CONTEXT FREE GRAMMAR

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- Three areas of theory of computation
  - Automata
  - Computability
  - Complexity
- Linked by the question
  - What are the fundamental capabilities and limitations of computers?

- Automata
  - Automaton - a machine made in imitation of a human being
  - DFA, NFA
  - Context-free grammar (CFG), pushdown automata (PDA)
- Computability
  - Decidability
  - What can or cannot be solved
- Complexity
  - Tractability
  - What can or cannot be solved "efficiently"
  - Time complexity: P, NP, NP-complete, NP-hard
  - Space complexity: PSPACE

- Finite Automata
  - DFA, NFA
  - Limited amount of memory
  - Applications in compilers, control units of hardware
- Context-free grammar
  - More expressive than finite automata
  - Applications in compilers, AI
- Turing Machine
  - Even more powerful
  - Can simulate a computer!
  - Problems Turing machine cannot solve are beyond theoretical limits of computation

Regular Languages

- Regular languages
  - Languages recognized by finite automata - DFA, NFA
  - Languages described by regular expressions
- Limitations
  - Finite number of states
  - Hence finite amount of memory
- An example of a non-regular language
  - $B = \{0^n 1^n | n \geq 0\}$

Context-free Languages

- Context-Free Languages
  - Languages described by context-free grammars (CFG)
  - Languages recognized by pushdown automata (PDA)
- Extensively used in compilers (parsers)
- First used in study of human languages

An Informal Example

- Language of palindromes
- Palindrome
  - A string that reads the same backward and forward
  - 0110, 11011, ϵ
- Recursive definition for palindromes (over binary alphabet)
  - 0, 1 and ϵ are palindromes
  - if w is a palindrome, then $0w0$ and $1w1$ are palindromes

An Informal Example

- A CFG for palindromes
  - $P \rightarrow \epsilon$
  - $P \rightarrow 0$
  - $P \rightarrow 1$
  - $P \rightarrow 0P0$
  - $P \rightarrow 1P1$

Linked Terminals

- Terminals may be linked to one another in that they have the same number of occurrences (or a related number).
- Example 1: { $0^n 1^n$ | n >= 0 }
- CFG ?
- S → 0 S 1 | ε

Linked Terminals

- Example 1: { $0^n 1^{2n}$ | n >= 0 }
- CFG ?
- S → 0 S 11 | 011

Banalnced Paranthesis

- { w | w is a string of balanced parentheses } over w = { (,) }

- **Base Case:** the empty string is balanced
- **Recursive Step:** Find out the closing parenthesis that matches the first opening parenthesis. Removing the first parenthesis and the matching parenthesis forms two new strings of balanced parentheses.

$$S \rightarrow (S) S \mid \varepsilon$$

or,

$$S \rightarrow SS \mid (S) \mid \varepsilon$$

CFG Practice

L = any string

$S \rightarrow 0S \mid 1S \mid \varepsilon$

L = any string with only even no of 1's and no 0's

$S \rightarrow 1S1 \mid \varepsilon$

L = { w | w contains 100 as substring }

$S \rightarrow P100P$
$P \rightarrow 0P \mid 1P \mid \varepsilon$

L = all strings that start and end with the same symbol

$S \rightarrow 0P0 \mid 1P1 \mid 0 \mid 1$
$P \rightarrow 0P \mid 1P \mid \varepsilon$

L = { w | the length of w is even }

$S \rightarrow 0S0 \mid 0S1 \mid 1S0 \mid 1S1 \mid \varepsilon$

L = { w | the length of w is odd and its middle symbol is a 0 }

$S \rightarrow 0 \mid 0S0 \mid 1S0 \mid 0S1 \mid 1S1$

L = { w | w is a palindrome }

$S \rightarrow \varepsilon \mid 0 \mid 1 \mid 0S0 \mid 1S1$

Practice

- L = { 0* }

  $S \rightarrow 0S \mid \varepsilon$

- L = { 0*1 }

  $S \rightarrow 0\,S \mid 1$

- L = { $0^n 1^n$ | n >= 0 }

  $S \rightarrow 0S1 \mid \varepsilon$

- L = { $0^n 1^n$ | n>=1 }

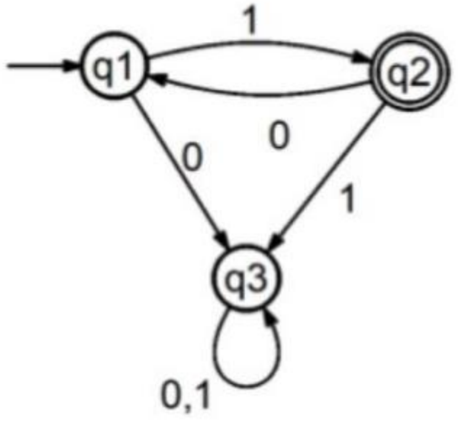
  $S \rightarrow 0S1 \mid 01$

- L = { $0^{2n} 1^{3n}$ | n>=0 }

  $S \rightarrow 00S111 \mid \varepsilon$

Converting DFA to CFG

- For each state q_i in the DFA, create a variable R_i for your CFG
- For each transition rule δ(qi, a) = q_k in your DFA, add the rule $R_i \rightarrow aR_k$ to your CFG
- For each accept state q_a in your DFA, add the rule $R_a \rightarrow \varepsilon$
- If q_0 is the start state in your DFA, then R_0 is the starting variable in your CFG.



CFG rules:

$R_1 \rightarrow 0\ R_3 \mid 1\ R_2$

$R_2 \rightarrow 0\ R_1 \mid 1\ R_3 \mid \varepsilon$

$R_3 \rightarrow 0\ R_3 \mid 1\ R_3$

Example: DFA to CFG

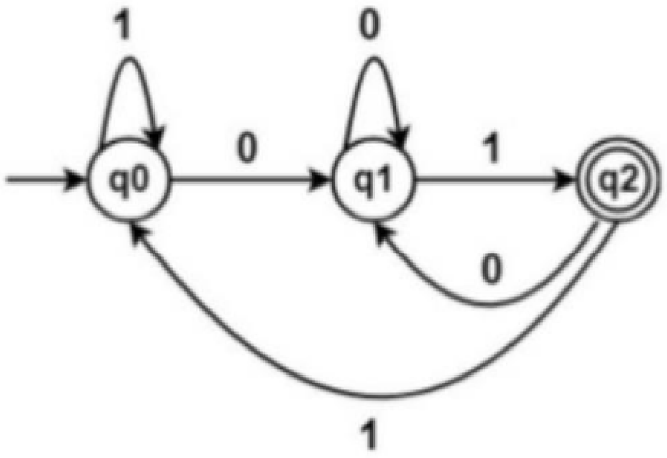
- L(M) = { w | w ends with 01 }

CFG rules:

$Q_0 \rightarrow 0\ Q_1 \mid 1\ Q_0$

$Q_1 \rightarrow 0\ Q_1 \mid 1\ Q_2$

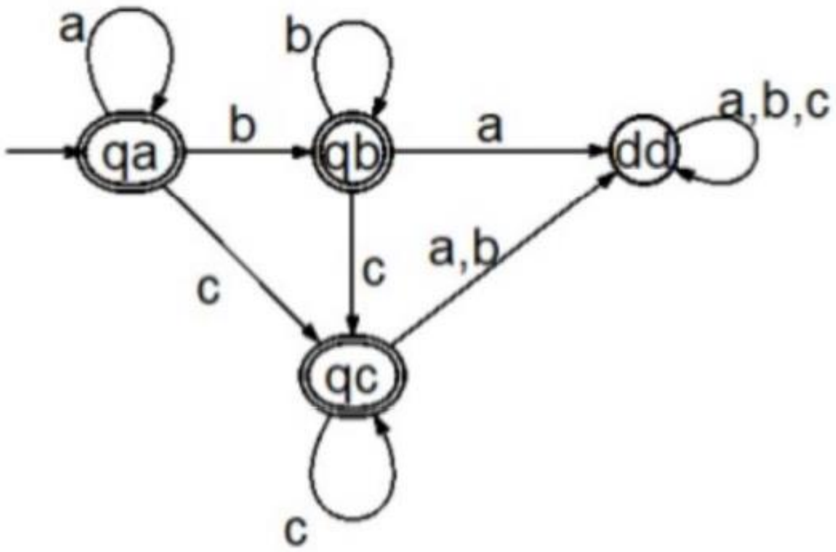
$Q_2 \rightarrow 0\ Q_1 \mid 1\ Q_0 \mid \epsilon$



DFA to CFG Practice

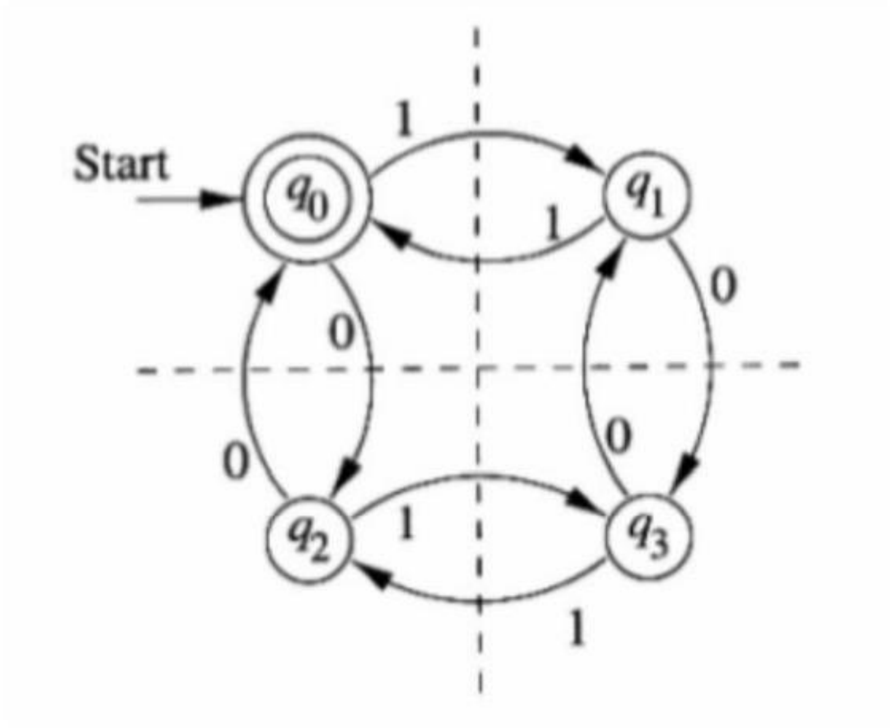
- L(M) = { $a^n b^m c^l$ | n, m, l >= 0 }

- Let's try it



DFA to CFG Practice

- L(M2) = { All binary strings with both an even number of zeros and an even number of ones }



- Let's try it