Information Theory, Fall 2014 | Iftach Haitner

Problem set 1

November 9, 2014 | Due: Nov 18

- Please submit the handout in class, or email the grader.
- Write clearly and shortly using sub-claims if needed. The emphasize in most questions is on the proofs (no much point is writing a “solution" w/o proving its correctness)
- For Latex users, a solution example can be found in the course web site.
- In it ok to work in (small) groups, but please write the id list of your partners in the solution file, and each student should write his solution by *himself* (joint effort is only allowed in the “thinking phase")

1. Let $(p_1, \ldots, p_m)$ and $(q_1, \ldots, q_m)$ be probability distributions (i.e., $p_i \geq 0$ for all i and $\sum_i p_i = 1$). Prove that
$$- \sum_i p_i \log p_i \leq - \sum_i p_i \log q_i$$

2. Prove the chain-rule for mutual information stated in class.

3. What is larger:

   (a) $I(X;Y|Z)$ or $I(X;Y)$?
   (b) $H(X|Y)$ or $H(f(X)|Y)$?
   (c) $H(X|Y)$ or $H(X|g(Y))$?
   (d) $H(X|Y)$ or $H(f(X,Y)|Y)$?
   (e) $H(X|Y)$ or $H(X|g(X,Y))$?

4. For a finite set $\mathcal{S}$ of random variables, let $H(\mathcal{S})$ denote the joint entropy of all variables in $\mathcal{S}$. Prove that for any two finite sets of random variables $\mathcal{S}$ and $\mathcal{U}$, it holds that $H(\mathcal{S} \cup \mathcal{U}) + (\mathcal{S} \cap \mathcal{U}) \leq H(\mathcal{S}) + H(\mathcal{U})$.