Table of Contents


```
clc; clear all; close all;

% Followed the lecture notes and used FK of the equivalent Stewart-Gough
% platform for validation of the IK solution. Visualizations added for your
% viewing pleasure
```

Desired Pose (Given)

```
P = [0; 10; 100; deg2rad(5); deg2rad(5); deg2rad(0)]; % Note- this is near a
 singular configuration!
```

Robot Parameters (Given)

```
Rf = 480/2;
Rm = 300/2;
alpha = pi/3;
beta = pi/3;
l1 = 20;
l2 = 70;
l3 = 100;
```

Calculating lower joint positions w.r.t. the lower coordinate frame

```
u1 = [-Rf*cos(alpha); Rf*sin(alpha); 0];
u2 = [Rf*cos(alpha); Rf*sin(alpha); 0];
u3 = [-Rf; 0; 0];
u4 = [Rf; 0; 0];
u5 = [-Rf*cos(alpha); -Rf*sin(alpha); 0];
u6 = [Rf*cos(alpha); -Rf*sin(alpha); 0];
```

```
u = [u1, u2 , u3 , u4 , u5 , u6];
```

Calculating upper joint positions w.r.t. the upper coordinate frame

```
s11 = [-Rm*cos(beta); Rm*sin(beta); 0];
s12 = [Rm*cos(beta); Rm*sin(beta); 0];
s13 = [-Rm; 0; 0];
s14 = [Rm; 0; 0];
s15 = [-Rm*cos(beta); -Rm*sin(beta); 0];
s16 = [Rm*cos(beta); -Rm*sin(beta); 0];

s1 = [s11 , s12 , s13 , s14 , s15 , s16];
```

Extracting Position and Euler Angle Information from the given desired pose

```
O = P(1:3,1);
a = P(4);
b = P(5);
c = P(6);
```

Calculating Rotation Matrix from Euler Angles

```
R1 = [1, 0,       0;
      0, cos(a), -sin(a);
      0, sin(a),  cos(a)];

R2 = [cos(b),  0, sin(b);
      0,       1, 0;
      -sin(b), 0, cos(b)];

R3 = [cos(c), -sin(c), 0;
      sin(c),  cos(c), 0;
      0,        0,     1];

R = R1*R2*R3;
```

IK of Radially Symmetric Hexapod Robot

```
L1 = zeros(3,6);
L1i = zeros(3,1);
s2 = zeros(3,6);
s2i = zeros(3,1);
L2i = zeros(3,1);
legLengths = zeros(6,1);
normL2i = 0;
q = zeros(3,6); % alpha, beta, gamma of each leg
for i = 1:6
```

First and Second Loop Closure

```
    L1i = O + R * s1(:, i) - u(:, i);
    L1(:,i) = L1i;
    q(1, i) = atan(L1i(2)/L1i(1));
    s2i = [s1(1, i) + ((-1)^i)*l1*cos(q(1, i));
           s1(2, i) + ((-1)^i)*l1*sin(q(1, i));
           s1(3,i)];
    s2(:,i) = s2i;
```

Third and Fourth Loop Closure

```
    L2i = O + R * s2i - u(:, i);
    normL2i = norm(L2i);
    legLengths(i) = norm(L1i);
    q(2, i) = acos(((l2^2) + (normL2i^2) - (l3^2)) / (2*l2*normL2i)) -
 (asin(L2i(3)/normL2i) + asin((L2i(3) - L1i(3))/l1));
    q(3, i) = pi - acos(((l2^2) + (l3^2) - normL2i^2)/(2*l2*l3));

end
```

Visualizations

```
figure
hold on;

plotCircle(Rf, zeros(3,1), eye(3));
plotCircle(Rm, O, R);

% Plot s
sInGlobal = [O + R*s1(:,1), O + R*s1(:,2), O + R*s1(:,3), O + R*s1(:,4), O +
 R*s1(:,5), O + R*s1(:,6)];
line([O(1), sInGlobal(1,1)], [O(2), sInGlobal(2,1)], [O(3),
 sInGlobal(3,1)],'Color','magenta');
line([O(1), sInGlobal(1,2)], [O(2), sInGlobal(2,2)], [O(3),
 sInGlobal(3,2)],'Color','magenta');
line([O(1), sInGlobal(1,3)], [O(2), sInGlobal(2,3)], [O(3),
 sInGlobal(3,3)],'Color','magenta');
line([O(1), sInGlobal(1,4)], [O(2), sInGlobal(2,4)], [O(3),
 sInGlobal(3,4)],'Color','magenta');
line([O(1), sInGlobal(1,5)], [O(2), sInGlobal(2,5)], [O(3),
 sInGlobal(3,5)],'Color','magenta');
line([O(1), sInGlobal(1,6)], [O(2), sInGlobal(2,6)], [O(3),
 sInGlobal(3,6)],'Color','magenta');

% Plot u
line([0, u(1,1)], [0, u(2,1)], [0, u(3,1)],'Color','blue');
line([0, u(1,2)], [0, u(2,2)], [0, u(3,2)],'Color','blue');
line([0, u(1,3)], [0, u(2,3)], [0, u(3,3)],'Color','blue');
line([0, u(1,4)], [0, u(2,4)], [0, u(3,4)],'Color','blue');
line([0, u(1,5)], [0, u(2,5)], [0, u(3,5)],'Color','blue');
line([0, u(1,6)], [0, u(2,6)], [0, u(3,6)],'Color','blue');
```

```
% Plot leg vector
line([u(1,1), L1(1,1)+u(1,1)], [u(2,1), L1(2,1)+u(2,1)], [u(3,1),
 L1(3,1)+u(3,1)],'Color','green');
line([u(1,2), L1(1,2)+u(1,2)], [u(2,2), L1(2,2)+u(2,2)], [u(3,2),
 L1(3,2)+u(3,2)],'Color','green');
line([u(1,3), L1(1,3)+u(1,3)], [u(2,3), L1(2,3)+u(2,3)], [u(3,3),
 L1(3,3)+u(3,3)],'Color','green');
line([u(1,4), L1(1,4)+u(1,4)], [u(2,4), L1(2,4)+u(2,4)], [u(3,4),
 L1(3,4)+u(3,4)],'Color','green');
line([u(1,5), L1(1,5)+u(1,5)], [u(2,5), L1(2,5)+u(2,5)], [u(3,5),
 L1(3,5)+u(3,5)],'Color','green');
line([u(1,6), L1(1,6)+u(1,6)], [u(2,6), L1(2,6)+u(2,6)], [u(3,6),
 L1(3,6)+u(3,6)],'Color','green');

s2InGlobal = [O + R*s2(:,1), O + R*s2(:,2), O + R*s2(:,3), O + R*s2(:,4), O +
 R*s2(:,5), O + R*s2(:,6)];

% Plot Leg 1
line([sInGlobal(1,1), s2InGlobal(1,1)], [sInGlobal(2,1), s2InGlobal(2,1)],
 [sInGlobal(3,1), s2InGlobal(3,1)],'Color','red');
line([sInGlobal(1,2), s2InGlobal(1,2)], [sInGlobal(2,2), s2InGlobal(2,2)],
 [sInGlobal(3,2), s2InGlobal(3,2)],'Color','red');
line([sInGlobal(1,3), s2InGlobal(1,3)], [sInGlobal(2,3), s2InGlobal(2,3)],
 [sInGlobal(3,3), s2InGlobal(3,3)],'Color','red');
line([sInGlobal(1,4), s2InGlobal(1,4)], [sInGlobal(2,4), s2InGlobal(2,4)],
 [sInGlobal(3,4), s2InGlobal(3,4)],'Color','red');
line([sInGlobal(1,5), s2InGlobal(1,5)], [sInGlobal(2,5), s2InGlobal(2,5)],
 [sInGlobal(3,5), s2InGlobal(3,5)],'Color','red');
line([sInGlobal(1,6), s2InGlobal(1,6)], [sInGlobal(2,6), s2InGlobal(2,6)],
 [sInGlobal(3,6), s2InGlobal(3,6)],'Color','red');

nL1InG = [(s2InGlobal(:,1)-sInGlobal(:,1))/norm(s2InGlobal(:,1)-
sInGlobal(:,1)), (s2InGlobal(:,2)-sInGlobal(:,2))/norm(s2InGlobal(:,2)-
sInGlobal(:,2)), ...
          (s2InGlobal(:,3)-sInGlobal(:,3))/norm(s2InGlobal(:,3)-
sInGlobal(:,3)), (s2InGlobal(:,4)-sInGlobal(:,4))/norm(s2InGlobal(:,4)-
sInGlobal(:,4)), ...
          (s2InGlobal(:,5)-sInGlobal(:,5))/norm(s2InGlobal(:,5)-
sInGlobal(:,5)), (s2InGlobal(:,6)-sInGlobal(:,6))/norm(s2InGlobal(:,6)-
sInGlobal(:,6))];

rots = [cross(nL1InG(:,1), R(1:3,3)), cross(nL1InG(:,2), R(1:3,3)),
 cross(nL1InG(:,3), R(1:3,3)), ...
        cross(nL1InG(:,4), R(1:3,3)), cross(nL1InG(:,5), R(1:3,3)),
 cross(nL1InG(:,6), R(1:3,3))];


leg2 = [l2 .* rodrigues(nL1InG(:,1), rots(:,1), q(2, 1)), l2 .*
 rodrigues(nL1InG(:,2), rots(:,2), q(2, 2)), l2 .* rodrigues(nL1InG(:,3),
 rots(:,3), q(2, 3)), ...
        l2 .* rodrigues(nL1InG(:,4), rots(:,4), q(2, 4)), l2 .*
 rodrigues(nL1InG(:,5), rots(:,5), q(2, 5)), l2 .* rodrigues(nL1InG(:,6),
 rots(:,6), q(2, 6))];
```

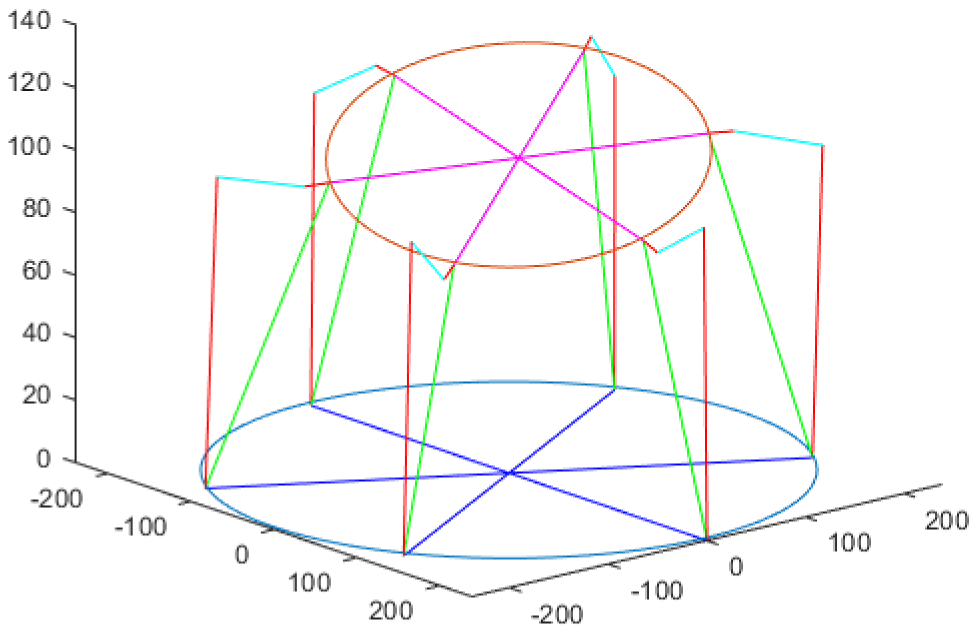
```
% % Plot the axis that you rotate beta (used for debugging)
% line([s2InGlobal(1,1), s2InGlobal(1,1) + l2 .* rots(1,1)], [s2InGlobal(2,1),
 s2InGlobal(2,1) + l2 .* rots(2,1)], [s2InGlobal(3,1), s2InGlobal(3,1) + l2 .*
 rots(3,1)],'Color','black');
% line([s2InGlobal(1,2), s2InGlobal(1,2) + l2 .* rots(1,2)], [s2InGlobal(2,2),
 s2InGlobal(2,2) + l2 .* rots(2,2)], [s2InGlobal(3,2), s2InGlobal(3,2) + l2 .*
 rots(3,2)],'Color','black');
% line([s2InGlobal(1,3), s2InGlobal(1,3) + l2 .* rots(1,3)], [s2InGlobal(2,3),
 s2InGlobal(2,3) + l2 .* rots(2,3)], [s2InGlobal(3,3), s2InGlobal(3,3) + l2 .*
 rots(3,3)],'Color','black');
% line([s2InGlobal(1,4), s2InGlobal(1,4) + l2 .* rots(1,4)], [s2InGlobal(2,4),
 s2InGlobal(2,4) + l2 .* rots(2,4)], [s2InGlobal(3,4), s2InGlobal(3,4) + l2 .*
 rots(3,4)],'Color','black');
% line([s2InGlobal(1,5), s2InGlobal(1,5) + l2 .* rots(1,5)], [s2InGlobal(2,5),
 s2InGlobal(2,5) + l2 .* rots(2,5)], [s2InGlobal(3,5), s2InGlobal(3,5) + l2 .*
 rots(3,5)],'Color','black');
% line([s2InGlobal(1,6), s2InGlobal(1,6) + l2 .* rots(1,6)], [s2InGlobal(2,6),
 s2InGlobal(2,6) + l2 .* rots(2,6)], [s2InGlobal(3,6), s2InGlobal(3,6) + l2 .*
 rots(3,6)],'Color','black');

% Plot Leg 2
line([s2InGlobal(1,1), s2InGlobal(1,1) + leg2(1,1)], [s2InGlobal(2,1),
 s2InGlobal(2,1) + leg2(2,1)], [s2InGlobal(3,1), s2InGlobal(3,1) +
 leg2(3,1)],'Color','cyan');
line([s2InGlobal(1,2), s2InGlobal(1,2) + leg2(1,2)], [s2InGlobal(2,2),
 s2InGlobal(2,2) + leg2(2,2)], [s2InGlobal(3,2), s2InGlobal(3,2) +
 leg2(3,2)],'Color','cyan');
line([s2InGlobal(1,3), s2InGlobal(1,3) + leg2(1,3)], [s2InGlobal(2,3),
 s2InGlobal(2,3) + leg2(2,3)], [s2InGlobal(3,3), s2InGlobal(3,3) +
 leg2(3,3)],'Color','cyan');
line([s2InGlobal(1,4), s2InGlobal(1,4) + leg2(1,4)], [s2InGlobal(2,4),
 s2InGlobal(2,4) + leg2(2,4)], [s2InGlobal(3,4), s2InGlobal(3,4) +
 leg2(3,4)],'Color','cyan');
line([s2InGlobal(1,5), s2InGlobal(1,5) + leg2(1,5)], [s2InGlobal(2,5),
 s2InGlobal(2,5) + leg2(2,5)], [s2InGlobal(3,5), s2InGlobal(3,5) +
 leg2(3,5)],'Color','cyan');
line([s2InGlobal(1,6), s2InGlobal(1,6) + leg2(1,6)], [s2InGlobal(2,6),
 s2InGlobal(2,6) + leg2(2,6)], [s2InGlobal(3,6), s2InGlobal(3,6) +
 leg2(3,6)],'Color','cyan');

% Plot Leg 3
line([s2InGlobal(1,1) + leg2(1,1), u(1,1)], [s2InGlobal(2,1) + leg2(2,1),
 u(2,1)], [s2InGlobal(3,1) + leg2(3,1), u(3,1)],'Color','red');
line([s2InGlobal(1,2) + leg2(1,2), u(1,2)], [s2InGlobal(2,2) + leg2(2,2),
 u(2,2)], [s2InGlobal(3,2) + leg2(3,2), u(3,2)],'Color','red');
line([s2InGlobal(1,3) + leg2(1,3), u(1,3)], [s2InGlobal(2,3) + leg2(2,3),
 u(2,3)], [s2InGlobal(3,3) + leg2(3,3), u(3,3)],'Color','red');
line([s2InGlobal(1,4) + leg2(1,4), u(1,4)], [s2InGlobal(2,4) + leg2(2,4),
 u(2,4)], [s2InGlobal(3,4) + leg2(3,4), u(3,4)],'Color','red');
line([s2InGlobal(1,5) + leg2(1,5), u(1,5)], [s2InGlobal(2,5) + leg2(2,5),
 u(2,5)], [s2InGlobal(3,5) + leg2(3,5), u(3,5)],'Color','red');
line([s2InGlobal(1,6) + leg2(1,6), u(1,6)], [s2InGlobal(2,6) + leg2(2,6),
 u(2,6)], [s2InGlobal(3,6) + leg2(3,6), u(3,6)],'Color','red');
```

```
disp(norm(s2InGlobal(:,1)+leg2(:,1)-u(:,1))) % check that leg 3 is 100mm long

view([50, 22])
```



Validation: Compute FK

```
P0 = [0; 0; 100; 0; 0; 0];
dl = 1;

while dl > 0.001
        aFK = P0(4);
        bFK = P0(5);
        cFK = P0(6);

        B = [1, 0,        sin(bFK);
             0, cos(aFK), -sin(aFK)*cos(bFK);
             0, sin(aFK), cos(aFK)*cos(bFK)]; % XYZ

        T = [eye(3),     zeros(3,3)
             zeros(3,3), B];

        R1FK = [1, 0,        0;
                0, cos(aFK), -sin(aFK);
                0, sin(aFK), cos(aFK)];
```

```
        R2FK = [cos(bFK),  0, sin(bFK);
                0,         1, 0;
                -sin(bFK), 0, cos(bFK)];

        R3FK = [cos(cFK), -sin(cFK), 0;
                sin(cFK), cos(cFK), 0;
                0,        0,     1];

        RFK = R1FK * R2FK * R3FK; % XYZ

        LFK = zeros(3,6);
        lFK = zeros(6,1);
        nFK = zeros(3,6);
        for leg = 1:6
            LFK(:,leg) = P0(1:3,1) + (RFK * s1(:, leg)) - u(:, leg);
            lFK(leg, 1) = norm(LFK(:,leg),2);
            nFK(:,leg) = LFK (:, leg)/lFK(leg,1);
        end

        J = [nFK(:,1)' , cross(RFK * s1(:,1), nFK(:,1))';
             nFK(:,2)' , cross(RFK * s1(:,2), nFK(:,2))';
             nFK(:,3)' , cross(RFK * s1(:,3), nFK(:,3))';
             nFK(:,4)' , cross(RFK * s1(:,4), nFK(:,4))';
             nFK(:,5)' , cross(RFK * s1(:,5), nFK(:,5))';
             nFK(:,6)' , cross(RFK * s1(:,6), nFK(:,6))'];

        JRP = J * T;
        Dl = legLengths - lFK;

        P0 = P0 + pinv(JRP) * Dl;
        dl = norm(Dl , 2);
end

disp(P); % verify solution
disp(P0);

         0
   10.0000
  100.0000
    0.0873
    0.0873
         0

   -1.8112
    8.7914
   98.6535
    0.0912
    0.0831
   -0.0934
```

Auxillary Functions

```
function plotCircle(radius, offset, rotationMatrix)
    % Original points, original plane
    t = linspace(0,2*pi);
    x = (radius*cos(t));
    y = (radius*sin(t));
    z = ones(1,size(t,2));
    pnts = rotationMatrix * [x;y;z] + offset;
    plot3(pnts(1,:), pnts(2,:), pnts(3,:));
end

function vrot = rodrigues(v, k, theta)
    vrot = v*cos(theta) + cross(k,v)*sin(theta) + k*(dot(k,v))*(1-cos(theta));
end
```

```
  100.0127
```

Published with MATLAB® R2021b