Application of Information Theory, Lecture 12

Accessible Entropy and Statistically Hiding Commitments

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Section 1

Commitment Schemes

Motivation

- Digital analogue of a safe
- Numerous applications (e.g., zero-knowledge, coin-flipping, secure computations)

Definition

μ is negligible, denoted $\mu(n) = \mathsf{neg}(n)$, if $\forall p \in \mathsf{poly}\ \exists n' \in \mathbb{N}$ s.t. $\mu(n) < \frac{1}{p(n)}$ forall $n > n'$.

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An efficient two-stage protocol (S, R).

- Commit stage: The sender S has private input bit $b \in \{0, 1\}$ and a common input is 1^n. Let trans be the transcript of this stage.
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Hiding: Let $V_n^{\mathsf{R}^*}(b)$ be R^*'s *view* in (the commit stage of) $(\mathsf{S}(b), \mathsf{R}^*)(1^n)$.
Then for any R^*: $\Delta^{\mathsf{R}^*}(V_n^{\mathsf{R}^*}(0), V_n^{\mathsf{R}^*}(1)) = \mathsf{neg}(n)$.

Binding: The following happens with negligible probability for any S^*:

> $\mathsf{S}^*(1^n)$ *interacts with* $\mathsf{R}(1^n)$ *in the commit stage resulting in transcript* trans*. Then* S^* *outputs two strings* r_0 *and* r_1 *such that* $\mathsf{R}(\mathsf{trans}, r_0, 0) = \mathsf{R}(\mathsf{trans}, r_1, 1) = \texttt{Accept}$.

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Alternative Binding definition: Assume that following the interaction S^* outputs a pair (r, b) with $\mathsf{R}(\mathsf{trans}, r, b) = \mathtt{Accept}$. Let V^{S^*} be S^*'s view in (the commit stage of) $(\mathsf{S}^*, \mathsf{R}^*)(1^n)$. Then $\mathsf{H}(b|V^{\mathsf{S}^*}) = \mathsf{neg}(n)$.

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- We focus on computationally binding, and statistically hiding commitments (SHC)

Section 2

Inaccessible Entropy

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Section 3

Manipulating Inaccessible Entropy

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For $\ell \in \text{poly}$ let $G^{\bigotimes \ell}$ be the following $(\ell - 1) \cdot m$-bit generator

$$G^{\bigotimes \ell}(x_1, \ldots, x_\ell, i) = G(x_1)_i, \ldots, G(x_1)_m, \ldots, G(x_\ell)_1, \ldots, G(x_\ell)_{i-1}$$

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  1. For any $i \in [(\ell - 1) \cdot m]$ and $(g_{\leq i-1}) \in \text{Supp}(G^{\bigotimes \ell}_{\leq i-1})$:

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- Assume $k_R \geq k_A + 1$, then for $\ell = m + 2$, it holds that $k_R^{\bigotimes \ell} \geq k_A^{\bigotimes \ell} + 1$

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- Assume accessible entropy of G is (at most) k_A, then the accessible entropy of G is at most $k_A^\ell = \ell k_A$.
- Assume $H(G_i|G_{\leq i-1}) = k_R$ for any $i \in [m]$, then for any $i \in [m]$ and $(g^\ell_{\leq i-1}) \in \mathsf{Supp}(G^\ell_{\leq i-1})$ it holds that

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- If $k_A \leq k_R - 1$, then $\forall n \in \mathsf{poly}\ \exists \ell \in \mathsf{poly}$ such that $\ell k^\ell_{min} > k^\ell_A + n$

Section 4

Inaccessible Entropy from OWF

The generator

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Given a function $f\colon \{0,1\}^n \mapsto \{0,1\}^n$, let G be the $(n+1)$-block generator

$$G(x) = f(x)_1, \ldots, f(x)_n, x$$

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Assume that f is a OWF then G has accessible entropy at most $n - \log n$.

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- Proof idea

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Let $\widetilde{G}$ be a PPT, and assume $\Pr\left[\mathsf{AccH}_{G,\widetilde{G}}(\widetilde{T}) \geq n - \log n\right] \geq \varepsilon = \frac{1}{\mathrm{poly}(n)}$.
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1. For $i = 1$ to n, do the following for n^2/ε times:
   1.1 Sample r_i uniformly at random and let g_i be the i'th output block of $\widetilde{G}(r_1, \ldots, r_i)$.
   1.2 If $g_i = z_i$, move to next value of i.
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  1. $\mathrm{AccH}_{G,\widetilde{G}}(\mathbf{t}) \geq n - \log n$, and

$\widetilde{T}$ vs. $\widehat{T}$ cont.

- $\mathbf{t} = (r_1, g_1, \ldots, r_{n+1}, g_{n+1}) \in \mathrm{Supp}(\widetilde{T})$
- $\Pr_{\widehat{T}}[\mathbf{t}] = \frac{\Pr\left[f(U_n)=g_{\leq n}\right]\cdot\Pr\left[\widetilde{G}_{n+1}=g_{n+1}|\widetilde{R}_{\leq n}=r_{\leq n}\right]}{2^{-\mathrm{AccH}_{G,\widetilde{G}}(\mathbf{t})}} \cdot \Pr_{\widetilde{T}}[\mathbf{t}]$
- Note that $\Pr\left[f(U_n) = g_{\leq n}\right] \cdot \frac{1}{\left|f^{-1}(g_{\leq n})\right|} = 2^{-n}$
- Hence, for $\mathbf{t}$ with
  1. $\mathrm{AccH}_{G,\widetilde{G}}(\mathbf{t}) \geq n - \log n$, and
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  It holds that

$$\Pr_{\widetilde{T}}[\mathbf{t}] \ge \frac{\alpha}{n} \cdot \Pr_{\widehat{T}}[\mathbf{t}] \tag{1}$$

Inv's success probability

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Let $\mathcal{S} \subseteq \mathsf{Supp}(\widetilde{T})$ denote the set of transcripts $\mathbf{t} = (r_1, g_1, \ldots, r_{n+1}, g_{n+1})$ with

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- $\Pr_{\widetilde{T}}[\mathcal{S}] \geq \Pr\left[\mathrm{AccH}_{G,\widetilde{G}}(T) \geq n - \log n\right] - 2 \cdot \frac{\varepsilon}{4} \geq \frac{\varepsilon}{2}$

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- $\Pr_{\widetilde{T}}\left[\exists i \in [n] \colon H_{\widetilde{G}_i | \widetilde{R}_{\leq i-1}}(g_i \mid r_{\leq i-1}) > \log(\frac{4n}{\varepsilon})\right] \leq n \cdot \frac{\varepsilon}{4n} = \varepsilon/4$
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- $\Pr_{\widetilde{T}}\left[\exists i \in [n] \colon H_{\widetilde{G}_i|\widetilde{R}_{\leq i-1}}(g_i \mid r_{\leq i-1}) > \log(\frac{4n}{\varepsilon})\right] \leq n \cdot \frac{\varepsilon}{4n} = \varepsilon/4$
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- $\Pr_{\widetilde{T}}[\mathcal{S}] \geq \Pr\left[\mathsf{AccH}_{G,\widetilde{G}}(T) \geq n - \log n\right] - 2 \cdot \frac{\varepsilon}{4} \geq \frac{\varepsilon}{2}$
- By Eq. (1): $\Pr_{\widehat{T}}[\mathcal{S}] \geq \frac{\varepsilon/4}{n} \cdot \Pr_{\widetilde{T}}[\mathcal{S}] \geq \frac{\varepsilon^2}{8n} \ldots$

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- For $z \in \{0,1\}^n$ for which $\exists (r_1, z_1, \ldots, r_n, z_n, \ldots) \in \mathcal{S}$:
  $\Pr[\mathsf{Inv}(z) \text{ aborts }] \leq n \cdot (1 - \frac{\varepsilon}{4n})^{n^2/\varepsilon} = O(n \cdot 2^{-n}) \leq \frac{1}{2}$

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- $\Pr_{\widetilde{T}}\left[\exists i \in [n] \colon H_{\widetilde{G}_i|\widetilde{R}_{\leq i-1}}(g_i \mid r_{\leq i-1}) > \log(\frac{4n}{\varepsilon})\right] \leq n \cdot \frac{\varepsilon}{4n} = \varepsilon/4$
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- $\Pr_{\widetilde{T}}[\mathcal{S}] \geq \Pr\left[\mathrm{AccH}_{G,\widetilde{G}}(T) \geq n - \log n\right] - 2 \cdot \frac{\varepsilon}{4} \geq \frac{\varepsilon}{2}$
- By Eq. (1): $\Pr_{\widehat{T}}[\mathcal{S}] \geq \frac{\varepsilon/4}{n} \cdot \Pr_{\widetilde{T}}[\mathcal{S}] \geq \frac{\varepsilon^2}{8n} \ldots$

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- For $z \in \{0,1\}^n$ for which $\exists (r_1, z_1, \ldots, r_n, z_n, \ldots) \in \mathcal{S}$:
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- Hence, $\Pr_{\widehat{T}}[\mathcal{S}] \geq \frac{\varepsilon^2}{16n}$

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Let $\mathcal{S} \subseteq \mathrm{Supp}(\widetilde{T})$ denote the set of transcripts $\mathbf{t} = (r_1, g_1, \ldots, r_{n+1}, g_{n+1})$ with

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2. $H_{\widetilde{G}_i \mid \widetilde{R}_{\leq i-1}}(g_i \mid r_{\leq i-1}) \leq \log(\frac{4n}{\varepsilon})$ for all $i \in [n]$,
3. $H_{\widetilde{G}_{n+1} \mid \widetilde{R}_{\leq n}}(g_{n+1} \mid r_{\leq n}) \leq \log(\frac{4}{\varepsilon} \cdot \left|f^{-1}(g_{\leq n})\right|$.

- $\Pr_{\widetilde{T}}\left[\exists i \in [n] \colon H_{\widetilde{G}_i \mid \widetilde{R}_{\leq i-1}}(g_i \mid r_{\leq i-1}) > \log(\frac{4n}{\varepsilon})\right] \leq n \cdot \frac{\varepsilon}{4n} = \varepsilon/4$
- $\Pr_{\widetilde{T}}\left[H_{\widetilde{G}_{n+1} \mid \widetilde{R}_{\leq n}}(g_{n+1} \mid r_{\leq n}) > \log(\frac{4}{\varepsilon} \cdot \left|f^{-1}(g_{\leq n})\right|\right] \leq \varepsilon/4$
- $\Pr_{\widetilde{T}}[\mathcal{S}] \geq \Pr\left[\mathrm{AccH}_{G,\widetilde{G}}(T) \geq n - \log n\right] - 2 \cdot \frac{\varepsilon}{4} \geq \frac{\varepsilon}{2}$
- By Eq. (1): $\Pr_{\widehat{T}}[\mathcal{S}] \geq \frac{\varepsilon/4}{n} \cdot \Pr_{\widetilde{T}}[\mathcal{S}] \geq \frac{\varepsilon^2}{8n} \ldots$

Back the bounded version of Inv.

- For $z \in \{0,1\}^n$ for which $\exists (r_1, z_1, \ldots, r_n, z_n, \ldots) \in \mathcal{S}$:
  $\Pr[\mathrm{Inv}(z) \text{ aborts }] \leq n \cdot (1 - \frac{\varepsilon}{4n})^{n^2/\varepsilon} = O(n \cdot 2^{-n}) \leq \frac{1}{2}$
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3. $H_{\widetilde{G}_{n+1}|\widetilde{R}_{\leq n}}(g_{n+1} \mid r_{\leq n}) \leq \log(\frac{4}{\varepsilon} \cdot \left|f^{-1}(g_{\leq n})\right|$.

- $\Pr_{\widetilde{T}}\left[\exists i \in [n] \colon H_{\widetilde{G}_i|\widetilde{R}_{\leq i-1}}(g_i \mid r_{\leq i-1}) > \log(\frac{4n}{\varepsilon})\right] \leq n \cdot \frac{\varepsilon}{4n} = \varepsilon/4$
- $\Pr_{\widetilde{T}}\left[H_{\widetilde{G}_{n+1}|\widetilde{R}_{\leq n}}(g_{n+1} \mid r_{\leq n}) > \log(\frac{4}{\varepsilon} \cdot \left|f^{-1}(g_{\leq n})\right|\right] \leq \varepsilon/4$
- $\Pr_{\widetilde{T}}[\mathcal{S}] \geq \Pr\left[\mathrm{AccH}_{G,\widetilde{G}}(T) \geq n - \log n\right] - 2 \cdot \frac{\varepsilon}{4} \geq \frac{\varepsilon}{2}$
- By Eq. (1): $\Pr_{\widehat{T}}[\mathcal{S}] \geq \frac{\varepsilon/4}{n} \cdot \Pr_{\widetilde{T}}[\mathcal{S}] \geq \frac{\varepsilon^2}{8n} \ldots$

Back the bounded version of Inv.

- For $z \in \{0,1\}^n$ for which $\exists(r_1, z_1, \ldots, r_n, z_n, \ldots) \in \mathcal{S}$:
  $\Pr[\mathrm{Inv}(z) \text{ aborts }] \leq n \cdot (1 - \frac{\varepsilon}{4n})^{n^2/\varepsilon} = O(n \cdot 2^{-n}) \leq \frac{1}{2}$
- Hence, $\Pr_{\widehat{T}}[\mathcal{S}] \geq \frac{\varepsilon^2}{16n} \implies \Pr_{x \leftarrow \{0,1\}^n}\left[\mathrm{Inv}(f(x)) \in f^{-1}(f(x))\right] \geq \frac{\varepsilon^2}{16n}$

Section 5

Statistically Hiding Commitment from Inaccessible Entropy Generator

High-level description

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- Entropy equalization + gap amplification to get generator that has the same min-entropy in each block and whose accessible entropy is n-bit smaller than the sum of the min entropies.

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- Use a a random block to mask the committed bit, to get a weakly binding SHC

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- Use "hashing protocol" to get a “generator” with zero accessible entropy block
- Use a a random block to mask the committed bit, to get a weakly binding SHC
- Amplify the above into full-fledged SHC