

**Problem set 1***October 27, 2015*

Due: Nov 10

- Please submit the handout in class, or email the grader (omer.rotem1 at gmail.com ).
- Write clearly and shortly using sub-claims if needed. The emphasize in most questions is on the proofs (no much point is writing a “solution” w/o proving its correctness)
- For Latex users, a solution example can be found in the course web site.
- It is allowed to work in (small) groups, but please write the id list of your partners in the solution file, and each student should write his solution by *himself* (joint effort is only allowed in the “thinking phase”)

1. Prove that the (Shanon) entropy function satisfies the (basic) grouping axioms  $A3$  (also verify for yourself that it satisfies the axioms  $A1, A2, A4$ ).
2. In Lecture 1, slide 10, we prove that  $H^*(p_1, p_2, p_3) = H(p_1, p_2, p_3)$  for any *rational*  $p_1, p_2, p_3$ , and say that the non-rational case follows by the continuity of  $H^*$ . Prove it.
3. Let  $(p_1, \dots, p_m)$  and  $(q_1, \dots, q_m)$  be probability distributions (i.e.,  $p_i \geq 0$  for all  $i$  and  $\sum_i p_i = 1$ ). Prove that

$$-\sum_i p_i \log p_i \leq -\sum_i p_i \log q_i$$

4. For random variables  $X$  and  $Y$ , what is larger, prove your answers: (you can use any of the inequalities stated in first two lectures.)
  - (a)  $H(X|Y)$  or  $H(f(X)|Y)$ ?
  - (b)  $H(X|Y)$  or  $H(X|g(Y))$ ?
  - (c)  $H(X|Y)$  or  $H(f(X, Y)|Y)$ ?
  - (d)  $H(X|Y)$  or  $H(X|g(X, Y))$ ?
5. For a finite set  $\mathcal{S}$  of random variables, let  $H(\mathcal{S})$  denote the joint entropy of all random variables in  $\mathcal{S}$ . Prove that for any two finite sets of random variables  $\mathcal{S}$  and  $\mathcal{U}$ , it holds that  $H(\mathcal{S} \cup \mathcal{U}) + H(\mathcal{S} \cap \mathcal{U}) \leq H(\mathcal{S}) + H(\mathcal{U})$ .