

$$p(\phi_{ks} | w, z, \theta, \Phi_{ks}, \alpha, \beta, \gamma, \Pi, l)$$

$$\propto p(\phi_{ks}, w, z, \theta, \Phi_{ks}, \Pi, l | \alpha, \beta, \gamma)$$

$$= p(\Phi | \beta) p(\theta | \alpha) p(w | z, \Phi, l) p(z | \theta, \alpha, l) \\ \times p(l | \Pi, \gamma) p(\Pi | \gamma)$$

$$\propto \prod_{k=1}^K \prod_{s=1}^S p(\phi_{ksv} | \beta_v) \prod_{d=1}^D \left\{ \prod_{n=1}^{N_d} p(w_{dn} | z_{dn}, l_{dn}, \Phi) \right\}$$

$$\propto \prod_{k=1}^K \prod_{s=1}^S \prod_{v=1}^V \phi_{ksv}^{\beta_v - 1} \prod_{d=1}^D \prod_{n=1}^{N_d} \prod_{k=1}^K \prod_{v=1}^V \prod_{s=1}^S \phi_{ksv}^{\delta(z_{dn}=k, l_{dn}=s, w_{dn}=v)}$$

$$= \prod_{k=1}^K \prod_{s=1}^S \left\{ \prod_{v=1}^V \phi_{ksv}^{(N_{ksv} + \beta_v - 1)} \right\}$$

N_{ksv} は、

k の s 単語 v

の総出現数

$$p(\theta_{ds} | w, z, \mathbb{W}_{ds}, \Phi, \alpha, \beta, \gamma, \Pi, \ell)$$

$$\propto p(\theta_{ds}, w, z, \mathbb{W}_{ds}, \Phi, \Pi, \ell | \alpha, \beta, \gamma)$$

$$= p(\mathbb{W} | \alpha) p(z | \mathbb{W}, \alpha, \ell) p(w | z, \Phi, \ell) p(\Phi | \beta) \\ \times p(\ell | \Pi, \gamma) p(\Pi | \gamma)$$

$$\propto \prod_{d=1}^D \prod_{s=1}^S p(\theta_{ds} | \alpha) \prod_{d=1}^D \prod_{n=1}^{N_d} p(z_{dn} | \theta_d, \ell_{dn})$$

$$\propto \prod_{d=1}^D \prod_{s=1}^S \left\{ \prod_{k=1}^K \theta_{dks}^{\alpha-1} \right\} \prod_{d=1}^D \prod_{n=1}^{N_d} \prod_{s=1}^S \left\{ \prod_{k=1}^K \theta_{dks}^{\delta(z_{dn}=k, \ell_{dn}=\ell)} \right\}$$

$$= \prod_{d=1}^D \prod_{s=1}^S \left\{ \prod_{k=1}^K \theta_{dks}^{N_{d,k,s} + \alpha - 1} \right\}$$

$N_{d,k,s}$ は、文書 d に s 回 k が ℓ 回出現する回数は

割りと振る総単語数

$$p(\pi_d | w, z, \Theta, \Phi, \ell, \alpha, \beta, \gamma, \pi_d)$$

$$\propto p(\pi_d, w, z, \Theta, \Phi, \pi_d, \ell | \alpha, \beta, \gamma)$$

$$= p(\pi | \gamma) p(\ell | \pi, \gamma) p(\Theta | \alpha) p(w | z, \Phi, \ell) \\ \times p(\Phi | \beta) p(z | \Theta, \alpha, \ell)$$

$$\propto \prod_{d=1}^D \{ p(\pi_d | \gamma) \prod_{n=1}^{N_d} p(\ell_{dn} | \pi_d) \}$$

$$\propto \prod_{d=1}^D \left\{ \sum_{s=1}^{\gamma} \pi_{ds}^{\gamma-1} \prod_{n=1}^{N_d} \pi_{ds}^{\delta(\ell_{dn}=s)} \right\}$$

$$= \prod_{d=1}^D \left\{ \sum_{s=1}^{\gamma} \pi_{ds}^{N_{ds} + \gamma - 1} \right\}$$

$N_{d,s}$ は、文書 d において、 s 番目の語に割り当てられた

総単語数

$$p(z_{dn} = k, l_{dn} = s \mid z_{1dn}, l_{1dn}, w, \alpha, \beta, \gamma)$$

$$\propto p(w_{dn} = v \mid w_{1dn}, z_{dn} = k, z_{1dn}, l_{dn} = s, l_{1dn}, \beta)$$

$$\times p(z_{dn} = k \mid z_{1dn}, l_{dn} = s, l_{1dn}, \alpha) \times p(l_{dn} = s \mid l_{1dn}, \gamma)$$

$$\propto \int_{\phi} p(w_{dn} = v \mid \phi_{ks}) p(\phi_{ks} \mid w_{1dn}, z_{1dn}, l_{1dn}, \beta) d\phi_{ks}$$

$$\times \int_{\theta} p(z_{dn} = k \mid \theta_{ds}) p(\theta_{ds} \mid z_{1dn}, l_{1dn}, \alpha) d\theta_{ds}$$

$$\times \int_{\pi} p(l_{dn} = s \mid \pi_d) p(\pi_d \mid l_{1dn}, \gamma) d\pi_d.$$

$$= \mathbb{E}_{p(\phi_{ks} \mid z_{1dn}, l_{1dn}, \beta)} [\phi_{ks} v] \times \mathbb{E}_{p(\theta_{ds} \mid z_{1dn}, l_{1dn}, \alpha)} [\theta_{dks}]$$

$$\times \mathbb{E}_{p(\pi_d \mid l_{1dn}, \gamma)} [\pi_{ds}]$$

$$= \frac{N_{ksv}^{1dn} + \beta}{N_{ks}^{1dn} + V\beta} \cdot \frac{N_{d,ks}^{1dn} + \alpha}{N_{d,s}^{1dn} + K\alpha} \cdot \frac{N_{ds}^{1dn} + \gamma}{N_d^{1dn} + S\gamma}$$