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Worksheet-1

MCQs

1) b → $P(k) = m^{(k)} + 5$
2) c → Trivial Proof
3) d → $m^3 + 3m$
4) d → if not S then not H.

Descriptive Questions:

1) $2^{2n} - 1$ is divisible by 3

S1) Let statement is true for $n = 1$

$= 2^{2 \times 1} - 1$

$= 2^2 - 1 = 4 - 1$

$\Rightarrow 3$ divisible by 3

S2) Let it is true for $n = k$

S3) Let $2^k - 1 = 3q$, divisible by 3

$n = k+1$

$= 2^{2k+2} - 1$

$= 2^{2k} \cdot 4 - 1 = 3 \cdot 2^{2k} + 3q$

$= 3(2^{2k} + q) = 3m$

$\therefore P(k+1)$ is true when $P(k)$ is true

2) $a^2 = 3K$, a^2 divisible by 3

$a = (3K+1)$ or $(3K+2)$ a divisible by 3

Let a^2 is divisible by 3, $a^2 = 3K$

a is divisible by 3, $a = (3K+1)$ or $(3K+2)$

True

$a^2 = (3K+1)^2$ or $(3K+2)^2$

$a^2 = (9K^2+6K+1)$ or $(9K^2+\cancel{12}K+4)$ $\cancel{+12}$

$a^2 = 3\underbrace{(3K^2+2K)}_{C}+1$ or $3\underbrace{(3K^2+4K+1)}_{K}+1$

$a^2 = 3C+1$ or $3K+1$

a^2 is not divisible by 3

a is divisible by 3

3) Assume following cases

a	b	a + b
odd	odd	even
even	odd	odd
odd	even	odd
even	even	even

let a = odd, b = odd

$a = 2K-1$

$b = 2K-1$

$a+b = 2K-1+2K-1$

$= 4K-1$

$= 2(2K \cdot 1)$ is even.

∴ a+b is odd if either of them is odd

4)

$P = 2n^2 - 16n + 31$

put $n = 4$

$= 2 \times 4^2 - 16 \times 4 + 31$

$= -1 < n$

$\therefore$ It is not always true

5) $n \geq 5$, $2^n > n^2$

Let it is true for $n > 5$

$2^{n+1} = 2 \times 2^n = 2 \times n^2 = (n+1)^2$ — ①

We know,

$(n-1)^2 = 4^2 > 2$

$(n-1)^2 > 2$

$n^2 - 2n + 1 > 2$

$2n^2 - 2n - 1 > n^2$

$2n^2 > n^2 + 2n + 1 = (n+1)^2$

which is same is ①

Hence proved.

6) For $n = 1$

$LHS = 1^2 = 1$

$RHS = \frac{1(1+1)(2+1)}{6} = \frac{2 \times 3}{6} = 1$

Assume statement is true for $n = k$

i.e. $1^2 + 2^2 + 3^2 + \dots + k^2 = \frac{k(k+1)(k+2)}{6}$

Let $P(K)$ is true, prove true for $n = k+1$

$$1^2 + 2^2 + 3^2 + \cdots\cdots = \frac{(K+1)(K+2)(2K+3)}{6}$$

$$= \frac{K(K+1)(2K+1)}{6} + (K+1)^2$$

$$= \frac{K(K+1)(2K+1)}{6} + \frac{6(K+1)^2}{6} = \frac{(K+1)K(2K+1) + 6(K+1)}{6}$$

$$= \frac{(K+1)(K+2)(2K+3)}{6}$$

True for $n=1$, $n=K$

& true for $n = K+1$

$$\sum_{r=1}^{n} r^2 = 1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Am

Worksheet - 2

MCQs

1) (i) Set of all strings starting & Ending with '10' & any number of 1's in between '10'.

2) (ii) $L = (0+1)^* 1001 (0+1)^*$

3) (iii) (ii) and (iii)

4) (i) Regular languages

5) c) The set of all strings containing at least two 0's.

Part-B

1) Language general by RE $0^+(101)^*11$ string → is any no. of 0's followed by (101) any no. of times & end with 11.

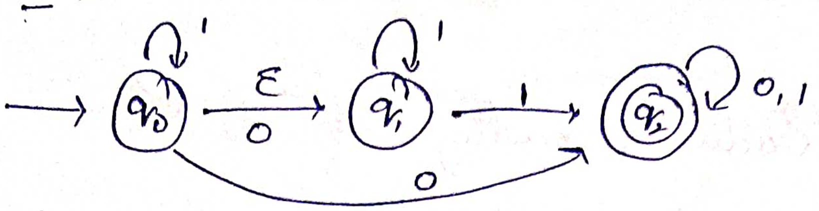
   Eg: (011) or (01011 0111)

2) Identify the RE for →
   any combination of 0's & 1's begning & Ending with '01'.

   $RE = (01)(0+1)^*(01)$

3) RE for set of all strings which contains repeated substrings of any length.
   The FSA can not store any memory to check for repeating symbol so we can write a R.E.

4) Given,



E - closure of q_0

$\varepsilon^* \; 0 \; \varepsilon^*$ $\qquad\qquad$ $\varepsilon^* \; 1 \; \varepsilon^*$

$q_0 \rightarrow q_1 \rightarrow \phi \rightarrow \phi$ $\qquad\qquad$ $q_0 \rightarrow q_1 \rightarrow q_1 \rightarrow q_1$

so ε^* of $q_0 \rightarrow \{q_0, q_1\}$

5) 5 Tuples structures of NFA & DFA

i) DFA $\rightarrow \{Q, \Sigma, q_0, F, \delta\}$

$Q \rightarrow$ Set of all states

$\Sigma \rightarrow$ input symbols

$q_0 \rightarrow$ initial state

$F \rightarrow$ final state

$\delta \rightarrow$ Transition funct^n

$f: n \rightarrow Q \times \Sigma = Q$

ii) NFA $\rightarrow \{Q, \Sigma, q_0, F, \delta\}$

Definition same as DFA

$f: Q \times \Sigma = 2^Q$

Worksheet-3

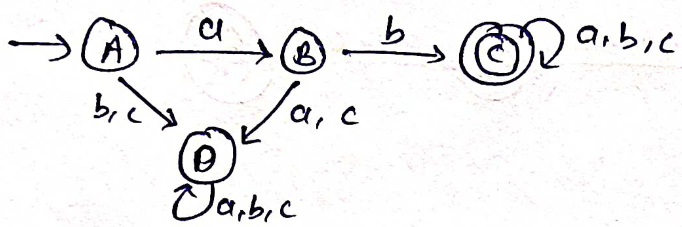
MCQs

1) b → 2
2) ~~a → h~~
3) ~~e →~~ i a → n
4) c → 15
5) a → increase computation
6) c → 2n
7) c → I is false & II is true
8) d → DFA is more powerful
9) a → 5
10)

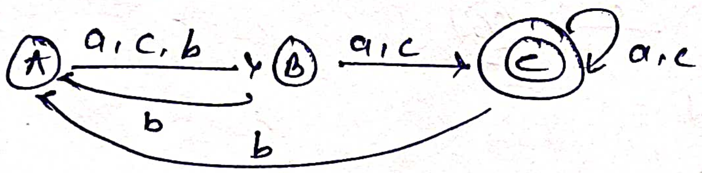
Part B

1) Pass start with 'ab'

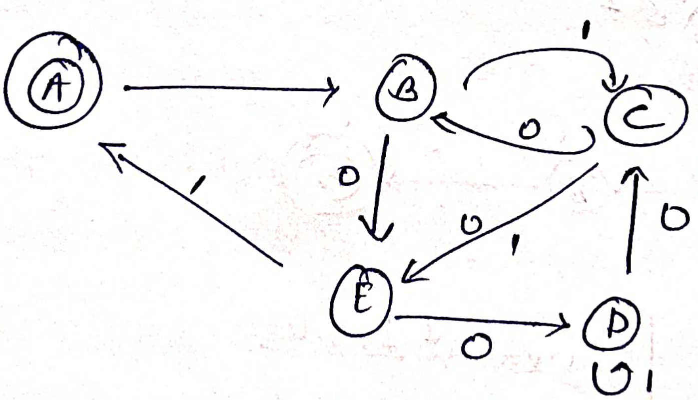
a)



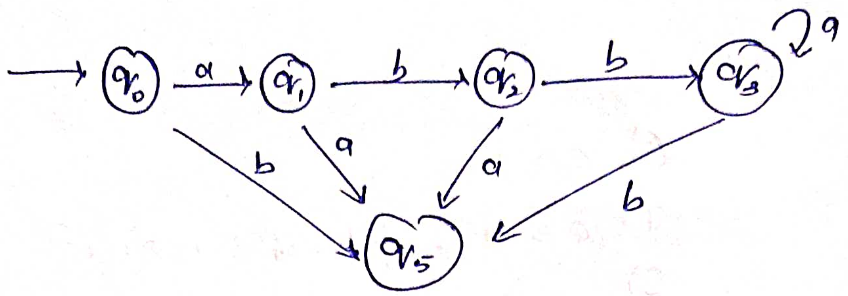
b) not end with 'bb



2) Multiple of 5

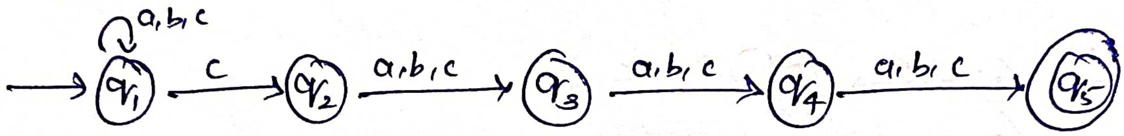


3) DFA

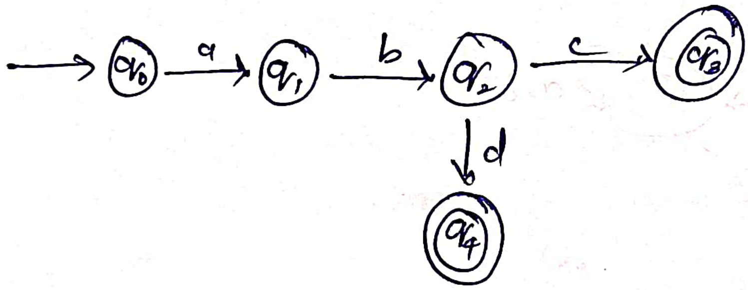


NFA

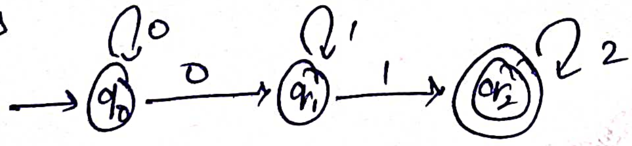
4) NFA



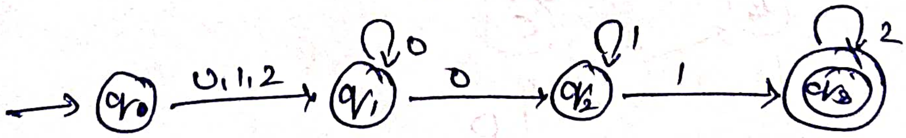
7) for string 'abc' & 'abd'



5) yes



6) yes

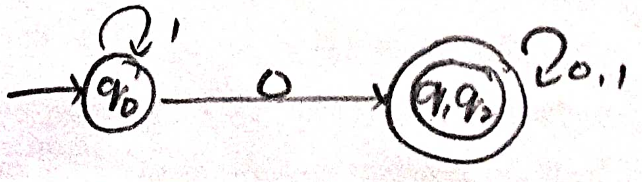


8).

δ	0	1
q_0	$\{q_1, q_2\}$	$\{q_0\}$
q_1	$\{q_1, q_2\}$	ϕ
q_2	$\{q_1\}$	$\{\phi, q_1\}$

Equivalent table

δ	0	1
q_0	$q_1 q_2$	q_0
$q_1 q_2$	$q_1 q_2$	$q_1 q_2$



Worksheet 4

MCQs

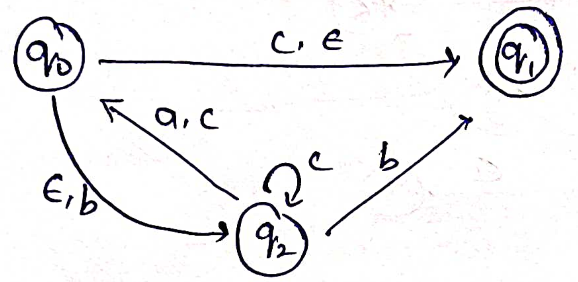
1) Diff between DFA & NFA is that we get multiple choices in NFA So if we have a language satisfying a DFA. it will also satisfy its Equivalent NDFA.

2) 2^n 3) $n \cdot 2^n$ 4). $\delta = Q \times (\Sigma \cup \epsilon) \to P(Q)$

5) Same ~~init~~ final states as ϵ-NFA

Part-B

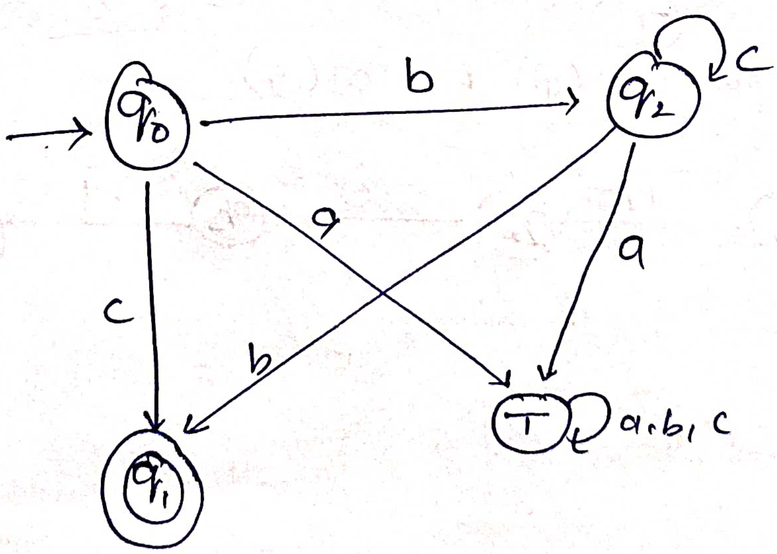
1)



	a	b	c
q_0	ϕ	q_2	q_1
$*q_1$	ϕ	ϕ	ϕ
q_2	ϕ	q_1	q_2

DFA-Table:

	a	b	c
q_0	T	q_2	q_3
q_1	T	T	T
q_2	T	q_3	q_3

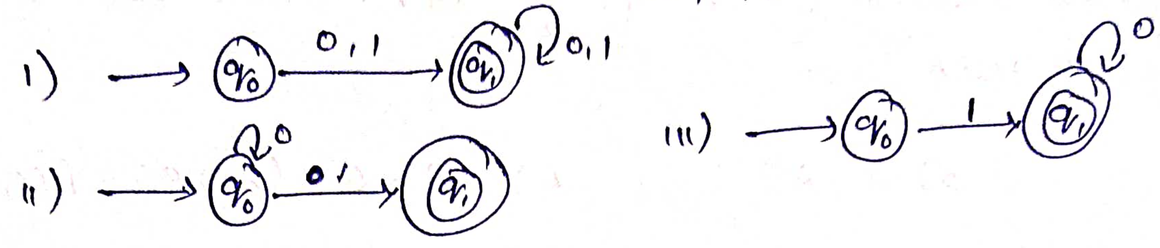


2) Given $L_1 = 01^*$ $L_2 = 0^*1$

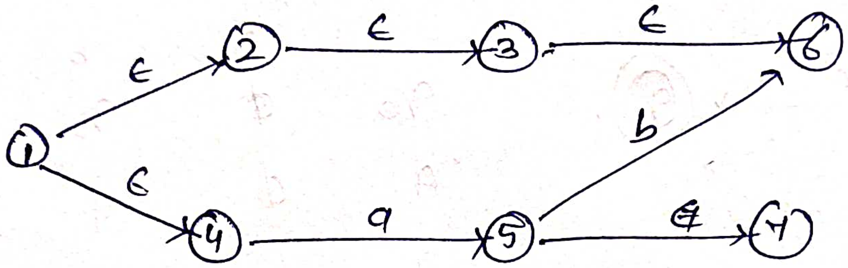
Design Automata for → i) $(01^* + 0^*1)$

ii) 0^*1

iii) 10^*



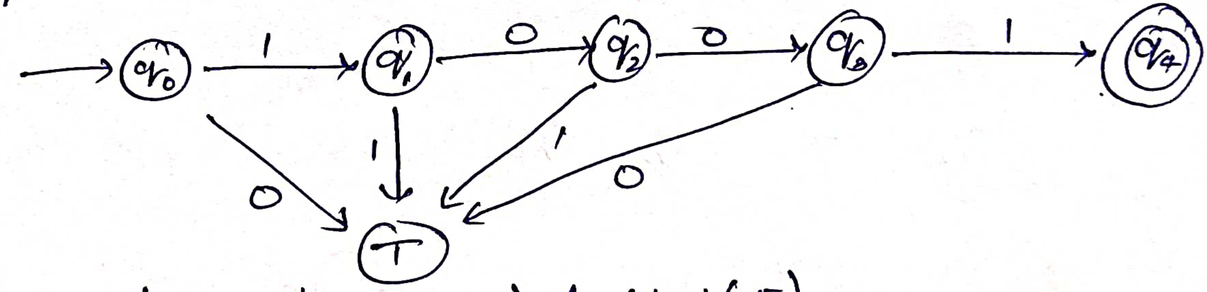
3) Find ε-Closure



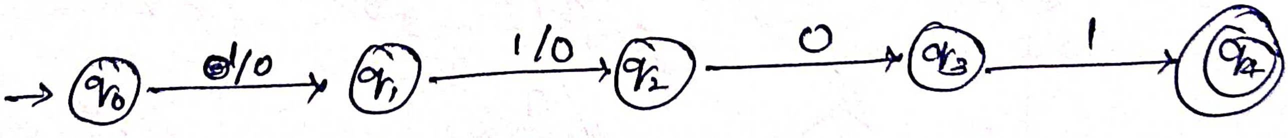
ε-closure (1) = {1, 2, 3, 6, 4}

4) Binary of : 1001

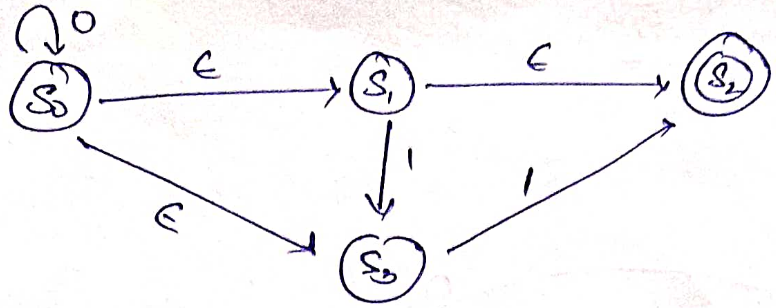
DFA:



NFA that accepts 1001 (9) & 0101 (5)



5)

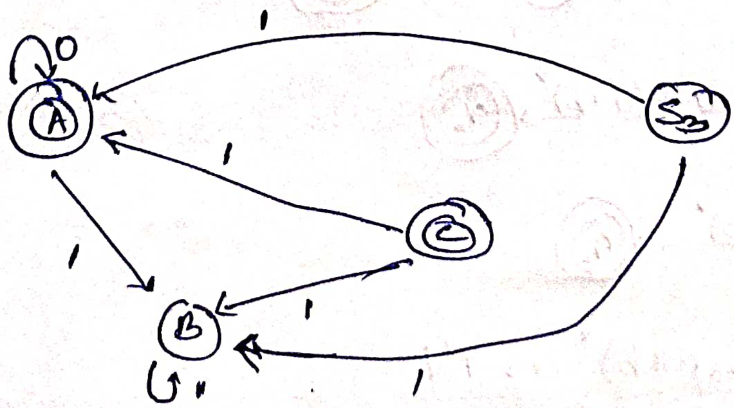


	ϵ^*	0	ϵ^*
S_0	s_0	s_0	$\to s_0$
	s_1	ϕ	$\to s_1$
	s_3	ϕ	$\to s_3$
S_1	$\to s_1$	ϕ	ϕ
	$\to s_2$	ϕ	ϕ
S_2	s_2	ϕ	ϕ
S_3	s_3	ϕ	ϕ

	ϵ^*	1	ϵ^*
S_0	s_0	ϕ	
	s_1	s_3	$\to s_3$
	s_3	s_2	$\to s_2$
S_1	$\to s_1$	s_3	s_3
	$\to s_2$	ϕ	ϕ
$s_2 \to$	s_2	ϕ	ϕ
$s_3 \to$	s_3	s_2	s_2

	0	1
S_0	$\{S_0, S_1, S_2, S_3\}$	$\{S_2, S_3\}$
S_1	$\{\phi\}$	$\{S_3\}$
S_2	$\{\phi\}$	$\{\phi\}$
S_3	$\{\phi\}$	$\{S_2\}$

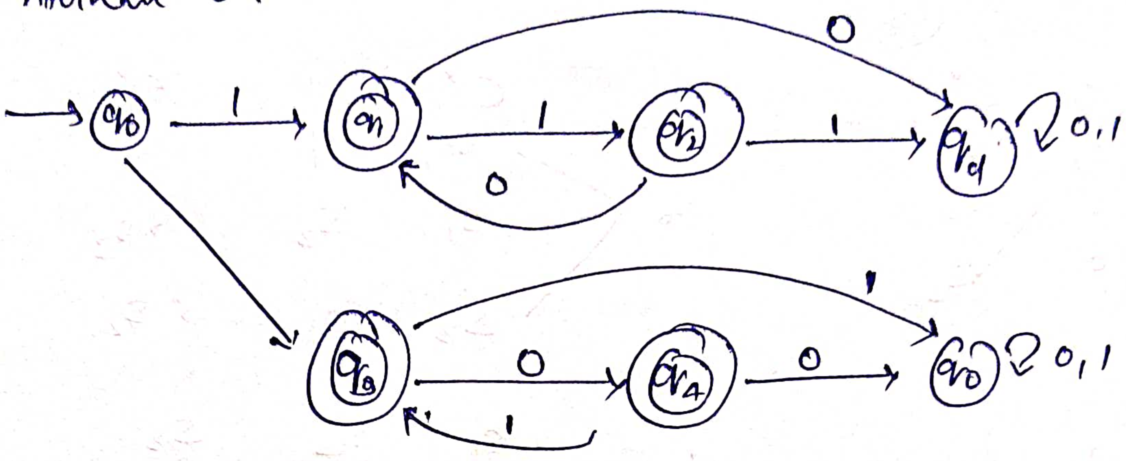
NFA:



Worksheet · 5

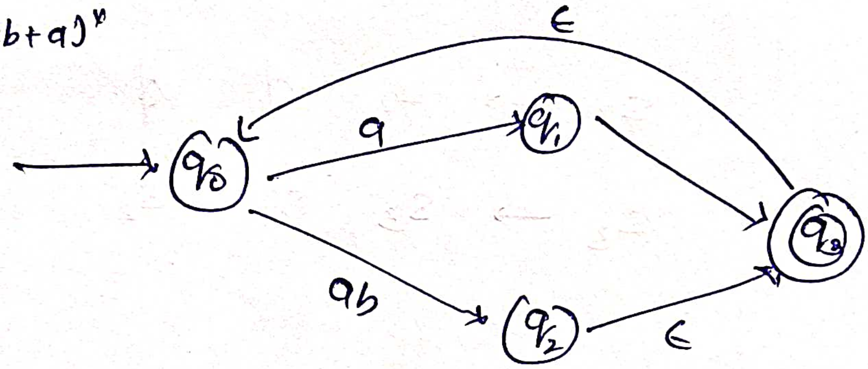
~~NFAs~~ Part B

1) Alternate of 1

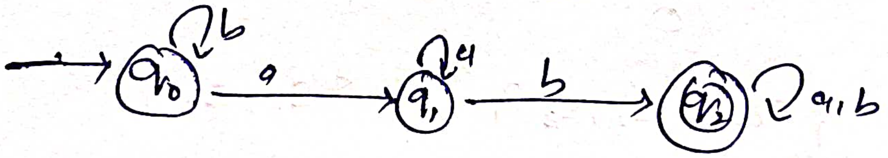


2) $L = \{0^n . 1^n (n \geq 1)\}$ is not a Regular Expression

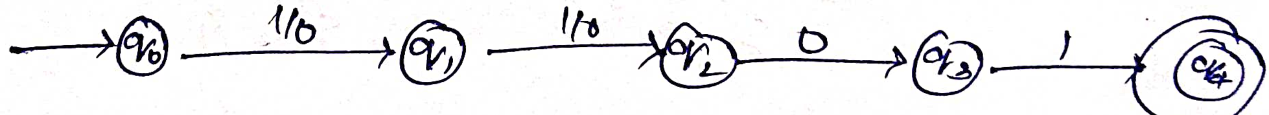
3) $(ab+a)^*$



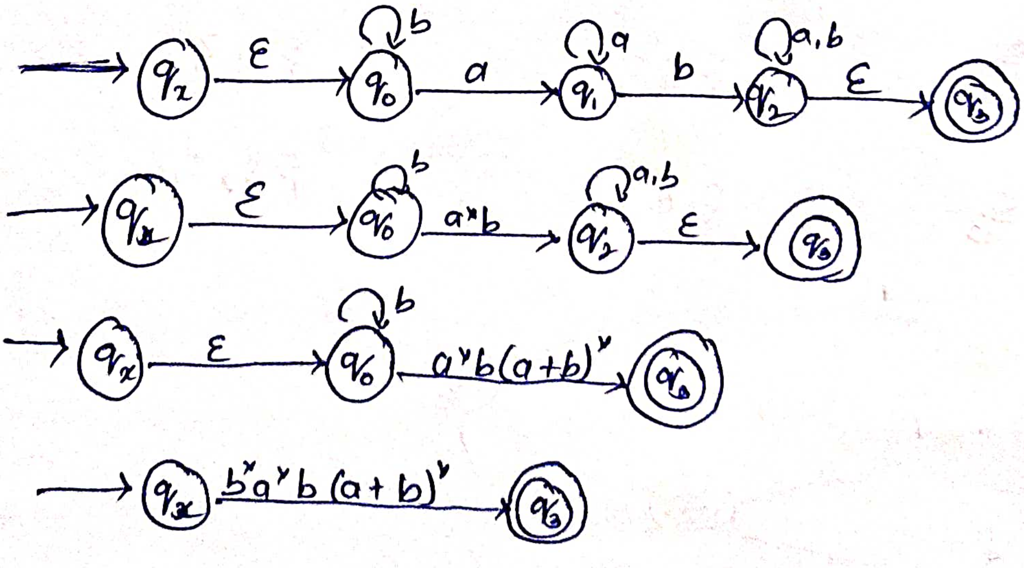
5)



8)

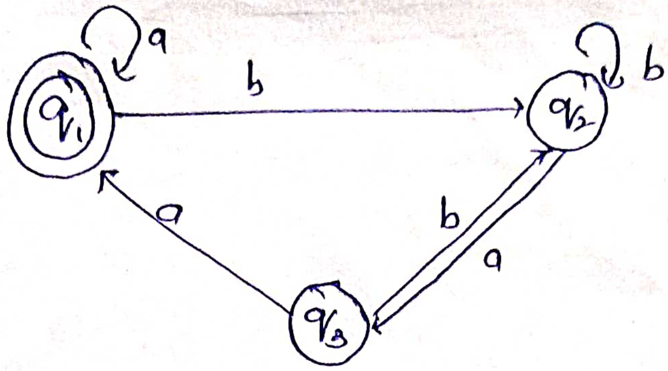


6)



$\therefore\ b^*a^*b(a+b)^*$ is R·E

7)



$q_1 = \varepsilon + q_1 a + q_3 a$ — (I)

$q_2 = q_1 b + q_2 b + q_3 b$ — (II)

$q_3 = q_2 a$ — (III)

using (III) & (I)

$q_1 = \varepsilon + q_1 a + q_2 aa$

$q_2 = \varepsilon + \underbrace{q_2 aa}_{Q} + \underbrace{q_1}_{R}\underbrace{a}_{P}$

$q_1 = (\varepsilon + a, aa) a^*$

using (I) & (III) in (II)

$q_2 = q_1 b + q_2 b + q_3 b$

$= (\varepsilon * q_2 \cdot aa) a^* + q_2 b + q_2 ab$

$= (\varepsilon * q_2)(a^* + b + ab)$

$q_2 = (\varepsilon * (\varepsilon \cdot (a^* + b + ab^*) \cdot aa)) a^*$

4) Thompson's Rule → Not discussed