MAT 341 | N. Battista | Computational Math
HW 3 | | Fall 2019

HW Goals:

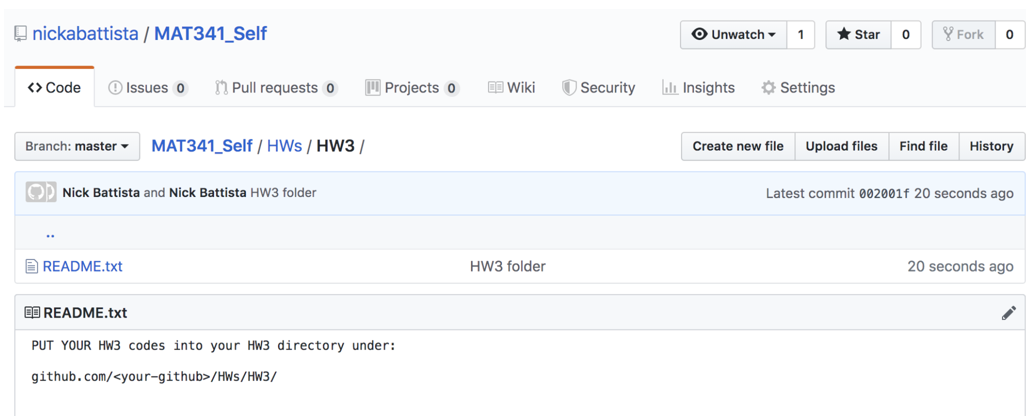
The purpose of this homework is to continue familiarizes ourselves with `for-loops`*,* `while-loops`*, and* `if-statements` *for mathematical purposes, here focused on some number theory and probability estimates (via Monte Carlo methods)*

HW DUE:

Friday, October 11 by 11:59pm

WHERE:

Put all of the codes for this homework into your personal GitHub: `MAT341_Self`→`HWs`→`HW3`. That is, you will have to `add`, `commit` and `push` these scripts to your GitHub for me to pull, see below for example:



1. Write a script called `calculate_LCM.m` that takes two inputs, two integers, x, y and returns the *least common multiple* between the two inputs, call it lcm_val. Note you do not need to explicitly define x, y as integers anywhere in the code, just only test the code on integer values, e.g.,

   function **lcm_val** = calculate_LCM(x,y)

   Within this script, write a code that will determine the *least common multiple* of the two inputs x, y and returns it.

2. Define a square grid $[0,1] \times [0,1]$ and a coin of radius, r. Answer the following:

   (a) What is the probability that when tossed into the square, the coin is contained *fully* within the square, e.g., it does not cross nor touch the boundaries of the square.

   Write a Monte Carlo scheme to estimate the probability after $N = 1e5$ **trials for a coin of radius** $r = 0.1$. Test your estimated probability for a few different N. Does $N = 1e5$ seem like a reasonable amount of trials? Why or why not? What makes this more difficult than the "coin flip" case we did in class?

   Call your script the following:

   function prob = estimate_Coin_In_Square_Probability(r,N)

   (to estimate the probability (output: *prob*) of a coin (of radius, r) landing in the square after N trials.)

   (b) We wish to find how does this probability changes for different radii. That is, write a script that will loop over different possible radii values, e.g.,

   $$rVec = [0.01{:}0.005{:}0.10 \;\; 0.1{:}0.01{:}0.5];$$

   Make a plot of the Estimated probabilities for a particular radius vs. *radius, r*. Make sure the plot is labeled appropriately. Assume $N = 1e5$. **Do this in a script called:**

   function vary_Radii_Plot()

   Approximately what values of r give estimated probabilities of 0.5, 0.1, and 0.01?

Write your responses as comments in the header of each code for $2a$ and $2b$ on how you approached the problem

3. Write a Monte Carlo script to estimate the probability for the following situation: Consider a circle of radius 2 centered at the origin. Choose three points randomly along the circle. What is the probability that the center of the circle $(0,0)$ is contained within the triangle formed between the three points?

   Write a script that takes as input N, the number of trials to perform, and name the script as follows:

   function prob = estimate_Triangle_Center_Circle_Probability(N)

   Hints:

   (a) You can randomly pick an x value between [-2,2], e.g., pick values of a, b so that $[0,1] \rightarrow$[-2,2] using `a·rand()-b` since `rand()`$\in [0,1]$

   (b) Get its corresponding y value using the equation of a circle (You *will* have to randomize the sign on y's value so you don't always choose the positive $\sqrt{\ }$)

   (c) Or you could also work in *polar coordinates* randomly choosing an angle $\theta \in [0, 2\pi]$ by choosing a, b such that `a·rand()-b` $\in [0, 2\pi]$

   (d) Draw out a couple scenarios where $(0,0)$ is contained within the triangle and a couple scenarios where it is not contained. Look for geometric patterns in how the points are distributed along the circle. Base your criteria for determining whether it is contained off of this.