Information Theory, Fall 2015	Iftach Haitner

Problem set 3

November 24, 2015 — Due: Dec 8

- Please submit the handout in class, or email the grader (omer.rotem1 at gmail.com).
- Write clearly and shortly using sub-claims if needed. The emphasize in most questions is on the proofs (no much point is writing a “solution" w/o proving its correctness)
- For Latex users, a solution example can be found in the course web site.
- It is allowed to work in (small) groups, but please write the id list of your partners in the solution file, and each student should write his solution by *himself* (joint effort is only allowed in the “thinking phase")

1. Prove that Shanon theorem for Binary Symmetric Channels (Thm1, Lecture 5) follows from his theorem for general channels (Slide 13, Lecture 5)

2. Prove that $H(X) \leq H(0.9, \frac{0.1}{3}, \frac{0.1}{3}, \frac{0.1}{3})$ for any rv over $\{1, 2, 3, 4\}$ with $\Pr[X = 1] \geq 0.9$.

3. Prove that for any discrete X there exists density function f with $h(f) = H(X)$.

   Use it to argue that there exists f with $h(f) = \infty$.

4. Let $Q\colon \{0, 1\} \mapsto \{0, 1\} \cup \{\perp\}$ be the random function with $\Pr[Q(x) = \perp] = p$ and $\Pr[Q(x) = x] = 1 - p$ for any $x \in \{0, 1\}$. Find the capacity of the channel described by Q?

   That is, find the right value of C_p for which the natural adjustment of Shannon's theorem (Theorem 1 in lecture 5) for the noise model described by Q (i.e., Q is applied independently to each transmitted bit) can be proven.

5. Let G be the graph with set of nodes $\{0, 1, 2\}^n$, where two nodes $(x, y) \in \{0, 1, 2\}^n$ are connected by an edge iff they differ in exactly one coordinate. State and prove an isoperimetric inequality for the graph G. (Similar to the isoperimetric inequality for the hyper-cube we did in class).

6. Prove or give a counter example: For every rv's X_1, X_2, X_3, X_4:

$$H(X_1, X_2, X_3) + H(X_2, X_3, X_4) + H(X_3, X_4, X_1) + H(X_4, X_1, X_2)$$
$$\leq \frac{3}{2}\left[H(X_1, X_2) + H(X_2, X_3) + H(X_3, X_4) + H(X_4, X_1)\right]$$