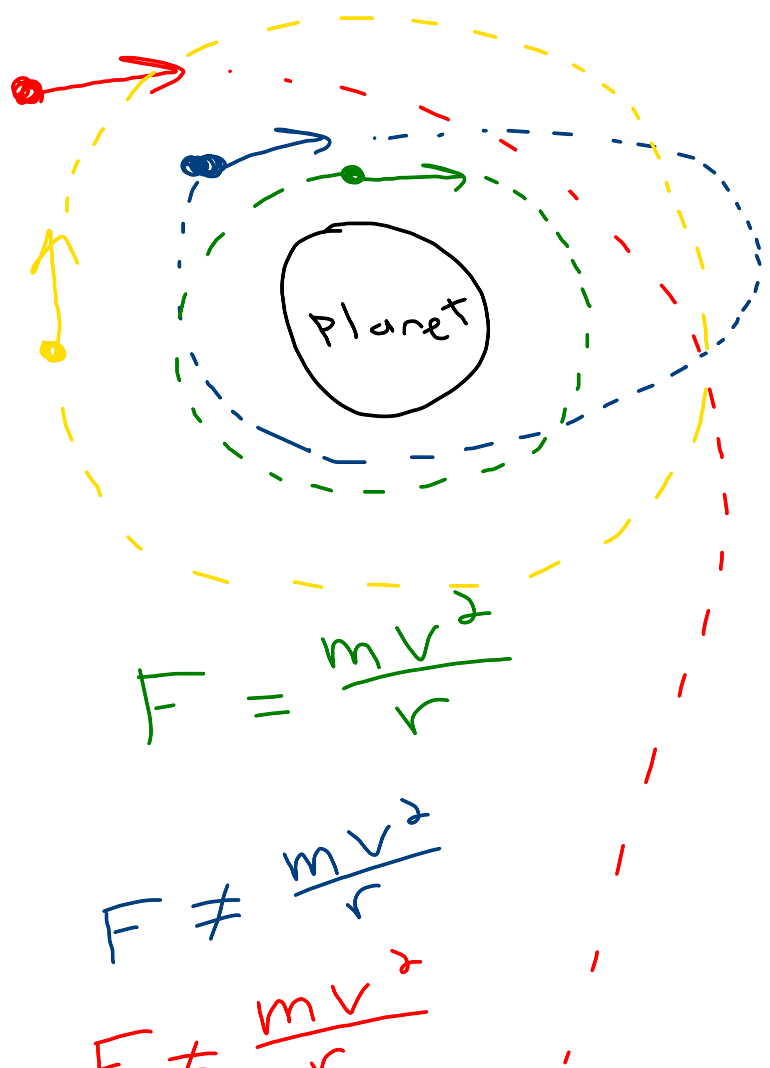


$F = \frac{mv^2}{r}$

$F \neq \frac{mv^2}{r}$

$F \neq \frac{mv^2}{r}$

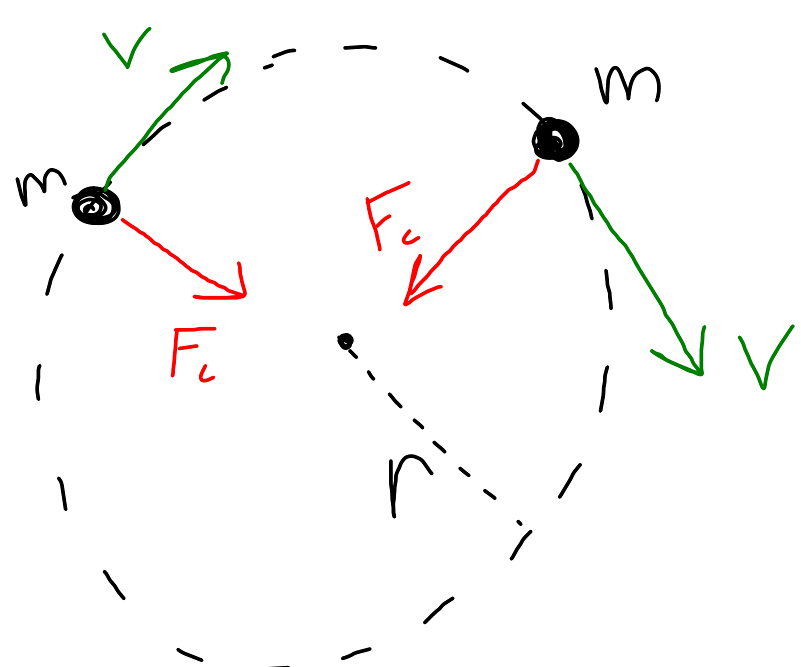
$F = \frac{mv^2}{r}$

$$\boxed{F_{centripetal} = \frac{mv^2}{r}}$$

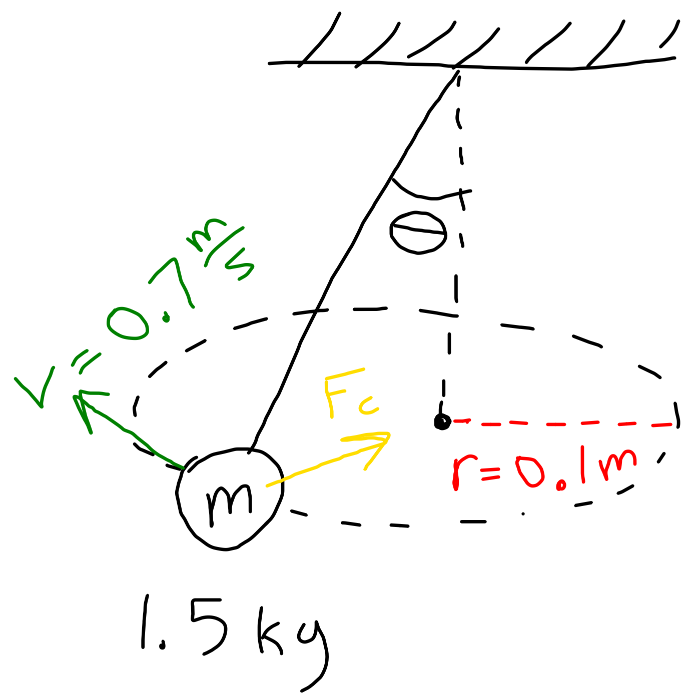
condition for circular motion

* Additional requirements:

- F_c must point towards the center of the circle
- v must always be tangent to the circle

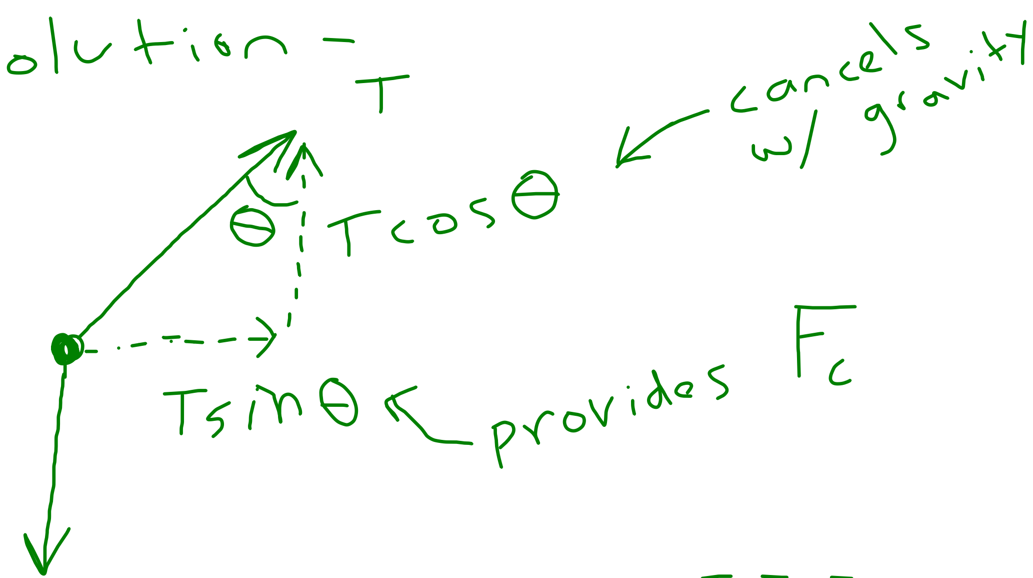


Example: "The conical pendulum"



- what is the tension in the rope?
- what is θ?

Solution -



$$\Longrightarrow \quad T\cos\theta = mg$$

$$T\sin\theta = \frac{mv^2}{r}$$

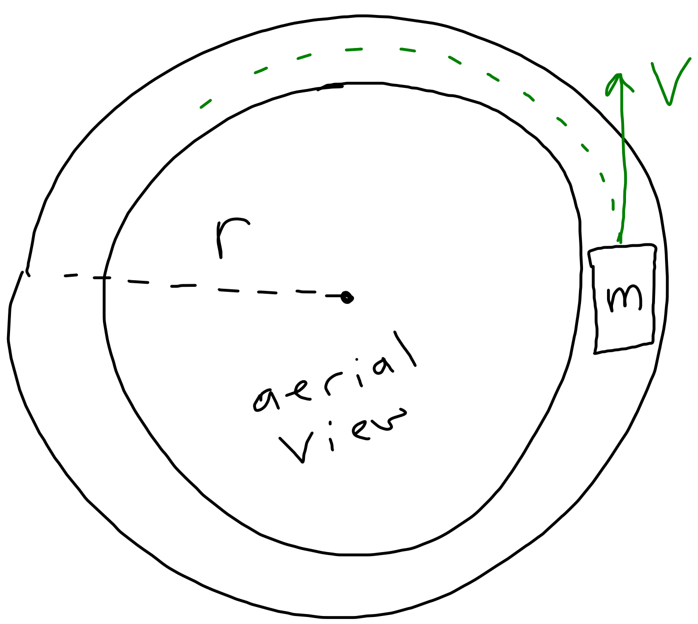
take ratio!

$$\tan\theta = \frac{\cancel{m}v^2}{r} \cdot \frac{1}{\cancel{m}g}$$

$$\theta = \arctan\left(\frac{v^2}{rg}\right) = \boxed{\theta = 26.56^\circ}$$

$$T = \frac{mg}{\cos\theta} = \boxed{T = 16.44\,N}$$

Example: car driving at constant speed around circular track:



if $m = 1500\ kg$

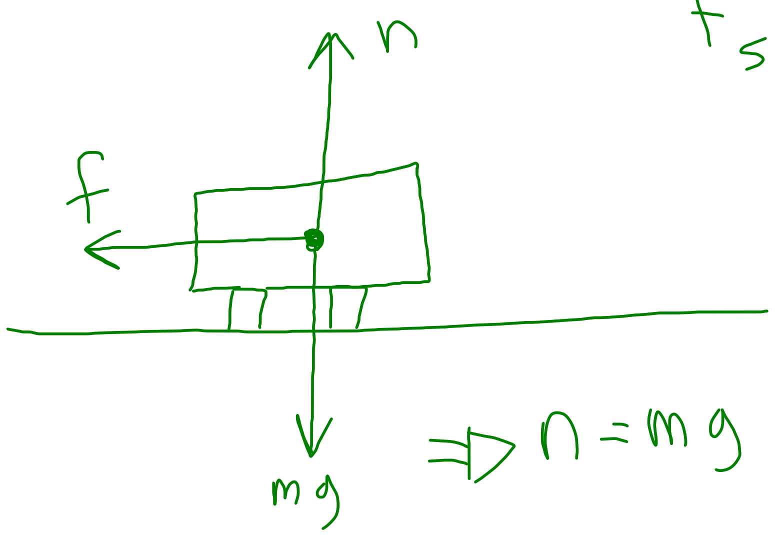
$r = 35\ m$

$\mu_s = 0.523$

• Find V_{max}.

solution –

rear view:



$\Rightarrow n = mg$

$$f_{s,max} = \mu_s n$$

$$= \mu_s (mg)$$

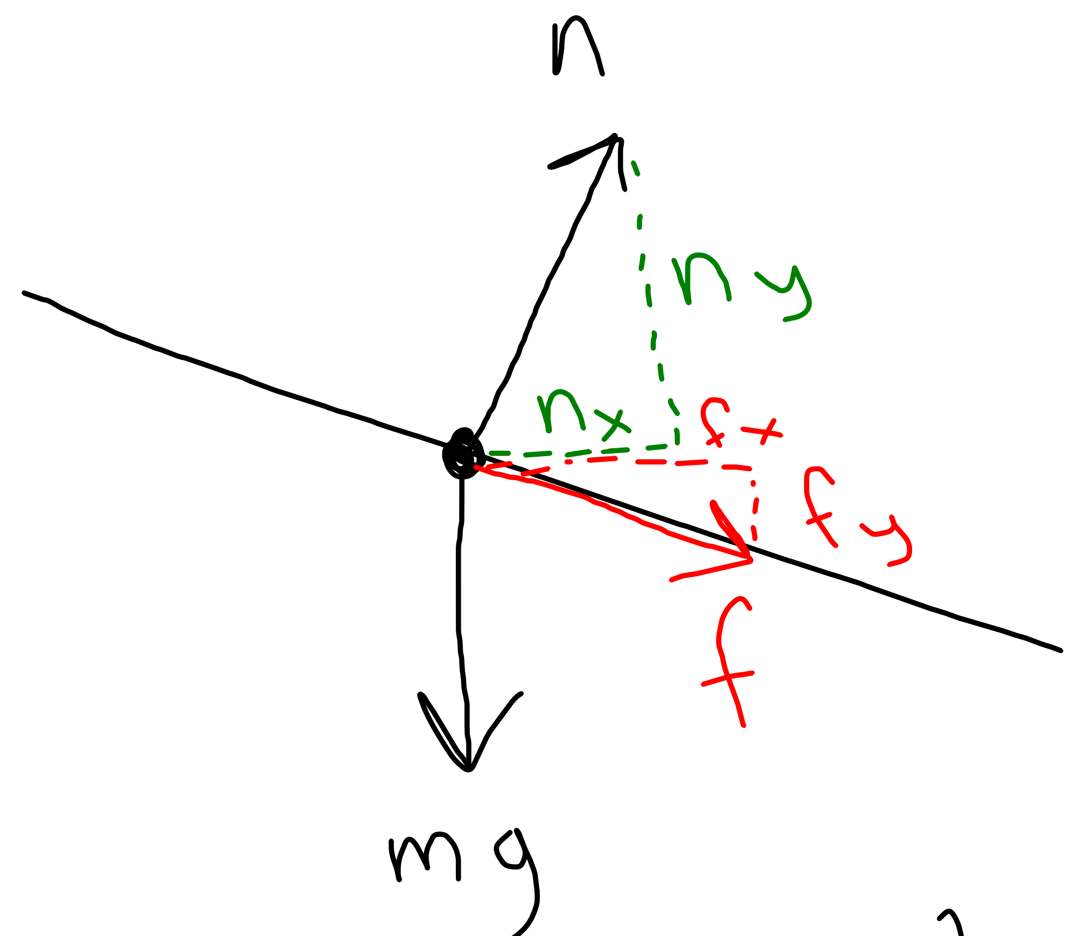
this provides the centripetal force

$$\mu_s (\cancel{m} g) = \frac{\cancel{m} V_{max}^2}{r}$$

$$V_{max} = \sqrt{\mu_s g r}$$

$$\boxed{V_{max} = 13.4\ m/s}$$

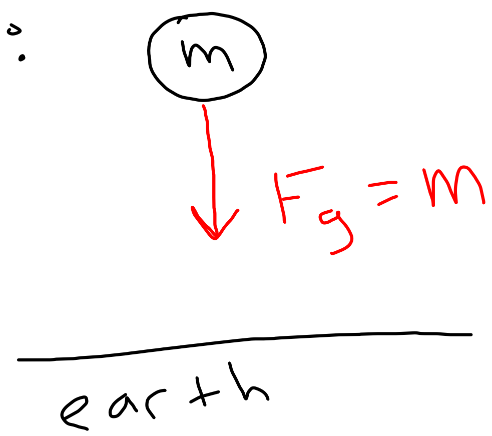
banked turns



$$n_x + f_x = \frac{mv^2}{r}$$

Gravity

Recall:



$F_g = mg$ where $g = 9.8 \frac{m}{s^2}$

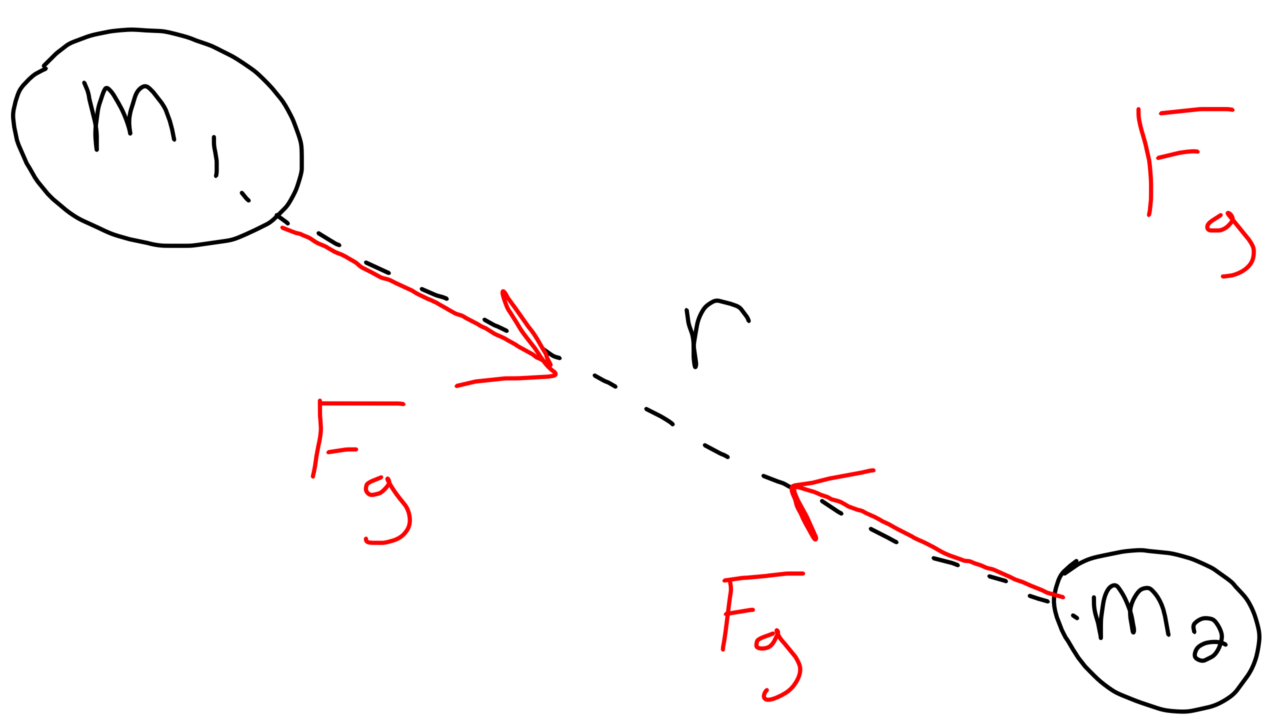
* approximation near the surface of the earth. Not good enough for larger distances (e.g. sun-earth distance).

$$\boxed{F_g = G\frac{M_1 M_2}{r^2} \quad \text{(magnitude)} \qquad \text{direction: attractive}}$$

Newton's Universal Law of Gravitation

G is called the Gravitational constant and has value:

$$G = 6.67 \times 10^{-11}\ N \cdot m^2 / kg^2$$



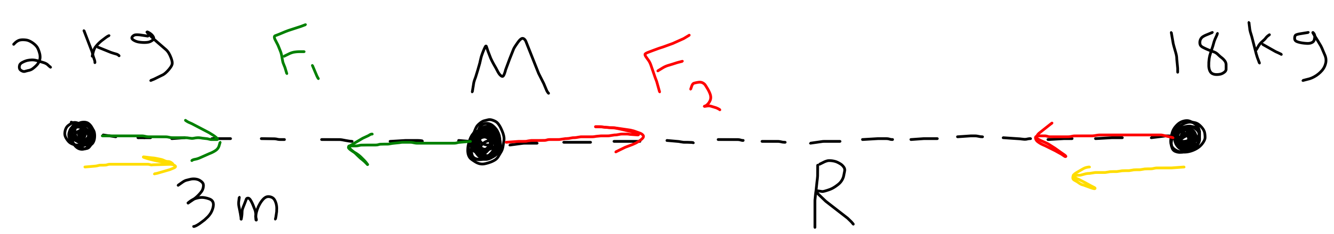
$$F_g = G\frac{m_1 m_2}{r^2}$$

Note: $$F_g = m\underbrace{\left(\frac{G M_{earth}}{R_{earth}^2}\right)}_{\dfrac{\left(6.67\times10^{-11}\frac{m^{\cancel{3}}}{\cancel{kg}\cdot s^2}\right)\left(5.972\times10^{24}\cancel{kg}\right)}{\left(6.371\times10^{6}\cancel{m}\right)^2}}$$

$$= 9.8\frac{m}{s^2}$$

Altitude	a_g
0 km	$9.83 \frac{m}{s^2}$
Mt. Everest (8.8 km)	$9.80 \frac{m}{s^2}$
Space shuttle in orbit (400 km)	$8.70 \frac{m}{s^2}$
Satellite (35,700 km)	$0.225 \frac{m}{s^2}$

Example:



If F_{total} on $M = 0$, what is R?

$$F_1 = G\frac{(2kg)M}{(3m)^2} \qquad F_2 = G\frac{(18kg)M}{R^2}$$

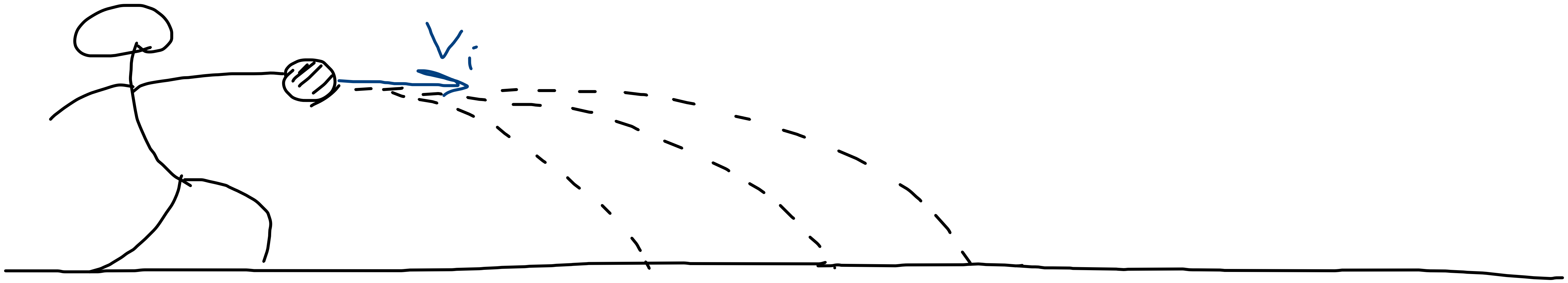
$$F_1 = F_2$$

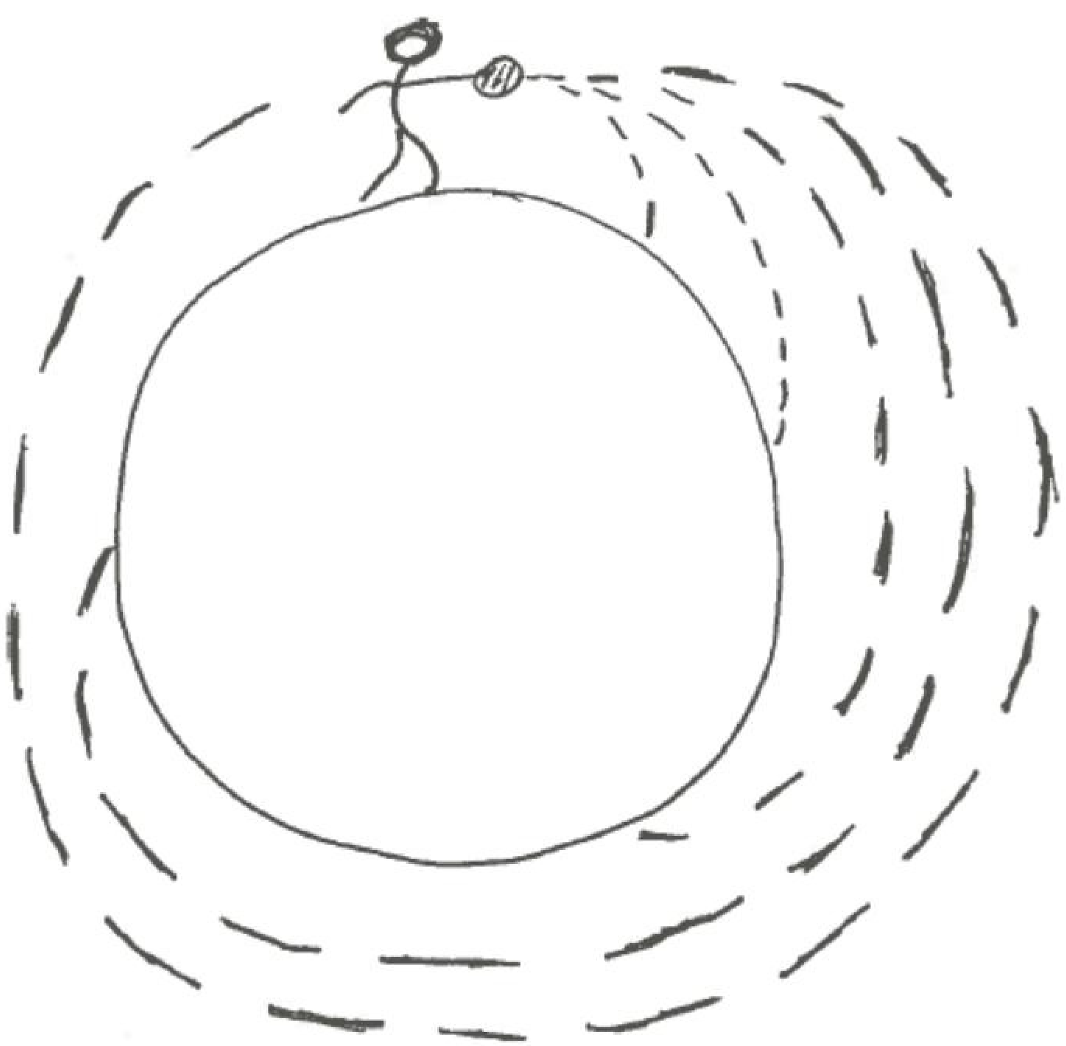
$$\cancel{G}\frac{(2)\cancel{M}}{9} = \cancel{G}\frac{(18)\cancel{M}}{R^2}$$

$$\frac{2}{9} = \frac{18}{R^2} \Longrightarrow 2R^2 = (9)(18)$$

$$R = \sqrt{\frac{(9)(18)}{2}} = \boxed{9\ m}$$

Orbital motion





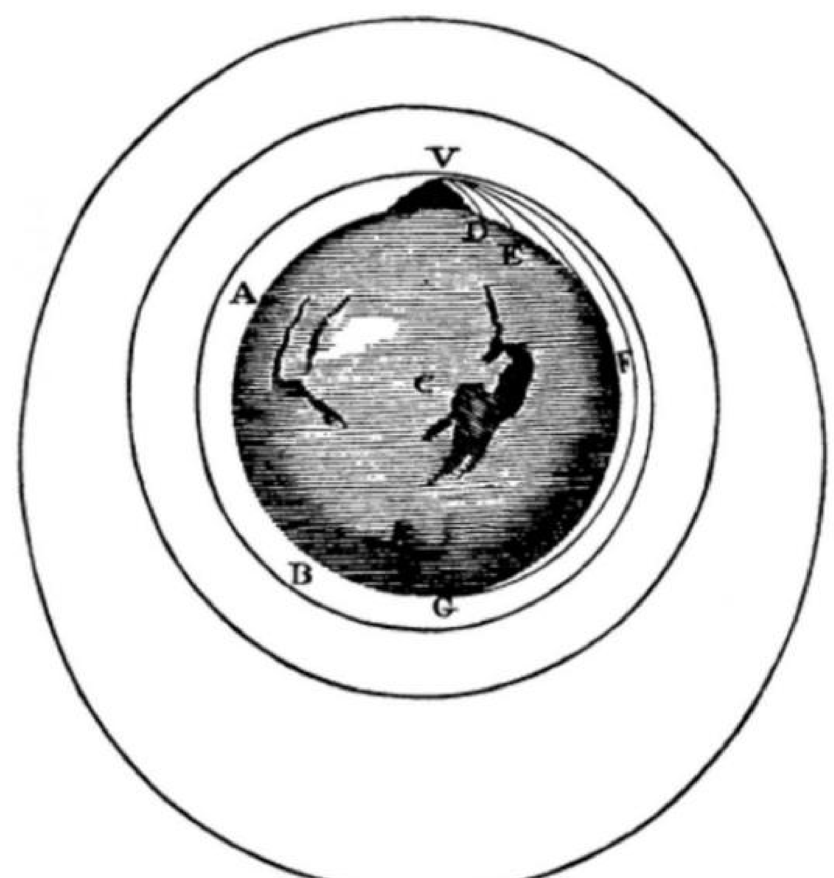


Image from Newton's Principia

for circular motion

$$F_c = \frac{mv^2}{r}$$

force of gravity

$$F_g = \frac{GmM}{r^2}$$

If gravity is to provide the centripetal force:

$$\frac{\cancel{m}v^2}{\cancel{r}} = \frac{G\cancel{m}M}{r^{\cancel{2}}}$$

$$\boxed{v^2 = \frac{GM}{r}}$$

condition for planetary/satellite circular orbit around a mass M.