Application of Information Theory, Lecture 12

Accessible Entropy and Statistically Hiding Commitments

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Section 1

Commitment Schemes

Motivation

- Digital analogue of a safe
- Numerous applications (e.g., zero-knowledge, coin-flipping, secure computations,)

Definition

Definition 1 (Commitment scheme)

An efficient two-stage protocol (S, R).

- Commit stage: The sender S has private input $\sigma \in \{0,1\}^*$ and the common input is 1^n. The commitment stage results in a **joint** output c, the commitment, and a **private** output d of S, the decommitment.
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Binding: The following happens with negligible prob. for any S^*:

> $\mathsf{S}^*(1^n)$ *interacts with* $\mathsf{R}(1^n)$ *in the commit stage resulting in a commitment* c*. Then* S^* *outputs two pairs* (d, σ) *and* (d', σ') *with* $\sigma \neq \sigma'$ *and* $\mathsf{R}(c, d, \sigma) = \mathsf{R}(c, d', \sigma') = \mathtt{Accept}$.

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Section 2

Inaccessible Entropy

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- In the actual construction, we sometimes measure the (real) entropy of some of the output blocks.

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- G has inaccessible entropy d, if the accessible entropy of any PPT $\widetilde{G}$ is smaller be at least d than its real entropy

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Section 3

Manipulating Inaccessible Entropy

Entropy equalization

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- Assume $k_R \geq k_A + 1$, then for $\ell = m + 2$, it holds that $k_R^{\otimes \ell} \geq k_A^{\otimes \ell} + 1$

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- If $k_A \leq k_R - 1$, then $\forall n \in \text{poly}$ $\exists \ell \in \text{poly}$ such that $\ell k_{min}^\ell > k_A^\ell + n$

Section 4

Inaccessible Entropy from OWF

The generator

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Assume that f is a OWF then G has accessible entropy at most $n - \log n$.

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- Recall f is OWF if
  $\Pr_{x \leftarrow \{0,1\}^n}\left[\mathsf{Inv}(f(x)) \in f^{-1}(f(x))\right] = \mathrm{neg}(n)$ for any PPT Inv.

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We start by assuming that Inv is unbounded (i.e., the test on Line 1.3 is removed)

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- $\Pr_{\widehat{T}}[\mathbf{t}] = \Pr[f(U_n) = (g_1, \ldots, g_n)] \cdot \Pr\left[\widetilde{G}_{n+1} = g_{n+1}\right] \cdot P(\mathbf{t})$

$\widetilde{T}$ vs. $\widehat{T}$

Fix $\mathbf{t} = (r_1, g_1, \ldots, r_{n+1}, g_{n+1}) \in \operatorname{Supp}(\widetilde{T})$

- Let $P(\mathbf{t}) = \prod_{i=1}^{n+1} \Pr\left[\widetilde{R}_i = r_i | (\widetilde{R}_{1,\ldots,i-1}, \widetilde{G}_i) = (r_{1,\ldots,i-1}, g_i)\right]$
-

$$\Pr_{\widetilde{T}}[t] = \Pr[\widetilde{G}_1 = g_1] \cdot \Pr[\widetilde{R}_1 = r_1 | \widetilde{G}_1 = g_1] \cdot \Pr[\widetilde{G}_2 = g_2 | \widetilde{R}_1 = r_1] \cdot \Pr[\widetilde{R}_2 = r_2 | \widetilde{G}_2 = g_2] \ldots$$

$$= 2^{-\sum_{i=1}^{m} H_{\widetilde{G}_i | \widetilde{R}_1, \ldots, \widetilde{R}_{i-1}}(g_i | r_1, \ldots, r_{i-1})} \cdot P(\mathbf{t})$$

$$= 2^{-\operatorname{AccH}_{G,\widetilde{G}}(\mathbf{t})} \cdot P(\mathbf{t})$$

- $\Pr_{\widehat{T}}[\mathbf{t}] = \Pr[f(U_n) = (g_1, \ldots, g_n)] \cdot \Pr\left[\widetilde{G}_{n+1} = g_{n+1}\right] \cdot P(\mathbf{t})$

$\widetilde{T}$ vs. $\widehat{T}$

Fix $\mathbf{t} = (r_1, g_1, \ldots, r_{n+1}, g_{n+1}) \in \mathrm{Supp}(\widetilde{T})$

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-

$$\Pr_{\widetilde{T}}[t] = \Pr[\widetilde{G}_1 = g_1] \cdot \Pr[\widetilde{R}_1 = r_1 | \widetilde{G}_1 = g_1] \cdot \Pr[\widetilde{G}_2 = g_2 | \widetilde{R}_1 = r_1] \cdot \Pr[\widetilde{R}_2 = r_2 | \widetilde{G}_2 = g_2] \ldots$$
$$= 2^{-\sum_{i=1}^{m} H_{\widetilde{G}_i | \widetilde{R}_1, \ldots, \widetilde{R}_{i-1}}(g_i | r_1, \ldots, r_{i-1})} \cdot P(\mathbf{t})$$
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$$\Pr_{\widetilde{T}}[t] = \Pr[\widetilde{G}_1 = g_1] \cdot \Pr[\widetilde{R}_1 = r_1 | \widetilde{G}_1 = g_1] \cdot \Pr[\widetilde{G}_2 = g_2 | \widetilde{R}_1 = r_1] \cdot \Pr[\widetilde{R}_2 = r_2 | \widetilde{G}_2 = g_2] \ldots$$
$$= 2^{-\sum_{i=1}^{m} H_{\widetilde{G}_i | \widetilde{R}_1, \ldots, \widetilde{R}_{i-1}}(g_i | r_1, \ldots, r_{i-1})} \cdot P(\mathbf{t})$$
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- $\Pr_{\widehat{T}}[\mathbf{t}] = \Pr[f(U_n) = (g_1, \ldots, g_n)] \cdot \Pr\left[\widetilde{G}_{n+1} = g_{n+1}\right] \cdot P(\mathbf{t}) \geq 2^{-n} \cdot P(\mathbf{t})$
- In particular, for $\mathbf{t}$ with $\mathrm{AccH}_{G, \widetilde{G}}(\mathbf{t}) \geq n - \log n$:

$$\Pr_{\widetilde{T}}[t] \geq \Pr_{\widehat{T}}[t] / n \tag{1}$$

Inv's success probability

Inv's success probability

Let $\mathcal{S} \subseteq \mathrm{Supp}(\widetilde{T})$ denote the set of transcripts $\mathbf{t} = (r_1, g_1, \ldots, r_{n+1}, g_{n+1})$ with

1. $\mathrm{AccH}_{G,\widetilde{G}}(\mathbf{t}) \geq n - \log n$, and
2. $H_{\widetilde{G}_i|\widetilde{G}_1,\ldots,\widetilde{G}_{i-1}}(g_i \mid g_1, \ldots, g_{i-1}) \leq \log(\frac{2n}{\varepsilon})$ for all $i \in [n]$.

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$$\Pr_{\widetilde{T}}[\mathcal{S}] \geq \Pr\left[\mathrm{AccH}_{G,\widetilde{G}}(T) \geq n - \log n\right] - \Pr_{(g_1,\ldots,g_{n+1})\leftarrow(\widetilde{G}_1,\ldots,\widetilde{G}_{n+1})}\left[\exists i \in [n]\colon H_{\widetilde{G}_i|\widetilde{G}_1,\ldots,\widetilde{G}_{i-1}}(g_i \mid g_1, \ldots, g_{i-1}) > \log(\tfrac{2n}{\varepsilon})\right]$$

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$$\Pr_{\widetilde{T}}[\mathcal{S}] \geq \Pr\left[\mathrm{AccH}_{G,\widetilde{G}}(T) \geq n - \log n\right]$$
$$- \Pr_{(g_1,\ldots,g_{n+1})\leftarrow(\widetilde{G}_1,\ldots,\widetilde{G}_{n+1})}\left[\exists i \in [n] : H_{\widetilde{G}_i|\widetilde{G}_1,\ldots,\widetilde{G}_{i-1}}(g_i \mid g_1, \ldots, g_{i-1}) > \log(\tfrac{2n}{\varepsilon})\right]$$
$$\geq \varepsilon - n \cdot \frac{\varepsilon}{2n} = \varepsilon/2$$

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$$\begin{aligned}
\Pr_{\widetilde{T}}[\mathcal{S}] &\geq \Pr\left[\text{AccH}_{G,\widetilde{G}}(T) \geq n - \log n\right] \\
&\quad - \Pr_{(g_1,\ldots,g_{n+1}) \leftarrow (\widetilde{G}_1,\ldots,\widetilde{G}_{n+1})}\left[\exists i \in [n] \colon H_{\widetilde{G}_i | \widetilde{G}_1,\ldots,\widetilde{G}_{i-1}}(g_i \mid g_1, \ldots, g_{i-1}) > \log(\tfrac{2n}{\varepsilon})\right] \\
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\end{aligned}$$

It follows that $\Pr_{\widehat{T}}[\mathcal{S}] \geq \varepsilon/2n$.

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Back to the bounded version of Inv.

Inv's success probability

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$$\Pr_{\widetilde{T}}[\mathcal{S}] \geq \Pr\left[\mathsf{AccH}_{G,\widetilde{G}}(T) \geq n - \log n\right]$$
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It follows that $\Pr_{\widetilde{T}}[\mathcal{S}] \geq \varepsilon/2n$.

Back to the bounded version of Inv.

- For $z \in \{0,1\}^n$ for which $\exists (r_1, z_1, \ldots, r_n, z_n, \ldots) \in \mathcal{S}$:
  $\Pr[\mathsf{Inv}(z) \text{ aborts }] \leq n \cdot (1 - \frac{\varepsilon}{2n})^{n^2/\varepsilon} = O(n \cdot 2^{-n}) \leq \frac{1}{2}$

Inv's success probability

Let $\mathcal{S} \subseteq \mathrm{Supp}(\widetilde{T})$ denote the set of transcripts $\mathbf{t} = (r_1, g_1, \ldots, r_{n+1}, g_{n+1})$ with

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$$\Pr_{\widetilde{T}}[\mathcal{S}] \geq \Pr\left[\mathrm{AccH}_{G,\widetilde{G}}(T) \geq n - \log n\right]$$
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$$\Pr_{\widetilde{T}}[\mathcal{S}] \geq \Pr\left[\mathrm{AccH}_{G,\widetilde{G}}(T) \geq n - \log n\right] - \Pr_{(g_1,\ldots,g_{n+1})\leftarrow(\widetilde{G}_1,\ldots,\widetilde{G}_{n+1})}\left[\exists i \in [n] \colon H_{\widetilde{G}_i|\widetilde{G}_1,\ldots,\widetilde{G}_{i-1}}(g_i \mid g_1, \ldots, g_{i-1}) > \log(\tfrac{2n}{\varepsilon})\right]$$
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- For $z \in \{0,1\}^n$ for which $\exists (r_1, z_1, \ldots, r_n, z_n, \ldots) \in \mathcal{S}$:
  $\Pr[\mathrm{Inv}(z) \text{ aborts }] \leq n \cdot (1 - \frac{\varepsilon}{2n})^{n^2/\varepsilon} = O(n \cdot 2^{-n}) \leq \frac{1}{2}$
- Hence, (in the bounded version of Inv) $\Pr_{\widetilde{T}}[\mathcal{S}] \geq \varepsilon/4n$

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Let $\mathcal{S} \subseteq \text{Supp}(\widetilde{T})$ denote the set of transcripts $\mathbf{t} = (r_1, g_1, \ldots, r_{n+1}, g_{n+1})$ with

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$$\Pr_{\widetilde{T}}[\mathcal{S}] \geq \Pr\left[\text{AccH}_{G,\widetilde{G}}(T) \geq n - \log n\right]$$
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- For $z \in \{0,1\}^n$ for which $\exists (r_1, z_1, \ldots, r_n, z_n, \ldots) \in \mathcal{S}$:
  $\Pr[\text{Inv}(z) \text{ aborts }] \leq n \cdot (1 - \frac{\varepsilon}{2n})^{n^2/\varepsilon} = O(n \cdot 2^{-n}) \leq \frac{1}{2}$
- Hence, (in the bounded version of Inv) $\Pr_{\widetilde{T}}[\mathcal{S}] \geq \varepsilon/4n$

$$\implies \Pr_{x\leftarrow\{0,1\}^n}\left[\text{Inv}(f(x)) \in f^{-1}(f(x))\right] \geq \varepsilon/4n$$

Section 5

Statistically Hiding Commitment from Inaccessible Entropy Generator

High-level description

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- Entropy equalization + gap amplification to get generator that has the same min-entropy in each block and whose accessible entropy is n-bit smaller than the sum of the min entropies.

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- Use target-collision-resistant hash family (a non-interactive cryptographic tool implied by OWF) to get weakly binding SHC
- Amplify the above into full-fledged SHC

Hashing protocol

Let $\mathcal{T} \subseteq \{0,1\}^\ell$ be 2^k-size set.
Let $\mathcal{H}^1$ be ℓ-wise independent family mapping ℓ-bit strings to k-bit strings
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Protocol 6 ((S, R))

1. S selects $x \in \mathcal{T}$
2. R sends $h^1 \leftarrow \mathcal{H}^1$ to S
3. S sends $y^1 = h^1(x)$ to R
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Let $\widetilde{\mathsf{S}}$ be an arbitrary algorithm and let Y^1, Y^2, H^1, H^2 be value of y^1, y^2, h^1, h^2 in a random execution of $(\widetilde{\mathsf{S}}, \mathsf{R})$.

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Claim 7

$\Pr\left[\exists x \neq x' \in \mathcal{T} \colon H^1(x) = H^1(x') = Y^1 \wedge H^2(x) = H^3(x') = Y^3\right] \in 2^{-\Omega(n)}$.

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Proof: ?

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Proof: ? Can we do it in a single round?

“Generator” with zero accessible entropy block

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Let G be m-block generator of block size ℓ and input length s. Let $\mathcal{H}^1$ be ℓ-wise function family mapping ℓ-bit strings of k-bit strings. Let $\mathcal{H}^2$ be 2-universal function family mapping ℓ-bit strings to n-bit strings.

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Protocol 8 ($G' = (\mathsf{S}, \mathsf{R})$)

S sets $x \leftarrow \{0,1\}^s$

For $i = 1$ to m:

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2. S sends $y_i^1 = h_i^1(G(x)_i)$ to R
3. R sends $h_i^2 \leftarrow \mathcal{H}^2$ to S
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5. S sends $g_i = G(x)_i$ to R

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- We view G' as an m-block “interactive generator" (the blocks are $g_1, \ldots, g_m$).

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- Assume G has accessible entropy mk, then w.p. $1 - negl(n)$ in an execution of G' exists block with accessible entropy 0:

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S sets $x \leftarrow \{0,1\}^s$

For $i = 1$ to m:

1. R sends $h_i^1 \leftarrow \mathcal{H}^1$ to S
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- We view G' as an m-block "interactive generator" (the blocks are $g_1, \ldots, g_m$).
- Assume the blocks of G has real min-entropy $(k + n + t)$, then the blocks of G' has real min-entropy roughly t
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"Generator" with zero accessible entropy block

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A function family $\mathcal{H} = \{\mathcal{H}_n\}$ is target collision resistant, if

$$\Pr_{(x,a)\leftarrow A_1(1^n);h\leftarrow\mathcal{H}_n;x'\leftarrow A_2(a,h)}\left[x \neq x' \wedge h(x) = h(x')\right] = \operatorname{neg}(n)$$

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Theorem 10

OWFs imply efficient compressing TCRs.

Weakly binding statistically hiding commitment

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  2. If $i^* = i$, we have binding

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