Foundation of Cryptography (0368-4162-01), Lecture 9

Secure Multiparty Computation

Iftach Haitner, Tel Aviv University

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Section 1

The Model

Multiparty Computation

- Multiparty Computation – computing a functionality f

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- Secure Multiparty Computation: compute f in a “secure manner"

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Examples: coin-tossing, broadcast, electronic voting, electronic auctions

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What is a secure protocol for a given task?
We focus on protocol Π for computing a two-party functionality $f\colon \{0,1\}^* \times \{0,1\}^* \times \{0,1\}^* \times \{0,1\}^*$

Real Model Execution

Let $\overline{\mathsf{A}} = (\mathsf{A}_1, \mathsf{A}_2)$ be a pair of algorithms, and $x_1, x_2 \in \{0, 1\}^*$. Define $\mathsf{REAL}_{\overline{\mathsf{A}}}(x, y)$ as the joint outputs of $(\mathsf{A}_1(x_1), \mathsf{A}_2(x_2))$

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- An honest party follows the prescribed protocol and outputs of the protocol
- A semi-honest party follows the protocol, but might output additional information

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Let $\overline{\mathrm{B}} = (\mathrm{B}_1, \mathrm{B}_2)$ be a pair of oracle-aided algorithms. An execution of $\overline{\mathrm{B}}$ in the ideal model on inputs $x_1, x_2 \in \{0,1\}^*$, denoted $\mathrm{IDEAL}_{f,\overline{\mathrm{B}}}(x, y)$, is the joint output of the parties in the end of the following experiment:

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1. The input of B_i is x_i ($i \in \{0,1\}$)
2. Each party send the value y_i to the *trusted party* (possibly $\perp$)
3. Trusted party send $f_i(y_0, y_1)$ to B_i (sends $\perp$, if $\perp \in \{y_0, y_1\}$)
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Definition 1 (secure computation)

a protocol π securely computes f (in the malicious model), if $\forall$ real model, admissible PPT $\overline{\mathrm{A}} = (\mathrm{A}_1, \mathrm{A}_2)$, exists an ideal-model admissible pair PPT $\overline{\mathrm{B}} = (\mathrm{B}_1, \mathrm{B}_2)$, s.t.

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where the enumeration is over all $x_1, x_2 \in \{0, 1\}^*$ with $|x_1| = |x_2|$.

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- Auxiliary inputs
- We focus on semi-honest adversaries

Section 2

Oblivious Transfer

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A protocol for securely realizing the functionality $f: (\{0,1\}^* \times \{0,1\}^*) \times \{0,1\} \mapsto \{0,1\}^* \times \perp$, where $f_1((x_0, x_1), i) = x_i$ and $f_2(\cdot) = \perp$.

- “Complete" for multiparty computation

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- “Complete" for multiparty computation
- We focus on bit strings

Oblivious Transfer from Trapdoor Permutations

- We define a protocol $\pi = (\mathsf{S}, \mathsf{R})$ where R's input is $i \in \{0, 1\}$, and S inputs is $\sigma_0, \sigma_1 \in \{0, 1\}$. Both parties gets a common input 1^n.

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- Can be easily modified to the standard definition of two-party computation
- Let (G, f, Inv) be a family of trapdoor permutations and let b be an hardcore predicate for f.

Protocol 2 ((S, R))

Common input: 1^n
S's **input:** $\sigma_0, \sigma_1 \in \{0,1\}$
R's **input:** $i \in \{0,1\}$

1. S chooses $(e,d) \leftarrow G(1^n)$, and sends e to R
2. R chooses $x_0, x_1 \leftarrow \{0,1\}^n$, sets $y_i = f_e(x_i)$ and $y_{1-i} = x_{1-i}$, and sends y_0, y_1 to S
3. S sets $c_j = b(\mathrm{Inv}_d(y_j)) \oplus \sigma_j$, for $j \in \{0,1\}$, and sends (c_0, c_1) to R
4. R outputs $c_i \oplus b(x_i)$.

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Claim 3

Protocol 2 securely realizes f (in the semi -honest model.

Proving Claim 3

We need to prove that $\forall$ real model, semi-honest, admissible PPT $\overline{\mathsf{A}} = (\mathsf{A}_1, \mathsf{A}_2)$, exists an ideal-model, admissible pair PPT $\overline{\mathsf{B}} = (\mathsf{B}_1, \mathsf{B}_2)$ s.t.

$$\{\mathsf{REAL}_{\overline{\mathsf{A}}}(1^n, (\sigma_0, \sigma_1), i\} \approx_c \{\mathsf{IDEAL}_{f,\overline{\mathsf{B}}}(1^n, (\sigma_0, \sigma_1), i\}, \tag{1}$$

where $n \in \mathbb{N}$ and $\sigma_0, \sigma_1, i \in \{0, 1\}$

Semi-honest S

For $\overline{\mathsf{A}} = (\mathsf{S}', \mathsf{R})$ where S' is a semi-honest implementation of S, let $\overline{\mathsf{B}} = (\mathsf{S}'_{\mathcal{I}}, \mathsf{R}_{\mathcal{I}})$ be the following ideal-model protocol:

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Algorithm 4 ($\mathsf{S}'_{\mathcal{I}}$)

input: $1^n, \sigma_0, \sigma_1$

1. Send (σ_0, σ_1) to the trusted party
2. Emulate $\mathsf{S}'(1^n, \sigma_0, \sigma_1)$, acting as $\mathsf{R}(1^n, 0)$
3. Output the same output that S' does

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Claim 5

Equation (1) holds with respect to $\overline{\mathsf{A}}$ and $\overline{\mathsf{B}}$.

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Proof?

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Algorithm 6 ($\mathsf{R}'_{\mathcal{I}}$)

input: 1^n,$i \in \{0, 1\}$

1. Send i to the trusted party, and let σ be its answer.
2. Emulate $\mathsf{R}'(1^n, i)$, acting as $\mathsf{S}(1^n, \sigma_0, \sigma_1)$, where $\sigma_i = \sigma$, and $\sigma_{1-i} = 0$
3. Output the same output that R' does

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Claim 7

Equation (1) holds with respect to $\overline{\mathsf{A}}$ and $\overline{\mathsf{B}}$.

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Proof?

Section 3

Yao Grabbled Circuit