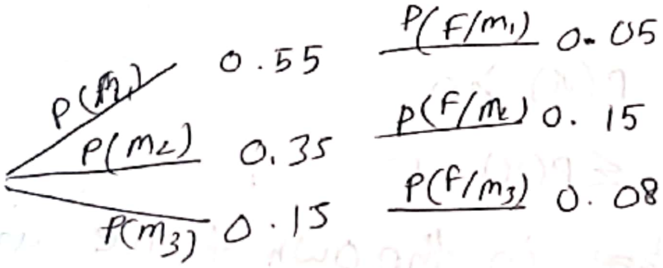
Sub :

Day

Time : Date : / /

Bayes Theonum:

Bayes theonum is the extended and generaliged ~~fnom~~ form of coditional probability.



$P(m_2/F) = ??$

$$P(m_2/F) = \frac{P(m_2 \cap F)}{P(F)}$$

$$= \frac{P(m_2) \times P(F/m_2)}{P(m_1) \times P(F/m_1) + P(m_2) \times P(F/m_2) + P(m_3) \times P(F/m_3)}$$

$$= 0.574$$

$$P(m_1/F) = \frac{P(m_1 \cap F)}{P(F)}$$

$$= \frac{P(m_1) \times P(F/m_1)}{P(m_1) \times P(F/m_1) + P(m_2) \times P(F/m_2) + P(m_3) \times P(F/m_3)}$$

$$= \frac{0.55 \times 0.05}{(0.55 \times 0.05) + (0.35 \times 0.15) + (0.15 \times 0.08)}$$

$$= 0.2989$$

Coditional probability:

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{P(A) \times P(B/A)}{P(B)}$$

$\boxed{1}$ Probability distribution:

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

The probability distribution represents all the outcomes of an event associated with corresponding probability

x	0	1	2
$P(x=x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

where as: (i) $P(x) \geq 0$

(ii) $\sum P(x) = 1$

Events: A ~~dice~~ coin is thrown twice and the number of head is counted / recoded the corresponding probability distribution is

x	0	1	2
$P(x=x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

A fair coin is thrown three times and the number of heads are recounted. Find the corresponding probability distribution.

HTT — $\frac{1}{8}$
THT — $\frac{1}{8}$
TTH — $\frac{1}{8}$
$3/8$

x	0	1	2	3
$P(x=x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

1 dice thrown once:

x	1	2	3	4	5	6
$P(X=x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Expected value / Mean value / Average Value:

$$E(x) = \frac{\cancel{\Sigma P.x}\ \Sigma x P(x)}{\Sigma P(x)} = \frac{\Sigma x P(x)}{1}$$

$$= \Sigma x P(x)$$

$$= x_1 P_1 + x_2 P_2 + x_3 P_3 + \cdots + x_n P_n$$

$$= \left(1 \times \frac{1}{6}\right) + \left(2 \times \frac{1}{6}\right) + \left(3 \times \frac{1}{6}\right) + \left(4 \times \frac{1}{6}\right) + \left(5 \times \frac{1}{6}\right) + \left(6 \times \frac{1}{6}\right)$$

$$= \frac{1}{6}\left[1 + 2 + 3 + 4 + 5 + 6\right]$$

$$= \frac{1}{6} \times 21$$

$$= \frac{21}{6} = 3.5 \text{ Ans.}$$

Sub :

Day

Time : Date : / /

Variance = Var(x) = σ^2 = $\frac{\sum x^2 P(x)}{1} - \{E(x)\}^2$

$$= \frac{1}{6}(1^2+4+9+16+25+36) - (3.5)^2$$

$$= \frac{1}{6}(91) - (3.5)^2$$

$$= \frac{35}{12} = 2.417$$

x	$P(x)$	$x^2 \cdot P(x)$
1	$\frac{1}{6}$	
2	$+\frac{1}{6}$	
3	$+\frac{1}{6}$	
4	$+\frac{1}{6}$	
5	$\frac{1}{6}$	
6	$\frac{1}{6}$	

Probability distribution:

(i) $P(x) \geq 0$

(ii) $\sum P(x) = 1$

Expected value and variance:

x	x_1	x_2	----	x_n
$P(x)$	$P(x_1)$	$P(x_2)$	----	$P(x_n)$

Expected value: $\mu = E(x) = \sum x P(x) = x_1 P(x_1) + x_2 P(x_2) + \cdots + x_n P(x_n)$

Variance: $\sigma^2 = \sum x^2 P(x) - \{E(x)\}^2$

Ex:

x	1	2	3	4
$P(x)$	0.3	a	b	0.25

Given that, $E(x) = 2.25$

Find the valus of a and b and find also the variance of the distribution

$0.3 + a + b + 0.25 = 1$

$\Rightarrow a + b = 1 - 0.55$

$\Rightarrow a + b = 0.45$ --- (i)

$E(X) = 2.25$

$1 \times 0.3 + 2a + 3b + 1 = 2.25$

$\Rightarrow 0.3 + 2a + 3b + 1 = 2.25$

Sub :

Day

Time : Date : / /

$\Rightarrow 2a + 3b = 0.95 \cdots$ (ii)

$a = 0.4$

$b = 0.05$

x	1	2	3	4
$P(X=x)$	0.3	0.4	0.05	0.25

Variance (σ^2) $= Var(x) = \sigma^2 = \sum x^2 P(x) - \{E(x)\}^2$

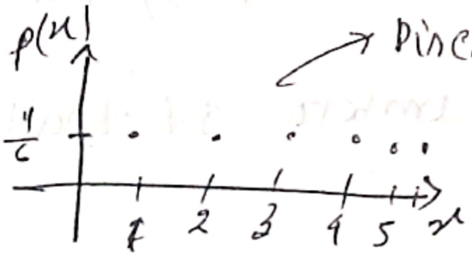
$= 1^2 \times 0.3 + 2^2 \times 0.4 + 3^2 \times 0.5 + 4^2 \times 0.25 - (2.25)^2$

$\sigma^2 = \cancel{1.0375}\ 1.2875$

Probability distribution

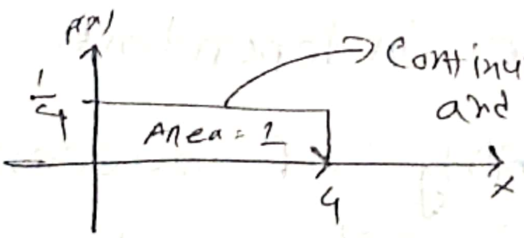
Discrete probability distribution

Continuous probability distribution



Discrete uniform

$\boxed{0 \le x \le 4}$

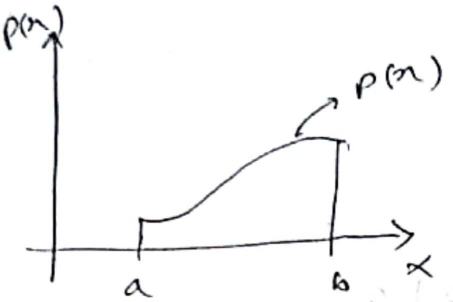


Continuous and uniform

Sub :

Day

Time : Date : / /



When it will be $p(x)$ distribution.

$$\int_a^b p(x)\,dx = 1$$

Discrete distribution:

(i) Binomial distribution

(ii) Poisson "

Continuous probability distribution

(i) Uniform distribution

(ii) Normal "

Binomial Distribution:

It is one kind of discrete probability distribution.

(i) Has two outcome (Success or failure)

True / false

pass / fail

(ii) Has fixed number of trails (pass/fail)

(iii) Trails are independent.

(iv) The probability of success (p) / failure (q) of each ~~event~~ outcome is constant

Sub :

Day

Time : Date : / /

if a discrete random variable X, is binominally distributed. than, it is demoted by $X \sim B(n, P)$

n & p are the parameter of B. d^n

n → Number of trails

p → success rate

$p + q = 1$ q = failure

* One coine throw 10 times and success rate is $\frac{1}{2}$.

$X \sim B(10, \frac{1}{2})$

$P(x = 7) = ??$

$(H+T)^1 = H + T$

$(H+T)^2 = H^2 + 2HT + T^2$

$= HH + 2HT + TT$

$(H+T)^3 = H^3 + 3H^2T + 3HT^2 + T^3$

$= HHH + 3HHT + 3HTT + TTT$

$P(x = r) = {}^nC_r \, p^r q^{n-r}$ | $r = 0, 1, 2 \cdots n$

$X \sim B(10, \frac{1}{2})$

$P(x = 7) = {}^{10}C_7 (\frac{1}{2})^7 (\frac{1}{2})^3 =$

Sub :

Day

Time : Date : / /

Probability function:

$P(x = r) = {}^{n}C_{r}\, p^{r}\, q^{n-r}$

$X \sim B(12, 0.75)$

Find: (i) $P(x = 7)$

(ii) $P(x \geq 1)$

(iii) $P(x < 2)$

(i) $P(x = 7) = {}^{12}C_{7}\,(0.75)^{7}\,(0.25)^{5}$

$= 0.1032$

(ii) $P(x \geq 1) = 1 - P(0)$

$= 1 - {}^{12}C_{0}\,(0.75)^{0}\,(0.25)^{12}$

$= 1 - (0.25)^{12}$

$= 0.99$

(iii) $P(x < 2) = P(0) + P(1)$

$= (0.25)^{12} + {}^{12}C_{1}\,(0.75)^{1}\,(0.25)^{11}$

$= 2.205 \times 10^{-6}$

Mean (expected value) and variance of a Binomial distribution

$x \sim \beta(n, p)$

Expected value $E(x) = \mu = np$

Variance $Var(x) = \sigma^2 = npq$

$x \sim \beta(10, 0.6)$

$\mu = np = 10 \times 0.6 = 6$

$\sigma^2 = npq = 10 \times 0.6 \times 0.4 = 2.4$

Sub :

Day

Time : Date : / /

$X \sim B(20, \frac{1}{2})$

$\mu = np = 20 \times \frac{1}{2} = 10$

The mean and variance of $x \sim B(n, p)$ is 9 and 3.6 nespectively find.

(i) The value of n and P

(ii) and hence the value of $P(x=6)$

Soln:

(i) given, $\mu = 9$

We know,

$\mu = np$

$\therefore np = 9$ ---- (i)

$\sigma^2 = npq = 3.6$ ---- (ii)

from (i) and (ii)

$9 \times q = 3.6$

$\Rightarrow q = \frac{3.6}{9} = 0.4$

$p + q = 1$

$\Rightarrow p + 0.4 = 1$

$\Rightarrow p = 0.6$

from, (i)

$n \cdot p = 9$

$\Rightarrow n \times 0.6 = 9$

$\Rightarrow n = \frac{9}{0.6}$

$= 15$

(ii) $P(x=6) = {}^{15}C_6 (0.6)^6 \times (0.9)^9$

$= 6.12 \times 10^{-2}$

Sub : Day Time : Date : / /

poisson Distribution :

poisson Distribution is one kind of discrete probability distribution.

If a random variable x; follow a poisson distribution then it is denoted by $x \sim P_o(\lambda)$

λ = Parameter = rate = mean value.

Properties :

(i) Occurs singly

(ii) Events are independent

(iii) occurs at a rate at a given interval of time and space

The probability function for poisson distribution

$x \sim P_o(\lambda)$

$$P(x=r) = \frac{e^{-\lambda} \times \lambda^r}{r!}$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^r}{r!} + \dots$$

$x = \lambda$

$$e^\lambda = 1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \dots + \frac{\lambda^r}{r!} + \dots$$

$$\frac{e^\lambda}{e^\lambda} = \frac{1}{e^\lambda} + \frac{\lambda}{e^\lambda \times 1!} + \frac{\lambda^2}{e^\lambda \times 2!} + \frac{\lambda^3}{e^\lambda \times 3!} + \dots + \frac{\lambda^r}{e^\lambda \times r!} + \dots$$

$$1 = e^{-\lambda} + \frac{e^{-\lambda} \times \lambda}{1!} + \frac{e^{-\lambda} \times \lambda^2}{2!} + \dots + \frac{e^{-\lambda} \times \lambda^r}{r!} + \dots$$

Sub :

Day

Time : Date : / /

$x \sim P_o(3.5)$

find, (i) $P(x=5)$

(ii) $P(x \leq 1)$

(iii) $P(x > 1)$

Sol^n:

(i) given, $x \sim P_o(3.5)$

$$\lambda = 3.5$$

$$P(x=5) = \frac{e^{-\lambda} \times \lambda^{n}}{n!}$$

$$= \frac{e^{-3.5} \times 3.5^{5}}{5!}$$

$$= 0.132$$

(ii) $P(x \leq 1)$

$$= \frac{e^{-\lambda} \times \lambda^{0}}{0!} + \frac{e^{-\lambda} \times \lambda^{1}}{1!}$$

$$= \frac{e^{-3.5} \times 3.5^{0}}{0!} + \frac{e^{-3.5} \times 3.5^{1}}{1!}$$

$$= 0.135$$

(iii) $P(x > 1) = 1 - P(0) + P(1)$

$$= 1 - 0.135$$

$$= 0.865$$

23-4-2022

Sub :

$x \sim B(n, p)$

$\mu = np$

$\sigma^2 = npq$

$P(x=r) = {}^nC_r p^r q^{n-r}$

$x \sim P_o(\lambda)$

$\mu = \sigma^2 = \lambda$

$P(x=r) = \frac{e^{-\lambda} \times \lambda^r}{r!}$

Note that the mean and variance of poisson distribution are same.

$\mu = \sigma^2 = \lambda$

$x \sim P_o(4)$

$\mu = 4$

$\sigma^2 = 4$

Standard deviation $= \sqrt{\sigma^2} = \sqrt{4} = 2$

Ans.

23-4-2022

Sub :

Day

Time : Date : / /

Poisson Distributions

1. Computer breakdown occur randomly on every 48 hours of use.

(i) Calculate the probality that there will be fewer than 4 breakdowns in 60 hours of use.

$X \sim Po(1)$

$\lambda = \frac{1}{48} \times 60$

$= 1.25$

$P(X<4)$

$= P(0) + P(1) + P(2) + P(3)$

$= e^{-\lambda}\left[1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!}\right]$

$= 0.96$

(ii) There will be no breakdown in 24 hours of use.

$\lambda = \frac{1}{48} \times 24 = 0.5$

$P(X=0) = e^{-0.5}$

Sub : Day Time : Date : / /

② Between 7 p.m. and 11 p.m arrivals of patients at the casualty department of a hospital occur at random at an average rate of 6 per hour.

(i) Find the ~~value~~ probability that during any period of one hour between 7p.m and 11p.m. exactly 5 people will arrive.

$$\lambda = 6$$

$$P(x=5) = \frac{e^{-6} \times e^{5}}{3!}$$

$$= 0.161$$

(ii) A patient arrive ~~at~~ exactly 10.15 p.m. Find the probality that a last one more patient arrives before 10.35 P.m

$$\frac{6}{60} \times 20 = 2$$

$$\lambda = 2$$

$$P(x \geq 1)$$

$$= 1 - P(0)$$

$$= 1 - e^{-\lambda}$$

$$= 1 - e^{-2}$$

$$= 0.865$$

3) The number of radioactive particles emitted per second by certain metal in random and has mean 1.7, the - - - per second in 0.6.
Find the probability that the total number of particles emitted in the next 3 second in 6 or 7 or 8.

Ans:-

given data,

$$\lambda = (1.7 + 0.6) \times 3$$

$$= 2.3 \times 3$$

$$= 6.9$$

$$P(6 \text{ or } 7 \text{ or } 8)$$

$$= P(6) + P(7) + P(8)$$

$$= e^{-6.9}\left(\frac{(6.9)^6}{6!} + \frac{(6.9)^7}{7!} + \frac{(6.9)^8}{8!}\right)$$

$$= 0.4284$$

Normal /symmeticical distribution:

Normal distribution is one kind of continus probability distribution.

Properties:

(i) Bell shapped symmetricall about mean.

(ii) mean = mode = median.

(iii) Area under the graph is unity (1)

(iv) Range from $-\infty$ to ∞

if a random variable X is nanmally distribution then it in denoted by $X \sim N(\mu, \sigma^2)$

(mean Veriance)

Example:

$$X \sim N(20, 4^2)$$

$$P(x > 22) = ??$$

Sub :

Day

Time : Date : / /

Standard Normal distribution:

A standard normal distribution has mean zero(0) and standard deviation 1. and denoted by

$$z \sim N(0, 1^2)$$

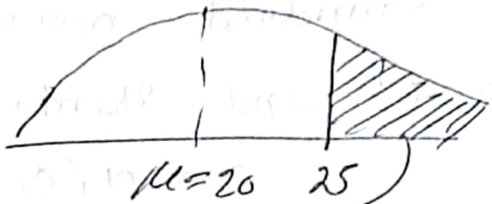
$$x \sim N(\mu, \sigma^2), \quad z \sim N(0, 1^2)$$

convention x to z $\Big\}$ $z = \dfrac{x-\mu}{\sigma}$

Normal distribution:

$x \sim N(\mu, \sigma^2)$

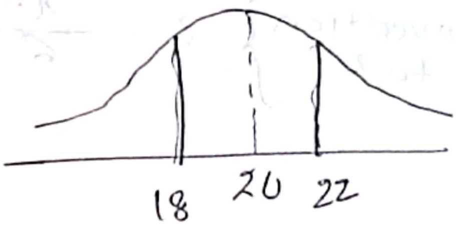
$x \sim N(20, 4^2)$



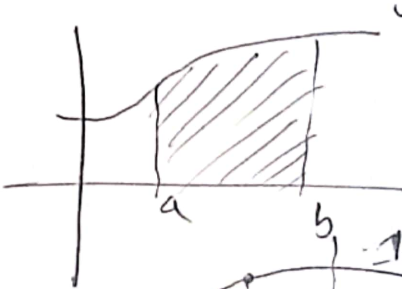
(i) $P(x>25) = ?$

(ii) $P(18<x<22) = ??$

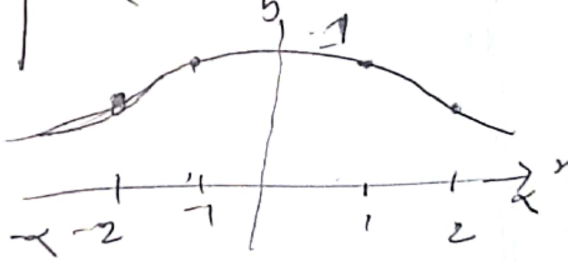
Area of this shaded region $= P(x>25)$



$$\text{Area} = \int_a^b y\,dx$$



$$y = e^{-x^2}$$



$x=0, \quad y=1$

$x=1, \quad y=e^{-1}=\frac{1}{e}$; $x=2 \quad y=e^{-4}=\frac{1}{e^4}$

$x=-1, \quad y=e^{-1} \quad \frac{1}{e}$; $x=-2 \quad y=e^{-4}=\frac{1}{e^4}$

$$\int_{-\infty}^{\infty} e^{-x^2} dx \neq 1$$

Sub: $\frac{1}{\sigma\sqrt{2\pi}}$

Day

Time: Date: / /

$$\boxed{f(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \qquad -\infty < x < \infty}$$

μ = mean

σ = S.D

$\pi \approx 3.14159....$

$e = 2.71828 ...$

$$\boxed{\int_{-\infty}^{\infty} f(x)\,dx = 1}$$

Normal distribution ⟶ Standard normal distribution

$X \sim N(\mu, \sigma^2)$

$Z \sim N(0, 1^2)$

$$\boxed{Z = \frac{x-\mu}{\sigma}}$$

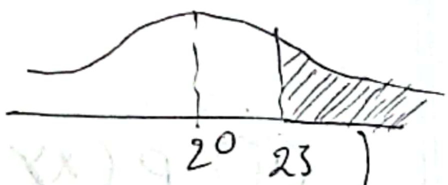
$X \sim N(20, 4^2)$

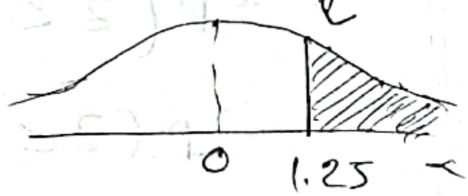
$P(x > 25)$

$= P\left(Z > \frac{25-20}{4}\right)$

$= P\left(Z > \frac{5}{4}\right)$

$= P(Z > 1.25)$





SN-এ.D তে convent করতে হবে Area = 1 থাকবে

Sub : 　　　　 Day 　 Time : 　 Date : / /

$$\int_{1.25}^{\alpha} f(x)\,dx$$

$$= \int_{1.25}^{\alpha} \frac{1}{6\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$$

Qus!

$X \sim N(12, 3^2)$

(i) $P(x > 9) = ?$

(ii) $P(x < 14) = ?$

(iii) $P(10 < x < 15) = ?$

(i) $P(x > 9)$

$= P\left(z > \frac{9-12}{3}\right)$

$= P(z > -1)$

(ii) $P(x < 14)$

$= P\left(z < \frac{14-12}{3}\right)$

$= P\left(z < \frac{2}{3}\right)$

$= P(z < 0.667)$

(iii) $P(-0.667 < z < 1)$

Sub : Day Time : Date : / /

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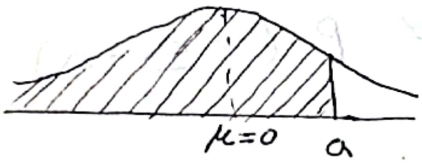
$Z \sim N(0, 1^2)$

$P(Z \geq 0) = 0.5$

$P(Z \leq 0) = 0.5$

$P(Z \leq a)$



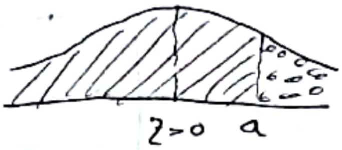
$P(Z \leq a) = \phi(a)$

$\phi(1.5) = P(Z \leq 1.5) = 0.8749.$

$P(Z \leq 0.46) = 0.6772$

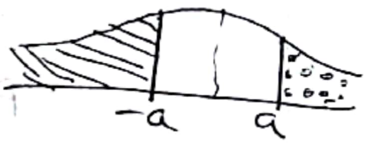
Z	0.0	0.01	0.02
0.0			
0.01			
0.02			

$P(z \leq a) = \phi(a)$



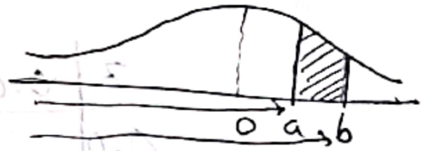
$P(z \geq a) = 1 - P(z \leq a) = 1 - \phi(a)$

$P(z \leq -a) = P(z \geq a) = 1 - \phi(a)$



$P(a \leq x \leq b) = P(z \leq b) - P(z \leq a)$

$= \phi(b) - \phi(a)$



$P(1.15 < z < 1.63)$

$= \phi(1.63) - \phi(1.15)$

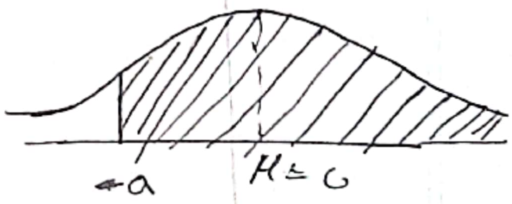
$= 0.9484 - 0.8749$

$= 0.0735$

$\phi(z \geq -a)$

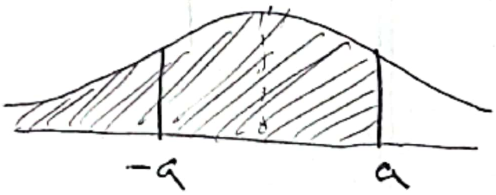
$= P(z \leq a)$

$= \phi(a)$



$\phi(-a) = P(z \leq -a)$

$= 1 - \phi(a)$



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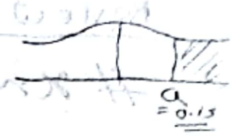


$P(Z > 0.15)$

$= 1 - P(Z \leq 0.15)$

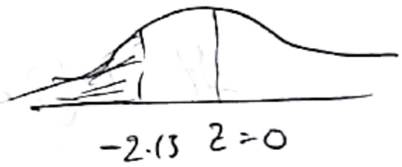
$= 1 - 0.5596 = 0.4404$

$P(1.1 \leq Z \leq 1.35)$

$\cancel{= P(Z \leq 1.13}$

$= \Phi(1.35) - \Phi(1.1) = 0.9115 - 0.8643 = 0.0472$

$P(Z < -2.15)$



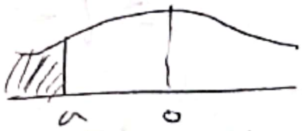
$= \Phi(-2.15)$

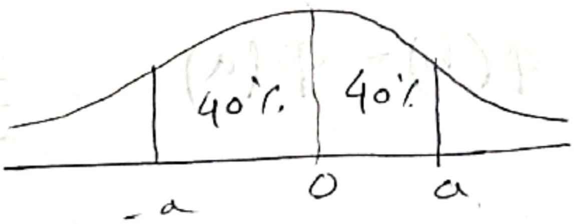
$= 1 - \Phi(2.15) = 1 - 0.9842$

$= 0.0158$

$P(Z \geq a) = 5\% = 0.05$

$P(Z \leq a) = 10\% = 0.1$





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Day

Time : Date : / /

Review

$x \sim N(\mu, \sigma^2)$

$z \sim N(0, 1^2)$

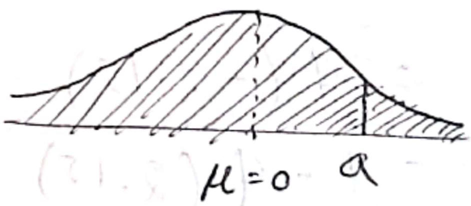
$$z = \frac{x-\mu}{\sigma}$$

$x \sim N(15, 4^2)$

$P(x > 20)$

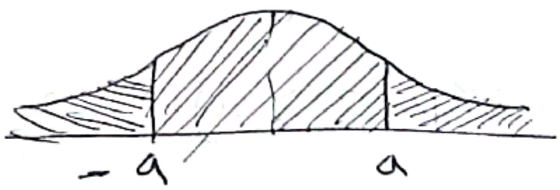
$$P\left(z > \frac{20-15}{4}\right) = P(z > 1.25)$$

$P(z<a) = \phi(a)$



$P(z>a) = 1-\phi(a)$

$P(z<-a)$

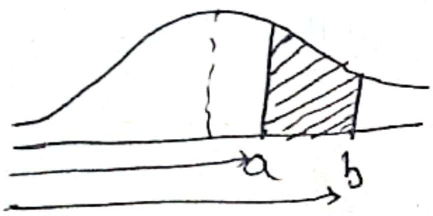


$= P(z>a)$

$= 1 - \phi(a)$

$P(z>-a) = P(z<a) = \phi(a)$

$P(a<x<b) = \phi(b) - \phi(a)$



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1. # given that $x \sim N(20, 16)$. find the following probabilitiesn.

(i) $P(x \le 26)$

(ii) $P(x > 30)$

(iii) $P(x < 15)$

(iv) $P(16 < x < 25)$

2. # $X \sim N(\mu, \frac{1}{9}\mu^2)$

Find $P(x > 1.5\mu)$

3. # $X \sim N(\mu, \sigma^2)$

$P(x \geq 9.81) = 0.1587$

$P(x \le 8.82) = 0.0116$

Find the value of μ, σ

1.

(i) $P\left(z \le \frac{26-20}{4}\right) = P(z \le 1.5) = 0.9332$ ~~$1 - \phi(1.5) = 1 - 0.9332$~~ $= 0.0668$

(ii) $P\left(z > \frac{30-20}{4}\right) = P(z > 2.5) = \cancel{0.9938}\ 1 - \phi(2.5)$
$= 1 - 0.9938 =$

(iii) $P\left(z < \frac{15-20}{4}\right) = P(z < -1.25) = 1 - \phi(1.25)$
$= 1 -$

(iv) $P\left(\frac{16-20}{4} < z < \frac{25-20}{4}\right) = P(-1 < z < 1.25)$

$= \phi(1.25) - \phi(-1)$

$= \phi(1.25) - [1 - \phi(1)]$

$= 0.8944 - (1 - 0.8413)$

1.684

44

4

Sub : Day Time : Date : / /

$\therefore X \sim N(\mu, \frac{1}{4}\mu^2) \qquad \sigma = \frac{\mu}{2}$

$\therefore P(x > 1.5\mu)$

$= P\left(z > \frac{1.5\mu - \mu}{\mu/2}\right)$

$= P(z > 1)$

$= 1 - \Phi(1)$

$= 1 - 0.8413 =$

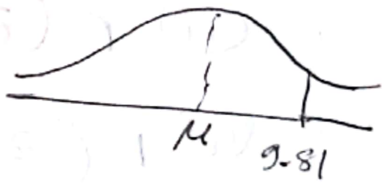
$X \sim N(\mu, \sigma^2)$

$P(x \geq 9.81) = 0.1587$

$P(x \leq 8.82) = 0.0116$

$\cancel{P(9.81 \leq x \leq 8.82)}$

$P(x \geq 9.81) = 0.1587$



$\Rightarrow P\left(z \geq \frac{9.81 - \mu}{\sigma}\right) = 0.1587$

$\Rightarrow 1 - \Phi\left(\frac{9.81 - \mu}{\sigma}\right) = 0.1587$

$\Rightarrow \Phi\left(\frac{9.81 - \mu}{\sigma}\right) = 0.8413$

$\Rightarrow \Phi\left(\frac{9.81 - \mu}{\sigma}\right) = \Phi(1)$

$\Rightarrow \frac{9.81 - \mu}{\sigma} = 1$

$\Rightarrow 9.81 - \mu = \sigma \quad \text{--- (1)}$

≤ মানে μ থেকে পিছনে থাকে
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Sub :

Day

Time : Date : / /

$P(x \leq 8.82) = 0.0116$

$\Rightarrow P\left(z \leq \frac{8.82 - \mu}{\sigma}\right) = 0.0116$

$\Rightarrow P\left(z \leq -\frac{\mu - 8.82}{\sigma}\right) = 0.0116$

$\Rightarrow P\left(-\frac{\mu - 8.82}{\sigma}\right) = 0.0116$

$\Rightarrow 1 - \Phi\left(\frac{\mu - 8.82}{\sigma}\right) = 0.0116$

$\Rightarrow \Phi\left(\frac{\mu - 8.82}{\sigma}\right) = 0.9884$

$\Rightarrow \frac{\mu - 8.82}{\sigma} = 2.27$

$\Rightarrow \mu - 8.82 = 2.27\sigma$ ------- (ii)

$P(z \le -a)$
$\Phi(-a)$
$= 1 - \Phi(a)$

(i) $\Rightarrow 9.81 - \mu = \sigma$

$\Rightarrow \sigma + \mu = 9.81$

(ii) $\Rightarrow \mu - 8.82 = 2.27\sigma$

$\Rightarrow \mu - 2.27\sigma = 8.82$

$\mu = 9.5228 \quad \sigma = 0.287$

$\# \; P(-1 < z < 1)$

$\Rightarrow \phi(1) - \phi(-1)$

$\Rightarrow \phi(1) - (1 - \phi(1))$

$= \phi(1) - 1 + \phi(1)$

$\Rightarrow 2 \times \phi(1) - 1 \quad = 2 \times 0.8413 - 1$

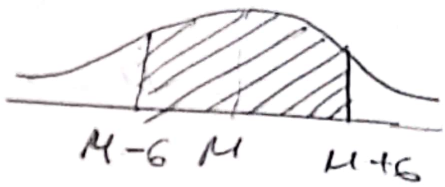
$\cancel{\Rightarrow 2 \times 0.5398 - 1} \quad = 0.6826$

$\cancel{\Rightarrow 1.0796 - 1}$

$\cancel{= 0.796}$

ans always

$\# \; P(\mu - \sigma < x \leq \mu + \sigma)$



within one S.D

$$P\left(\frac{\mu - \sigma - \mu}{\sigma} < z < \frac{\mu + \sigma - \mu}{\sigma}\right)$$

$= P(-1 < z < 1)$

$\# \; P(\mu - 2\sigma < x < \mu + 2\sigma)$

$= P(-2 < x < 2)$

within two - S.D $= 0.95$

Sub : Day Time : Date : / /

$P(\mu - 3\sigma < x < \mu + 3\sigma)$

within 3 standard deviation

$2\times \phi(1) - 1 = 0.68$

$2\times\phi(2) - 1 = 0.9544$

$2\times\phi(3) - 1 = 0.9974$

$P(x \geq 59.1) = 0.0218$

$P(x \geq 29.2) = 0.9345$

$P(x \geq 59.1) = 0.0218$

$\Rightarrow P\left(z \geq \frac{59.1 - \mu}{\sigma}\right) = 0.0218$

$\Rightarrow 1 - \phi\left(\frac{59.1-\mu}{\sigma}\right) = 0.0218$

$= \frac{59.1 - \mu}{\sigma} = 0.9782$

$= 59.1 - \mu = 2.026$ --- (1)

$P(x \geq 29.2) = 0.9345$

$\Rightarrow P\left(z \geq \frac{29.2 - \mu}{\sigma}\right) = 0.9345$

$\Rightarrow \phi\left(-\frac{29.2 - \mu}{\sigma}\right) = 0.9345$

$\Rightarrow \phi\left(-\frac{\mu - 29.2}{\sigma}\right) = 0.9345$

$\Rightarrow 1 - \phi\left(\frac{\mu - 29.2}{\sigma}\right) = 0.9345$

$\Rightarrow \phi$

Normal distribution as an approximation to Binomial and poisson distribution :

$X \sim B(60, 0.3)$ | $Y \sim Po(3.5)$

$P(x \geq 26)$ | $P(x \geq 15)$

$U \sim N(N, P)$

$X \sim B(n, p) \approx Y \sim N(\mu, \sigma^2)$

* <u>Conditions:</u>

$$\left.\begin{array}{l} np > 5 \\ nq > 5 \end{array}\right\}$$ Binomial can be approximated by normal

* $\begin{cases} \text{mean} = \mu = np \\ \text{Variance} = \sigma^2 = npq \end{cases}$

* discrete $\longrightarrow$ continuous $\Rightarrow$ half continuity convention

$x \sim B(60, 0.3)$

$np = 60 \times 0.3 = 18$
$nq = 60 \times 0.7 = 42$

$P(x \geq 26)$

$\mu = np = 18$

$\sigma^2 = npq = 18 \times 0.7 = 12.6$

$x \sim B(60, 0.3)$

$\approx y \sim N(18, 12.6)$

$= P(y \geq 25.5)$

$= P\left(z \geq \frac{25.5 - 18}{\sqrt{12.6}}\right)$

$= P(z \geq 2.11)$

$1.5 \leq x < 2.5$

$\downarrow$

z

#

$x \sim P_0(\lambda) \approx y \sim N(\mu, \sigma^2)$

<u>Condition:</u> $\lambda > 10$

$$\mu = \sigma^2 = \lambda$$

$$x \sim P_0(\lambda) \approx y \sim N(\lambda, \lambda)$$

(*) $X \sim P_0(12.5)$

$P(x \cancel{\leq} 20)$

<u>Ans:-</u>

$$X \sim P_0(12.5) \approx y \sim N(12.5, 12.5)$$

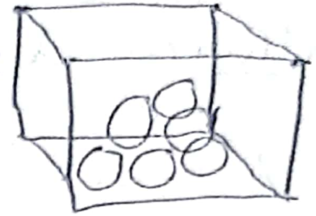
$$P(x < 20) = P(y < 19.5)$$

$$= P\left(z < \frac{19.5 - 12.5}{\sqrt{12.5}}\right)$$

$$= P(z < 1.98)$$

Linear Combination of random Variable:

X is a discreate random variable



$\Rightarrow \mu = E(x)$, $Var(x) = \sigma^2$

$E(ax + b) = aE(x) + b$ ← Expected

$E(x) = 3.5$, $Var(x) = 11.75$

$y = 2x + 3$

$E(y) = E(2x+3) = 2E(x) + 3$

$= 2 \times 3.5 + 3$

$= 7 + 3$

$= 10$

$\Rightarrow Var(ax \pm b) = a^2 Var(x)$

$Var(y) = Var(2x + 3)$

$= 2^2 Var(x)$

$= 4 \times 11.75$

$= 47$

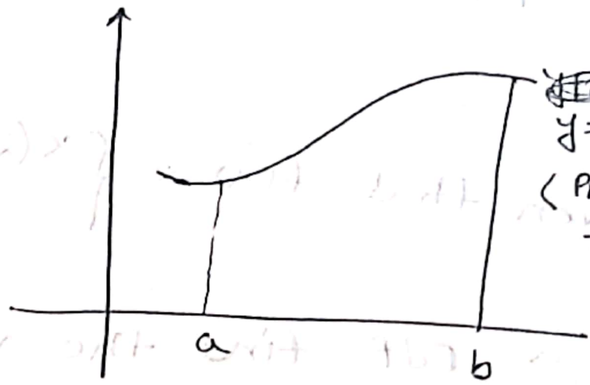
→ Here 3 will be ~~vetre~~ delete because Here 3 is continus so that var is 0

$$X_1 \sim N(\mu_1, \sigma_1^2)$$
$$X_2 \sim N(\mu_2, \sigma_2^2)$$
} x_1 & x_2 are independent

$$E(ax_1 \pm bx_2) = a_1 E(x_1) \pm a_2 E(x_2)$$

$$Var(ax_1 \pm bx_2) = a^2 Var(x_1) + b^2 Var(x_2)$$

Continuous random variable :

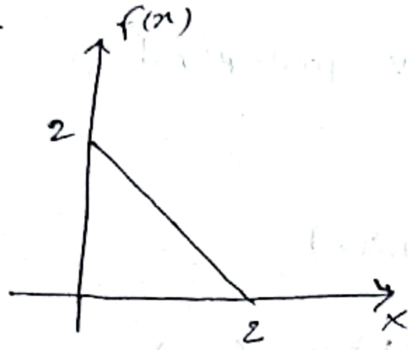


(*) if $\int_a^b f(x)\, dx = 1$, $f(x) \geq 0$ when the condition fulfill then we can say that it is ~~continuous~~ probability density function.

if X is a Continuous random variable with p.d.f $f(x)$ and $f(x) \geq 0$ for all x; then

$$\int_{-\infty}^{\infty} f(x)\, dx = 1$$

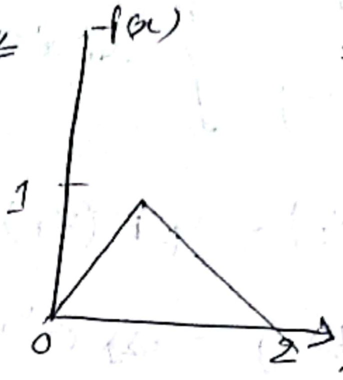
and $$P(a < x < b) = \int_a^b f(x)\, dx.$$

#


$A = \frac{1}{2} \times 2 \times 2$

$= 2 \neq 1$

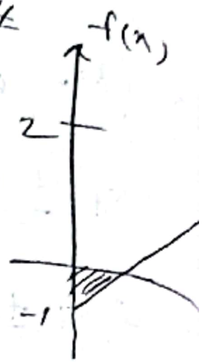
not pdf

#


$\frac{1}{2} \times 2 \times 1$

$= 1$

$=$ P.d.f

#


Not Pdf

$f(x) \leq$

Given that $f(x) = \begin{cases} k(2-x) & ; -2 \leq x \leq 2 \\ 0 & ; \text{otherwise.} \end{cases}$

is a pdf find the value of k

$\int_{-2}^{2} k(2-x)\,dx = 1$

$\Rightarrow k\left[2x - \frac{x^2}{2}\right]_{-2}^{2} = 1$

$\Rightarrow k(4-2) - k(-4-2) = 1$

$\Rightarrow 2k + 6k = 1$

$\Rightarrow 8k = 1$

$\Rightarrow k = \frac{1}{8}$

Given that: $f(x) = \begin{cases} k(4-x); & 1 \le x \le 4 \\ 0 & ; \text{otherwise} \end{cases}$ is a p.d.f

(i) find the value of k

(ii) find $P(2 \le x \le 3)$

(i) $\displaystyle\int_1^4 k(4-x)\,dx = 1$

$\Rightarrow k\left[4x - \frac{x^2}{2}\right]_1^4 = 1$

$\Rightarrow k(16-8) - k\left(4 - \frac{1}{2}\right) = 1$

$\Rightarrow 8k - 3.5k = 1$

$\Rightarrow 4.5k = 1$

$\Rightarrow k = \frac{1}{4.5} = \frac{2}{9}$

$\therefore f(x) = \frac{2}{9}(4-x)\,dx$

~~(ii) P(~~

(ii) $P(2 \le x \le 3)$

$= \displaystyle\int_2^3 \frac{2}{9}(4-x)\,dx$

$= \frac{2}{9}\left[4x - \frac{x^2}{2}\right]_2^3$

$= \frac{2}{9}\left(12 - \frac{9}{2}\right) - \frac{2}{9}(8-2)$

$= \frac{5}{3} - \frac{4}{3} = \frac{1}{3}$ Ans.

Another: $E(x) = \sum x\, p(x)$
$var(x) = \sum x^2 p(x) - \{E(x)\}^2$

\# If $f(x)$

\# If x is continuous random variable with p.d.f $f(x)$ then $E(x) = \int_{-\infty}^{\infty} x f(x)\, dx$ = mean value

$$var(x) = \int_{-\infty}^{\infty} x^2 f(x)\, dx - \{E(x)\}^2$$

\# $f(x) = \begin{cases} \frac{2}{9}(4-x); & 1 \le x \le 4 \\ 0 & ; \text{otherwise} \end{cases}$

$$E(x) = \int_1^4 x \cdot f(x)\, dx = \int_1^4 x \cdot \frac{2}{9}(4-x)\, dx$$

$$= \cancel{\frac{2}{9} x \Big|_1^4 \; 4x - ^2}$$

$$= \frac{2}{9}\int_1^4 (4x - x^2)\, dx$$

$$= \frac{2}{9}\left[2x^2 - \frac{x^3}{3}\right]_1^4$$

$$= \frac{2}{9}\left(\left(32 - \frac{64}{3}\right) - \left(2 - \frac{1}{3}\right)\right)$$

$$= \frac{9}{2}$$

$$var(x) = \int_{-\infty}^{\infty} x^2 f(x)\, dx - \{E(x)\}^2$$

$$= \int_1^4 x^2 \cdot$$

1 703 No Room

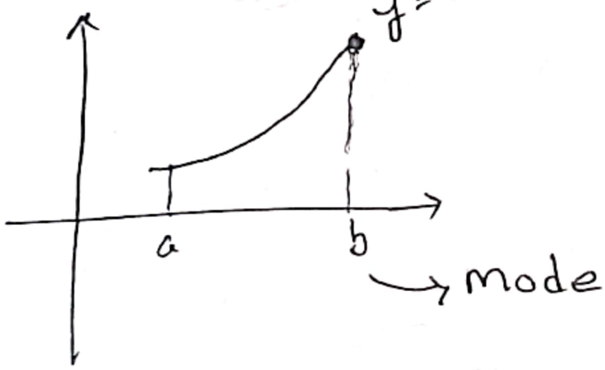
Mean, Variance, Mode, and Median of a continuous random variable:

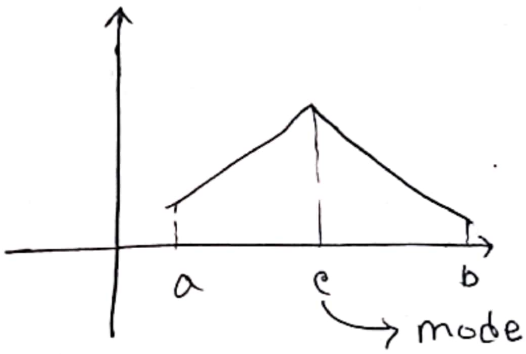
$y = f(x) \rightarrow$ P.d.f

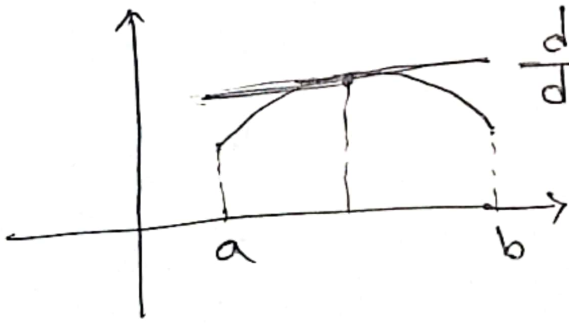
$$E(x) = \int_{\alpha}^{-\alpha} x f(x)\, dx$$

$$Var(x) = \int_{\alpha}^{-\alpha} x^2 f(x)\, dx - \{E(x)\}^2$$

Mode:







$$\frac{dy}{dx} = 0$$

The continuous random variable x has p.d.f given by

$$f(x) = \begin{cases} 4x - 4x^3 \ ; & 0 \le x \le 1 \\ 0 \ ; & \text{otherwise} \end{cases}$$

(a) find the mode of x

(b) find $P(0.1 < x < 0.6)$

(c) find the median of x $\longrightarrow$ $\int_0^m f(x)\,dx = \int_m^1 f(x)\,dx = \frac{1}{2}$

(a) $f(x) = 4x - 4x^3$

$\therefore f'(x) = 4 - 12x^2 = 0$

$\Rightarrow 4 - 12x^2 = 0$

$\Rightarrow 1 - 3x^2 = 0$

$\Rightarrow x^2 = \frac{1}{3}$

$\Rightarrow x = \frac{1}{\sqrt{3}} = 0.577$

(b) $\int_{0.1}^{0.6} 4x - 4x^3 \, dx$

$= \int_{0.1}^{0.6} 4(x - x^3)\, dx$

$= 4\int_{0.1}^{0.6} (x - x^3)\, dx$

$= 4\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_{0.1}^{0.6}$

$= 4[0.1476 - 0.004975]$

$= 0.5705$

ii) $\int_0^m 4x - 4x^3 \, dx = \frac{1}{2}$

$\Rightarrow \left[2x^2 - x^4\right]_0^m = \frac{1}{2}$

$\boxed{m^2 = a}$

$\Rightarrow 2m^2 - m^4 = \frac{1}{2}$

$\Rightarrow 4m^2 - 2m^4 = 1$

$\therefore 2m^4 - 4m^2 + 1 = 0$ / $2a^2 - 4a + 1 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\frac{4 \pm \sqrt{16 - 8}}{4}$

$\therefore$

$a =$

Hypothesis test :

what is hypothesis testing?

⇒ Hypothesis testing is method or statistical procedure that is used to draw a conclusion about the population parameter based on test-statistic.

Steps of Hypothesis test :

① Specify the null Hypothesis

② Specify the alternative hypothesis

③ Set the significance level

④ Draw the conclusion.

NULL Hypothesis :

A statement that is made about the parameter. The null hypothesis is denoted by H_0.

$$H_0 : \mu = 3.52$$

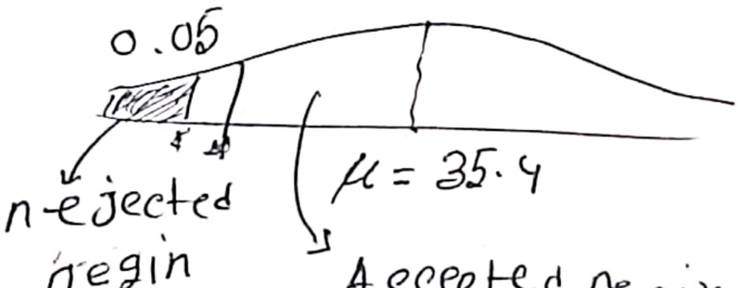
Alternative hypothesis : An alternative hypothesis is the decision taken when H_0 (null hypothesis) is rejected. Alternative hypothesis is denoted by H_1.

$H_0 : \mu = 60$	$H_0 : P = \frac{1}{2}$
$H_1 : \mu > 60$	$H_1 : P > \frac{1}{2}$
$\quad , \mu < 60$	

Singhificance lavel:

Canelusion:

$H_0: \mu = 35.4$
$H_1: \mu < 35.4$ } 5% len of sighifine



$P(x < 35.4) = 0.063$

Ques 1

8 A random variable x has p.d.f given by

$$f(x) = \begin{cases} k(1-x) ; & 0 \le x \le 1 \\ 0 ; & \text{othewise} \end{cases}$$

k is a constant.

(i) find the value of k

(ii) find E(x)

(iii) Show that ~~variable~~ variance of x i.e $var(x) = \frac{1}{8}$

(iv) find $P(x > \mu)$

Soln:

(i) given

$$f(x) = \begin{cases} k(1-x) ; & 0 \le x \le 1 \\ 0 ; & \text{otherwise} \end{cases}$$

We know, $\int_{-\alpha}^{\alpha} f(x)\, dx = 1$

$\Rightarrow \int_0^1 k(1-x)\, dx = 1$

$\Rightarrow k \int_0^1 1-x\, dx = 1$

$\Rightarrow k \left[x - \frac{x^2}{2}\right]_0^1 = 1$

$\Rightarrow k \left[\frac{2x - x^2}{2}\right]_0^1 = 1$

$\Rightarrow k \left[\frac{2\times1 - 1}{2}\right] = 1$

$\Rightarrow k \left[\frac{1}{2}\right] = 1$

$\Rightarrow k = 2$ Ans.-

(ii) $E(x) = \int_0^1 x \cdot 2(1-x)\,dx$

$= 2\int_0^1 (x-x^2)\,dx$

$= 2\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$

$= 2\left[\frac{1}{2} - \frac{1}{3} - 0\right]$

$= \frac{1}{3}$

(iii) $var(x) = \int_0^1 x^2 \cdot 2(1-x)\,dx - \{E(x)\}^2$

$= 2\int_0^1 (x^2-x^3)\,dx - \left(\frac{1}{3}\right)^2$

$= 2\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1 - \frac{1}{9}$

$= 2\left[\frac{1}{3} - \frac{1}{4} - 0\right] - \frac{1}{9}$

$= \frac{1}{6} - \frac{1}{9}$

$= \frac{1}{8}$ (proved)

(iv) $P(x > \mu)$

$= P(x > \frac{1}{3})$

$= \int_{\frac{1}{3}}^1 2(1-x)\,dx$

$= 2\int_{1/3}^1 (1-x)\,dx$

$= 2\left[x - \frac{x^2}{2}\right]_{1/3}^1$

$= 2\left[1 - \frac{1}{2} - \frac{1}{3} - \frac{\frac{1}{9}}{2}\right]$

$= 2\left[\frac{1}{2} - \frac{1}{3} - \frac{1}{9} \times \frac{1}{2}\right]$

$= 2\left[\frac{1}{2} - \frac{1}{3} + \frac{1}{18}\right]$

$= 2\left(\frac{9-6+1}{18}\right)$

$= \cancel{2} \times \frac{4}{18_9}$

$= \cancel{\frac{4}{18}}$

$= \frac{4}{9}$

Type I and Type II. error!

Ho	Accept	Reject
True	~~Accept~~ Correct dession	Wrong dession → Type I error
False	~~Acc~~ Wrong dession	Correct dession

Type II error

Hypothesis test:

H_0 :
H_1 :
} one - tail
two tail

Level of significance

Calculating test statistic:

z - Value
P - Value
Critical value

Estimation:

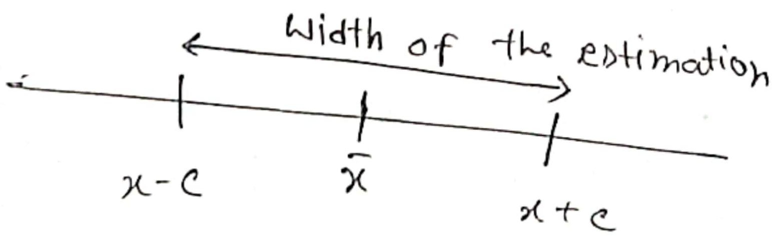
→ point estimation
→ Interval "

$\bar{x}$ - sample mean
μ - population
σ - S.D
n - sample site

Interval estimation:

Is the range in which the estimated sample will be using with a certain percentage (level of confidence)

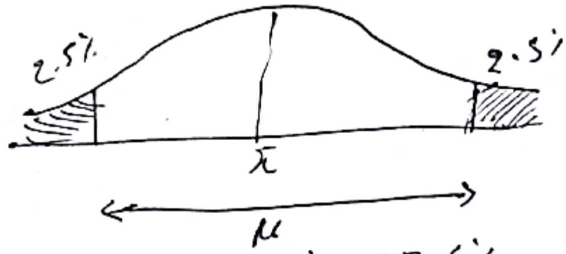
$$\bar{x} - z \times \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z \frac{\sigma}{\sqrt{n}}$$

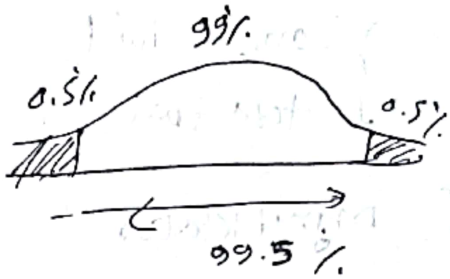


$$c = z \times \frac{\sigma}{\sqrt{n}}$$

$\bar{x} = 10$
$c = 1.5$
$8.5 < \mu < 11.5$

95% Confidance Interval!





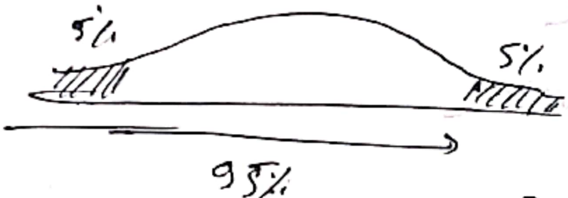
P=0.
z=?

95% 97.5%

$z = 99.5\% = ?$

$z_{97.5} = ?$ $p = 0.975$

$z = ?$



$x + z \times \frac{\sigma}{\sqrt{n}}$

$x - z \times \frac{\sigma}{\sqrt{n}}$

1) $\bar{x} = \frac{\Sigma x}{n}$

$\bar{t} = \frac{\Sigma t}{n} = \frac{4820}{60}$

$\sigma^2 = \frac{\Sigma x^2}{n} - (\bar{x})^2$

$\sigma^2 = \frac{\Sigma t^2}{n} - (\bar{t})^2$

$t - c < \mu < t + c$

$t - z \times \frac{\sigma}{\sqrt{n}} < \mu < t + z \times \frac{\sigma}{\sqrt{n}}$

(17) (a) $H_0 = 42.4$

$H_1 = \mu \neq 42.4$

(b) level of singnifient = 2.5%

$P(x > 45.6)$

$= (z > \frac{45.6 - 42.4}{\sqrt{38.2}}$

$= P(z > 0.6795)$

$= 1 - \phi(0.6795)$

$= 1 - \phi(0.6722)$

$= 1 - 0.7456$

$= 0.2544$

$P(x < 45.6)$

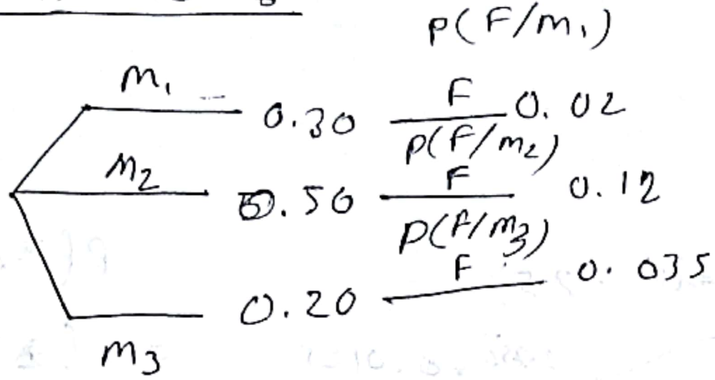
$= P(z < \frac{45.6 - 42.4}{\sqrt{38.2}})$

$= P(z < 0.6745)$

$= \phi(0.67955)$

$= 0.7456$

Bay's theorem:



~~P(F/m2)~~

$P(m_2/F)$

conditional probability:

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A) \times P(B/A)}{P(B)}$$

$$= \frac{P(m_2 \cap F)}{P(F)}$$

$$= \frac{P(m_2) \times P(F/m_2)}{P(m_1) \times P(F/m_1) + P(m_2) \times P(F/m_2) + P(m_3) \times P(F/m_3)}$$

Discrete random variable:

x	x_1	x_2	x_3	......	x_n
$P(X=x)$	P_1	P_2	P_3	......	P_n

(i) $P(x) \geq 0$

(ii) $\sum P(x) = 1$

$$E(x) = \sum x P(x) = x_1 P_1(x) + x_2 P_2(x) + \cdots + x_n P_n(x)$$

$$\text{Variance}(x) = \sum x^2 P(x) - \{E(x)\}^2$$

(3)

1	2	3	4
4k	6k	6k	4k

$$4k + 6k + 6k + 4k = 1$$

$$k = \frac{1}{20}$$

Binomal distribution:

* Two outcomes
* Fixed Number of trails
* Trails are independent
* success /Failure ~~...~~ has constant probability.

$X \sim B(n,p)$

$p(X=r) = {}^nC_r\, p^r \cdot (1-p)^{n-r}$

Expected value = Mean value = μ = np

Variance = npq

Poisson distribution:

$X \sim Po(\lambda)$

$p(x=r) = \dfrac{e^{-\lambda} \times \lambda^r}{r!}$, $r = 0, 1, 2, 3 \ldots$

mean = veriance = λ

Normal distribution:

$X \sim N(\mu, \sigma^2)$

$z = \dfrac{x-\mu}{\sigma}$

$P(z \le a) = \phi(a)$

$P(z \ge a) = 1 - \phi(a)$

$P(z < -a) = P(z \ge a) = 1 - \phi(a) = (\phi(-a))$

$P(z > -a) = P(z < a) = \phi(a)$

$P(a < z < b) = \phi(b) - \phi(a)$

Normal as an approximation to Binomial

$X \sim B(n, p) \approx Y \sim N(\mu, \sigma^2)$

$$\left.\begin{matrix} np > 5 \\ np > 5 \end{matrix}\right\} \qquad \begin{matrix} \mu = np \\ \sigma^2 = npq \end{matrix}$$

* $\frac{1}{2}$ continuous correction (± 0.5)