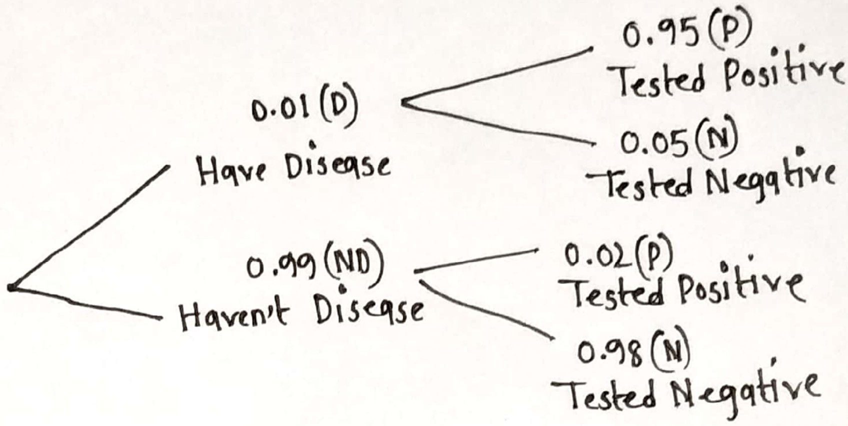
1. 1% of a population have a certain disease and the remaining 99% are free this disease. A test used to detect the disease. This test is positive in 95% of the people with the disease and is also (falsely) positive in 2% of people free from disease. If a person, selected at random from the population, has positive, what is the probability that she/ has the disease?

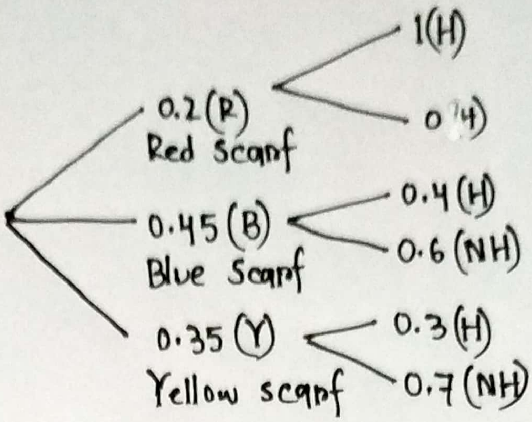
→



$$\therefore P(D/P) = \frac{P(P/D) \times P(D)}{P(P)} = \frac{(0.95 \times 0.01)}{(0.01 \times 0.95) + (0.99 \times 0.02)}$$

$$= 0.3242 \text{ (Ans).}$$

2. Georgie has a red scarf, a blue scarf, a yellow scarf. Each day she wears exactly one of these scarves. The probability for the three colors are 0.2, 0.45 and 0.35 respectly. When she wear a red scarf, she always a ~~red~~ hat. When she wears a blue scarf, she wears a hat with probability 0.4. When she wears a yellow scarf she wears a hat with probability 0.3.



(a) Find the probability that on a randomly chosen day Georgie wears a hat.

→ $P(H) = \dfrac{(0.2\times 1)+(0.4\times 0.45)+(0.3\times 0.35)}{1}$

$= 0.485$ (Ans).

(b) Find the probability that on a random chosen day Georgie wears a yellow scarf given that she doesn't wear a hat.

→ $P(Y/NH) = \dfrac{P(NH/Y)\times P(Y)}{P(NH)} = \dfrac{0.7\times 0.35}{1-0.485}$

$= 0.4757$ (Ans).

3. The random variable x takes the values 1, 2, 3, 4 only. The probability that takes the value x is $kx(5-x)$, where k is constant.

(a) Draw up the probability distribution table for x in term of k.

→

x	1	2	3	4
$P(X=x)$	4k	6k	6k	4k

(b) Show that, $Var(x) = 1.05$

→ $4k+6k+6k+4k = 1$

$\Rightarrow k = 1/20$

$\therefore E(x) = 1/5 + 6/10 + 9/10 + 4/5 = 5/2$

$\therefore Var(x) = 1/5 + 12/10 + 27/10 + 16/5 - (5/2)^2 = 1.05$ (Showed).

4. Sharma knows that she has 3 tins of carrots, 2 tins of peas and 2 tins of sweetcorn in her cupboard. All the tins are the same shape and size, but the labels have all been removed, so Sharma does not which each tin contains. Sharma wants carrots for her meal, and she starts opening the tins one at a time, choosen randomly, until she opens a tin of carrots. The random varriable X is the number of tins that she needs to open.

(a) Show that $P(x=3) = 6/35$.

→ $P(x=3) = 4/7 \times 3/6 \times 3/5 = \frac{6}{35}$ (showed).

(b) Draw up the probability distribution table for X.

→

X	1	2	3	4	5
$P(X=x)$	$4/7$	$2/7$	$6/35$	$3/35$	$1/35$

(c) Find Var(x)

→ $E(x) = 4/7 + 4/7 + 18/35 + 12/35 + 5/35 = 2$

$\therefore Var(x) = 4/7 + 8/7 + 54/35 + 48/35 + 25/35 - 2^2$

$= 1.2$ (Ans).

5. The probability distribution for the random variable X is given by the following table.

(a) Find the value of a

→

$0.03 + 2a + 0.32 + a + 0.05 = 1$

$\Rightarrow 3a = 0.6$

$\Rightarrow a = 0.2$

(b) calculate E(x) and Var(x)

→ $E(x) = 0.03 \times 0 + 0.4 + 0.64 + 0.6 + 0.2 = 1.84$ (Ans).

$Var(x) = 0.4 + 1.28 + 1.8 + 0.8 - (1.84)^2 = 0.8944$ (Ans).

(c) Hence find E(4x-1) and Var(4x-1)

→ $E(4x-1) = 4E(x) - 1 = 4 \times 1.84 - 1 = 6.36$

$Var(4x-1) = 4^2 Var(x) = 14.3104$

6. In a certain town 60% of the home have internet connection

(a) In a random sample of 20 homes in town, find the probability that,

(i) Exactly 12 have internet connection.

→ $X \sim B(20, 0.6)$

$\therefore P(x=12) = {}^{20}C_{12} (0.6)^{12} (0.4)^{8} = 0.1797$ (Ans).

(ii) Exactly 5 do not have internet connection.

→ $P(x \neq 5) = 1 - P(x=5)$

$= 1 - {}^{20}C_{5} (0.6)^{5} (0.4)^{15}$

$= 0.998$ (Ans).

7. On average, 1 in 400 microchips made at a certain factory are faulty. The number of faulty microchips in a random sample of 1000 is denoted by X.

(a) State the distribution of x, given the values of any parameters.

→ $X \sim P_o(2.5)$

(b) Use this approximate distribution to find each of following:

(i) $P(x=4)$

→ $P(x=4) = \dfrac{e^{-2.5} \cdot (2.5)^4}{4!} = 0.1336$

(ii) $P(x \le 2 \le 4)$

→ $P(2 \le x \le 4) = P(2) + P(3) + P(4)$

$$= e^{-2.5}\left[\frac{(2.5)^2}{2} + \frac{(2.5)^3}{3!} + \frac{(2.5)^4}{4!}\right]$$

$= 0.60388$ (Ans).

8. Accidents of two factories occur randomly and indepently. On average, the number of accidents per month are 3.1 at factory A and 1.7 at factory B.

Find the probability that total number of accidents in two factories during a 2 month is more than 3.

→ $\lambda = 2(1.7 + 3.1) = 9.6$

$\therefore P(x>3) = 1 - P(0) - P(1) - P(2) - P(3)$

$$= 1 - e^{-9.6}\left[\frac{(9.6)^0}{1} + \frac{(9.6)^1}{1} + \frac{(9.6)^2}{2} + \frac{(9.6)^3}{6}\right]$$

$= 0.98617$ (Ans).

9. The time spent by shoppers in a large shopping center has a normal distribution with mean 96 minutes and standard deviation 18 minutes.

(a) Find the probability that a shopper choosen at random spends between 85 and 100 minutes in the shopping centre.

$\rightarrow X \sim N(96, 18^2)$

$$P(85 < x < 100) = P(-0.611 < z < 0.22)$$
$$= \phi(0.22) - 1 + \phi(0.61)$$
$$= 0.58706 - 1 + 0.74537$$
$$= 0.33243 \text{ (Ans).}$$

(b) 88% of shoppers spend more than t minutes. Find the value of t.

$\rightarrow P(x > t) = 0.88$

$$\Rightarrow P\left(z > \frac{t-96}{18}\right) = 0.88$$
$$\Rightarrow 1 - \phi\left(\frac{t-96}{18}\right) = 0.88$$
$$\Rightarrow 1 - 1 + \phi\left(\frac{-t+96}{18}\right) = 0.88$$
$$\Rightarrow \phi\left(\frac{-t+96}{18}\right) = \phi(1.17)$$
$$\Rightarrow \frac{-t+96}{18} = 1.17$$
$$\Rightarrow t = 74.94 \text{ (Ans).}$$

10. A company produces a particular type of rod. The lenghts of these are normally distributted with mean 25.2 cm and standard deviation 0.4 cm. A random sample of 500 of these rods is choosen. How many rods you expect to have a lenght that within 0.5 cm of the mean lenght?

$$\rightarrow P(25.2 - 0.5 < x < 25.2 + 0.5) = P\left(\frac{-0.5}{0.4} < z < \frac{0.5}{0.4}\right)$$
$$= 2\phi(1.25) - 1$$
$$= 0.7888$$

$\therefore$ Expected rods $= 500 \times 0.7888$

$= 394.4$ (Ans).

11. The weights of bags of sugar are normally distributed with mean 1.04 kg and standard deviation σ kg. In random sample of 2000 bags of sugar, 72 weighed more than 1.10 kg. Find the value of σ.

$\rightarrow \quad P(x > 1.10) = \frac{72}{2000} = 0.036$

$\Rightarrow P\left(z > \frac{0.06}{\sigma}\right) = 0.036$

$\Rightarrow 1 - \phi\left(\frac{0.06}{\sigma}\right) = 0.036$

$\Rightarrow \phi\left(\frac{0.06}{\sigma}\right) = 0.964$

$\Rightarrow \frac{0.06}{\sigma} = 1.8$

$\therefore \sigma = 0.033$ (Ans).

12. The diagram shows the graph of the probability density function, f, of a random variable X.

(a) Find the value of constant k

$$\rightarrow \quad f(x) = \begin{cases} \frac{1}{2}(k-x), & 0 \le x \le k \\ 0, & \text{othewise} \end{cases}$$

$$\int_0^k \frac{1}{2}(k-x)\,dx = 1$$

$$\Rightarrow \frac{1}{2}\left[kx - \frac{x^2}{2}\right]_0^k = 1$$

$$\Rightarrow \frac{1}{2}\left[k^2 - \frac{k^2}{2}\right] = 1$$

$$\Rightarrow \frac{k^2}{4} = 1$$

$$\Rightarrow k^2 = 4$$

$\therefore k = 2$ (Ans).

(b) Using the value of k, find $f(x)$ for $0 \le x \le k$ and hence find $E(x)$

→

$$f(x) = \begin{cases} \frac{1}{2}(2-x) & , 0 \le x \le 2 \\ 0 & , \text{otherwise} \end{cases}$$

$$\therefore E(x) = \int_0^2 \frac{1}{2}(2-x)x\,dx$$

$$= \frac{1}{2}\int_0^2 2x - x^2\,dx$$

$$= \frac{1}{2}\left[\frac{2x^2}{2} - \frac{x^3}{3}\right]_0^2$$

$$= 0.667 \text{ (Ans)}.$$

(c) Find the value of p such that $P(p < x < 1) = 0.25$

→

$$\int_p^1 \frac{1}{2}(2-x) = 0.25$$

$$\Rightarrow \frac{1}{2}\left[2x - \frac{x^2}{2}\right]_p^1 = 0.25$$

$$\Rightarrow 2 - \frac{1}{2} - 2p + \frac{p^2}{2} = 0.5$$

$$\Rightarrow \frac{p^2 - 4p}{2} = -1$$

$$\Rightarrow p^2 - 4p + 2 = 0$$

$$\therefore p = \frac{-(-4) \pm \sqrt{4^2 - 4 \cdot 1 \cdot 2}}{2 \cdot 1}$$

$$= \cancel{3.4142}, 0.5857 \text{ (Ans)}.$$

13. The random variable X takes the values range $1 \le x \le p$, where p is constant. The graph of the probability density function of X is shown in the diagram.

→ (a) Show that $p = 2$

→ $\frac{1}{2} \times p \times (p-1) = 1$

$\Rightarrow \frac{p^2 - p}{2} = 1$

$\Rightarrow p^2 - p - 2 = 0$

$\Rightarrow p^2 - 2p + p - 2 = 0$

$\Rightarrow p(p-2) + 1(p-2) = 0$

$\Rightarrow (p-2)(p+1) = 0$

$\therefore p = 2, \cancel{-1}$ (Showed).

(b) $E(x) = ?$

→ $E(x) = \int_1^2 x(2x-2)\,dx$

$= \int_1^2 (2x^2 - 2x)\,dx$

$= \left[\frac{2x^3}{3} - x^2\right]_1^2$

$= 5/3$ (Ans).

$y = mx + c$

$m = \frac{2}{2-1} = 2$

$\therefore y = 2x - 2$

14. The probability density function, f, of a random variable X is given by

$$f(x) = \begin{cases} k(6x - x^2) & 0 \le x \le 6 \\ 0 & \text{otherwise} \end{cases}$$

State the value of $E(x)$ and show that $Var(x) = 9/5$

→ $\int_0^6 k(6x - x^2)\,dx = 1$

$\Rightarrow k\left[3x^2 - x^3/3\right]_0^6 = 1$

$\Rightarrow k = 1/36$

$\therefore E(x) = 1/36 \int_0^6 (6x^2 - x^3)\,dx$

$= 1/36 \left[2x^3 - x^4/4\right]_0^6$

$= 3$ (Ans)

$\therefore Var(x) = 1/36 \int_0^6 x^2(6x - x^2)\,dx - E(x)^2$

$= 1/36 \int_0^6 (6x^3 - x^4)\,dx - 3^2$

$= 1/36 \left[6x^4/4 - x^5/5\right]_0^6 - 3^2$

$= 1.8$ (Ans).

15. Alethia models the lenght of time, in minutes, by which her train is late on any day by the random variable X with probability desity function given by

$$f(x) = \begin{cases} \frac{3}{8000}(x-20)^2 & 0 \le x \le 20 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the probability that the train is more than 10 minutes late on each of two random chosen days.

$\rightarrow$ $P(x>10) = \int_{10}^{20} \frac{3}{8000}(x-20)^2 \, dx$

$$= \frac{3}{8000} \int_{10}^{20} (x^2 - 40x + 4000) \, dx$$

$$= \frac{3}{8000} \left[\frac{x^3}{3} - \frac{40x^2}{2} + 4000x \right]_{10}^{20}$$

$$= \frac{109}{8} \text{ (Ans).}$$

(b) The median of x is denoted by m.

Show that, m satisfies the equation $(m-20)^3 = -4000$

$\rightarrow$ $\int_0^m \frac{3}{8000}(x-20)^2 \, dx = 0.5$

$$\Rightarrow \int_0^m (x-20)^2 \, dx = \frac{4000}{3}$$

$$\Rightarrow \left[\frac{(x-20)^3 - 8000}{3} \right]_0^m = \frac{4000\cancel{0}}{3}$$

$$\Rightarrow \frac{(m-20)^3}{3} = \frac{4000}{3} - \frac{8000}{3}$$

$$\Rightarrow (m-20)^3 = -4000 \text{ (Showed).}$$

16. An archited wishes to investigate whethen the building in a centain city ane highen, on avenage, than building in othen cities. He takes a lange nandom sample of building from the city and finds the mean height of building in the sample. He calculates the value of the test statistic, z, and find $z = 2.41$.

(a) Explain bniefly whethen he should use a one-tail test on two tail test.

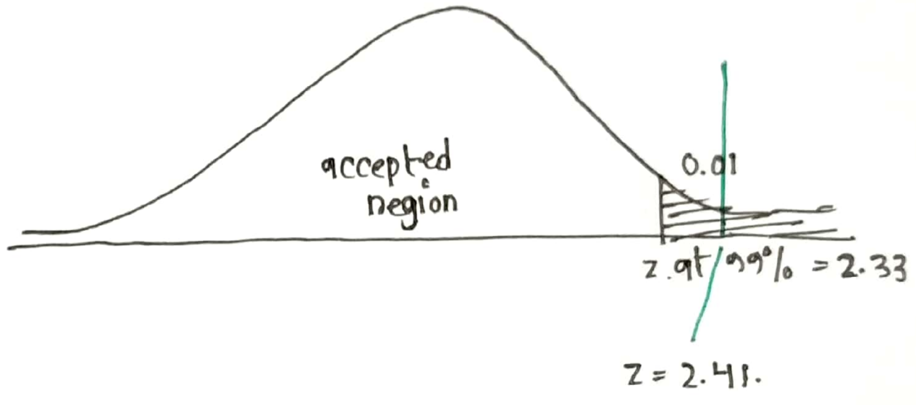
→ $H_0 = M = \lambda$

$H_1 = M > \lambda$

So, the test is one tail test

(b) Canny out the test at the 1% signficance level.

→



So, thene enough evidense to neject the null hypothesis. (Ans).

17. The time, in minutes, spent by customer at a particular gym has the distribution $N(\mu, 38.2)$. In the past the value of μ has been 42.4. Following the installation of some new equipment the management wishes to test whether the value of μ has changed.

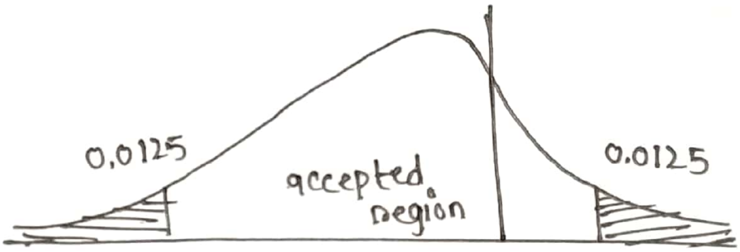
(a) State what is meant by Typ I error in this context.

→ If the mean value ($\mu = 42.4$) hasn't changed, but the null null hypothesis got rejected.

(b) The mean time for a sample of 20 customers is found to be 45.6. Test at the 2.5% significance level whether the value of μ has changed.

→ $H_0 = \mu = 42.4$

$H_1 = \mu \neq 42.4$



$$P(x > 45.6) = P\left(z > \frac{45.6 - 42.4}{\sqrt{38.2}}\right)$$

$$= 1 - \phi(0.52)$$

$$= 0.3$$

So, there are not enough evidense to reject the null hypothesis.

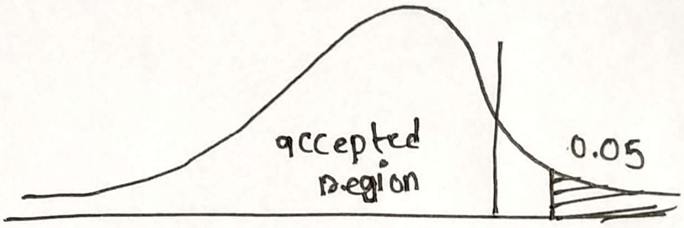
18. It is known that 8% of adults in a certain town own a chantor car. After an advertising campaign, a car dealer wishes to investigate whether this proportion has increased. He choses a random sample of 25 adults from the town and notes how many of them own a Chantor car.

(a) He finds that 4 of the 25 adults own a chantor car. Carry a hypothesis test at the 5% significance level.

→

$H_0 = \mu = 0.08$

$H_1 = \mu > 0.08$



$$P(x \geq 4) = 1 - P(0) - P(1) - P(2) - P(3)$$

$$= 1 - {}^{25}C_0 (0.08)^0 (0.92)^{25} - {}^{25}C_1 (0.08)^1 (0.92)^{24} - {}^{25}C_2 (0.08)^2 (0.92)^{23} - {}^{25}C_3 (0.08)^3 (0.92)^{22}$$

$$= 1 - 0.124 - 0.2\cancel{4}7 - 0.\cancel{0}28 - 0.19$$

$$= 0.139$$

There is no enough evidence to reject the null hypothesis.

(b) Type1: If the mean remain unchanged, but the null hypothesis got rejected.

Type2: If the null hypothesis is false but the mean ~~no~~ got changed.

19. A contruction company notes the time, t days, that it takes to build each house of a certain design. The result for a random sample of 60 such houses are summarised as follows.

→ $\bar{t} = \frac{4820}{60} = 80.33$ $\sigma^2 = \frac{399250}{60} - (80.33)^2 = 81.258$

$\therefore \sigma = 9.01$

$\therefore \bar{t} - z\times\frac{\sigma}{\sqrt{n}} \le x \le \bar{t} + z\times\frac{\sigma}{\sqrt{n}}$ | z at 99% is 2.33

$= 80.33 - 2.33\frac{9.01}{\sqrt{60}} \le x \le 80.33 + \frac{2.33\times 9.01}{\sqrt{60}}$

$= 77.61 \le x \le 83.04$ (Ans).

20. The heights, h centimetres, of a random sample of 100 fully grown animals are measured. The result are summarized below.

→ (a) $\bar{h} = \frac{7570}{100} = 75.7$ $\sigma^2 = \frac{588050}{100} - (75.7)^2 = 150.01$

$\therefore \sigma = 12.248$

(b) $\bar{h} - z\frac{\sigma}{\sqrt{n}} \le h \le \bar{h} + z\frac{\sigma}{\sqrt{n}}$ | z at 99.5% is 2.58

$= 75.7 - \frac{2.58\times 12.248}{\sqrt{100}} \le h \le 75.7 + \frac{2.58\times 12.248}{\sqrt{100}}$

$= 72.54 \le h \le 78.85999$ (Ans).