SIT215-Artificial and Computational Intelligence
PBL task 3 documentation

The Game of Nim

Prepared by:

Iftekhar Qureshi (218676618)

Lecturer: Dr Somaiyeh MahmoudZadeh

19th September 2020

Contents

Introduction

In this documentation, I am going to discuss about The Game of Nim which is a popular problem of Combinatorial Game Theory (CGT). Although the study on CGT started way back, since the ancient Greeks, it was formally developed in the early 20th century. Combinatorial game theory is different to the classical game theory because it is not completely probabilistic or does not include any winning by "chance". The players can win the game easily if they know the strategy for the winning moves. It can be proved using mathematics that either one player can force a win or both players can force a draw.

The Game of Nim is an impartial game. Impartial game is a part of CGT. Impartial games are two player games in which players take turns to make their own move, and the moves available from a given position does not depend on whose turn it is. When there are no more possible move from a position, such position is called a terminal position or end position which ends the game. Winner and loser is then declared according to the rule of the game.

Problem Recognition

There are two variants of the Nim game: the basic one is the one where the player who removes the last object wins and the other variant is the Misère play where the player who removes the last object loses. The play can start with a number of piles, heaps or stacks with objects in it. The objects can be anything that is singular and can be counted. In each turn, the players get to take out as many objects as they want from each of the piles but they cannot remove none. Let's look at an example and try to understand how the game is played and how normal play differentiated from the misère play with three stacks A, B and C and players Jane and Mary.

Sizes of stacks	Moves
A B C	
3 4 5	Jane takes 2 from A (at this point, after taking two, nim sum becomes zero which means its clear Jane would win)
1 4 5	Mary takes 3 from C
1 4 2	Jane takes 1 from B
1 3 2	Mary takes 1 from B
1 2 2	Jane takes entire A heap, leaving two 2s
0 2 2	Mary takes 1 from B
0 1 2	Jane takes 1 from C leaving two 1s. (*In misère play he would take 2 from C leaving (0,1,0).*)
0 1 1	Mary takes 1 from B
0 0 1	Jane takes entire C heap and wins

Above is an example of a normal play along with the misère play. As it can be seen, if it was misère play, one certain move (the move at (0,1,2)) could have changed the goal of the game. The strategy for misère play is to keep playing like the normal play until the opponent leaves a pile of size greater than one object. Then reducing the piles to sizes 1 or 0 leaves odd number of piles with only one object. We will have a clear winner.

The actual strategy of winning the normal nim game is calculating the nim-sum. For creating a winning position, or what we call in terms of CGT a **P-position**, we need to make the nim-sum zero. P-positions are any position where the previous player moving in can force a win. Contrary to that is **N-position,** where the next player moving away can force a win. Hence, we can conclude that from a N-position (losing position) it is possible to move to at least one P-position (winning position). But from P-position we can only move to N-position. Or else, the opponent would be able to counterattack with a different strategy to win even if the nim-sum was zero. We would always want to convert losing position to winning position in each move.

Solution

To calculate the nim sum, we have to convert the size of each pile into its binary representation. For example- if we have three piles of size 1, 4, 7, and if we add their binary numbers (can be represented as $1 \oplus 4 \oplus 7$) the nim-sum would be 2 which is not equal to zero. Now to find the winning move, we have to find a particular next move that will make the nim-sum zero. If the next player takes out 2 from the last pile, it becomes 1,4,5. If we do $1 \oplus 4 \oplus 5$, the nim-sum equates to zero and therefore, we have found our P-position. Since our move has created a p-position, the next player will not have any chance of creating a P-position for himself. This means, we can keep creating P-position until the end of the game and eventually win.

This binary method works extremely well even for piles of size reaching million. This is because the input size of nim game is logarithmic. Whether the nim sum is non-zero or actual winning move both can be found out in log space.

Some of the problems that arise from this strategy is that the players have to study the strategy before starting actual play. If we assume that both the players are professional and know the winning strategy, then the player going first will get the upper hand. The first player to make the move will try to create a P-position and therefore put the next player in a N-position for the rest of the game. If the first player keeps playing without making any mistake, he will eventually win. Certainly, the order of the objects in the piles matter. If the initial order is in losing position, then the player making the first move has to bring it to a winning position and vice versa.

Conclusion

Understanding the concept of the Nim game is very important to understand the impartial games in general. By using Sprague-Grundy Theorem, we can state that every impartial game in its normal play is equivalent to one heap game of Nim. This equivalence relationship is found using the Sprague Grundy function. Every impartial game can therefore be analysed using the Sprague Grundy function if we have the nim heap value.