

Problem set 2

March 27, 2018

Due: April 12

- Please submit the handout in class, or email the grader (quefumas at gmail.com).
- Write clearly and shortly using sub-claims if needed. The emphasize in most questions is on the proofs (no much point is writing a “solution” w/o proving its correctness)
- For Latex users, a solution example can be found in the course web site.
- It is allowed to work in (small) groups, but please write the id list of your partners in the solution file, and each student should write his solution by *himself* (joint effort is only allowed in the “thinking phase”)

1. Prove the chain rule from mutual information (see, Lecture 2, Slide 21)
2. For random variables X and Y , what is larger $I(X; Y|Z)$ or $I(X; Y)$?
3. Let $X \rightarrow Y \rightarrow Z$ be a Markov chain. Is it always the case that $I(Y; Z) \geq I(X; Z)$?
4. Formally define and prove the infinite case of Kraft inequality (Thm. 2, Lecture 4).
5. Let $X \sim (p_1, \dots, p_m)$ such that each p_i is a power of 2 (i.e., 2^{-k} for some $k \in \mathbb{Z}$).
 - (a) Prove that Huffman's code assigns a word $x \in \text{Supp}(X)$ of probability 2^{-i} , a codeword of length i .
 - (b) Prove that the average code length obtained by Huffman's code for X is (exactly) $H(X)$.
6. Use the above question and the optimality of Huffman's code to deduce that the average code length obtained by Huffman's code on any random variable X is at most $H(X) + 1$. (Do that without using the upper bound on the optimal code length proved in class).
 Can you prove it without the assumption that the binary expansion of each p_i is finite?
7. Prove Proposition 5 in Lecture 4.