MATH 201: Coordinate Geometry and Vector Analysis

"Lecture 4"

Chapter: 11.8

Cylindrical and Spherical Coordinates

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❑ Conversion formulas for coordinate systems

CONVERSION		FORMULAS	RESTRICTIONS
Cylindrical to rectangular	$(r, \theta, z) \to (x, y, z)$	$x = r\cos\theta, \quad y = r\sin\theta, \quad z = z$	$r \geq 0, \rho \geq 0$ $0 \leq \theta < 2\pi$ $0 \leq \phi \leq \pi$
Rectangular to cylindrical	$(x, y, z) \to (r, \theta, z)$	$r = \sqrt{x^2 + y^2}, \quad \tan\theta = y/x, \quad z = z$	
Spherical to cylindrical	$(\rho, \theta, \phi) \to (r, \theta, z)$	$r = \rho\sin\phi, \quad \theta = \theta, \quad z = \rho\cos\phi$	
Cylindrical to spherical	$(r, \theta, z) \to (\rho, \theta, \phi)$	$\rho = \sqrt{r^2 + z^2}, \quad \theta = \theta, \quad \tan\phi = r/z$	
Spherical to rectangular	$(\rho, \theta, \phi) \to (x, y, z)$	$x = \rho\sin\phi\cos\theta, \quad y = \rho\sin\phi\sin\theta, \quad z = \rho\cos\phi$	
Rectangular to spherical	$(x, y, z) \to (\rho, \theta, \phi)$	$\rho = \sqrt{x^2 + y^2 + z^2}, \quad \tan\theta = y/x, \quad \cos\phi = z/\sqrt{x^2 + y^2 + z^2}$	

Exercise -1: Convert $(4\sqrt{3}, 4, -4)$ from rectangular to cylindrical coordinates.

Exercise 1 : a) $(x, y, z) = (4\sqrt{3}, 4, -4)$

$$(x, y, z) \rightarrow (r, \theta, z)$$

$$r = \sqrt{x^2+y^2} = 8$$

$$\tan\theta = \frac{y}{x} \Rightarrow \theta = \tan^{-1}\left(\frac{4}{4\sqrt{3}}\right)$$

$$= \frac{\pi}{6}$$

$$z = z = -4$$

$$\therefore (r, \theta, z) = \left(8, \frac{\pi}{6}, -4\right)$$

Exercise -3: Convert $\left(8, \frac{3\pi}{4}, -2\right)$ from cylindrical to rectangular coordinates.

Exercise 3: b) $(r, \theta, z) = \left(8, \frac{3\pi}{4}, -2\right)$

$(r, \theta, z) \rightarrow (x, y, z)$

$x = r\cos\theta = 8\cos\frac{3\pi}{4} = 8\left(-\frac{1}{\sqrt{2}}\right)$

$= -4\sqrt{2}$

$y = r\sin\theta = 8\sin\frac{3\pi}{4} = 8\left(\frac{1}{\sqrt{2}}\right) = 4\sqrt{2}$

$z = z = -2$

$\therefore (x, y, z) = (-4\sqrt{2}, 4\sqrt{2}, -2)$

❑ **Exercise -7:** Convert $(2, \frac{3\pi}{2}, \frac{\pi}{2})$ from spherical to rectangular coordinates.

Exercise 7: (d) $(\rho, \theta, \phi) = (2, \frac{3\pi}{2}, \frac{\pi}{2})$

$$(\rho, \theta, \phi) \longrightarrow (x, y, z)$$

$$x = \rho \sin\phi \cos\theta = 2 \cdot \sin\frac{\pi}{2} \cos \cdot \frac{3\pi}{2}$$

$$= 0$$

$$y = \rho \sin\phi \sin\theta = 2 \cdot \sin\frac{\pi}{2} \sin\frac{3\pi}{2}$$

$$= -2$$

$$z = \rho \cos\phi = 2 \cos\frac{\pi}{2} = 0$$

$$\therefore (x, y, z) = (0, -2, 0)$$

Exercise -11: Convert $\left(5,\ \frac{\pi}{4}, \frac{2\pi}{3}\right)$ from spherical to cylindrical coordinates.

Exercise ~~0~~11: (a) $(\rho, \theta, \phi) = \left(5, \frac{\pi}{4}, \frac{2\pi}{3}\right)$

$(\rho, \theta, \phi) \rightarrow (r, \theta, z)$

$r = \rho \sin\phi = 5 \sin \frac{2\pi}{3} = 5 \cdot \frac{\sqrt{3}}{2}$

$\theta = \theta = \frac{\pi}{4}$

$z = \rho \cos\phi = 5 \cos \frac{2\pi}{3} = 5 \cdot \left(-\frac{1}{2}\right) = -\frac{5}{2}$

$\therefore (r, \theta, z) = \left(\frac{5\sqrt{3}}{2}, \frac{\pi}{4}, -\frac{5}{2}\right)$

THANK YOU