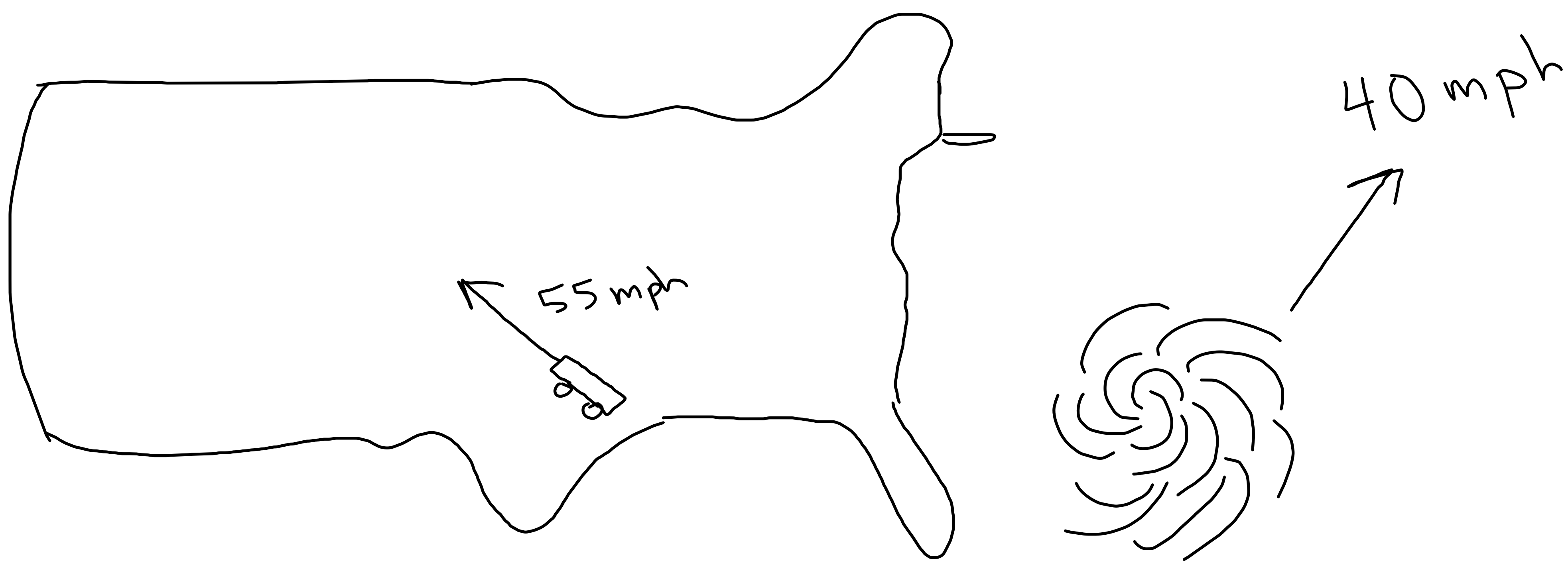
1.9 - Vectors

* So far we've worked with scalars, i.e. single numbers. For example
    * pi = 3.14159....
    * mass of a rock = 7.2 kg
    * speed = v = 22.5 m/s
* Many things in physics cannot be described by a single number. For example:



- the car is driving <u>55 mph</u> <u>northwest</u>
- the storm is moving <u>40 mph</u> <u>northeast</u>

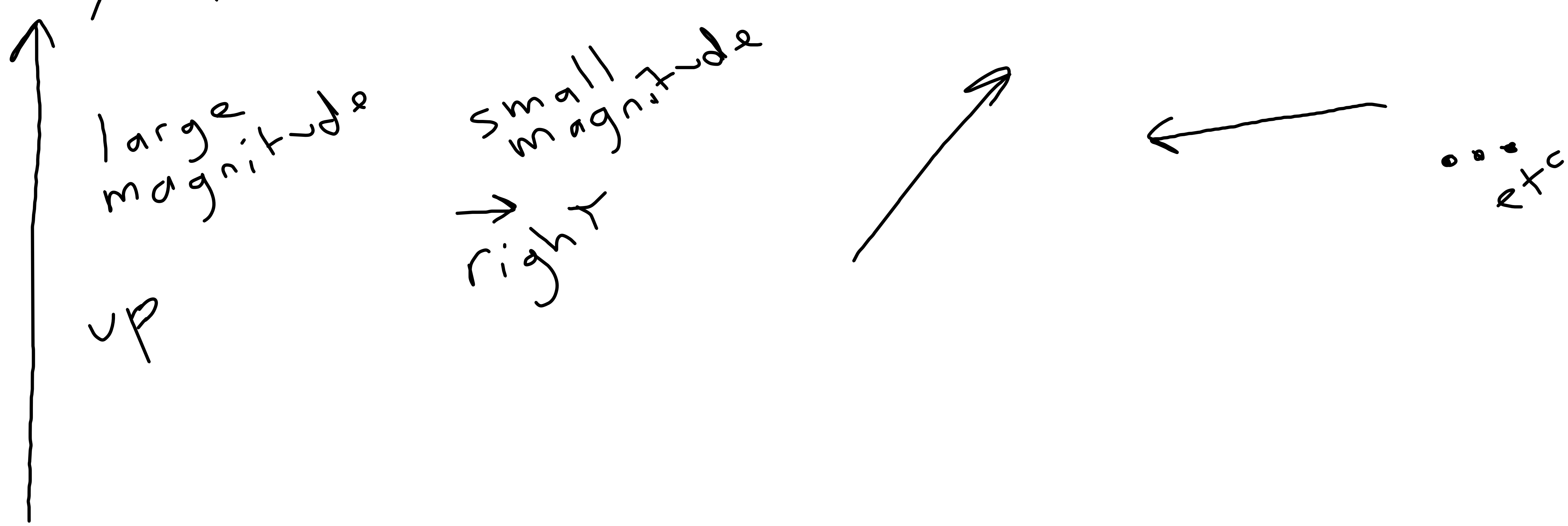
two pieces of information

(Optional: Look up the formal definition of a vector space (has nothing to do with magnitude and direction))

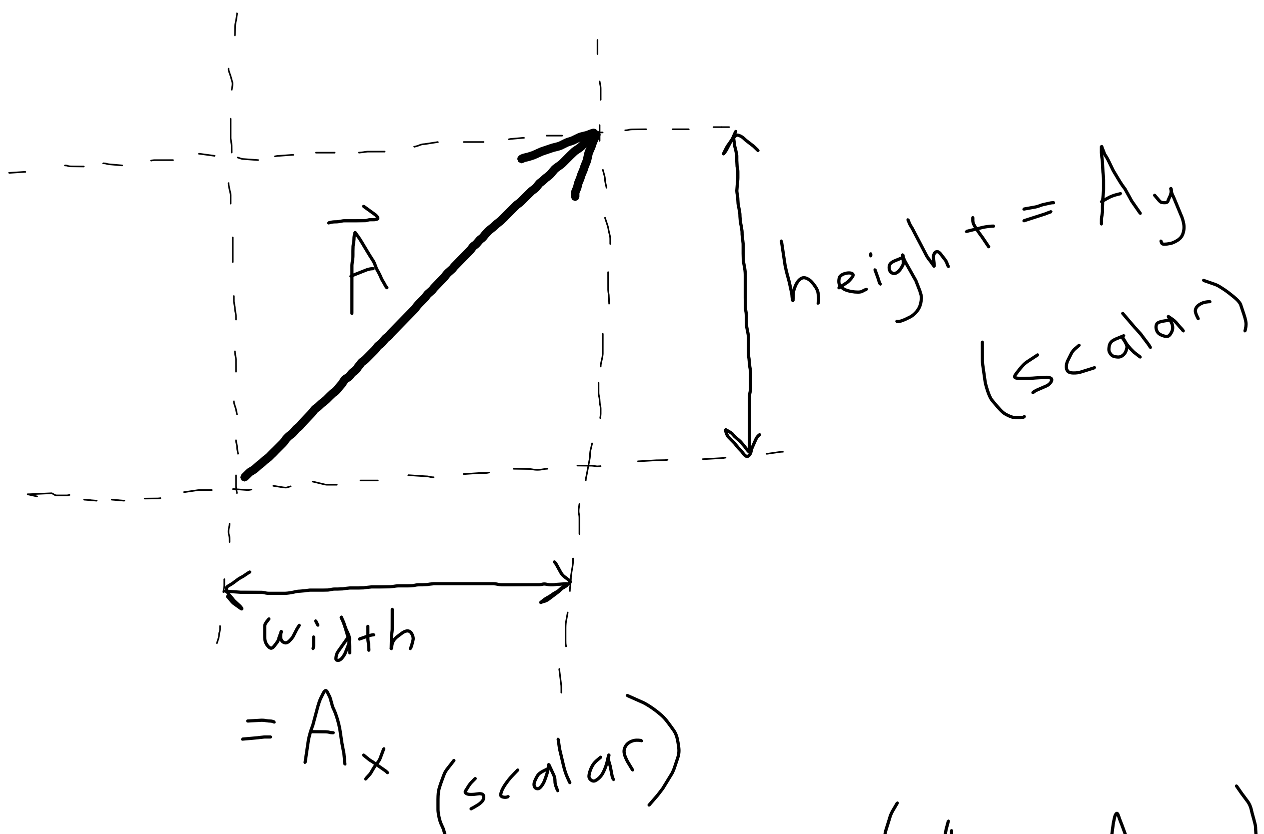
- The vector space we are going to use is $\mathbb{R}^2$ - it can be thought of as a set of object with <u>magnitude</u> <u>and</u> <u>direction</u>.

Basically arrows in the x-y plane.

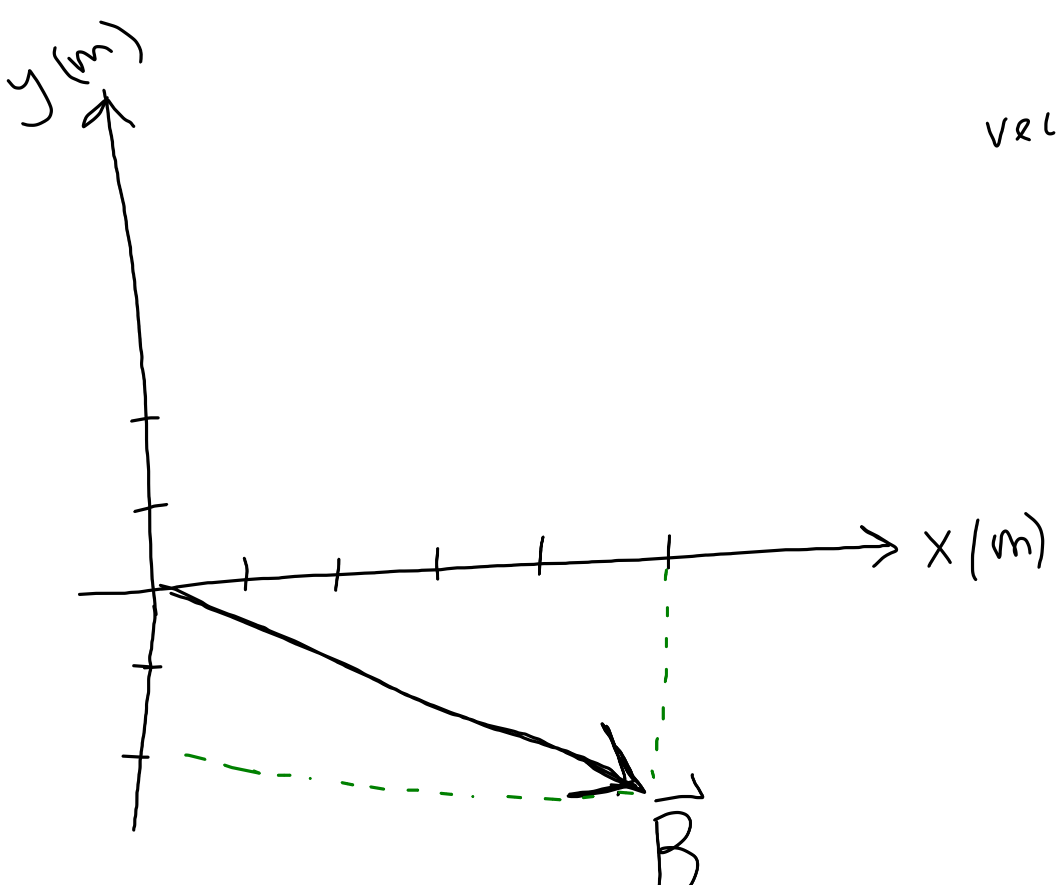
e.g.



All vectors (in $\mathbb{R}^2$) have a "width" (x-component) and a "height" (y-component). (can be +, −, 0)

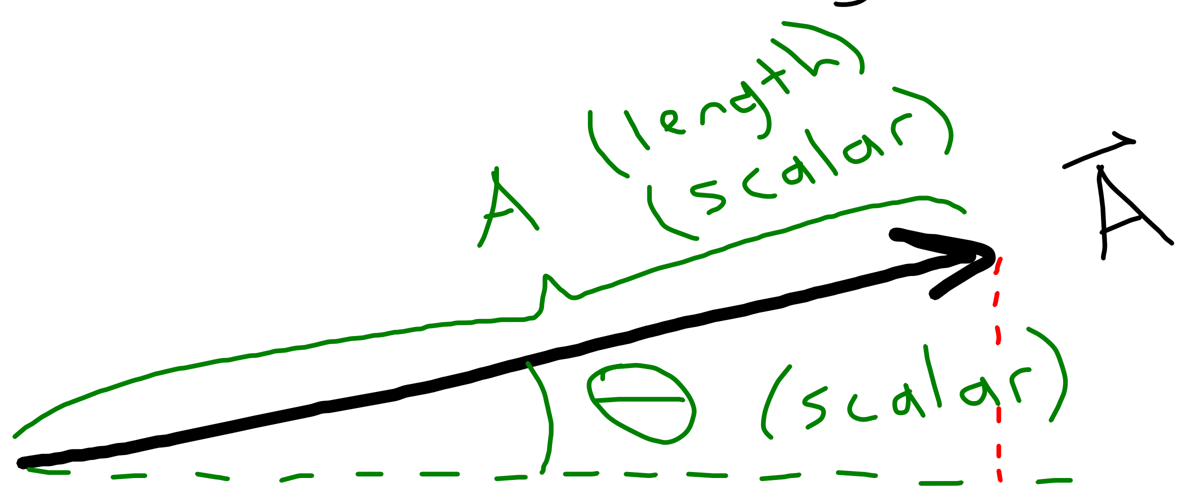


$$\underbrace{\vec{A}}_{\text{vector}} = \underbrace{(\underbrace{A_x}_{\text{scalar}}, \underbrace{A_y}_{\text{scalar}})}_{\text{vector}}$$



$$\vec{B} = (5\,\text{m}, -2\,\text{m})$$

Equivalently, a vector can be described by its length and angle:



basic trigonometry:

$$A_x = A\cos\theta$$

$$A_y = A\sin\theta$$

or

$$A = \sqrt{A_x^2 + A_y^2}$$

$$\theta = \arctan\left(\frac{A_y}{A_x}\right)$$

<u>Aside</u>:

$$\arctan(x) = \tan^{-1}(x)$$

$$\neq \frac{1}{\tan(x)}$$

$$y^{-1} = \frac{1}{y}$$

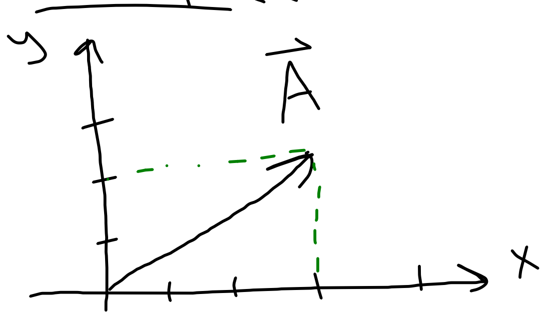
Adding and scaling vectors

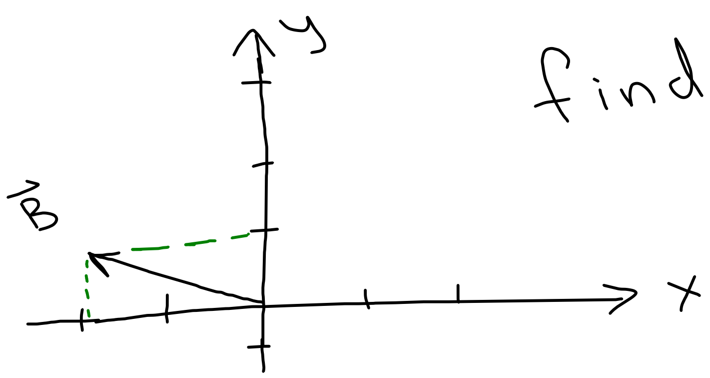
given $\vec{A}$ and $\vec{B}$, we can write

$$\vec{A} = (A_x, A_y) \qquad \vec{B} = (B_x, B_y)$$

$$\vec{A} + \vec{B} = (A_x + B_x, A_y + B_y)$$

example:





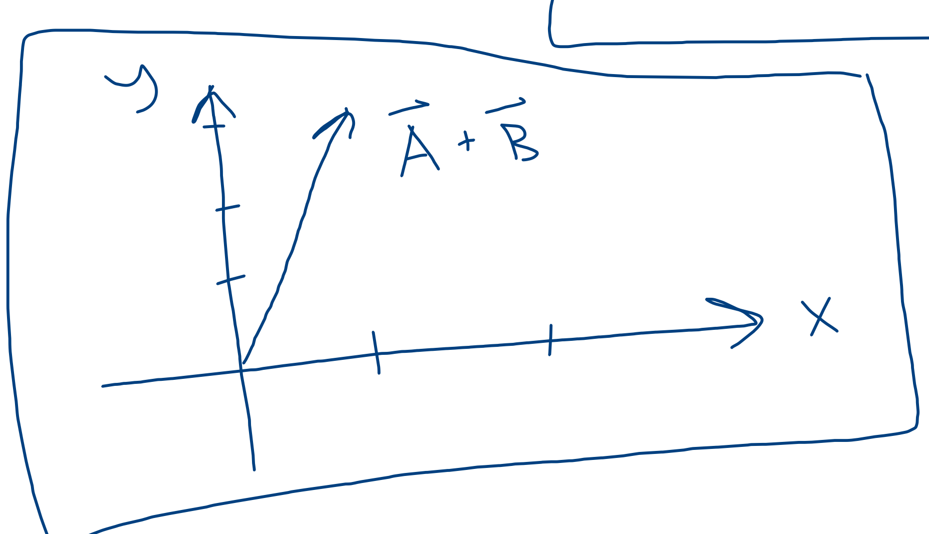
find $\vec{A} + \vec{B}$.

solution:

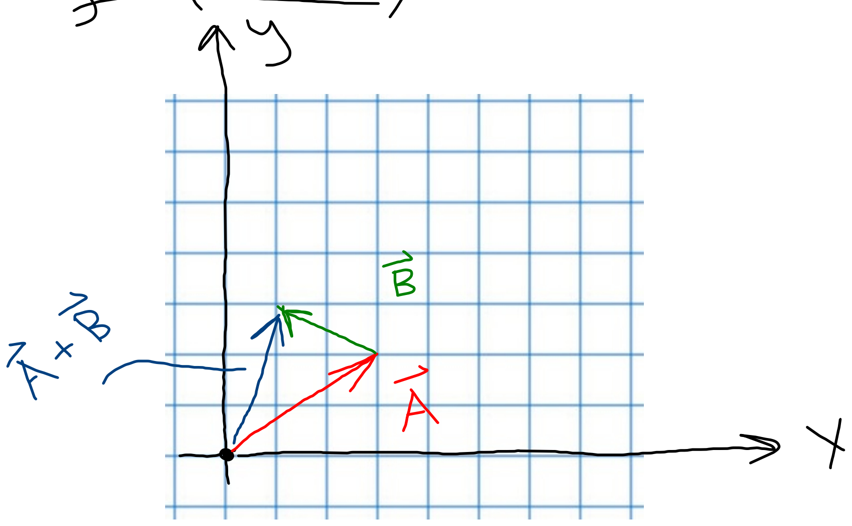
$$\vec{A} = (3, 2) \qquad \vec{B} = (-2, 1)$$

$$\Rightarrow \vec{A} + \vec{B} = (3-2, 2+1) = (1, 3)$$

$$\boxed{\vec{A} + \vec{B} = (1, 3)}$$



graphically:



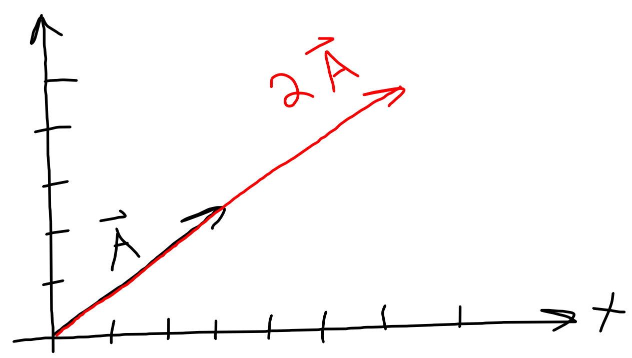
Slide $\vec{B}$ such that the tail of $\vec{B}$ is at the tip of $\vec{A}$.

Scaling a vector (scalar times a vector)

$$5\vec{A} = (5A_x, 5A_y)$$

scalar times vector

example:



$\vec{A} = (3,2)$

$2\vec{A} = (6,4)$

Activity

$\vec{A} = (3,2)$

$\vec{B} = (-2,3)$

find: $2\vec{A}+3\vec{B}$, $\vec{B}-\frac{1}{2}\vec{A}$, $2\vec{A}-2\vec{B}$

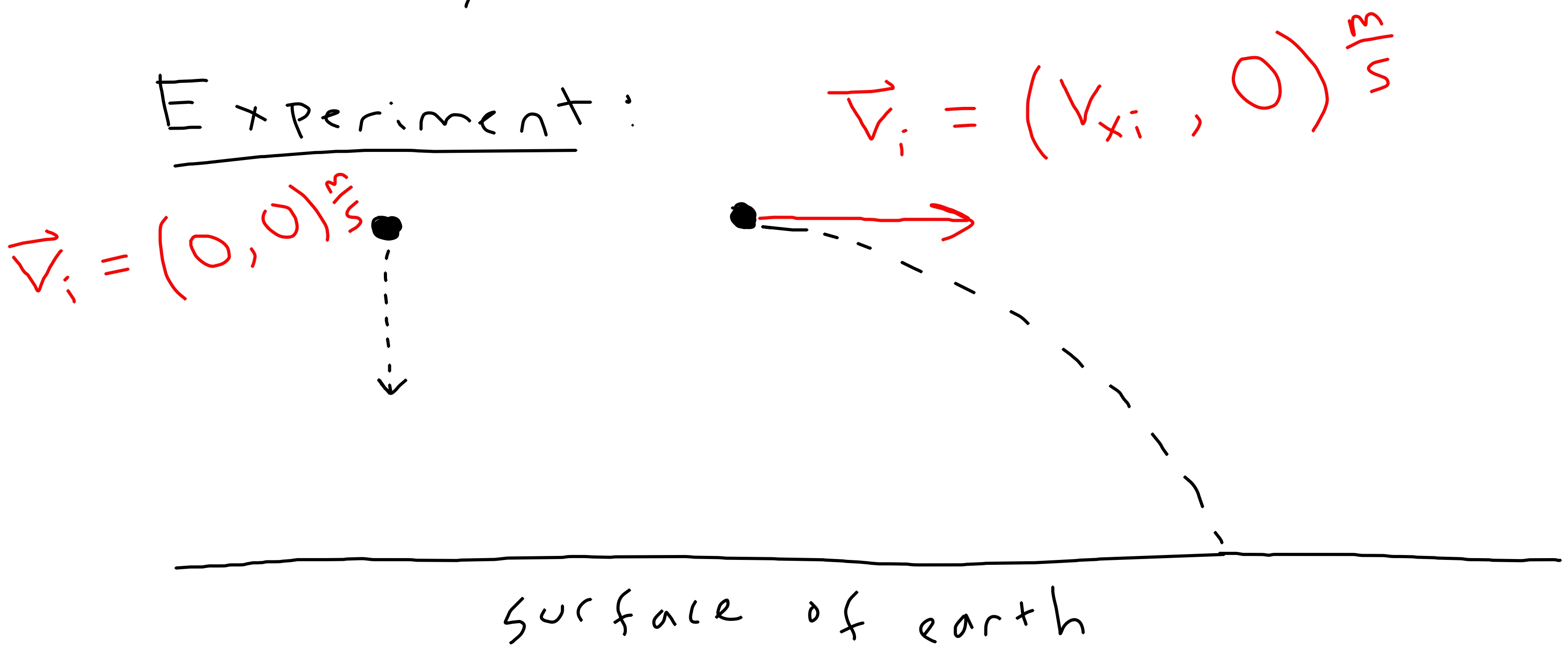
algebraically and graphically

2-D motion

- position, velocity, acceleration – vectors!
- time – still a scalar

* x and y are completely independent but linked by time

Experiment:

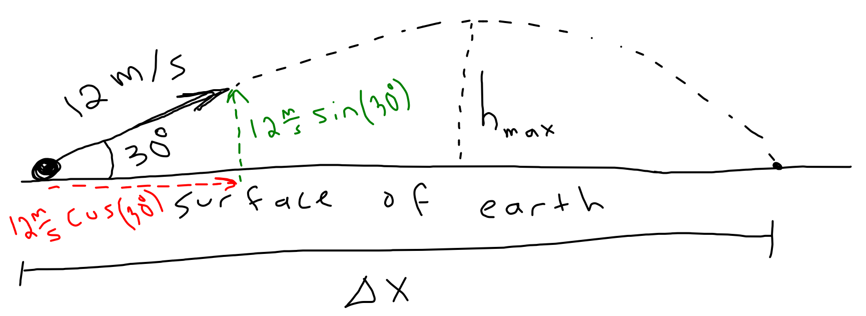


Which ball hits the ground first?

Experimental Result: Tie!

Example:

find h_{max} and Δx.



y-direction (find t when $y = h_{max}$)

$y_i = 0$

$V_i = 6$

$a = -9.8$

$V_f = 0$

$V_f = V_i + at$

$0 = 6 - 9.8t \Rightarrow t = 0.61\,s$

$y_f = y_i + V_i t + \frac{1}{2}at^2$

$y_f = 0 + 6(0.61) + \frac{1}{2}(-9.8)(0.61)^2$

$y_f = \boxed{1.84\,m = h_{max}}$

x-direction

$V_i = (12\frac{m}{s})\cos(30°) = 10.4\frac{m}{s}$

$a = 0$

$t = 0.61\,s \longrightarrow 1.22\,s$ (full width)

$X_i = 0$

$X_f = ?$

$X_f = X_i + V_i t + \frac{1}{2}at^2$

$= 0 + (10.4)(1.22) + \frac{1}{2}(0)(1.22)^2$

$= 12.68\,m$

$\boxed{\Delta X = 12.68\,m}$