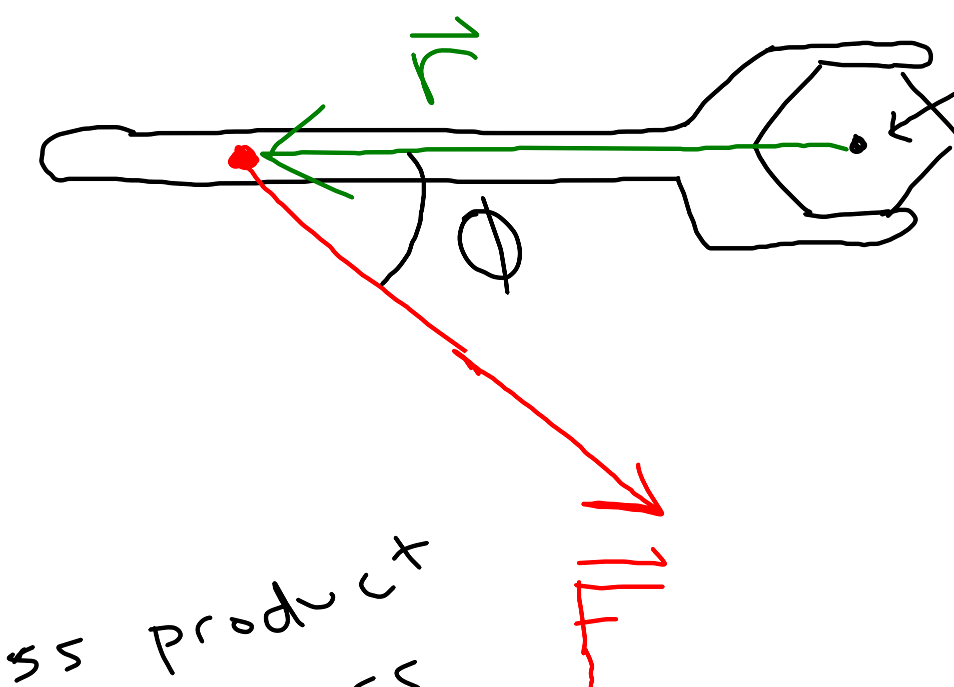
Torque

(see clicker quiz)



pivot point - make measurement with respect to this point.

$$\vec{\tau} = \vec{r} \times \vec{F}$$

cross product of 2 vectors

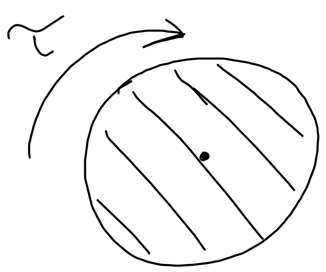
magnitude: $\tau = r F_\perp = r F \sin\phi$

direction: clockwise or counterclockwise

frictionless

$\Rightarrow$ produces acceleration, a

$$F = ma$$



$\Rightarrow$ produces angular acceleration, α

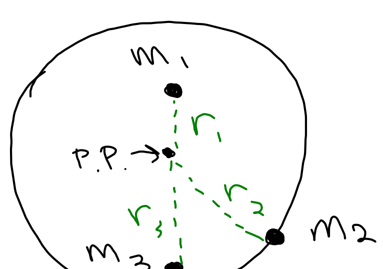
$$\tau = I\alpha$$

moment of inertia

mass - property of matter that describes how much it resists changes to motion

moment of inertia - property of a system that describes how much it resists changes to rotational motion.

depends on mass and geometry.

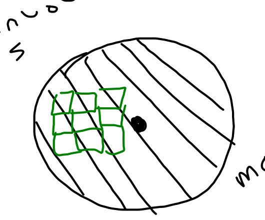


massless disk (for support)

$$\boxed{\begin{aligned} I &= \sum_j m_j r_j^2 \\ &= m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 \end{aligned}}$$

discreet point masses

continuous mass



massive disk

$\left(I = \int r^2\, dm\right)$

common shapes:

$$I_{disk} = \frac{1}{2} M R^2$$

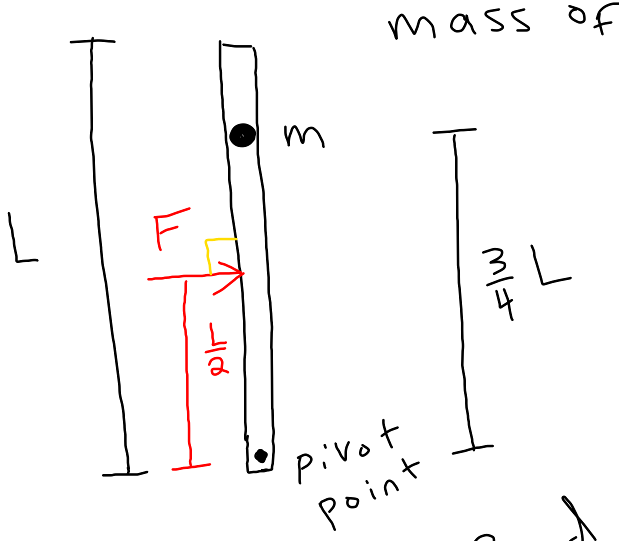
$$I_{hoop} = M R^2$$

etc.

see table in textbook

Example

mass of rod = M



$L = 0.9\ m$

$M = 1.2\ kg$

$m = 0.7\ kg$

$F = 4\ N$

find α.

$$I_{total} = \text{sum over all contributions}$$

$$= I_{rod} + I_m = \frac{1}{3}ML^2 + m\left(\frac{3}{4}L\right)^2$$

$$= \frac{1}{3}(1.2)(0.9)^2 + (0.7)\left((0.75)(0.9)\right)^2$$

$$= 0.643\ kg \cdot m^2$$

$$\tau = r F_\perp = \left(\frac{L}{2}\right) F \sin(90^\circ) = (0.45)(4)(1)$$

$$= 1.8\ N \cdot m$$

$$\alpha = \frac{\tau}{I} = \frac{1.8\ N \cdot m}{0.643\ kg \cdot m^2} = \frac{1.8\ \frac{\cancel{kg} \cdot \cancel{m}}{s^2} \cdot \cancel{m}}{0.643\ \cancel{kg} \cdot \cancel{m^2}}$$

$$\boxed{\alpha = 2.79\ \frac{rad}{s^2}}$$