathrm{Exp}^{\mathrm{CCA2}}_{\mathsf{A},n,z_n}(b)$)

1. $(e, d) \leftarrow G(1^n)$
2. $(x_0, x_1, s) \leftarrow \mathsf{A}_1^{E_e(\cdot), D_d(\cdot)}(1^n, z)$
3. $c \leftarrow \mathrm{E}_e(x_b)$
4. Output $\mathsf{A}_2^{E_e(\cdot), D_d^{\neg c}(\cdot)}(1^n, s, c)$

Definition 23 (private key CCA1/CCA2)

$(G, \mathsf{E}, \mathsf{D})$ has indistinguishable encryptions in the private-key model under $x \in \{\mathrm{CCA1}, \mathrm{CCA2}\}$ attack, if $\forall$ PPT $\mathsf{A}_1, \mathsf{A}_2$, and poly-bounded $\{z_n\}_{n\in\mathbb{N}}$:

$$|\Pr[\mathrm{Exp}^{x}_{\mathsf{A},n,z_n}(0) = 1] - \Pr[\mathrm{Exp}^{x}_{\mathsf{A},n,z_n}(1) = 1]| = \mathrm{neg}(n)$$

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- Constructing private-key CCA2 is not difficult
- Private key CCA2 from TPD, but highly non trivial (next class)