

Problem set 1*November 3, 2014*

Due: Nov 18

- Please submit the handout in class, or email the grader.
- Write clearly and shortly using sub-claims if needed. The emphasize in most questions is on the proofs (no much point is writing a “solution” w/o proving its correctness)
- For Latex users, a solution example can be found in the course web site.
- In it ok to work in (small) groups, but please write the id list of your partners in the solution file, and each student should write his solution by *himself* (joint effort is only allowed in the “thinking phase”)

1. Let (p_1, \dots, p_m) and (q_1, \dots, q_m) be probability distributions (i.e., $p_i \geq 0$ for all i and $\sum_i p_i = 1$). Prove that

$$-\sum_i p_i \log p_i \leq -\sum_i p_i \log q_i$$

2. Prove the chain-rule for mutual information stated in class.
3. What is larger:

- (a) $I(X; Y|Z)$ or $I(X; Y)$?
- (b) $H(X|Y)$ or $H(f(X); Y)$?
- (c) $H(X|Y)$ or $H(X; g(Y))$?
- (d) $H(X|Y)$ or $H(f(X, Y); Y)$?
- (e) $H(X|Y)$ or $H(X; g(X, Y))$?

4. For a finite set \mathcal{S} of random variables, let $H(\mathcal{S})$ denote the joint entropy of all variables in \mathcal{S} . Prove that

$$H(\mathcal{S} \cup \mathcal{U}) + H(\mathcal{S} \cap \mathcal{U}) \leq H(\mathcal{S}) + H(\mathcal{U})$$

for any two finite sets of random variables \mathcal{S} and \mathcal{U} .