Foundation of Cryptography, Lecture 7
Commitment Schemes

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Section 1

Commitment Schemes

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Digital analogue of a safe.

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Definition 1 (Commitment scheme)

An efficient two-stage protocol (S, R) .

Commit The sender S has private input $\sigma \in \{0,1\}^*$ and the common input is 1^n. The commitment stage result in a joint output c, the commitment, and a private output d to S, the decommitment.

Reveal S sends the pair (d, σ) to R, and R either accepts or rejects.

Completeness: R always accepts in an honest execution.

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Hiding:. In commit stage: $\forall$ PPT R^*, $m \in \mathbb{N}$ and $\sigma \neq \sigma' \in \{0,1\}^m$,
$\{\mathsf{View}_{\mathsf{R}^*}(\mathsf{S}(\sigma), \mathsf{R}^*)(1^n)\}_{n\in\mathbb{N}} \approx_c \{\mathsf{View}_{\mathsf{R}^*}(\mathsf{S}(\sigma'), \mathsf{R}^*)(1^n)\}_{n\in\mathbb{N}}$.

Commitment Schemes cont.

Binding: "Any" S^* succeeds in the following game with negligible probability in n:

> *On security parameter* 1^n, S^* *interacts with* R *in the commit stage resulting in a commitment* c, *and then output two pairs* (d, σ) *and* (d', σ') *with* $\sigma \neq \sigma'$ *such that*
> $\mathsf{R}(c, d, \sigma) = \mathsf{R}(c, d', \sigma') = \texttt{Accept}$

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- (non-uniform) OWFs imply statistically binding, and statistically hiding commitments

Perfectly Binding Commitment from OWP

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Protocol 2 ((S,R))

Commit:

S's input: $\sigma \in \{0,1\}$

S chooses a random $x \in \{0,1\}^n$, and sends $c = (f(x), b(x) \oplus \sigma)$ to R

Reveal:

S sends (x,σ) to R, and R accepts iff (x,σ) is consistent with c (i.e., $f(x) = c_1$ and $b(x) \oplus \sigma = c_2$)

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$$\Delta_n^{\mathsf{A}} = |\Pr[\mathsf{A}(f(U_n), b(U_n) \oplus 0) = 1] - \Pr[\mathsf{A}(f(U_n), b(U_n) \oplus 1) = 1]|$$

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Thus, Δ_n^{A} is negligible for any PPT

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Common input: 1^n

S's input: $\sigma \in \{0,1\}$

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1. R chooses a random $r \leftarrow \{0,1\}^{3n}$ to S
2. S chooses a random $x \in \{0,1\}^n$, and send $g(x)$ to S in case $\sigma = 0$ and $c = g(x) \oplus r$ otherwise.

Reveal: S sends (σ, x) to R, and R accepts iff (σ, x) is consistent with r and c

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