DC motor

formulas

$$U - E = R_m I + L_m \frac{dI}{dt} \tag{1.1}$$

U - motor voltage
E - voltage created by motor
R_m - motor resistance
I - current

$$E = K_e \omega_m \tag{1.2}$$

ω_m - angular velocity of the motor
K_e hopefully a constant

$$M = K_m (I - I_0) \tag{1.3}$$

M - moment from the motor
K_m - hopefully a constant
I_0 - no-load current (0.15A)

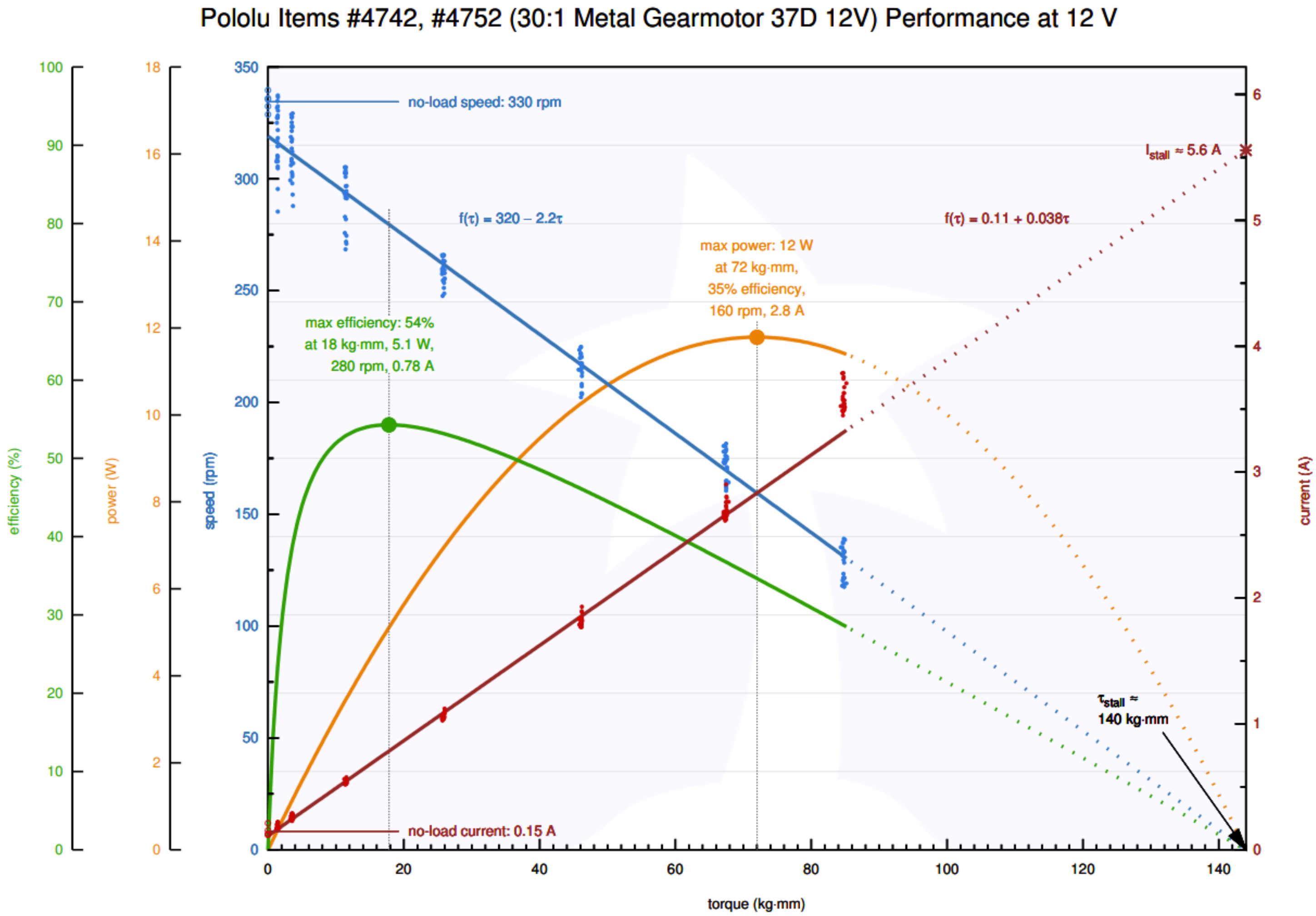
In case $L_m \approx 0$,

$$M = \frac{K_m}{R_m}(U - K_e \omega_m) \tag{1.4}$$

$$U = \frac{R_m}{K_m} M + K_e \omega_m \tag{1.5}$$

Motor parameters (from the plot)

from the specs of the motor : https://www.pololu.com/product/4691/specs
https://www.pololu.com/file/0J1736/pololu-37d-metal-gearmotors-rev-1-2.pdf



I_0 - no-load current 0.15A

$K_m \approx 0.03$

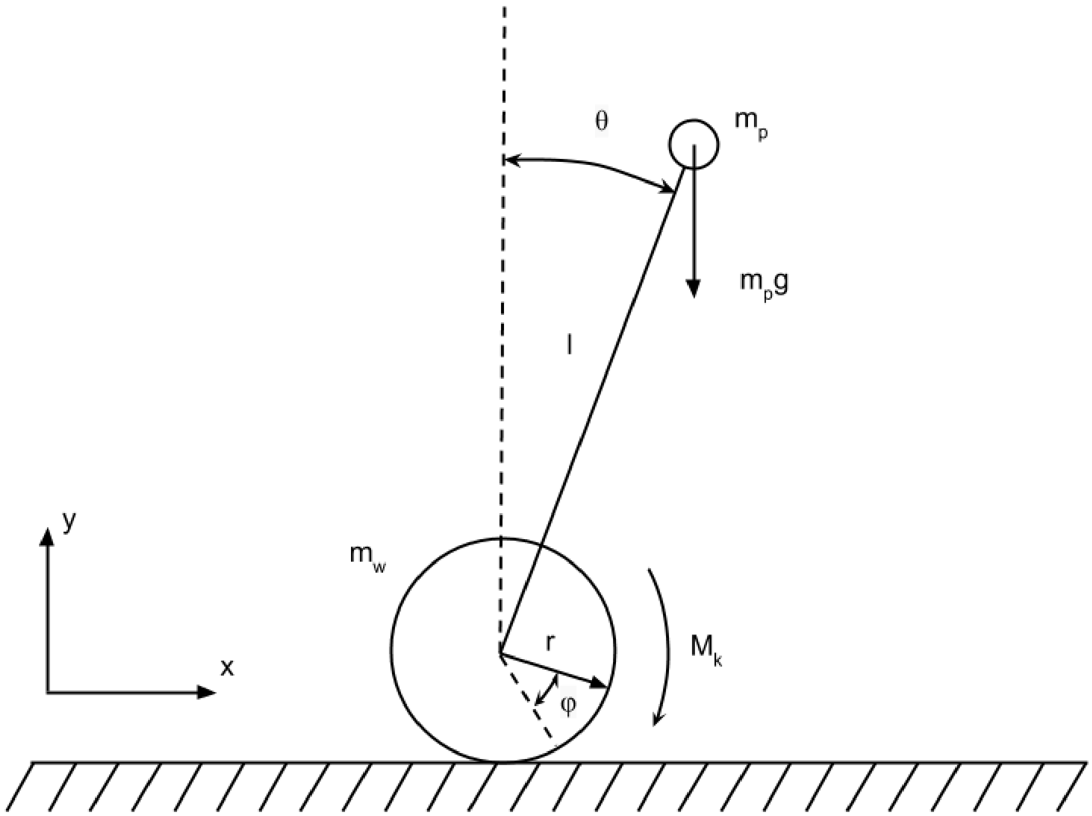
$K_e \approx 0.34$

$R_m \approx 2.18$ Om

pendulum

some of the info is taken from http://spin7ion.ru/ru/blog/balancerBuildSteps

lets consider "flat" (2D) model.



m_p robot mass (without wheels)
m_w mass of the wheels
l length (height?)
r wheel radius
θ angle between vertical and pendulum
φ wheel rotation angle
M DC motor moment
$w = w_p = \dot{\theta}$ some (part of the) rotation rate from the IMU
$w_w = \dot{\varphi}$ wheel rotation rate

wheel center

$$x = r\varphi$$
$$y = r$$

x, y coordinates of the center of the wheel.

x_p, y_p - coordinates of the "upper point":

$$x_p = x + lsin(\theta)$$
$$y_p = r + lcos(\theta)$$

Kinematic energy of the whole system:

$$T = T_w + T_p$$

$$T_w = \frac{m_w V_w^2}{2} + \frac{I_w w_w^2}{2} = \frac{m_w w_w^2}{2}(r^2 + r_i^2)$$

$V_w = \dot{x}$ velocity of the wheel
I_w wheel moment of inertia

$I_w = \sum m_i r_i^2$, for the ring it will be $I_w = m_w r_i^2$

In our case, r_i is some radius, probably $r_i > r$ (?). And there is a motor with a gearbox attached, BTW. We can try to estimate I_w, if we measure the motor current.

If θ is small, then $sin(\theta) \approx 0$ and formula is simple:

$$T_p = \frac{m_p V_p^2}{2} \approx \frac{m_p}{2}(rw_w + lw_p)^2$$

In case we have a noticiable θ:

$$T_p = \frac{m_p}{2}(\dot{y}_p^2 + \dot{x}_p^2) = \frac{m_p}{2}(l^2 sin^2(\theta) w_p^2 + (rw_w + lcos(\theta)w_p)^2)$$

$$T_p = \frac{m_p}{2}(l^2 sin^2(\theta) w_p^2 + r^2 w_w^2 + l^2 cos^2(\theta) w_p^2 + 2rlw_w w_p cos(\theta))$$
$$T_p = \frac{m_p}{2}(l^2 w_p^2 + r^2 w_w^2 + 2rlw_w w_p cos(\theta))$$

which is just a little bit smaller then in "simple case".

"full" kinetic energy:

$$T = \frac{m_w w_w^2}{2}(r^2 + r_i^2) + \frac{m_p}{2}(l^2 w_p^2 + r^2 w_w^2 + 2rlw_w w_p cos(\theta))$$

coordinates:

$$q = \begin{pmatrix} \varphi \\ \theta \end{pmatrix}$$

$$\frac{\delta T}{\delta \varphi} = 0$$

$$\frac{\delta T}{\delta \dot{\varphi}} = \frac{\delta T}{\delta w_w} = (m_w(r^2 + r_i^2) + m_p r^2) w_w + m_p r l w_p cos(\theta)$$

$$\frac{d}{dt}\frac{\delta T}{\delta \dot{\varphi}} = (m_w(r^2 + r_i^2) + m_p r^2)\varepsilon_w + m_p r l(\varepsilon_p cos(\theta) - w_p^2 sin(\theta))$$

$$\frac{\delta T}{\delta \theta} = -m_p r l w_w w_p sin(\theta)$$

$$\frac{\delta T}{\delta \dot{\theta}} = \frac{\delta T}{\delta w_p} = m_p l^2 w_p + m_p r l w_w cos(\theta)$$

Just in case that we need $\frac{d}{dt}\frac{\delta T}{\delta \theta}$:

$$(w_w w_p)'_t = \varepsilon_w w_p + \varepsilon_p w_w$$

$$((w_w w_p) sin(\theta))'_t = (\varepsilon_w w_p + \varepsilon_p w_w) sin(\theta) + w_w w_p^2 cos(\theta)$$

$$\frac{d}{dt}\frac{\delta T}{\delta \theta} = -m_p r l((\varepsilon_w w_p + \varepsilon_p w_w) sin(\theta) + w_w w_p^2 cos(\theta))$$

$$\frac{d}{dt}\frac{\delta T}{\delta \dot{\theta}} = m_p l^2 \varepsilon_p + m_p r l(\varepsilon_w cos(\theta) - w_w w_p sin(\theta))$$

forces:

$$\delta s_1 = r\delta\varphi$$

$$\delta A_1 = P_1 \delta\varphi$$

$$P_1 = M$$

$$\delta s_2 = l\delta\theta$$

$$\delta A_2 = P_2 \delta\theta = m_p g l \delta\theta sin(\theta)$$

$$P_2 = m_p g l sin(\theta)$$

the motion eq:

$$\frac{d}{dt}\frac{\delta T}{\delta \dot{q}} - \frac{\delta T}{\delta q} = Q$$

$$(m_w(r^2 + r_i^2) + m_p r^2)\varepsilon_w + m_p r l(\varepsilon_p cos(\theta) - w_p^2 sin(\theta)) = M$$

$$m_p l^2 \varepsilon_p + m_p r l(\varepsilon_w cos(\theta) - w_w w_p sin(\theta)) + m_p r l w_w w_p sin(\theta) = m_p g l sin(\theta)$$

$$m_p l^2 \varepsilon_p + m_p r l \varepsilon_w cos(\theta) - m_p g l sin(\theta) = 0$$

$$l\varepsilon_p + r\varepsilon_w cos(\theta) - g sin(\theta) = 0$$

from the http://spin7ion.ru/ru/blog/balancerBuildSteps, a bit different result:

$$r l m_p cos(\theta)\varepsilon_p + r^2(m_p + 2m_w)\varepsilon_w - r l m_p sin(\theta)\theta^2 = M_k$$

$$\varepsilon_w cos(\theta) l m_p r - m_p g l sin(\theta) + 2m_p l^2 \varepsilon_p = 0$$