Secure Multiparty Computation: Introduction

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Scenario 1: Private Dating

Alice and Bob meet at a pub

- If both of them want to date together – they will find out
- If Alice doesn't want to date – she won't learn his intentions
- If Bob doesn't want to date – he won't learn her intentions

Solution: use a trusted bartender

Scenario 2: Private Auction

Many parties wish to execute a private auction

- The highest bid wins
- Only the highest bid (and bidder) is revealed

Solution: use a trusted auctioneer

Scenario 3: Private Set Intersection

Intelligence agencies holds lists of potential terrorists

- The would like to compute the intersection
- Any other information must remain secret

Solution: use a trusted party

Mossad

MI5

FBI

Trust me

Scenario 4: Online Poker

Play online poker reliably

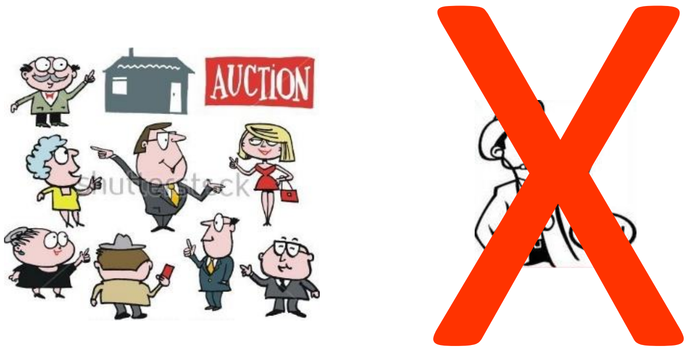
Solution: use a trusted party

Secure Multiparty Computation

- In all scenarios the solution of an external trusted third party works
- Trusting a third party is a very strong assumption
- Can we do better?
- We would like a solution with the same security guarantees, but without using any trusted party

Secure Multiparty Computation

Goal: use a protocol to emulate the trusted party



The Setting

- Parties $P_1, \dots, P_n$ (modeled as interactive TM)
- Party P_i has private input x_i
- The parties wish to jointly compute a (known) function $y = f(x_1, \dots, x_n)$
- The computation must preserve certain security properties, even is some of the parties collude and maliciously attack the protocol
- Normally, this is modeled by an external adversary $\mathcal{A}$ that corrupts some parties and coordinates their actions

Auction Example – Security Requirements

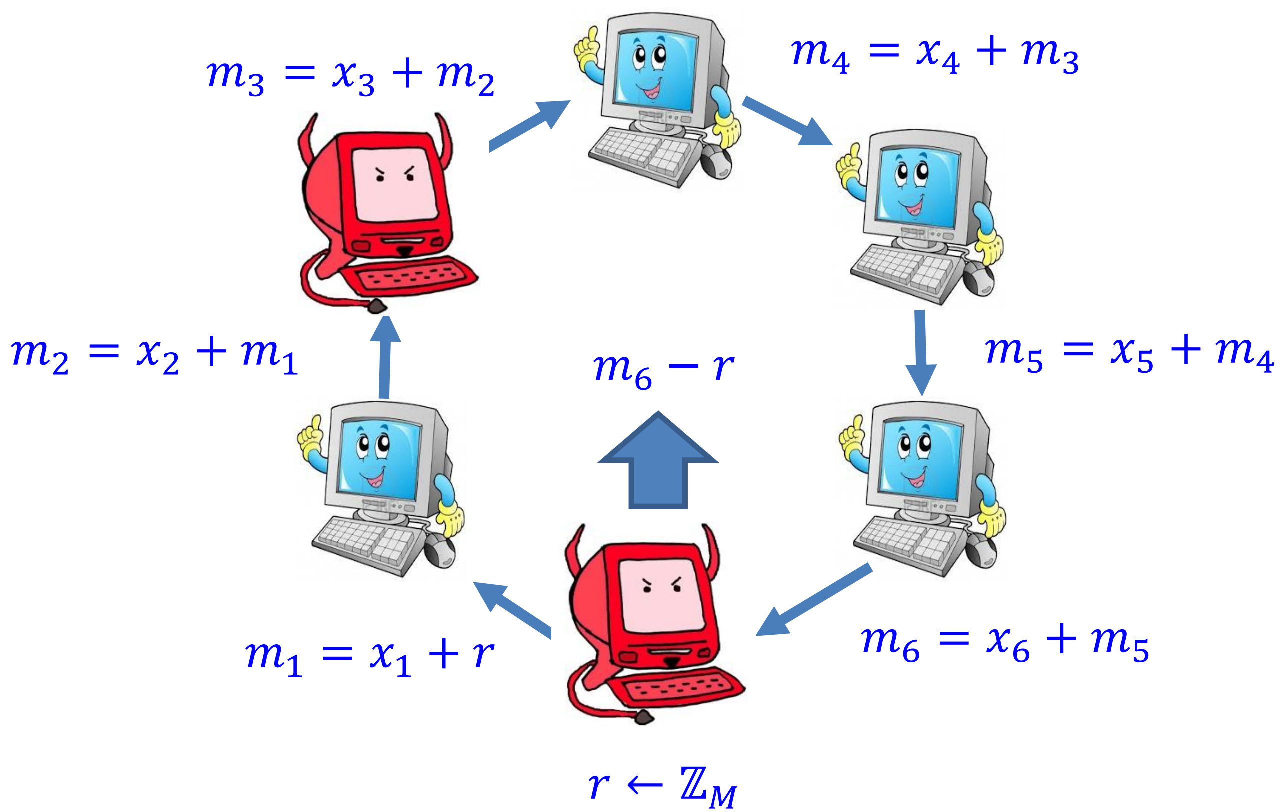
- **Correctness**: $\mathcal{A}$ can't win using lower bid than the highest
- **Privacy**: $\mathcal{A}$ learns an upper bound on all inputs, nothing else
- **Independence of inputs**: $\mathcal{A}$ can't bid one dollar more than the highest (honest) bid
- **Fairness**: $\mathcal{A}$ can't abort the auction if his bid isn't the highest (i.e., after learning the result)
- **Guaranteed output delivery**: $\mathcal{A}$ can't abort (stronger than fairness, no DoS attacks)

Security Requirements

- **Correctness**: parties obtain correct output (even if some parties misbehave)
- **Privacy**: only the output is learned (nothing else)
- **Independence of inputs**: parties cannot choose their inputs as a function of other parties' inputs
- **Fairness**: if one party learns the output, then all parties learn the output
- **Guaranteed output delivery**: all honest parties learn the output

Example – Computing Sum

- Each P_i has input $x_i < M$ (work modulo M)
- Want to compute $\sum x_i$
- Is the protocol is secure facing one corruption (semi-honest)?
- What about two corruptions?



How to Define Security

Option 1: property-based definition

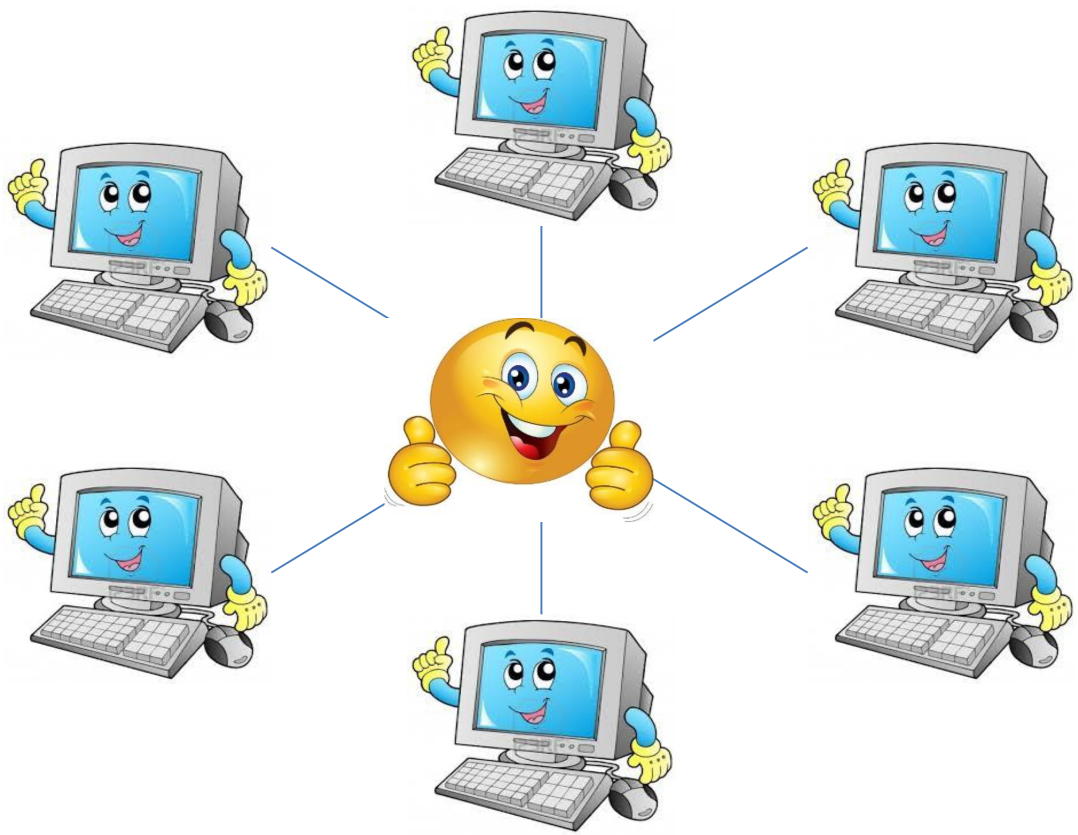
- Define a list of security requirements for the task
- Used for Byzantine agreement, coin flipping, etc.
- Difficult to analyze complex tasks
- How do we know if all concerns are covered?

Option 2: the real/ideal paradigm

- Whatever an adversary can achieve by attacking a real protocol can also be achieved by attacking an ideal computation involving a trusted party
- Formalized via a simulator

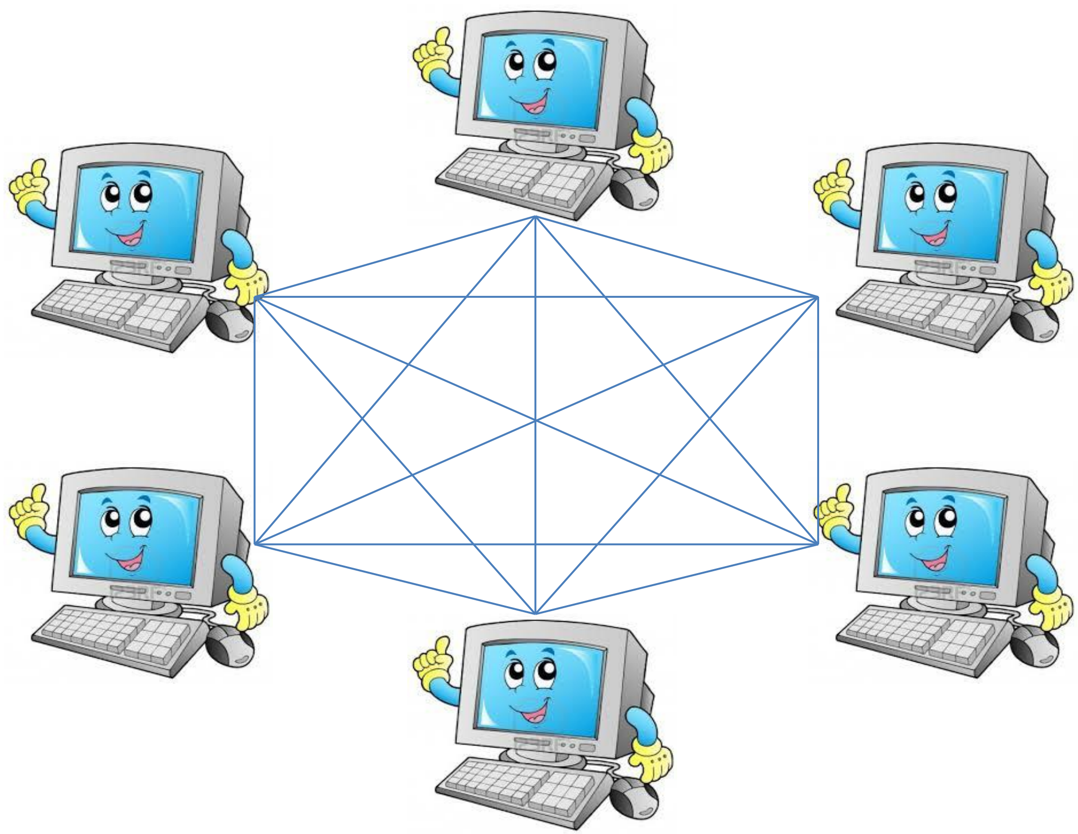
Ideal World

1) Each party sends its input to the trusted party
2) The trusted party computes $y = f(x_1, \ldots, x_n)$
3) Trusted party sends y to each party

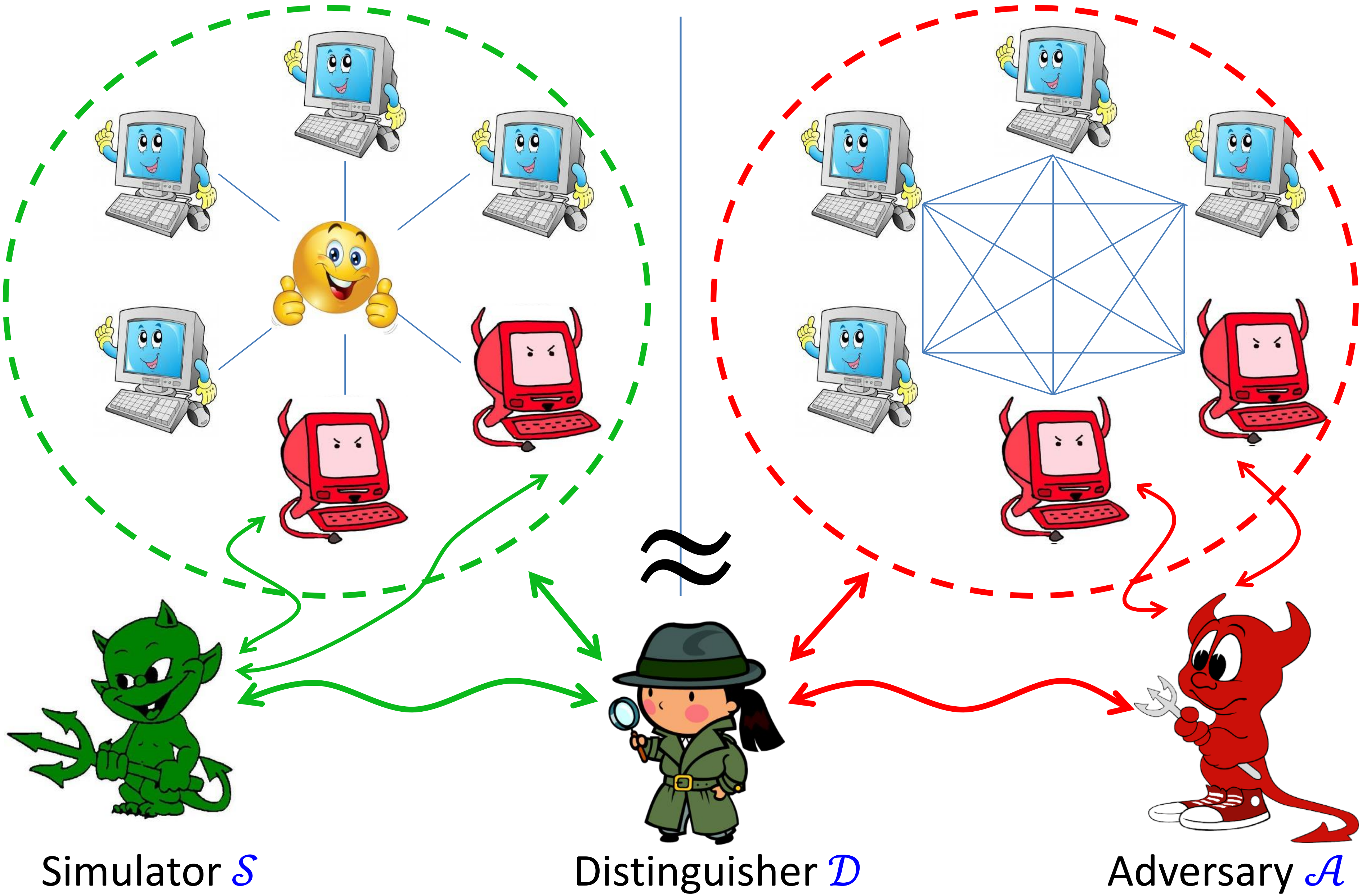


Real World

Parties run a protocol π on inputs $(x_1, \ldots, x_n)$



Simulation-Based Security

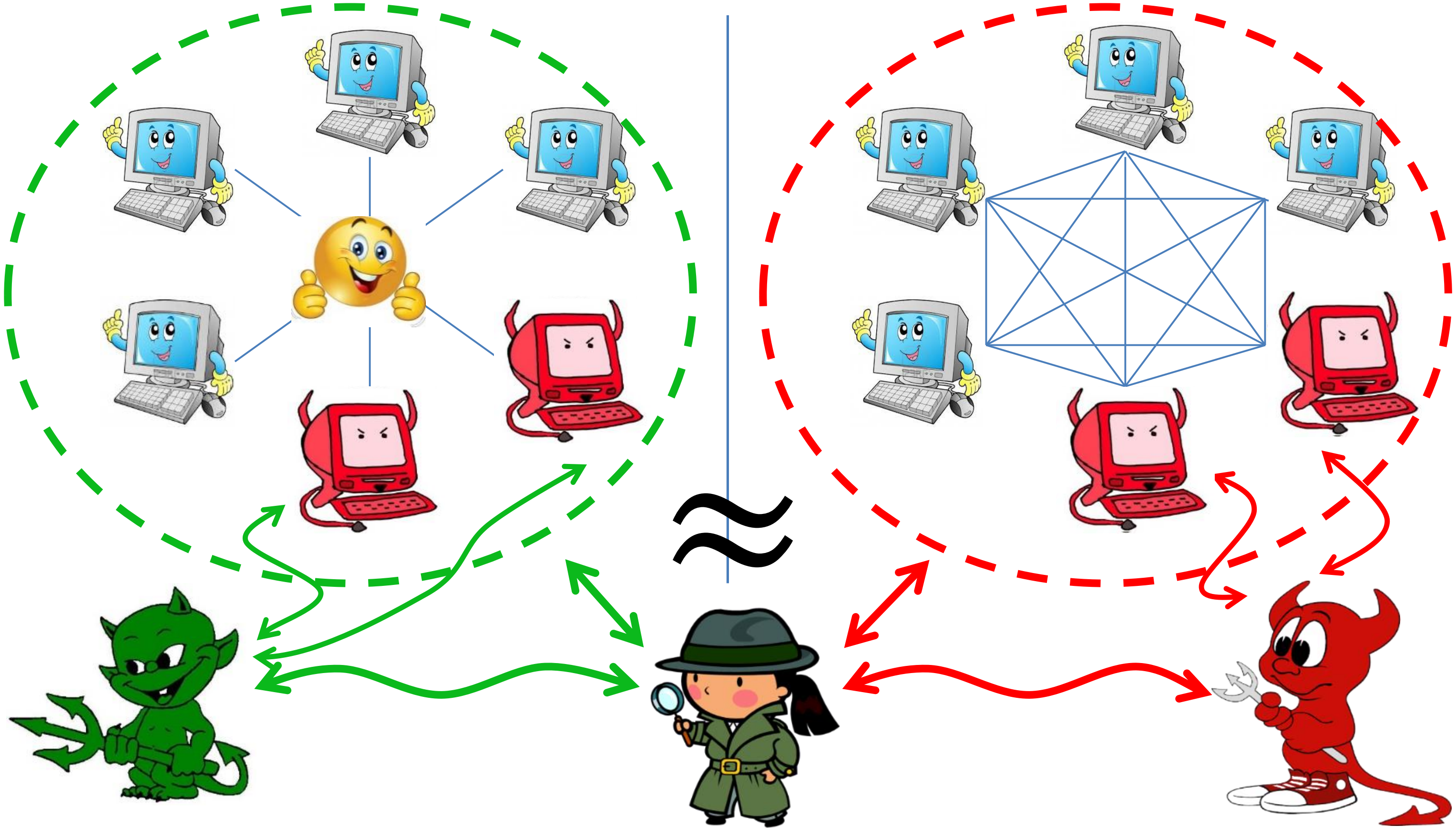


Simulation-Based Security

The distinguisher $\mathcal{D}$:

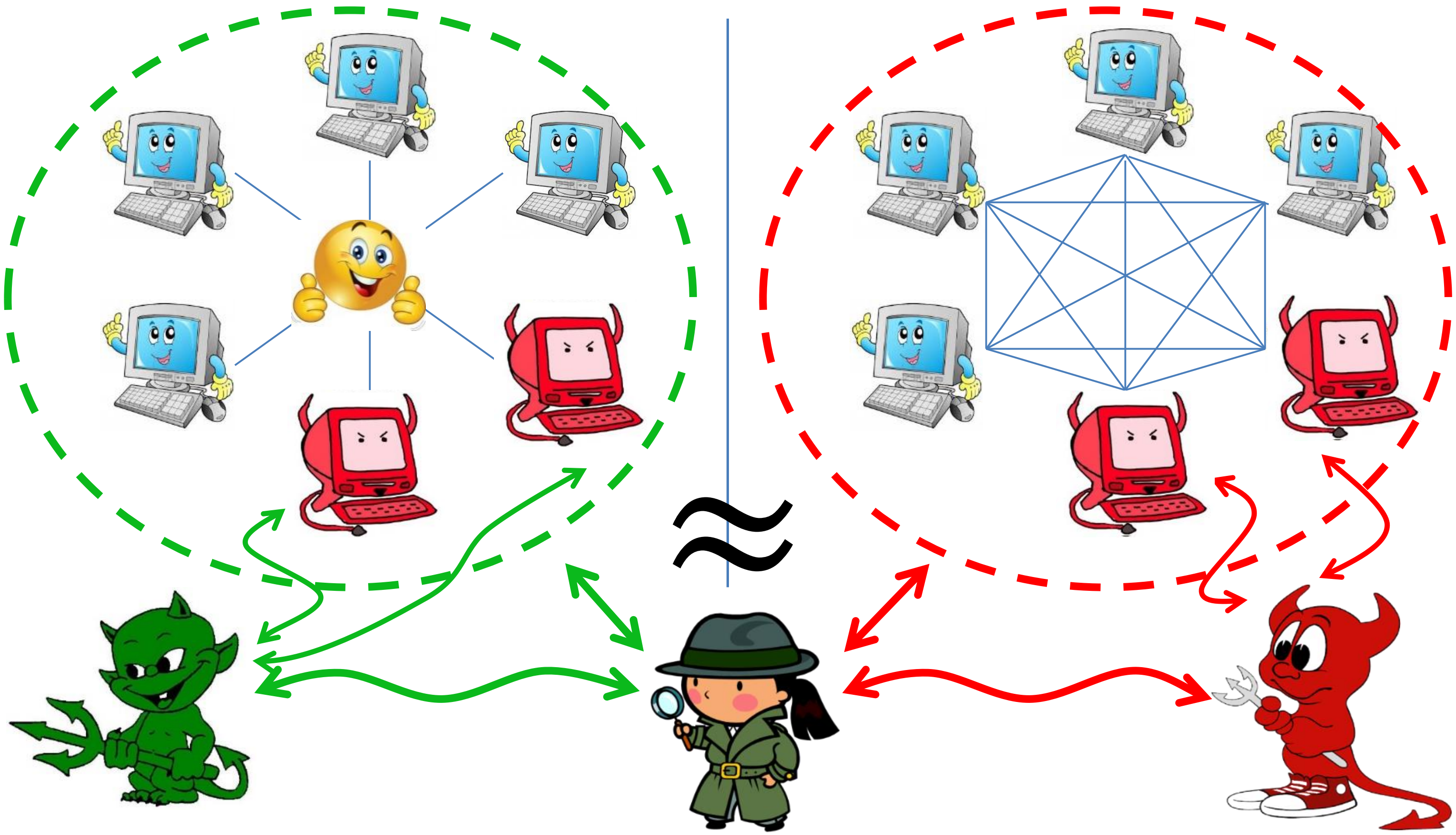
- Gives inputs to parties
- Gets back output from parties and from adversary/simulator
- Guesses which world it is real/ideal

Protocol π securely computes f if $\forall \mathcal{A}\ \exists \mathcal{S}\ \forall \mathcal{D}$ distinguishing success is "small"



Sanity check

- Correctness
- Privacy
- Independence of inputs
- Fairness
- Guaranteed output delivery



Advantages of this Approach

- Very general – captures any computational task
- The security guarantees are simple to understand
  Simply imagine a trusted party computes the task
- No security requirements are ”missed”
- Supports sequential modular composition
  - Security remains when secure protocols run sequentially
  - A single execution at a time
  - Arbitrary messages can be sent between executions
- Useful for modular design of protocols

Sequential Modular Composition

- Design a protocol in a hybrid model
  - Similar to the stand-alone real world
  - A trusted party helps to compute some functionality f
  - In rounds with calls to f no other messages are allowed
- Theorem (informal)
  - Protocol π securely computes g in the f-hybrid model
  - Protocol ρ securely computes f
  - Then, protocol π^ρ securely computes g in the real world

Replace ideal calls to f with real protocol ρ

The Definition Cont’d

A definition of an MPC task involves defining:

- Functionality: what do we want to compute?
- Security type: how strong protection do we want?
- Adversarial model: what do we want to protect against?
- Network model: in what setting are we going to do it?

The Functionality

- The code of the trusted party
- Captures inevitable vulnerabilities
- Sometimes useful to let the functionality talk to the ideal-world adversary (simulator)
- We will focus on secure function evaluation (SFE), the trusted party computes $y = f(x_1, \dots, x_n)$
  - Deterministic vs. randomized
  - Single public output vs. private outputs
  - Reactive vs. non-reactive

Security Type

- **Computational**: a PPT distinguisher
  - The real & ideal worlds are computationally indistinguishable
- **Statistical**: all-powerful distinguisher, negligible error probability
  - The real & ideal worlds are statistically close
- **Perfect**: all-powerful distinguisher, zero error probability
  - The real & ideal worlds are identically distributed

Adversarial Model (1)

- Adversarial behavior
  - **Semi honest**: honest-but-curious. corrupted parties follow the protocol honestly, $\mathcal{A}$ tries to learn more information. Models inadvertent leakage
  - **Fail stop**: same as semi honest, but corrupted parties can prematurely halt. Models crash failures
  - **Malicious**: corrupted parties can deviate from the protocol in an arbitrary way

Adversarial Model (2)

- Adversarial power
  - **Polynomial time**: computational security, normally requires cryptographic assumptions, e.g., encryption, signatures, oblivious transfer
  - **Computationally unbounded**: an all-powerful adversary, information-theoretic security

Adversarial Model (3)

- Adversarial corruption
  - **Static**: the set of corrupted parties is defined before the execution of the protocol begins. Honest parties are always honest, corrupted parties are always corrupted
  - **Adaptive**: $\mathcal{A}$ can decide which parties to corrupt during the coarse of the protocol, based on information it dynamically learns
  - **Mobile**: $\mathcal{A}$ can "jump" between parties
    Honest parties can become corrupted,
    corrupted parties can become honest again

Adversarial Model (4)

- Number of corrupted parties
  - Threshold adversary:
    Denote by $t \leq n$ an upper bound on # corruptions
    - No honest majority, e.g., two-party computation
    - Honest majority, i.e., $t < n/2$
    - Two-thirds majority, i.e., $t < n/3$
  - General adversary structure:
    Protection against specific subsets of parties

Communication Model (1)

- **Point-to-point**: fully connected network of pairwise channels.
  - Unauthenticated channels
  - Authenticated channels: in the computational setting
  - Private channels: in the IT setting
  - Partial networks: star, chain
- **Broadcast**: additional broadcast channel

Communication Model (2)

- Message delivery:
  - **Synchronous**: the protocol proceeds in rounds. Every message that is sent arrives within an known time frame
  - **Asynchronous (eventual delivery)**: the adversary can impose arbitrary (finite) delay on any message
  - **Fully Asynchronous**: the adversary has full control over the network, can even drop messages

Execution Environment

- **Stand alone**:
  - A single protocol execution at any given time (isolated from the rest of the world)
- **Concurrent general composition**:
  - Arbitrary protocols are executed concurrently
  - An Internet-like setting
  - Requires a strictly stronger definition
    Captured by the universal composability (UC) framework
  - Impossible in general without a trusted setup assumption (e.g., common reference string)

Relaxing the Definition

- Recall the ideal world (with guaranteed output delivery)
  1) Each party sends its input to the trusted party
  2) The trusted party computes $y = f(x_1, \dots, x_n)$
  3) Trusted party sends y to each party
- This ideal world is overly ideal
- In general, fairness cannot be achieved without an honest majority [Cleve'86]
- A relaxed definition is normally considered

Security with Abort

- Ideal world without fairness and guaranteed output delivery:

  1) Each party sends its input to the trusted party
  2) The trusted party computes $y = f(x_1, \ldots, x_n)$
  3) Trusted party sends y to the adversary
  4) The adversary responds with continue/abort
  5) If continue, trusted party sends y to all parties
     If abort, trusted party sends $\perp$ to all parties

- Correctness, privacy, independence of inputs are satisfied

Prevalent Models

- In the seminar we will consider:
  - Adversary: semi honest / malicious with static corruptions
  - Synchronous P2P network with a broadcast channel
  - Stand-alone setting
- Computational setting
  - PPT adversary & distinguisher (computational security)
  - Arbitrary number of corruptions $t < n$
  - Authenticated channels
- Information-theoretic setting
  - All powerful adversary & distinguisher (perfect/statistical)
  - Honest majority $t < n/2$ (if $t < n/3$ no need for broadcast)
  - Secure channels

Oblivious Transfer

Feasibility Results

- Malicious setting
  - For $t < n/3$, every f can be securely computed with perfect security [BGW'88,CCD'88]
  - For $t < n/2$, every f can be securely computed with statistical security [RB'89]
  - For $t < n$, assuming OT, every f can be securely computed with abort and computational security [GMW'87]
- Semi-honest setting
  - For $t < n/2$, every f can be securely computed with perfect security [BGW'88,CCD'88]
  - For $t < n$, assuming OT, every f can be securely computed with computational security [GMW'87]

Outline of the Seminar

- Lecture 2: definitions
- Lectures 3-7: semi-honest setting
  - Yao's garbled circuit
  - Oblivious transfer
  - GMW protocol [Goldreich, Micali, Wigderson'87]
  - BGW protocol [Ben-Or, Goldwasser, Wigderson'88]
  - BMR protocol (constant-round MPC) [Beaver, Micali, Rogaway'90]
- Lectures 8-11: malicious setting
  - GMW compiler
  - IKOS zero-knowledge proof
  - Cut and choose (Yao's protocol for malicious)
  - Sigma protocols
- Lecture 12: specific functionalities (median, PSI)

Summary

- Secure multiparty protocols emulate computations involving a trusted party
- Impressive feasibility results: every task that can be computed can also be computed securely
- Many different models and settings
- Exciting and active field – many open questions