Information Theory, Fall 2015	Iftach Haitner

Problem set 4

December 27, 2015	Due: December 22

- Please submit the handout in class, or email the grader (omer.rotem1 at gmail.com).
- Write clearly and shortly using sub-claims if needed. The emphasize in most questions is on the proofs (no much point is writing a “solution" w/o proving its correctness)
- For Latex users, a solution example can be found in the course web site.
- It is allowed to work in (small) groups, but please write the id list of your partners in the solution file, and each student should write his solution by *himself* (joint effort is only allowed in the “thinking phase")

1. Show that $\mathrm{SD}(p,q) = \max_{\mathcal{S} \subseteq [m]} \left(\sum_{i \in \mathcal{S}} p_i - \sum_{i \in \mathcal{S}} q_i \right)$ for any two distributions p, q over $[m]$.

2. Use the above to prove that $\mathrm{SD}(p,q) = \max_{\mathsf{D}} \left(\Pr_{X \sim p} [\mathsf{D}(X) = 1] - \Pr_{X \sim q} [\mathsf{D}(X) = 1] \right)$, where the max is over all deterministic distinguishers. Try and extend the above to randomized distinguishers?

3. Relative entropy is not symmetric: give two distributions p, q such that $D(p \| q) \neq D(q \| p)$, and $D(p \| q), D(q \| p) < \infty$.

4. Relative entropy does not obey the triangle inequality: give three distributions p_1, p_2, p_3 such that $D(p_1 \| p_2) + D(p_2 \| p_3) < D(p_1 \| p_3)$

5. Relative entropy is non-negative: given two distributions p, q, show that $D(p \| q) \geq 0$, with equality only if $p = q$.

6. Does Theorem 4 in Lecture 7 hold for any prefix code C with $\mathrm{E}_{i \leftarrow q} [|C(i)|] \leq H(q) + 1$?

   (and not only for a code C with $C(x) = \left\lceil \log \frac{1}{q(x)} \right\rceil$, as stated in the theorem)

7. Prove the data processing inequality of relative entropy (Claim 7 in Lecture 7) for randomized functions.