Confidential Transactions Theory Justification

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Abstract

[Iftach's Note: TODO]

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1 Introduction

[Iftach's Note: TODO]

2 Preliminaries

2.1 Notation

We use calligraphic letters to denote sets, uppercase for random variables, and lowercase for integers and functions. Let \mathbb{N} denote the set of natural numbers. For $n \in \mathbb{N}$, let $[n] := \{1, \ldots, n\}$ and $(n) := \{0, \ldots, n\}$. For a relation \mathcal{R} , let $\mathcal{L}(\mathcal{R})$ denote its underlying language, i.e., $\mathcal{L}(\mathcal{R}) := \{x : \exists w : (x, w) \in \mathcal{R}\}$.

2.2 Homomorphic Encryption

An homomorphic encryption is a triplet (KeyGen, Enc, Dec) of efficient algorithms, with the standard correctness and semantic security properties. In addition, there is addition operation denote + over any two (valid) ciphertexts such that for any validly generated public key pk and valid ciphertexts x_0, x_1 , it holds that $\operatorname{Enc}_{sk}(x_0) + \operatorname{Enc}_{pk}(x_1) \in \operatorname{Supp}(\operatorname{Enc}_{sk}(x_0 + x_1 \bmod q), \text{ where } q \in \mathbb{N} \text{ is efficiently determined by } pk$.

[Iftach's Note: Do we really need the homomorphic properties or only for the proofs?]

3 The Confidential Transaction Protocols

3.1 The Ideal Functionality

Functionality 3.1 ($\mathcal{F}_{ConfTrans}$: Confidential transactions).

Parties: Issuer I, chain holder C and users U_1, \ldots, U_n .

Init. Upon receiving init from all parties:

- 1. For each $i \in [n]$: AvlBlance_i, PndBalance_i $\leftarrow 0$ and $\log_i \leftarrow \emptyset$.
- 2. $\log \leftarrow \emptyset$.

Issue. Upon receiving (sid, issue, x, d) from C and I:

- 1. Assert $(x \in \mathbb{N} \text{ and } d \in [n])$.
- 2. PndBalance_d += x.
- 3. Set $\log \cup = (\text{sid}, \text{issue}, x, d)$.

Transfer. Upon receiving (sid, transfer, d) from C and U_s , with U_s using private input x.

- 1. Assert $(x \in \mathbb{N}, \text{AvIBlance}_s \ge x \text{ and } s, d \in [n])$.
- 2. AvlBlance $_s -= x$.

- 3. PndBalance_d $\cup = x$.
- 4. Set $\log_d \cup = (\text{sid}, \text{transfer}, s, x)$
- 5. Set $\log \cup = (\text{sid}, \text{transfer}, s, d)$

Rollover. Upon receiving (sid, rollover) from party U_i and C, party C

- 1. Set AvlBlance_i += PndBalance_i.
- 2. Set PndBalance_i $\leftarrow 0$.
- 3. Set $\log \cup = (\text{sid}, \text{rollover}, i)$

Withraw. Upon receiving (sid, withraw, x) from party U_i and C, party C

- 1. Assert $(x \in \mathbb{N}, AvlBlance_i \ge x \text{ and } i \in [n])$.
- 2. AvlBlance_i -= x.
- 3. Set $\log \cup = (\text{sid}, \text{withraw}, i, x)$

History. Upon receiving (sid, history) from party P_i and C:

Send (\log, \log_i) to P_i .

Audit. [Iftach's Note: Later]

3.2 The Protocol

Throughout, we fix a security parameter κ and omit is from the notation. We also fix an homomorphic encryption scheme (KeyGen, Enc, Dec) over \mathbb{Z}_q with randomness domain \mathcal{D} . We require that $\mathsf{Dec}_{sk}(A)$ outputs (a;r) such that $A = \mathsf{Enc}(a;r)$.

Protocol 3.2 ($\Pi_{ConfTrans}$: Confidential transactions).

Parameters: $p_{\mathsf{num}}, p_{\mathsf{size}} \in \mathbb{N}$.

Parties: Issuer I, chain-holder C and users U_1, \ldots, U_n .

Subprotocols: See below.

We use the of key-generation relation

$$\mathcal{R}_{\mathsf{KeyGen}} = \{(pk, w)\} : \mathsf{KeyGen}(w) = (\cdot, pk)\}.$$

Protocol 3.3 ($\Pi_{ConfTrans}$.Init).

Participating parties. All parties.

Protocols: $\Pi_{\mathcal{R}_{\mathsf{KeyGen}}}^{\mathsf{ZK-POK}}$. Algorithms: KeyGen .

Operation:

- 1. P_i , for all $i \in [n]$:
 - (a) Set $(pk_i, sk_i) \stackrel{\mathbb{R}}{\leftarrow} \mathsf{KeyGen}(r_i)$ for $r_i \stackrel{\mathbb{R}}{\leftarrow} \mathcal{D}$.
 - (b) Store sk_i .
 - (c) Let $\pi_i \stackrel{\mathbb{R}}{\leftarrow} \mathsf{P}^{\mathsf{ZK-POK}}_{\mathcal{R}_{\mathsf{KeyGen}}}(pk_i, r_i)$.
 - (d) Send (pk_i, π_i) to C.
- 2. C:
 - (a) Call $\{V_{\mathcal{R}_{\mathsf{KeyGen}}}^{\mathsf{ZK-POK}}(pk_i, \pi_i)\}_{i \in [n]}$. Abort and publish i, if the i^{th} proof is not verified.
 - (b) Store $\{pk_i\}_{i\in[n]}$.
- 3. C:
 - (a) Broadcast $\{p_i \leftarrow 0, P_i \stackrel{\mathbb{R}}{\leftarrow} \mathsf{Enc}_{pk_i}(0), B_i \leftarrow \emptyset\}_{i \in [n]}$.
 - (b) Broadcast $\log \leftarrow \emptyset$.

Protocol 3.4 ($\Pi_{ConfTrans}$. Issue).

Participating parties. I and C.

C's input. sid, $x \in \mathbb{N}$ and $i \in [n]$.

Operation:

- 1. I: Send (x, i) to C.
- 2. C:
 - (a) Assert $(x \in [p_{\mathsf{size}}] \text{ and } p_i \leq p_{\mathsf{num}})$
 - (b) Set $P_i += \operatorname{Enc}_{pk_i}(x)$).
 - (c) Publish $\log \cup = (\text{sid}, \text{issue}, x, i, P_i)$.

We use proofs for the following relations

$$\mathcal{R}_{\mathsf{rp}} = \{((pk, A), (a, r)) \colon \mathsf{Enc}_{pk}(a; r) = A \, \land \, a \in [p_{\mathsf{size}}]\}$$

$$\mathcal{R}_{\mathsf{eq}} = \{((pk_0, pk_1, A_0, A_1), (a, r_0, r_1)) \colon \forall i \in \{0, 1\} \; \mathsf{Enc}_{pk_i}(a; r_i) = A_i\}$$

$$\mathcal{R}_{\mathsf{lrg}} = \{((pk, A_0, A_1), (a, r_0, r_1)) \colon \forall i \in \{0, 1\} \; \mathsf{Enc}_{pk}(a_i; r_i) = A_i\} \; \land \; a_1 - a_0 \in [q] \; \}$$

Protocol 3.5 ($\Pi_{ConfTrans}$.Transfer).

Participating parties. P_s and C.

Protocols: $\Pi_{\mathcal{R}_{rp}}^{ZK\text{-POK}}$.

Algorithms: Dec.

Cpmmon input: $d \in [n]$.

 P_s 's private input. $x \in \mathbb{N}$.

Operation:

- 1. P_s:
 - (a) $X_d \stackrel{\mathbb{R}}{\leftarrow} \mathsf{Enc}_{pk_d}(x; r)$ for $r^d \stackrel{\mathbb{R}}{\leftarrow} \mathcal{D}$.
 - (b) $\pi^{\mathsf{rp}} \overset{\mathbb{R}}{\leftarrow} \mathsf{P}^{\mathsf{ZK-POK}}_{\mathcal{R}_{\mathsf{rp}}}((pk_d, X_s, p_{\mathsf{size}}), (x, r)).$
 - (c) $X_s \stackrel{\mathbb{R}}{\leftarrow} \mathsf{Enc}_{pk_s}(x;r)$ for $r^s \stackrel{\mathbb{R}}{\leftarrow} \mathcal{D}$.
 - $(\mathbf{d}) \ \pi^{\mathsf{eq}} \overset{\mathtt{R}}{\leftarrow} \mathsf{P}^{\mathsf{ZK}\text{-POK}}_{\mathcal{R}_{\mathsf{eq}}}((pk_s, pk_d, X_s, X_s), (x, r_s, r_d)).$
 - (e) $(b, r^b) \leftarrow \mathsf{Dec}_{sk_s}(B_s)$.
 - $(\mathbf{f}) \ \pi^{\mathsf{lrg}} \overset{\mathbb{R}}{\leftarrow} \mathsf{P}^{\mathsf{ZK\text{-}POK}}_{\mathcal{R}_{\mathsf{lrg}}}((pk_s, X_s, B_s), (x, r_s, r_b)).$
 - (g) Send $(X_s, X_d, \pi^{\mathsf{rp}}, \pi^{\mathsf{eq}}, \pi^{\mathsf{lrg}})$ to C.
- 2. C:
 - (a) Call $\mathsf{V}_{\mathcal{R}_{\mathsf{rp}}}^{\mathsf{ZK-POK}}((pk_d, X_s, p_{\mathsf{size}}), \pi^{\mathsf{rp}}), \qquad \mathsf{V}_{\mathsf{eq}}^{\mathsf{ZK-POK}}((pk_s, pk_d, X_s, X_s), \pi^{\mathsf{rp}})$ and $\mathsf{V}_{\mathcal{R}_{\mathsf{lrg}}}^{\mathsf{ZK-POK}}((pk_s, X_s, B_s), \pi^{\mathsf{rp}}).$
 - (b) Set $U_s = X_s$.
 - (c) Set $P_d += X_d$.
 - (d) Publish $\log \cup = (\text{sid}, \text{transfer}, s, d, U_s, P_d)$.

Protocol 3.6 ($\Pi_{ConfTrans}$.Update).

Participating parties. P_i and C.

Operation: C

- 1. $B_i += \sum_{(\cdot, X) \in H_i} X$.
- 2. $H_i \leftarrow \emptyset$.
- 3. $\log += (\text{sid}, \text{rollover}, i)$

Protocol 3.7 ($\Pi_{ConfTrans}$. History).

Participating parties. P_i and C.

Operation: C sends (\log, H_i) to P_i .