$\boldsymbol{Physics\ Jam}$

$\boldsymbol{Phys250}$:

$\boldsymbol{Vector}$: *Magnitude and Direction*

$\boldsymbol{Definition}$:

$$\vec{V}: P_1 \rightarrow P_2$$

$$\vec{V}: < x_1, y_1 > \rightarrow < x_2, y_2 >$$

$$\vec{V} = < x_2 - x_1, y_2 - y_1 >$$

$$\vec{C} = \vec{A} + \vec{B}$$

$$\vec{C} - \vec{A} = \vec{B}$$

$$\overrightarrow{x_2} = c * \overrightarrow{x_1}$$

$\boldsymbol{Component}$:

$$\vec{r} = \vec{x} + \vec{y} = < rcos\theta, 0 > + < 0, rsin\theta > = rcos\theta\hat{\imath} + rsin\theta\hat{\jmath}$$

$$|\vec{x}| = rcos\theta$$

$$|\vec{y}| = rsin\theta$$

$$|\vec{r}| = \sqrt{|\vec{x}|^2 + |\vec{y}|^2} = \sqrt{(rcos\theta)^2 + (rsin\theta)^2} = r$$

$\boldsymbol{Unit\ Vector}$: *length = 1*

$$\hat{u} = \frac{\vec{x}}{|\vec{x}|}$$

$\boldsymbol{Dot\ Product}$: *scalar*

$$\vec{A} = < x_1, y_1 >$$

$$\vec{B} = < x_2, y_2 >$$

$$\vec{A} * \vec{B} = ABcos\theta = |\vec{A}||\vec{B}|cos\theta = x_1x_2 + y_1y_2$$

$$\vec{A} * \vec{B} = 0 \rightarrow \theta = 90° \ or\ 270°$$

$\boldsymbol{Cross\ Product}$: *vector*

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = -\vec{B} \times \vec{A}$$

$$|\vec{A} \times \vec{B}| = ABsin\theta$$

Motion:

$$x \rightarrow Displacement$$

$$v \rightarrow Velocity$$

$$a \rightarrow Acceleration$$

$$\Delta x = x_2 - x_1$$

$$\Delta t = t_2 - t_1$$

Average:

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

Instantaneous:

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Motion with Constant Acceleration:

$$a = a$$

$$v = \int_0^t a\, dt = v_0 + at \rightarrow t = \frac{v - v_0}{a}$$

$$x = \int_0^t v\, dt = \int_0^t v_0 + atdt = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$v^2 = {v_0}^2 + 2a(x - x_0)$$

Freely Falling Body: $a = g = 9.8(m/s), v_0 = 0, x_0 = h$

$$a = g = 9.8$$

$$v = gt$$

$$h = \frac{1}{2} g t^2$$

Projectile Motion:

$y_0 = 0, a_x = 0, a_y = -g$

$$x = v_0 cos\theta * t$$

$$y = v_0 sin\theta t - \frac{1}{2} g t^2$$

$$v_x = v_0 cos\theta$$

$$v_y = v_0 sin\theta - gt$$

Entire Time:

$$t = \frac{2v_0 sin\theta}{g}$$

Time to the Highest Point:

$$t \ = \frac{v_0 sin\theta}{g}$$

Parametric Equation:

$$y = v_0 sin\theta \left(\frac{x}{v_0 cos\theta}\right) - \frac{1}{2} g \left(\frac{x}{v_0 cos\theta}\right)^2$$

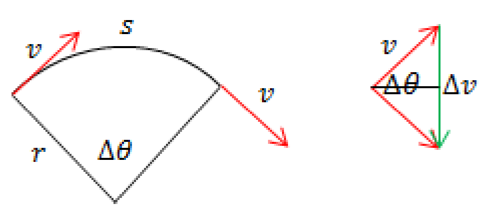
$$= (tan\theta)x - \left(\frac{g}{2v_0^2 cos^2\theta}\right) x^2$$

Uniform Circular Motion:

$$v = \frac{2\pi R}{T}$$

$$a_{rad} = \frac{v^2}{R}$$

Proof:



$$\Delta S \ = \ 2\pi R$$

$$v = \frac{\Delta S}{\Delta t} = \frac{2\pi R}{T}$$

$$\Delta S \ = \ v\Delta t \ = \ r\Delta\theta$$

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{v}{r} \rightarrow v = \ \omega r$$

$$sin\frac{\Delta\theta}{2} = \frac{\Delta v/2}{v}$$

$$\Delta v \ = \ 2v sin\frac{\Delta\theta}{2}$$

$$\frac{\Delta v}{\Delta\theta} = v \frac{sin\frac{\Delta\theta}{2}}{\frac{\Delta\theta}{2}}$$

$$\frac{dv}{d\theta} = \lim_{\Delta\theta \to 0} v \frac{\sin\frac{\Delta\theta}{2}}{\frac{\Delta\theta}{2}} = v$$

$$\frac{dv}{dt} = \frac{dv}{d\theta}\frac{d\theta}{dt} = v * \frac{v}{R} = \frac{v^2}{R}$$

$$\therefore a_c = a_{rad} = \frac{v^2}{R}$$

$$\because v = \omega R$$

$$\therefore a_c = \frac{v^2}{R} = \frac{(\omega R)^2}{R} = \omega^2 R$$

$$\because v = \frac{2\pi R}{T}$$

$$\therefore a_c = \frac{v^2}{R} = \frac{\left(\frac{2\pi R}{T}\right)^2}{R} = \frac{4\pi^2 R}{T^2}$$

$$\because v = \omega R = \frac{2\pi R}{T}$$

$$\therefore \omega = \frac{2\pi}{T}$$

$$\boxed{\begin{aligned} &\omega = \frac{2\pi}{T} \\ &v = \omega R = \frac{2\pi R}{T} \\ &a_c = \frac{v^2}{R} = \omega^2 R = \frac{4\pi^2 R}{T^2} \end{aligned}}$$

Force:

$$\overrightarrow{F_{net}} = \sum \overrightarrow{F_i}$$

$$\overrightarrow{F_x} = \sum \overrightarrow{F_{xi}}$$

$$\overrightarrow{F_y} = \sum \overrightarrow{F_{yi}}$$

Newton's First Law:

$$\overrightarrow{F_{net}} = \sum \overrightarrow{F_i} = 0$$

Newton's Second Law:

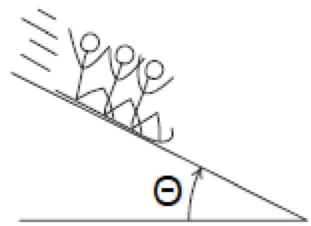
$$\overrightarrow{F_{net}} = \sum \overrightarrow{F_i} = m\vec{a}$$

$$\vec{w} = m\vec{g}$$

Newton's Third Law:

$$\overrightarrow{F_{A\ on\ B}} = -\overrightarrow{F_{B\ on\ A}}$$

Free Body Diagram: ***No Friction Force***



Force Perpendicular to the Hill:

$$\sum F_y = N - wcos\theta = 0$$

Force Parallel to the Hill:

$$\sum F_x = wsin\theta = ma_x$$

Sol:

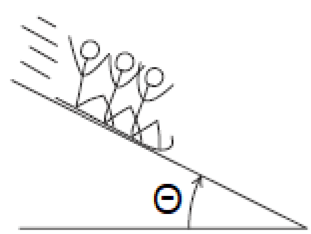
$$\because w = \ mg$$

$$wsin\theta \ = \ mgsin\theta = ma_x$$

$$\therefore a_x = gsin\theta$$

$$N = wcos\theta = mgcos\theta$$

Free Body Diagram: *With Friction Force, Equilibrium, Not Moving*



Force Perpendicular to the Hill:

$$\sum F_y = N - wcos\theta = 0$$

Force Parallel to the Hill:

$$\sum F_x = wsin\theta - f_s \ = 0$$

Sol:

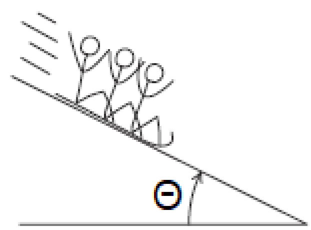
$$\because N = wcos\theta = mgcos\theta$$

$$f_s = N\mu_s = wsin\theta = mgsin\theta$$

$$\therefore \mu_s = \frac{f_s}{N} = \frac{mgsin\theta}{mgcos\theta} \ = \ tan\theta$$

$$\theta = tan^{-1}\mu_s$$

Free Body Diagram: *With Friction Force, Equilibrium, Moving*



Force Perpendicular to the Hill:

$$\sum F_y = N - wcos\theta = 0$$

Force Parallel to the Hill:

$$\sum F_x = wsin\theta - f_k \ = 0$$

Sol:

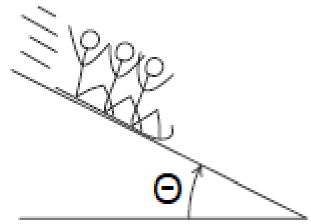
$$\because N = wcos\theta = mgcos\theta$$

$$f_k = N\mu_k = wsin\theta = mgsin\theta$$

$$\therefore \mu_k = \frac{f_k}{N} = \frac{mgsin\theta}{mgcos\theta} \ = \ tan\theta$$

$$\theta = tan^{-1}\mu_k$$

Free Body Diagram: *With Friction Force, Not Equilibrium, Moving*



Force Perpendicular to the Hill:

$$\sum F_y = N - w cos\theta = 0$$

Force Parallel to the Hill:

$$\sum F_x = w sin\theta - f_k = m a_x$$

Sol:

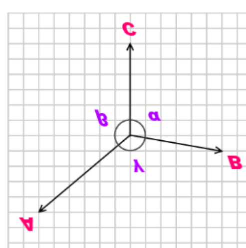
$$\because N = w cos\theta = mg cos\theta$$

$$f_k = N\mu_k = w sin\theta = mg sin\theta = mg cos\theta \mu_k$$

$$\therefore \mu_k = tan\theta$$

$$a_x = \frac{w sin\theta - f_k}{m} = \frac{mg sin\theta - mg cos\theta * \mu_k}{m} = g(sin\theta - \mu_k cos\theta)$$

Three Forces in Equilibrium:



Lami's Theorem:

$$\because Sine\ Rule$$

$$\frac{F_A}{sin(\pi - \alpha)} = \frac{F_B}{sin(\pi - \beta)} = \frac{F_C}{sin(\pi - \gamma)}$$

$$\therefore \frac{F_A}{sin\alpha} = \frac{F_B}{sin\beta} = \frac{F_C}{sin\gamma}$$

Work and Energy:

Work:

$$W = \vec{F} \cdot \vec{s} = Fs\cos\theta$$

$$W = \int_{p_1}^{p_2} \vec{F} \cdot d\vec{l} = \int_{p_1}^{p_2} F\cos\theta dl = \int_{p_1}^{p_2} F_l dl$$

Kinetic Energy:

$$K = \frac{1}{2}mv^2$$

Work − Energy Theorem:

$$W_{total} = K_f - K_i = \Delta K$$

Free Falling Object:

$$mgh = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh}$$

Spring:

Hook′s Law:

$$F = kx$$

$$k \rightarrow Force's\ Constant$$

$$x \rightarrow Spring's\ Elongation$$

$$W = \int_{x_i}^{x_f} F_x dx = \int_{x_i}^{x_f} kxdx = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$$

$$W = \int_0^x F_x dx = \int_0^x kxdx = \frac{1}{2}kx^2$$

Power:

Average Power:

$$P_{avg} = \frac{\Delta W}{\Delta t}$$

Instantaneous Power:

$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt} = \frac{d(\vec{F} \cdot \vec{s})}{dt} \overset{F\ is\ constant}{=} \vec{F}\frac{d\vec{s}}{dt} = \vec{F} \cdot \vec{v}$$

Potential Energy:

$$U = -\int_{p_i}^{p_f} \vec{F} \cdot d\vec{s}$$

$$W = -\Delta U$$

$$\therefore \boldsymbol{F} = -\nabla \boldsymbol{U}$$

Gravitational Potential Energy:

$$U_{grav} = -\int_0^h -mgdy = mgh$$

$$W_{grav} = -\Delta U_{grav} = -\left(U_{grav,f} - U_{grav,i}\right) = -(mgy_2 - mgy_1) = mgy_1 - mgy_2$$

Conservation of Mechanical Energy:

$$W_{tot} = \Delta K = K_f - K_i$$

$$= -\Delta U_{grav} = U_{grav,i} - U_{grav,f}$$

$$\therefore K_i + U_{grav,i} = K_f + U_{grav,f}$$

Free Falling Object:

$$0 + mgh = \frac{1}{2}mv^2 + 0$$

$$\therefore mgh = \frac{1}{2}mv^2$$

Elastic Potential Energy:

$$U_{elas} = -\int_0^x -kxdx = \frac{1}{2}kx^2$$

$$W_{elas} = U_{elas,i} - U_{elas,f} = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

$$K_i + U_{elas,i} = K_f + U_{elas,f}$$

$$\frac{1}{2}mv_{max}^2 = \frac{1}{2}kX^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$X \rightarrow Maximum\ Elongation$$

Momentum:

Newton's Second Law:

$$\overrightarrow{F_{net}} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d}{dt}(m\vec{v}) = \frac{d\vec{p}}{dt}$$

$$\vec{p} = m\vec{v}$$

Impluse:

$$d\vec{p} = \overrightarrow{F_{net}}dt$$

$$\vec{J} = \int_{p_i}^{p_f} d\vec{p} = \overrightarrow{p_f} - \overrightarrow{p_i} = \Delta\vec{p}$$

$$= \int_{t_i}^{t_f} \overrightarrow{F_{net}}\, dt \overset{F\ constant}{=} \overrightarrow{F_{net}}\Delta t$$

$$= \int_{t_i}^{t_f} \overrightarrow{F_{net}}\, dt = \overrightarrow{F_{avg}}\Delta t$$

Newton's Third Law: *Conservation of Momentum*

$$\overrightarrow{F_{A\ on\ B}} = -\overrightarrow{F_{B\ on\ A}}$$

$$\overrightarrow{F_{A\ on\ B}} + \overrightarrow{F_{B\ on\ A}} = 0$$

$$\frac{d\overrightarrow{p_A}}{dt} + \frac{d\overrightarrow{p_B}}{dt} = 0$$

$$\vec{p} = \overrightarrow{p_A} + \overrightarrow{p_B} = 0$$

$$\vec{p} = \sum_i \overrightarrow{p_i} = 0$$

Collision:

Elastic Collision: $\boldsymbol{\Delta K = 0}$

$$\because Conservation\ of\ Kinetic\ Energy$$

$$\therefore \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1{v_1'}^2 + \frac{1}{2}m_2{v_2'}^2\ \ldots\ Eq1$$

$$m_1\left({v_1}^2 - {v_1'}^2\right) = m_2\left({v_2'}^2 - {v_2}^2\right)$$

$$m_1(v_1 + v_1')(v_1 - v_1') = m_2(v_2' + v_2)(v_2' - v_2)\ \ldots\ Eq2$$

$$\because Conservation\ of\ Momentum$$

$$\therefore m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2' \ \ldots\ Eq3$$

$$m_1(v_1 - v_1') = m_2(v_2' - v_2) \ \ldots\ Eq4$$

$$\because Eq2\ /\ Eq4$$

$$\therefore v_1 + v_1' = v_2 + v_2'$$

$$v_2' = v_1 - v_2 + v_1' \ \ldots\ Eq5$$

$$\because Substitute\ Eq5\ into\ Eq3$$

$$\therefore m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2(v_1 - v_2 + v_1') = v_1'(m_1 + m_2) + m_2(v_1 - v_2)$$

$$v_1' = \frac{m_1 v_1 + m_2 v_2 - m_2(v_1 - v_2)}{m_1 + m_2}$$

$$= \frac{m_1 v_1 + 2m_2 v_2 - m_2 v_1}{m_1 + m_2}$$

$$= \frac{2m_1 v_1 + 2m_2 v_2 - m_1 v_1 - m_2 v_1}{m_1 + m_2}$$

$$= 2\left(\frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}\right) - \frac{v_1(m_1 + m_2)}{m_1 + m_2}$$

$$= 2v_c - v_1$$

$$v_2' = v_1 - v_2 + v_1'$$

$$= v_1 - v_2 + 2v_c - v_1$$

$$= 2v_c - v_2$$

$$\boxed{\begin{array}{l} v_c = \dfrac{m_1 v_1 + m_2 v_2}{m_1 + m_2} \ \rightarrow Velocity\ of\ Central\ Mass \\ v_1' = 2v_c - v_1 \\ v_2' = 2v_c - v2 \end{array}}$$

$$\boldsymbol{Case}: \boldsymbol{m_1 = m_2\ =\ m}$$

$$\boldsymbol{v_c} = \frac{mv_1 + mv_2}{m\ +\ m} = \frac{1}{2}(v_1 + v_2)$$

$$v_1' = 2v_c - v_1 = v_1 + v_2 - v_1 = v_2$$

$$v_2' = 2v_c - v_2 = v_1 + v_2 - v_2 = v_1$$

$$\boxed{\begin{aligned} &\boldsymbol{v_c} = \frac{1}{2}(v_1 + v_2) \\ &v_1' = v_2 \\ &v_2' = v_1 \end{aligned}}$$

$\boldsymbol{Inelastic\ Collision}$: *Only Conservation of Momentum*

$\boldsymbol{Completely\ Inelastic\ Collision}$: $\boldsymbol{v_1' = v_2' = v'}$

$$\because \boldsymbol{Conservation\ of\ Momentum}$$

$$m_1v_1 + m_2v_2 = (m_1 + m_2)v'$$

$$\therefore v' = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} = v_c$$

$\boldsymbol{Rotation}$:

$$\theta = \frac{s}{r}$$

$$s \to Arc\ Length$$

$$1\ rad \text{ is defined when } s = r$$

$$\because s = 2\pi r * \left(\frac{\theta}{360°}\right)$$

$$r = 2\pi r * \left(\frac{1\ rad}{360°}\right)$$

$$360° = 2\pi\ rad$$

$$\therefore \pi(rad) = 180°$$

$\boldsymbol{Average\ Angular\ Velocity}$:

$$\omega_{avg} = \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta\theta}{\Delta t}$$

$\boldsymbol{Instantaneous\ Angular\ Velocity}$:

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

$\boldsymbol{Average\ Angular\ Acceleration}$:

$$\alpha_{avg} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}$$

Instantaneous Angular Acceleration:

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} = \frac{d}{dt}\frac{d\theta}{dt} = \frac{d^2\theta}{dt^2}$$

Constant Angular Acceleration:

$$\omega = \omega_0 + \alpha t$$
$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$
$$\omega^2 = {\omega_0}^2 + 2\alpha(\theta - \theta_0)$$

Kinetic Energy:

$$K = \sum_i \frac{1}{2} m_i {v_i}^2 = \sum_i \frac{1}{2} m_i (\omega r_i)^2 = \frac{1}{2}\left(\sum_i m_i r_i^2\right)\omega^2 = \frac{1}{2} I\omega^2$$

$$I = \sum_i m_i r_i^2 = \int_0^M r^2 dm$$

Lists of Moment of Inertia:

Rod, axis through center: $I = \frac{1}{12} ML^2$
Rod, axis through end: $I = \frac{1}{3} ML^2$
Ring: $I = MR^2$
Solid Disk: $I = \frac{1}{2} MR^2$
Hollow Disk: $I = \frac{1}{2} M\left(R_{out}^2 + R_{in}^2\right)$
Solid Sphere: $I = \frac{2}{5} MR^2$
Hollow Sphere: $I = \frac{2}{3} MR^2$

Parallel Axis Theorem:

$$I_p = I_{cm} + Md^2$$

Stretch Rule:

$$I_{Solid\ Disk} = I_{Solid\ Cylinder}$$

$$I_{Hollow\ Disk} = I_{Hollow\ Cylinder}$$

Perpendicular Axis Theorem:

$$I_z = I_x + I_y$$

Proof:

$$dI_x = y^2 dm$$

$$dI_y = x^2 dm$$

$$dI_x + dI_y = (x^2 + y^2)dm$$

$$= r^2 dm = dI_z$$

$$\int dI_x + \int dI_y = \int dI_z$$

$$\therefore I_z = I_x + I_y = \int (x^2 + y^2)dm = \iint_S (x^2 + y^2)\sigma dA$$

Rotational Motion:

Torque:

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = rFsin\theta = F_{tan} r$$

$$\tau_{net} = \sum \tau_i = \sum F_{tan,i}\, r_i = \sum m_i\, a_i r_i = \left(\sum m_i\, r_i^2\right)\alpha \;=\; I\alpha$$

Energy:

$$K = K_{trans} + K_{rotat}$$

$$= \frac{1}{2} M v_{cm}^2 + \frac{1}{2} I_{cm}\omega^2$$

Proof:

$$K_i = \frac{1}{2} m_i \left(\overrightarrow{v_{cm}} + \overrightarrow{v_i'}\right)^2$$

$$= \frac{1}{2} m_i \left(v_{cm}^2 + 2\overrightarrow{v_{cm}} \cdot \overrightarrow{v_i'} + v_i'^2\right)$$

$$K = \sum_i K_i = \sum_i \frac{1}{2} m_i v_{cm}^2 + \sum_i m_i \overrightarrow{v_{cm}} \cdot \overrightarrow{v_i'} + \sum_i \frac{1}{2} m_i v_i'^2$$

$$= \frac{1}{2} M v_{cm}^2 + 0 + \frac{1}{2} I_{cm}\omega^2$$

$$= \frac{1}{2} M v_{cm}^2 + \frac{1}{2} I_{cm}\omega^2$$

$\boldsymbol{Work}$:

$$dW = Fds = FRd\theta = \tau d\theta$$

$$W = \int_{\theta_i}^{\theta_f} \tau \, d\theta$$

$$\because \tau d\theta = I\alpha d\theta = I\frac{d\omega}{dt}d\theta = I\frac{d\theta}{dt}d\omega = I\omega d\omega$$

$$\therefore W = \int_{\omega_i}^{\omega_f} I\omega d\omega = \frac{1}{2}I\left(\omega_f^2 - \omega_i^2\right)$$

$\boldsymbol{Power}$:

$$\because \frac{dW}{dt} = \tau\frac{d\theta}{dt}$$

$$\therefore p = \tau\omega$$

$\boldsymbol{Angular\ Momentum}$:

$$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v}$$

$$L = mvrsin\theta = mvl$$

$$\frac{d\vec{L}}{dt} = \left(\frac{d\vec{r}}{dt} \times m\vec{v}\right) + \left(\vec{r} \times m\frac{d\vec{v}}{dt}\right)$$

$$= \vec{v} \times m\vec{v} + \vec{r} \times m\vec{a}$$

$$= 0 + \vec{r} \times \vec{F}$$

$$= \vec{\tau}$$

$$L_i = m_i v_i r_i = m_i(\omega r_i)r_i = m_i r_i^2 \omega$$

$$L = \sum_i L_i = \left(\sum_i m_i r_i^2\right)\omega = I\omega$$

$$\vec{L} = I\vec{\omega}$$

$\boldsymbol{Conservation\ of\ Angular\ Momentum}$: *No External Torque*

$$\because \frac{d\overrightarrow{L_A}}{dt} = -\frac{d\overrightarrow{L_B}}{dt}$$

$$\frac{d\overrightarrow{L_A}}{dt} + \frac{d\overrightarrow{L_B}}{dt} = 0$$

$$\therefore \frac{d}{dt}\left(d\overrightarrow{L_A} + d\overrightarrow{L_B}\right) = \frac{d\vec{L}}{dt} = 0$$

Equilibrium:

$$\overrightarrow{F_{net}} = 0$$

$$\overrightarrow{\tau_{net}} = 0$$

Simple Harmonic Motion:

$$F_x = -kx = ma_x$$

$$a_x = \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

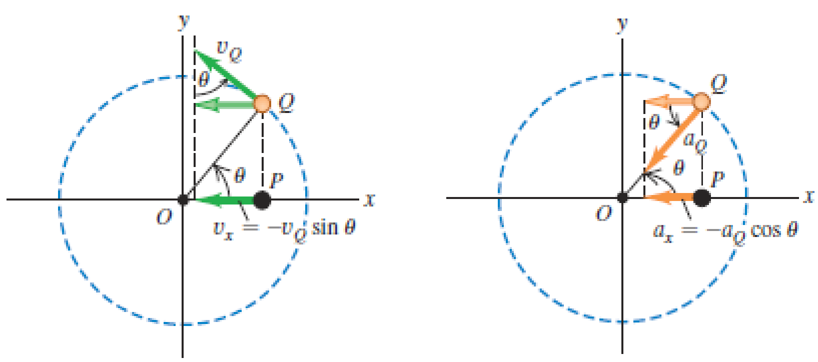
Definition of ODE:

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

$$\omega = \sqrt{\frac{k}{m}} = \frac{2\pi}{T}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Phasor Diagram:



$$x = R cos\theta$$

$$a_c = \omega^2 R$$

$$a_x = -a_c cos\theta = -\omega^2 R cos\theta = -\omega^2 x = -\frac{k}{m}x$$

$$\omega = \sqrt{\frac{k}{m}} = \frac{2\pi}{T}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Displacement:

$$x = Rcos\theta = Rcos(\omega t + \phi)$$

$$R \rightarrow Amplitude$$

$$\omega \rightarrow Angular\ Frequency$$

$$t \rightarrow time$$

$$\phi \rightarrow Phase\ Angle$$

Velocity:

$$v_x = \frac{dx}{dt} = -\omega Rsin(\omega t + \phi)$$

Acceleration:

$$a_x = \frac{dv_x}{dt} = -\omega^2 Rcos(\omega t + \phi)$$

Angular SHM:

$$\tau = -k\theta = I\alpha = I\frac{d^2\theta}{dt^2}$$

$$\frac{d^2\theta}{dt^2} + \frac{k}{I}\theta = 0$$

$$\omega = \sqrt{\frac{k}{I}} = \frac{2\pi}{T}$$

$$T = 2\pi\sqrt{\frac{I}{k}}$$

Simple Pendulum:

$$F = -mgsin\theta \overset{\theta \rightarrow 0}{=} -mg\theta = -mg\frac{x}{L} = -\frac{mg}{L}x = -kx$$

$$w = \sqrt{\frac{k}{m}} = \sqrt{\frac{mg/L}{m}} = \sqrt{\frac{g}{L}} = \frac{2\pi}{T}$$

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$\boldsymbol{Phys260}$:

$\boldsymbol{Electric\ Force}$:

$\boldsymbol{Coulomb's\ Law}$:

$$F = k\frac{q_1q_2}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}$$

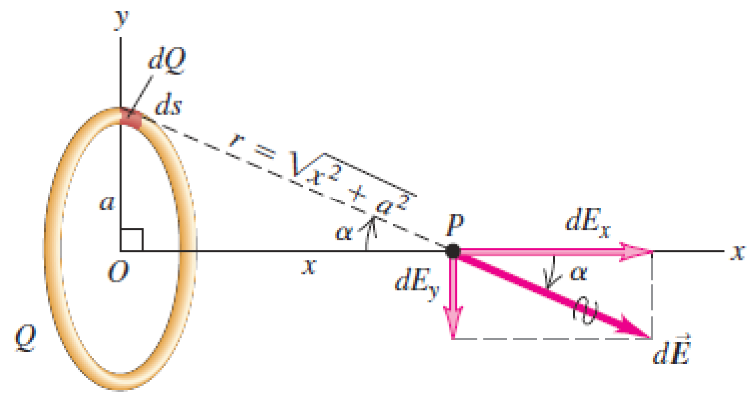
$\boldsymbol{Electric\ Field}$:

$$\vec{E} = \frac{\overrightarrow{F_0}}{q_0} = \frac{kq}{r^2}\hat{r} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

$\boldsymbol{Superposition\ of\ Electric\ Fields}$:

$$\vec{E} = \frac{\overrightarrow{F_0}}{q_0} = \sum_i \overrightarrow{E_\iota}$$

$\boldsymbol{Electric\ Field\ of\ Charged\ Ring}$:



$\boldsymbol{Sol}$:

$$dE = k\frac{dQ}{x^2 + a^2}$$

$$dE_x = dEcos\alpha = k\frac{dQ}{x^2 + a^2}\frac{x}{\sqrt{x^2 + a^2}}$$

$$Q = 2\pi a\lambda$$

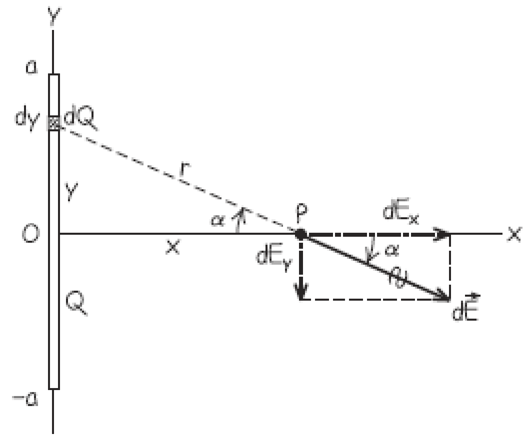
$$dQ = \lambda ds$$

$$E_x = \int dE_x = k\frac{\lambda x}{(x^2 + a^2)^{3/2}}\int_0^{2\pi a} ds = \frac{k\lambda x 2\pi a}{(x^2 + a^2)^{3/2}} = \frac{kQx}{(x^2 + a^2)^{3/2}}$$

$$E_y = 0$$

$$\vec{E} = E_x\hat{\imath} = \frac{kQx}{(x^2 + a^2)^{3/2}}\hat{\imath}$$

Electric Field of Charged Rod:



Sol:

$$dQ = \lambda dy = \frac{Q}{2a}dy$$

$$dE = k\frac{dQ}{r^2} = k\frac{Q}{2a}\frac{dy}{x^2 + y^2}$$

$$dE_x = k\frac{Q}{2a}\frac{xdy}{(x^2 + y^2)^{3/2}}$$

$$dE_y = k\frac{Q}{2a}\frac{ydy}{(x^2 + y^2)^{3/2}}$$

$$E_x = k\frac{Q}{2a} * 2\int_0^a \frac{xdy}{(x^2 + y^2)^{3/2}} = k\frac{Q}{a}\frac{a}{x\sqrt{x^2 + a^2}} = \frac{kQ}{x\sqrt{x^2 + a^2}}$$

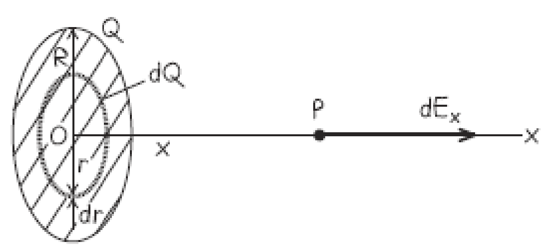
$$E_y = 0$$

$$\vec{E} = E_x\hat{\imath} = \frac{kQ}{x\sqrt{x^2 + a^2}}\hat{\imath}$$

Case: $\boldsymbol{a \gg x}$

$$\vec{E} = \frac{kQ}{xa}\hat{\imath} = \frac{2a\lambda}{4\pi\epsilon_0 xa}\hat{\imath} = \frac{\lambda}{2\pi\epsilon_0 x}\hat{\imath}$$

Electric Field of Charged Disk:



$\boldsymbol{Sol}$: $\boldsymbol{Using\ Ring}$

$$E_{ring} = \frac{kQ_{ring}x}{(x^2 + r^2)^{3/2}}$$

$$\because E_{ring} = dE_{disk}$$

$$Q_{ring} = dQ_{Disk} = \rho dA = \rho 2\pi r\, dr$$

$$\therefore dE_{disk} = \frac{k\rho 2\pi r x}{(x^2 + r^2)^{3/2}}\, dr$$

$$E = \int_0^R \frac{k\rho 2\pi r x}{(x^2 + r^2)^{3/2}} dr$$

$$= k\pi\rho x \int_0^R \frac{2r}{(x^2 + r^2)^{3/2}} dr$$

$$= -2k\pi\sigma x \left(\frac{1}{\sqrt{x^2 + r^2}} - \frac{1}{x}\right)$$

$$= -2\frac{\pi\sigma x}{4\pi\epsilon_0}\left(\frac{1}{\sqrt{x^2 + r^2}} - \frac{1}{x}\right)$$

$$= \frac{\sigma}{2\epsilon_0}\left(1 - \frac{1}{\sqrt{\left(\frac{R^2}{x^2}\right) + 1}}\right)$$

$\boldsymbol{Sol}$: $\boldsymbol{Using\ Particle}$

$$\because dQ = \rho dA = \rho r dr d\theta$$

$$\therefore dE = k\frac{dQ}{x^2 + r^2} = \frac{k\rho r}{x^2 + r^2} dr d\theta$$

$$E = \int_0^{2\pi}\int_0^R \frac{k\rho r}{x^2 + r^2}\frac{x}{\sqrt{x^2 + r^2}} dr d\theta$$

$$= k\pi\rho x \int_0^R \frac{2r}{(x^2 + r^2)^{3/2}}\, dr$$

$$= -2k\pi\sigma x \left(\frac{1}{\sqrt{x^2 + r^2}} - \frac{1}{x}\right)$$

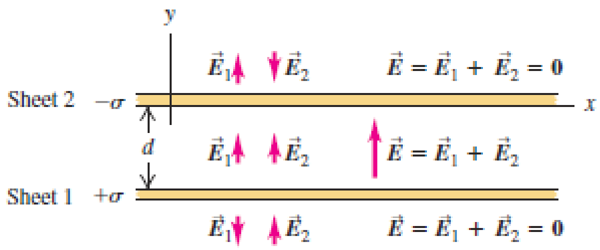
$$= -2\frac{\pi\sigma x}{4\pi\epsilon_0}\left(\frac{1}{\sqrt{x^2 + r^2}} - \frac{1}{x}\right)$$

$$= \frac{\sigma}{2\epsilon_0}\left(1 - \frac{1}{\sqrt{\left(\frac{R^2}{x^2}\right) + 1}}\right)$$

$\boldsymbol{Case: R \gg x}$

$$E = \frac{\sigma}{2\epsilon_0}$$

$\boldsymbol{Electric\ Field\ of\ Two\ Oppositely\ Charged\ Infinite\ Sheets}$:



$\boldsymbol{Sol}$:

$$\vec{E} = \overrightarrow{E_1} + \overrightarrow{E_2} = \begin{cases} 0 \rightarrow \ above\ Sheet2 \\ \dfrac{\rho}{\epsilon_0}\hat{j} \rightarrow between\ two\ sheets \\ 0 \rightarrow below\ Sheet1 \end{cases}$$

$\boldsymbol{Flux}$:

$$\frac{dV}{dt} = \vec{v} \cdot \vec{A} = vAcos\phi$$

$$V \rightarrow Volume$$

$$v \rightarrow Velocity$$

$\boldsymbol{Electric\ Flux}$:

$$\Phi_E = \vec{E} \cdot \vec{A}$$

$$\Phi_E = \int_s \vec{E} \cdot d\vec{A}$$

$\boldsymbol{Gauss's\ Law}$:

$$\Phi_E = \oint\!\!\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$\boldsymbol{Divergence\ Theorem}$:

$$\oint\!\!\oint \vec{E} \cdot d\vec{A} = \iiint_V \boldsymbol{\nabla} \cdot \boldsymbol{E}\, dV = \frac{Q_{encl}}{\epsilon_0} = \iiint_V \frac{\rho}{\epsilon_0} dV$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = \frac{\rho}{\epsilon_0}$$

Electric Field of Uniformly Charged Sphere:

$$\Phi_E = EA = E(4\pi r^2) = \frac{Q}{\epsilon_0}$$

Case: $\boldsymbol{r > R}$

$$\Phi_E = E(4\pi r^2) = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

Case: $\boldsymbol{r < R}$

$$\Phi_E = E(4\pi r^2) = \frac{Q}{\epsilon_0}\frac{r^3}{R^3}$$

$$E = \frac{Qr}{4\pi\epsilon_0 R^3} = \frac{Q}{4\pi\epsilon_0 R^2}\frac{r}{R}$$

Electric Field of Point Charge:

$$\because \text{Point Charge} \equiv \text{Charged Sphere where } r > R$$

$$\therefore E = \frac{Q}{4\pi\epsilon_0 r^2}$$

Electric Field of Conducting Sphere:

$$\because \text{All charges reside on the conducting surface}$$

$$\therefore E = 0 \text{ when } r < R$$

$$E = \frac{Q}{4\pi\epsilon_0 r^2} \text{ when } r \geq R$$

Electric Field of Uniformly Charged Cylinder:

$$\Phi_E = EA = E(2\pi rL) = \frac{Q}{\epsilon_0}$$

Case: $\boldsymbol{r \geq R}$

$$E = \frac{Q}{2\pi rL\epsilon_0} = \frac{\lambda L}{2\pi rL\epsilon_0} = \frac{\lambda}{2\pi\epsilon_0 r}$$

Case: $\boldsymbol{r < R}$

$$E = \frac{Q}{2\pi rL\epsilon_0}\frac{r^2}{R^2} = \frac{\lambda L r^2}{2\pi rL\epsilon_0 R^2} = \frac{\lambda r}{2\pi\epsilon_0 R^2}$$

$\boldsymbol{Electric\ Field\ of\ Conducting\ Cylinder}$:

$$\because All\ charges\ reside\ on\ the\ conducting\ surface$$

$$\therefore E\ =\ 0\ when\ r\ <\ R$$

$$E\ =\ \frac{\lambda}{2\pi\epsilon_0 r}\ when\ r\ \geq\ R$$

$\boldsymbol{Electric\ Field\ of\ Line\ Charge}$:

$$\because Line\ Charge\ \equiv Charged\ Cylinder\ where\ r\ >\ R$$

$$\therefore E\ =\ \frac{\lambda}{2\pi\epsilon_0 r}$$

$\boldsymbol{Electric\ Field\ of\ Conductor\ Surface}$:

$$\Phi_E = EA = \frac{\sigma A}{\epsilon_0}$$

$$E = \frac{\sigma}{\epsilon_0}$$

$$E_{gap} = 0$$

$\boldsymbol{Electric\ Field\ of\ Charged\ Sheet}$:

$$\Phi_E = E(2A) = \frac{\sigma A}{\epsilon_0}$$

$$E = \frac{\sigma}{2\epsilon_0}$$

$\boldsymbol{Electric\ Field\ of\ Parallel\ Plates}$:

$$E_{gap} = 0$$

$$E_{out} = 0$$

$$E_{betw} = 2E_{sheet} = 2\frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$$

$\boldsymbol{Electric\ Potential\ Energy}$:

$$W_{a\to b} = \int_a^b \vec{F}\cdot d\vec{l} = U_a - U_b = -(U_b - U_a) = -\Delta U$$

$$= \int_{r_a}^{r_b} F_r\,dr = \int_{r_a}^{r_b} \frac{kqq_0}{r^2}\,dr = kqq_0\left(\frac{1}{r_a} - \frac{1}{r_b}\right) = U_a - U_b$$

$$U = \frac{kqq_0}{r}$$

$\boldsymbol{Conservation\ of\ Mechanical\ Energy}$:

$$K_a + U_a = K_b + U_b$$

$\boldsymbol{Potential\ Energy\ for\ Multiple\ Charges}$:

$$U = k\sum_{i<j}\frac{q_i q_j}{r_{ij}}$$

$\boldsymbol{Electric\ Potential}$: $\boldsymbol{aka\ Voltage}$

$$V = \frac{U}{q_0} = \frac{kq}{r}$$

$$U = q_0 V$$

$\boldsymbol{Potential\ Difference}$:

$$V_{ab} = \frac{U_a - U_b}{q_0} = V_a - V_b$$

$\boldsymbol{Electric\ Potential\ for\ Multiple\ Charges}$:

$$V = k\sum_i \frac{q_i}{r_i} = k\int \frac{dq}{r}$$

$\boldsymbol{Electric\ Potential\ from\ Electric\ Field}$:

$$W_{a\to b} = \int_a^b \vec{F}\cdot d\vec{l} = \int_a^b q_0\,\vec{E}\cdot d\vec{l}$$

$$\frac{W_{a\to b}}{q_0} = \frac{U_a - U_b}{q_0} = V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l}$$

$$\boldsymbol{E} = -\boldsymbol{\nabla V}$$

$\boldsymbol{Electric\ Potential\ outside\ the\ Infinite\ Line\ Charge\ at\ distance\ r}$:

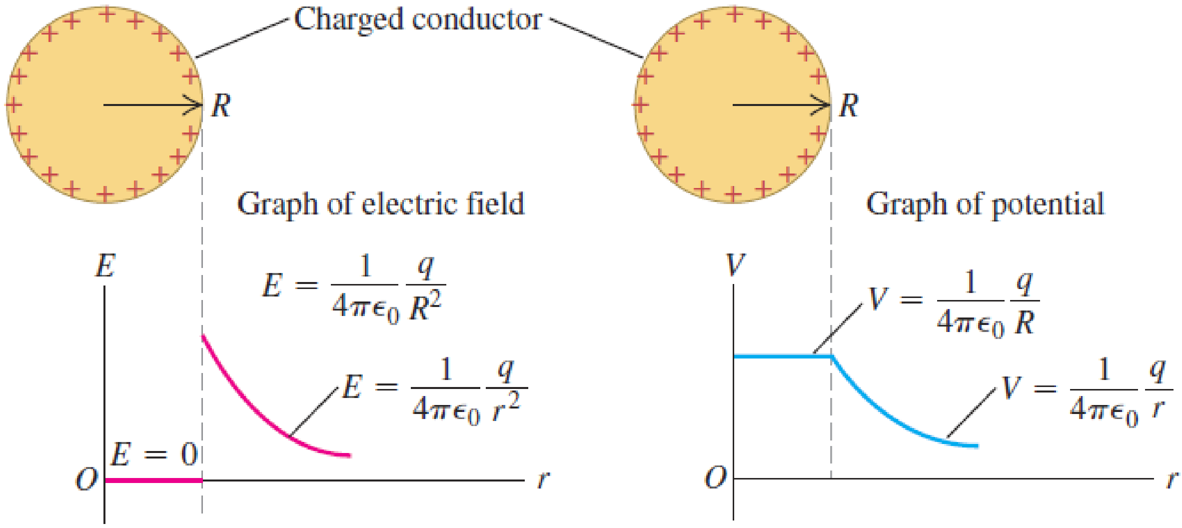
$\boldsymbol{Electric\ Field\ of\ Infinite\ Line\ Charge}$: $E = \dfrac{\lambda}{2\pi\epsilon_0 r}$

$$V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l} = \int_{r_a}^{r_b} Edr = \frac{\lambda}{2\pi\epsilon_0} ln\frac{r_b}{r_a}$$

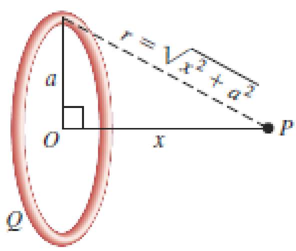
$\boldsymbol{Electric\ Potential\ outside\ the\ Sphere\ at\ distance\ r}$:

$\boldsymbol{Electric\ Field\ of\ Sphere}$: $E = \dfrac{Q}{4\pi\epsilon_0 r^2}$

$$V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l} = \int_{r_a}^{r_b} Edr = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

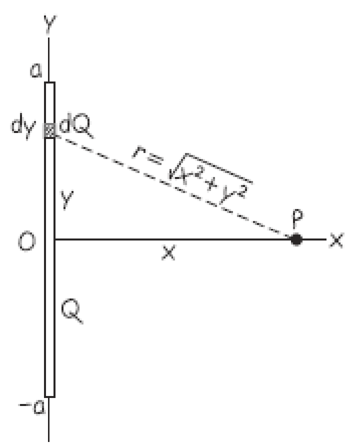


Electric Potential of Ring:



$$V = k\int \frac{dq}{r} = \frac{k}{\sqrt{x^2 + a^2}}\int dq = \frac{kQ}{x^2 + a^2}$$

Electric Potential of Rod:



$$dQ = \lambda dy$$

$$\lambda = \frac{Q}{2a}$$

$$dV = \frac{kdQ}{r} = \frac{k\lambda dy}{\sqrt{x^2 + y^2}}$$

$$V = 2\int_0^a \frac{k\lambda dy}{\sqrt{x^2 + y^2}}$$

$$= 2k\lambda \int_0^a \frac{dy}{\sqrt{x^2 + y^2}} = 2k\frac{Q}{2a} ln \frac{\sqrt{x^2 + a^2} + a}{x} = \frac{kQ}{a} ln \frac{\sqrt{x^2 + a^2} + a}{x}$$

Potential Gradient:

$$V_a - V_b = \int_b^a dV = -\int_a^b dV = \int_a^b \vec{E} \cdot d\vec{l}$$

$$-dV = \vec{E} \cdot d\vec{l} = E_x dx + E_y dy + E_z dz$$

$$\vec{E} = < E_x, E_y, E_z > = < -\frac{\partial V}{\partial x}, -\frac{\partial V}{\partial y}, -\frac{\partial V}{\partial z} >$$

$$= -\left(\hat{\imath}\frac{\partial V}{\partial x} + \hat{\jmath}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}\right)$$

$$= -\left(\hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)V$$

$$= -\vec{\nabla} V$$

$$\therefore \boldsymbol{E} = -\boldsymbol{\nabla V}$$

Vector Electric Field of a Point Charge:

$$\boldsymbol{V = \frac{q}{4\pi\epsilon_0 r}}$$

Sol: ***Radial Symmetry***

$$E_r = -\frac{\partial V}{\partial r} = -\frac{\partial}{\partial r}\left(\frac{q}{4\pi\epsilon_0 r}\right) = \frac{q}{4\pi\epsilon_0 r^2}$$

$$\overrightarrow{E_r} = E_r\hat{r} = \frac{q}{4\pi\epsilon_0 r^2}\hat{r}$$

Sol: ***Components***

$$V = \frac{q}{4\pi\epsilon_0 r} = \frac{q}{4\pi\epsilon_0}\frac{1}{\sqrt{x^2 + y^2 + z^2}}$$

$$\vec{\nabla} V = < \frac{-qx}{4\pi\epsilon_0 r^3}, \frac{-qy}{4\pi\epsilon_0 r^3}, \frac{-qz}{4\pi\epsilon_0 r^3} >$$

$$E = -\vec{\nabla} V = \left[\hat{\imath}\frac{qx}{4\pi\epsilon_0 r^3} + \hat{\jmath}\frac{qy}{4\pi\epsilon_0 r^3} + \hat{k}\frac{qz}{4\pi\epsilon_0 r^3}\right] = \frac{q}{4\pi\epsilon_0 r^2}\left(\frac{x\hat{\imath} + y\hat{\jmath} + z\hat{k}}{r}\right) = \frac{q}{4\pi\epsilon_0 r^2}\hat{r}$$

Capacitance:

$$C = \frac{Q}{V_{ab}}$$

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

$$V_{ab} = Ed = \frac{Qd}{\epsilon_0 A}$$

$$C = \frac{Q}{V_{ab}} = \epsilon_0 \frac{A}{d}$$

Spherical Capacitor:

$$V_{ab} = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right) = \frac{Q}{4\pi\epsilon_0}\frac{r_b - r_a}{r_a r_b}$$

$$C = \frac{Q}{V_{ab}} = 4\pi\epsilon_0 \frac{r_a r_b}{r_b - r_a}$$

Total Potential Difference across Series Combination:

$$V_{total} = \sum_i V_i$$

Cylindrical Capacitor:

$$V_{ab} = \frac{\lambda}{2\pi\epsilon_0} ln \frac{r_b}{r_a}$$

$$C = \frac{Q}{V_{ab}} = \frac{\lambda L}{\frac{\lambda}{2\pi\epsilon_0} ln \frac{r_b}{r_a}} = \frac{2\pi\epsilon_0 L}{ln \frac{r_b}{r_a}}$$

Capacitors in Series:

$$V_{ab} = \sum_i V_i = \sum_i \frac{Q}{C_i} = Q \sum_i \frac{1}{C_i}$$

$$\frac{V_{ab}}{Q} = \sum_i \frac{1}{C_i} = \frac{V_{ab}}{C_{eq} V_{ab}} = \frac{1}{C_{eq}}$$

$$\frac{1}{C_{eq}} = \sum_i \frac{1}{C_i}$$

Capacitors in Parallel:

$$Q = \sum_i Q_i = \sum_i C_i V = V \sum_i C_i$$

$$\frac{Q}{V} = \sum_i C_i = \frac{C_{eq} V}{V} = C_{eq}$$

$$C_{eq} = \sum_i C_i$$

Energy:

$$dW = v dq = \frac{q}{C} dq$$

$$W = \int_0^W dW = \frac{1}{C} \int_0^Q q dq = \frac{Q^2}{2C}$$

$$U = \frac{Q^2}{2C} = \frac{1}{2} C V^2 = \frac{1}{2} Q V$$

Energy Density:

$$u = \frac{\frac{1}{2} C V^2}{Ad} = \frac{\frac{1}{2}\left(\epsilon_0 \frac{A}{d}\right)(Ed)^2}{Ad} = \frac{1}{2} \epsilon_0 E^2$$

Dielectric:

$$C = K C_0$$

$$C = \frac{Q}{V} = K \frac{Q}{V_0}$$

$$V = \frac{V_0}{K}$$

$$\because V \propto E$$

$$\therefore E = \frac{E_0}{K}$$

Permitivity:

$$\epsilon = K \epsilon_0$$

$$C = K C_0 = K \epsilon_0 \frac{A}{d} = \epsilon \frac{A}{d}$$

$$u = \frac{1}{2} K \epsilon_0 E^2 = \frac{1}{2} \epsilon E^2$$

Resistance:

Current:

$$I = \frac{dQ}{dt} = nqv_dA$$

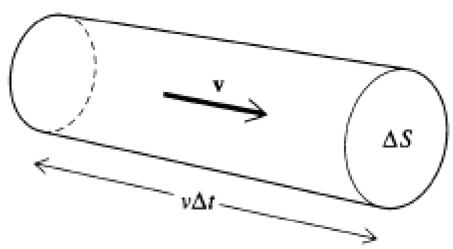
$$(A) = \left(\frac{1}{m^3}\right)(C)\left(\frac{m}{s}\right)(m^2) = \left(\frac{C}{t}\right)$$

Current Density:

$$J = \frac{I}{A} = nqv_d$$

$$\vec{J} = nq\overrightarrow{v_d}$$

Continuity Equation:



$$\Delta Q \;=\; \rho\Delta V \;=\; \rho v\Delta t\Delta S$$

$$\frac{\Delta Q}{\Delta t} = \rho\vec{v}\cdot\hat{n}\Delta S = \vec{J}\cdot\Delta\vec{A}$$

$$I \;=\; \lim_{\Delta t\to 0}\frac{\Delta Q}{\Delta t} = \iint_S -\vec{J}\cdot d\vec{A} = \iiint_V -\vec{\nabla}\cdot\vec{J}\,dV$$

$$I = \iiint_V \frac{\partial\rho}{\partial t}\,dV \;=\; \iiint_V -\vec{\nabla}\cdot\vec{J}\,dV$$

$$\frac{\partial\rho}{\partial t} = -\vec{\nabla}\cdot\vec{J}$$

$$\frac{\partial\rho}{\partial t} \;+\; \boldsymbol{\nabla}\cdot\boldsymbol{J} \;=\; 0$$

Resistivity:

$$\rho = \frac{E}{J} = \frac{1}{\sigma}$$

$$\rho \to Resistivity$$

$$\sigma \rightarrow Conductivity$$

$$\rho(T) = \rho_0[1 + \alpha(T - T_0)]$$

$$E = VL = \rho J = \rho \frac{I}{A}$$

$$\frac{V}{I} = \frac{\rho L}{A} = R$$

$\boldsymbol{Resistance}$:

$$R = \rho \frac{L}{A}$$

$$\because R \propto \rho$$

$$\therefore R(T) = R_0[1 + \alpha(T - T_0)]$$

$\boldsymbol{Electromotive\ Force}$: ***lower to higher potential***

$\boldsymbol{Ideal\ Source}$:

$$V_{ab} = \varepsilon = IR$$

$\boldsymbol{Nonideal\ Source}$:

$$V_{ab} = \varepsilon - Ir = IR$$

$$V_{ab} \rightarrow Terminal\ Voltage$$

$$r \rightarrow Internal\ Resistance$$

$$I = \frac{\varepsilon}{R + r}$$

$\boldsymbol{Power}$:

$$E = QV$$

$$P = \frac{dE}{dt} = Q\frac{dV}{dt} + V\frac{dQ}{dt} = 0 + IV = IV$$

$$P = IV = I^2R = \frac{V^2}{R}$$

$\boldsymbol{Power\ Output}$:

$$P = IV_{ab} = I(\varepsilon - Ir) = I\varepsilon - I^2r$$

$\boldsymbol{Power\ Input}$:

$$P = IV_{ab} = I(\varepsilon + Ir) = I\varepsilon + I^2r$$

Resistors in Series:

$$V_{ab} = \sum_i V_i = \sum_i IR_i = I\sum_i R_i = IR_{eq}$$

$$R_{eq} = \sum_i R_i$$

Resistors in Parallel:

$$I = \sum_i \frac{V}{R_i} = V\sum_i \frac{1}{R_i} = \frac{V}{R_{eq}}$$

$$\frac{1}{R_{eq}} = \sum_i \frac{1}{R_i}$$

Kirchhoff's Junction Rule:

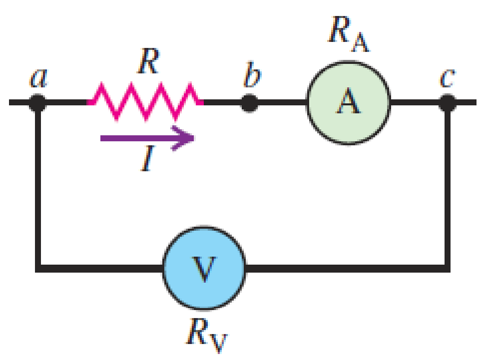
$$\sum I = 0$$

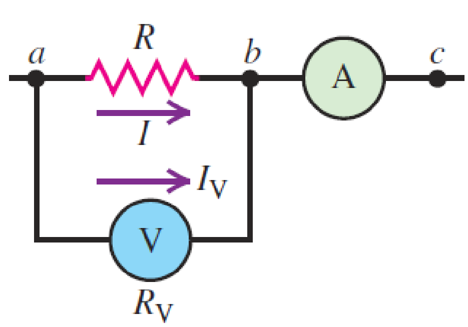
Kirchhoff's Loop Rule:

$$\sum V = 0$$

Voltmeter: ***In Parallel with Resistor***

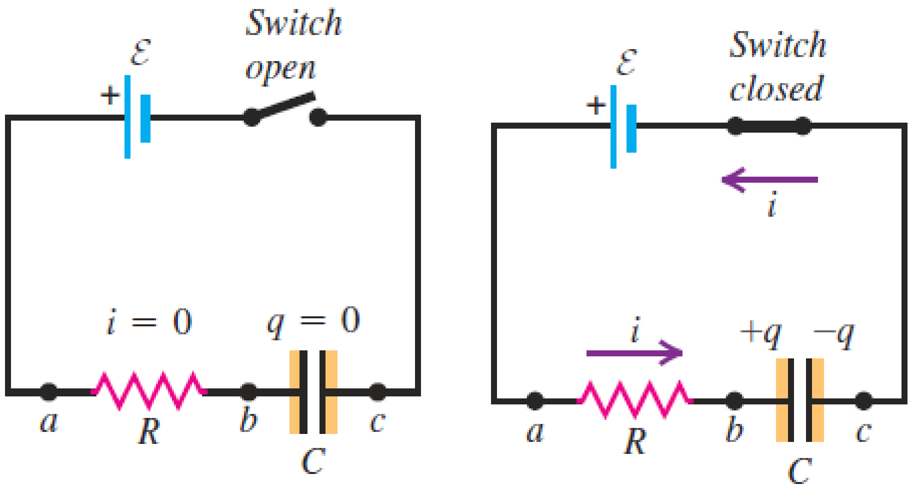
Ammeter: ***In Series with Resistor***





RC Circuit:

Charging:



$$V_{ab} = iR$$

$$V_{bc} = \frac{q}{C}$$

$$\sum V = \varepsilon - iR - \frac{q}{C} = 0$$

$$\frac{dq}{dt} = i = \frac{\varepsilon}{R} - \frac{q}{RC} = -\frac{1}{RC}(q - C\varepsilon)$$

$$\frac{dq}{q - C\varepsilon} = -\frac{dt}{RC}$$

$$\int_0^q \frac{dq}{q - C\varepsilon} = -\int_0^t \frac{dt}{RC}$$

$$ln\left(\frac{q - C\varepsilon}{-C\varepsilon}\right) = -\frac{t}{RC}$$

$$\frac{q - C\varepsilon}{-C\varepsilon} = e^{\frac{-t}{RC}}$$

$$q = C\varepsilon\left(1 - e^{\frac{-t}{RC}}\right) = Q_f\left(1 - e^{\frac{-t}{RC}}\right)$$

$$i = \frac{dq}{dt} = \frac{\varepsilon}{R}e^{\frac{-t}{RC}} = I_0 e^{\frac{-t}{RC}}$$

$When\ t = 0 \rightarrow i = \frac{\varepsilon}{R} = I_0$

$When\ i = 0 \rightarrow Q_f = C\varepsilon$

Discharging:

$$\sum V = -iR - \frac{q}{C} = 0$$

$$\frac{dq}{dt} = i = -\frac{q}{RC}$$

$$\int_{Q_0}^{q} \frac{dq}{q} = -\frac{1}{RC}\int_0^t dt$$

$$ln\frac{q}{Q_0} = \frac{-t}{RC}$$

$$q = Q_0 e^{\frac{-t}{RC}}$$

$$i = \frac{dq}{dt} = -\frac{Q_0}{RC}e^{\frac{-t}{RC}} = I_0 e^{\frac{-t}{RC}}$$

Time Constant:

$$\tau = RC$$

Magnetism:

Magnetic Field:

$$\vec{F} = q\vec{v} \times \vec{B}$$

$$F = qvBsin\theta$$

Charged Particle moves through both $\vec{E}$ and $\vec{B}$:

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

Magnetic Flux:

$$\Phi_B = \iint \vec{B} \cdot d\vec{A}$$

$$B = \frac{d\Phi_B}{dA_\perp}$$

Gauss's Law for Magnetism:

$$\oiiint_s \vec{B} \cdot d\vec{A} = 0$$

$$\oiiint_s \vec{B} \cdot d\vec{A} = \oiiint_v \vec{\nabla} \cdot \vec{B} dV = 0$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = \boldsymbol{0}$$

Charged Particle under Magnetic Field:

$$F = qvB = m\frac{v^2}{R}$$

$$R = \frac{mv}{qB}$$

$$\omega = \frac{v}{R} = \frac{qB}{m}$$

Velocity Selector:

$$qE = qvB$$

$$v = \frac{E}{B}$$

Charge to Mass Ratio:

$$\frac{1}{2}mv^2 = eV$$

$$v = \sqrt{\frac{2eV}{m}} = \frac{E}{B}$$

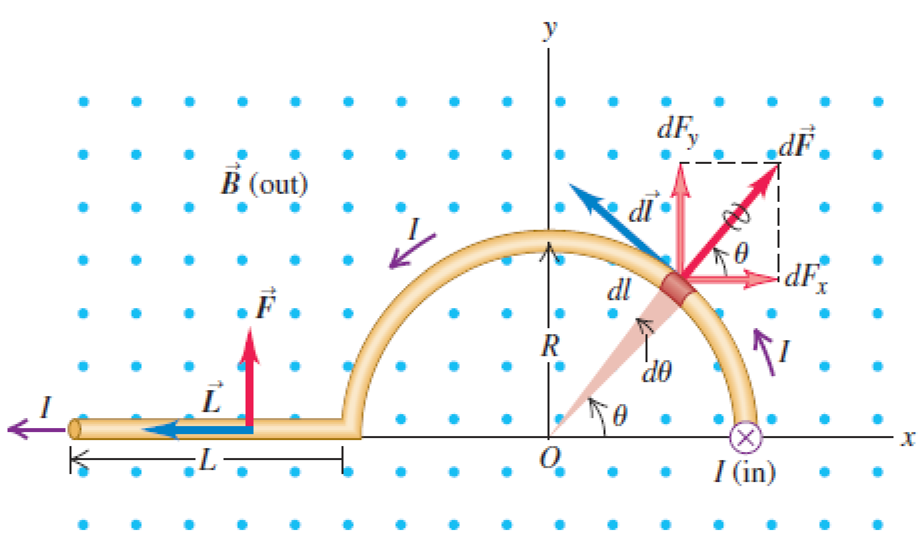
$$\frac{e}{m} = \frac{E^2}{2VB^2}$$

Magnetic Force on a Current − Carrying Conductor:

$$F = qvB = (nAl)(qv_dB) = (nqv_dA)(lB) = IlB$$

$$F = I\vec{l} \times \vec{B}$$

$$d\vec{F} = Id\vec{l} \times \vec{B}$$



$\boldsymbol{Biot - Savart\ Law}$:

$\boldsymbol{Magnetic\ Field\ of\ Moving\ Charge}$:

$$B = \frac{\mu_0}{4\pi}\frac{qvsin\theta}{r^2}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v} \times \hat{r}}{r^2}$$

$\boldsymbol{Magnetic\ Field\ of\ Current\ Element}$:

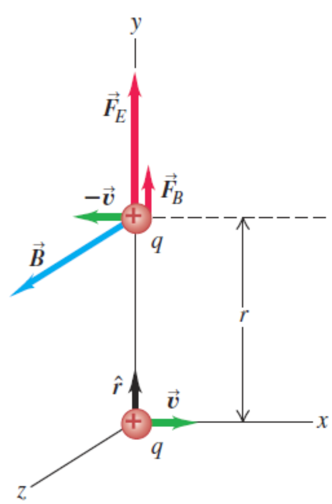
$$dQ = nqAdl$$

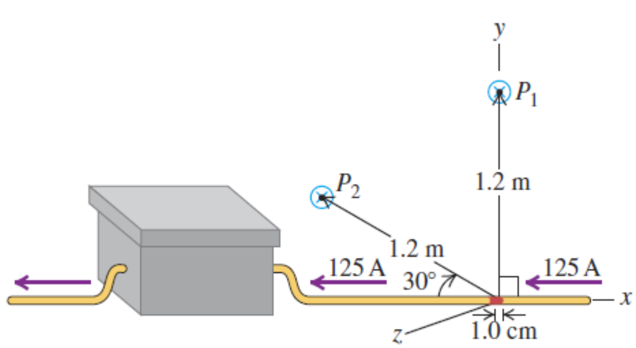
$$dB = \frac{\mu_0}{4\pi}\frac{dQv_d sin\theta}{r^2} = \frac{\mu_0}{4\pi}\frac{nqAv_d dlsin\theta}{r^2}$$

$$dB = \frac{\mu_0}{4\pi}\frac{Idlsin\theta}{r^2}$$

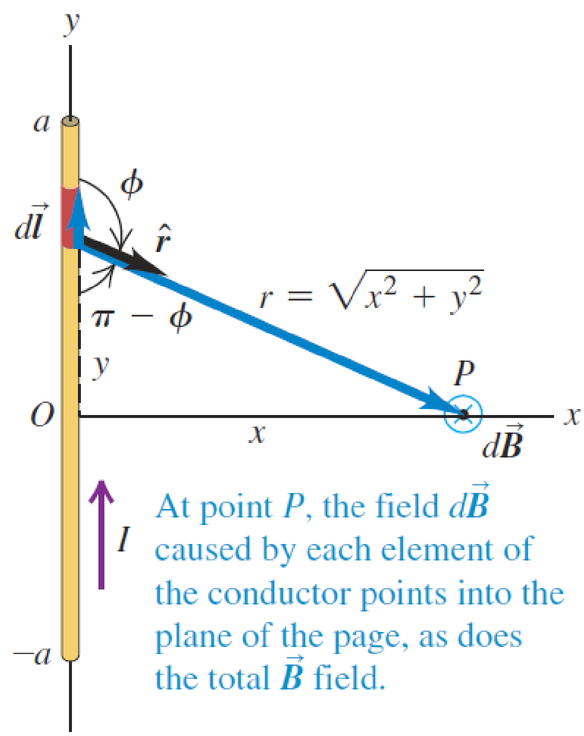
$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l} \times \hat{r}}{r^2}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\int_c \frac{Id\vec{l} \times \hat{r}}{r^2}$$





Magnetic Field of Rod:



$$sin\phi = sin(\pi - \phi) = \frac{x}{\sqrt{x^2 + y^2}}$$

$$dB = \frac{\mu_0}{4\pi}\frac{Idlsin\phi}{r^2} = \frac{\mu_0}{4\pi}\frac{Idy}{x^2 + y^2}\frac{x}{\sqrt{x^2 + y^2}}$$

$$B = \frac{\mu_0 I}{4\pi} * 2\int_0^a \frac{xdy}{(x^2 + y^2)^{3/2}}$$

$$= \frac{\mu_0 I}{2\pi}\frac{a}{x\sqrt{x^2 + a^2}}$$

Case: $\boldsymbol{a \gg x}$

$$B = \frac{\mu_o I}{2\pi x}$$

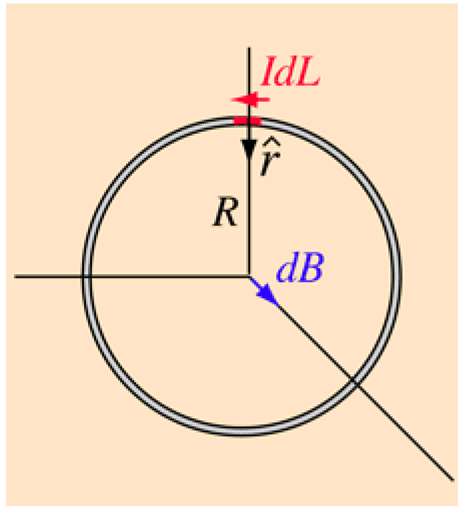
Magnetic Force between Two Parallel Wires:

$$F = I_1 LB = I_1 L\frac{\mu_0 I_2}{2\pi x} = \frac{\mu_0 I_1 I_2 L}{2\pi x}$$

Force Per Unit Lenght:

$$\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi x}$$

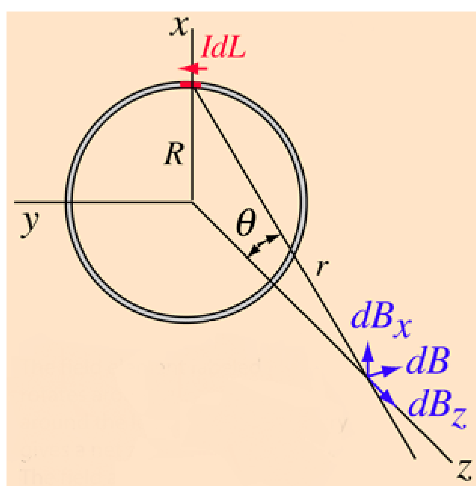
Magnetic Field at Center of Current Loop:



$$dB = \frac{\mu_0}{4\pi}\frac{Idl sin90°}{R^2} = \frac{\mu_0}{4\pi}\frac{Idl}{R^2}$$

$$B = \frac{\mu_0 I}{4\pi R^2}\oint_c dl = \frac{\mu_0 I}{4\pi R^2} * (2\pi R) = \frac{\mu_0 I}{2R}$$

Magnetic Field on Axis of Current Loop:



$$dB = \frac{\mu_0}{4\pi}\frac{Idl sin90°}{z^2 + R^2} = \frac{\mu_0}{4\pi}\frac{Idl}{z^2 + R^2}$$

$$dB_z = dB * cos\theta = \frac{\mu_0}{4\pi}\frac{Idl}{z^2 + R^2} * \left(\frac{R}{\sqrt{z^2 + R^2}}\right) = \frac{\mu_0}{4\pi}\frac{IRdl}{(z^2 + R^2)^{3/2}}$$

$$B_z = \frac{\mu_0 IR}{4\pi(z^2+R^2)^{3/2}} \oint_c dl$$

$$= \frac{\mu_0 IR}{4\pi(z^2+R^2)^{3/2}} * (2\pi R)$$

$$= \frac{\mu_0 IR^2}{2(z^2+R^2)^{3/2}}$$

Ampere's Law:

$$\oint_c \vec{B} \cdot d\vec{l} = \oint_c Bdl = B\oint_c dl = \frac{\mu_0 I}{2\pi r}(2\pi r) = \mu_0 I$$

$$\oint_c \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

$$\oint_c \vec{B} \cdot d\vec{l} = \oiint_s \vec{\nabla} \times \vec{B} dS = \mu_0 I_{encl} = \oiint_s \mu_0 J dS$$

$$\boldsymbol{\nabla \times B = \mu_0 J}$$

Magnetic Field of Long, Straight, Current − Carrying Conductor:

$$\oint_c \vec{B} \cdot d\vec{l} = \oint_c B\,dl = B(2\pi r) = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

Magnetic Field of Long Cylindrical Conductor:

Case: $\boldsymbol{r < R}$

$$\oint_c \vec{B} \cdot d\vec{l} = \oint_c Bdl = B\oint_c dl = B(2\pi r) = \mu_0 I * \left(\frac{r^2}{R^2}\right)$$

$$B = \frac{\mu_0 I}{2\pi} \frac{r}{R^2}$$

Case: $\boldsymbol{r > R}$

$$\oint_c \vec{B} \cdot d\vec{l} = \oint_c Bdl = B\oint_c dl = B(2\pi r) = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

Magnetic Field of Solenoid:

$$\oint_c \vec{B} \cdot d\vec{l} = \oint_c B dl = B \oint_c dl = BL = \mu_0 NI$$

$$n = \frac{N}{L}$$

$$B = \mu_0 ni$$

Magnetic Field of Toroidal Solenoid:

$$\oint_c \vec{B} \cdot d\vec{l} = \oint_c B dl = B \oint_c dl = B(2\pi r) = \mu_0 NI$$

$$B = \frac{\mu_0 NI}{2\pi r}$$