Information Theory, Fall 2014 Iftach Haitner

Problem set 2

November 18, 2014 Due: December 2

- Please submit the handout in class, or email the grader.
- Write clearly and shortly using sub-claims if needed. The emphasize in most questions is on the proofs (no much point is writing a “solution" w/o proving its correctness)
- For Latex users, a solution example can be found in the course web site.
- In it ok to work in (small) groups, but please write the id list of your partners in the solution file, and each student should write his solution by *himself* (joint effort is only allowed in the “thinking phase")

1. Prove theorem 5 from Lecture 3.
2. Prove theorem 11 from Lecture 3.
3. Let $X \sim (p_1, \ldots, p_m)$ such that each p_i is a power of 2 (i.e., 2^{-k} for some $k \in \mathbb{Z}$). Prove that the average code length obtained by Huffman's code for X is (exactly) $H(X)$.
4. Use the above question and the optimality of Huffman's code to deduce that the average code length obtained by Huffman's code on any random variable X is at most $H(X) + 1$. (Do that without using the upper bound on the optimal code length proved in class).
5. Prove Proposition 4 from Lecture 4.