Midterm Review

- Ch. 1 – no direct questions, but concepts will be ubiquitous in other problems.

- Ch. 2–3 –

$$V_{xf} = V_{xi} + a_x t$$
$$X_f = X_i + V_{ix} t + \tfrac{1}{2} a_x t^2$$
$$X_f = X_i + \tfrac{1}{2}(V_{ix} + V_{fx}) t$$
$$V_{fx}^2 = V_{ix}^2 + 2 a_x (X_f - X_i)$$

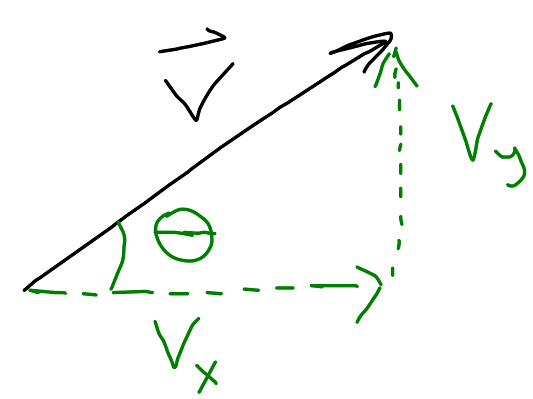
$$V_{yf} = V_{yi} + a_y t$$
$$y_f = y_i + V_{iy} t + \tfrac{1}{2} a_y t^2$$

etc.

same time, otherwise independent

$$V_{ave} = \frac{\Delta X}{\Delta t} = \frac{X_f - X_i}{t_f - t_i} \qquad a_{ave} = \frac{\Delta V}{\Delta t} = \frac{V_f - V_i}{t_f - t_i}$$

$$\vec{r} = (X, y)$$
$$\vec{V} = (V_x, V_y) \Longrightarrow$$
$$\vec{a} = (a_x, a_y)$$

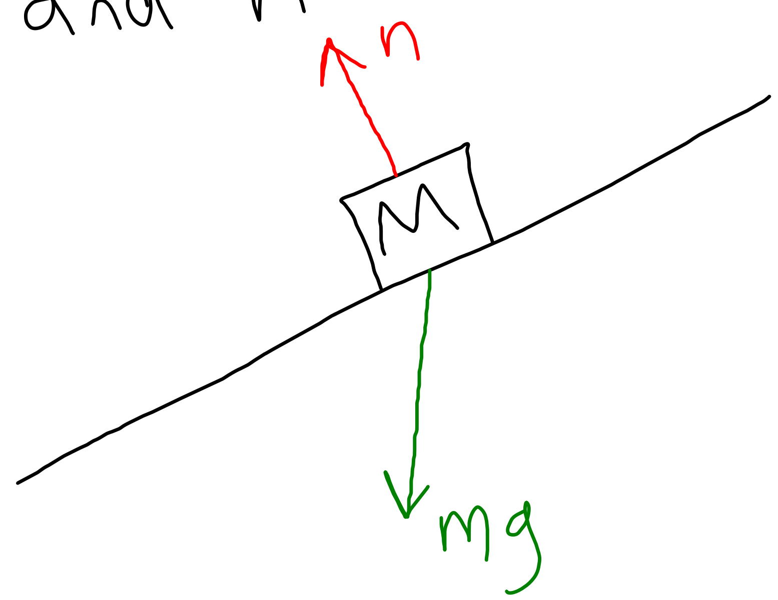


Concepts! e.g. $V_y = 0$ at max. height

• Ch. 4 - Forces and applications of Newton's Laws

$$\boxed{\sum \vec{F} = m\vec{a}}$$

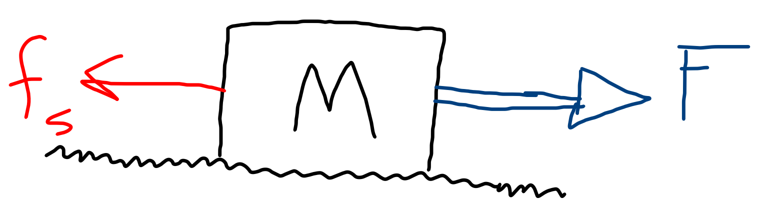
$\vec{F}_g$ and $\vec{n}$



static and kinetic friction

static

$V = 0$

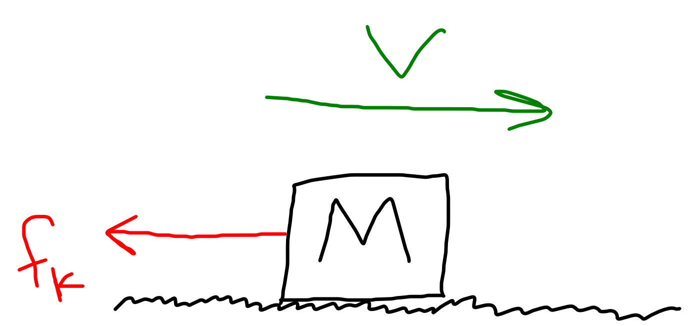


if $F \leq f_{s,max}$, where

$f_{s,max} = \mu_s n$, then

$f_s = F$.

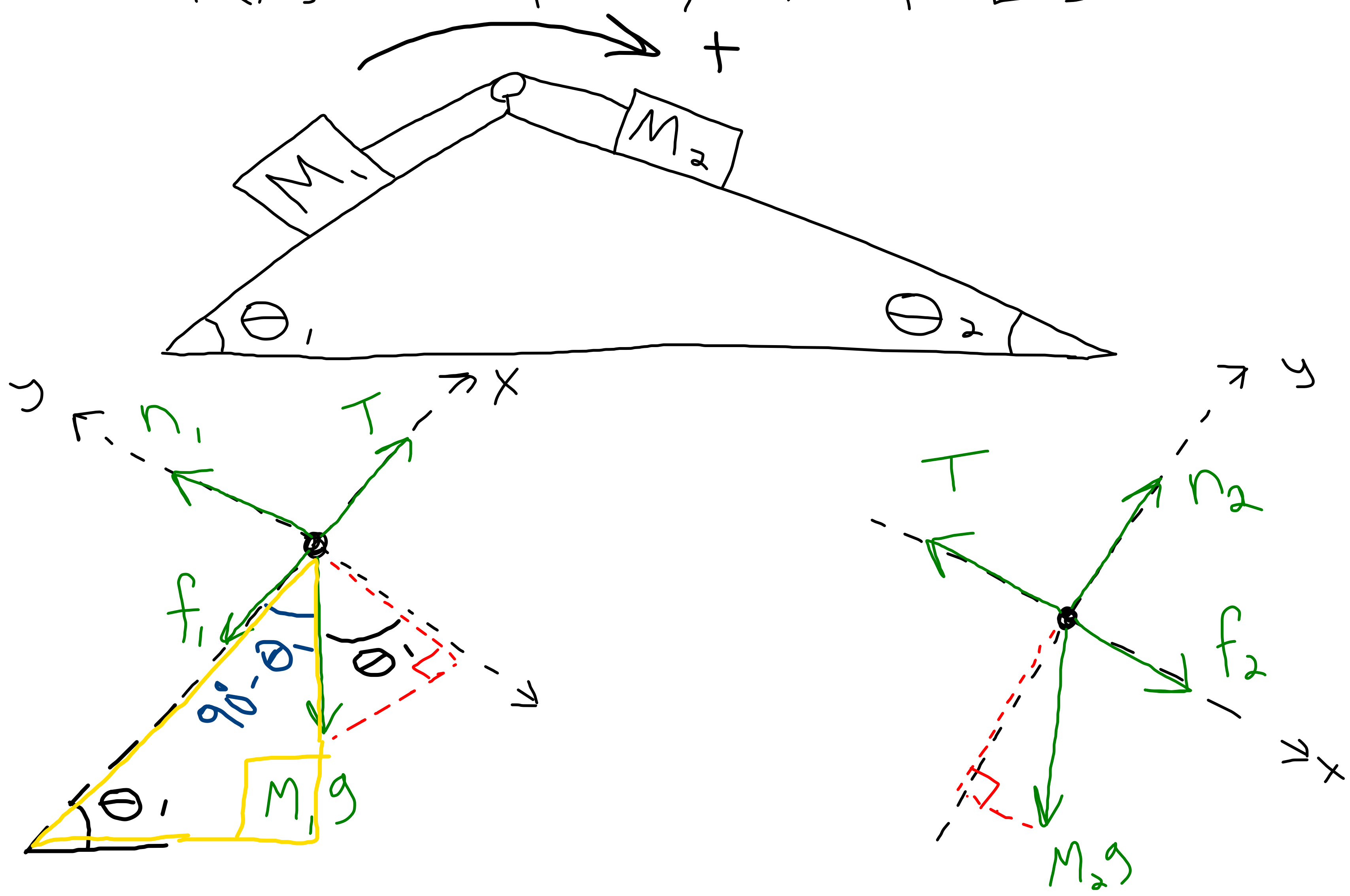
kinetic



$f_k = \mu_k n$

(n not necessarily $= mg$)

tension & pulleys, FBDs



same T
same a

$$T - f_1 - M_1 g \sin\theta_1 = M_1 a$$

$$n_1 - M_1 g \cos\theta_1 = 0$$

$$f_2 - T + M_2 g \sin\theta_2 = M_2 a$$

$$n_2 - M_2 g \cos\theta_2 = 0$$

Ch. 5 - Energy

$$\boxed{\begin{aligned} KE &= \tfrac{1}{2}mv^2 \\ PE_g &= mgh \\ PE_s &= \tfrac{1}{2}k(\Delta x)^2 \end{aligned}}$$

$$E_{total} = KE + PE_g + PE_s$$

$$(+ \ldots)$$

Energy conservation:

$$E_{total}^{final} = E_{total}^{initial} \quad (+ \text{Work added})$$

$$W_{total} = \Delta KE$$

$$W = (F)(\Delta x)\cos\theta$$
$$= (F_{\parallel})(\Delta x)$$

- Ch. 6 - Momentum

$$\boxed{\vec{P} = m\vec{V}} \quad \text{(single object)}$$

$$\vec{P}_{total} = m_1\vec{V}_1 + m_2\vec{V}_2 + m_3\vec{V}_3 + \ldots$$

momentum conservation:

$$\vec{P}^{\,initial}_{total} = \vec{P}^{\,final}_{total} \quad \text{(no outside forces)}$$

collisions: elastic - $\vec{P}$ and E conserved

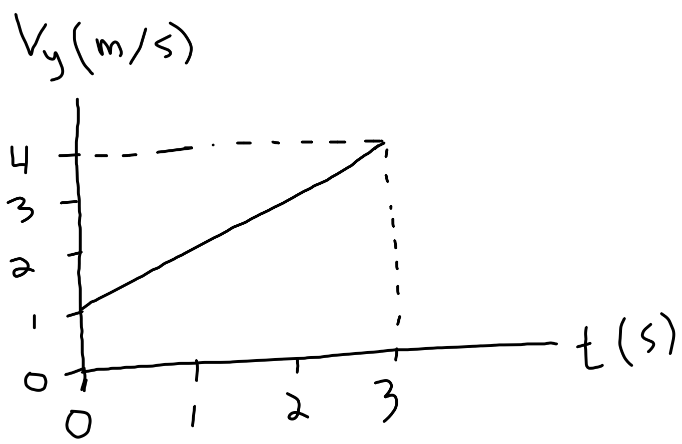
inelastic - $\vec{P}$ conserved

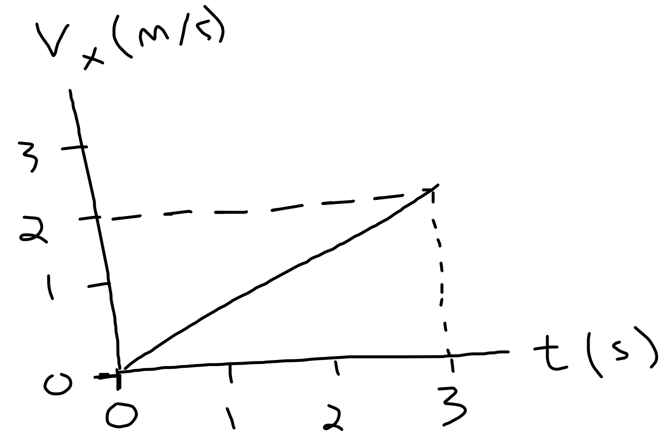
Power: $P = \dfrac{\text{Energy}}{\text{time}}$ $\quad [P] = \text{Watts (W)}$

$= J/s$

Example

given: $\vec{r}_i = (0,0)\ \text{m}$





find $\vec{r}_f$ after 3 s.

$$V_{xi} = 0\ \frac{m}{s}$$

$$V_{xf} = 2\ \frac{m}{s}$$

$$a_x = \frac{(2-0)\frac{m}{s}}{(3-0)\,s} = \frac{2}{3}\ \frac{m}{s^2}$$

$$a_y = \frac{(4-1)\frac{m}{s}}{(3-0)\frac{m}{s}} = 1\ \frac{m}{s^2}$$

$$x_f = x_i + \tfrac{1}{2}(V_{xi} + V_{xf})\,t$$

$$x_f = \tfrac{1}{2}(2)(3) = 3\ \text{m}$$

or,

$$x_f = x_i + V_{ix}t + \tfrac{1}{2}a_x t^2$$

$$= \tfrac{1}{\cancel{2}}\left(\tfrac{\cancel{2}}{\cancel{3}}\right)(3)^{\cancel{2}} = 3\ \text{m}$$

$$y_f = y_i + \tfrac{1}{2}(V_{yi} + V_{yf})\,t$$

$$= 0 + \tfrac{1}{2}(1+4)\,3$$

$$= 7.5\ \text{m}$$

or,

$$y_f = y_i + V_{iy}t + \tfrac{1}{2}a_y t^2$$

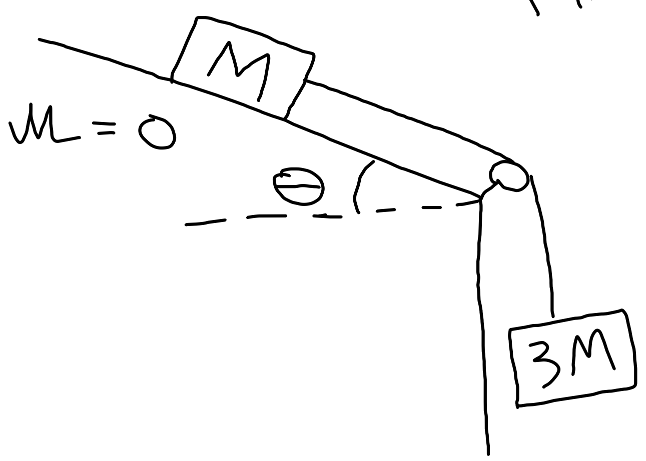
$$= 0 + 1(3) + \tfrac{1}{2}(1)(3)^2$$

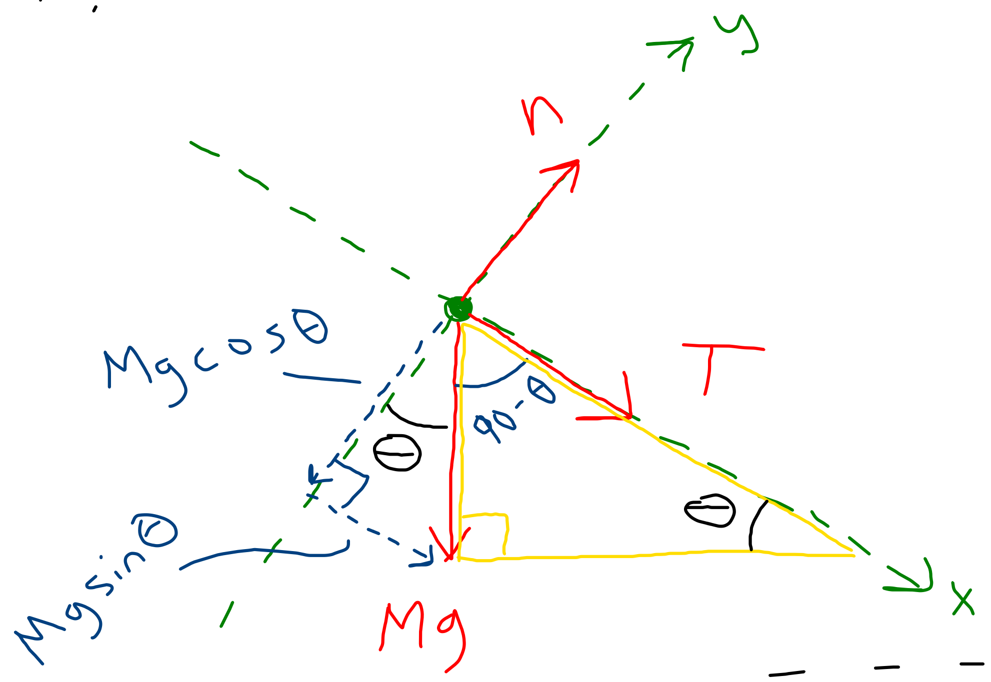
$$= 7.5\ \text{m}$$

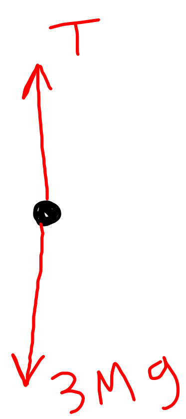
$$\boxed{\vec{r}_f = (3, 7.5)\ \text{m}}$$

Example

find a, T.







$$\boxed{T + Mg\sin\theta = Ma}$$

$$\boxed{3Mg - T = (3M)a}$$

add

$$3\cancel{M}g + \cancel{M}g\sin\theta = 4\cancel{M}a$$

$$\boxed{a = \frac{1}{4}g\left(3+\sin\theta\right)}$$

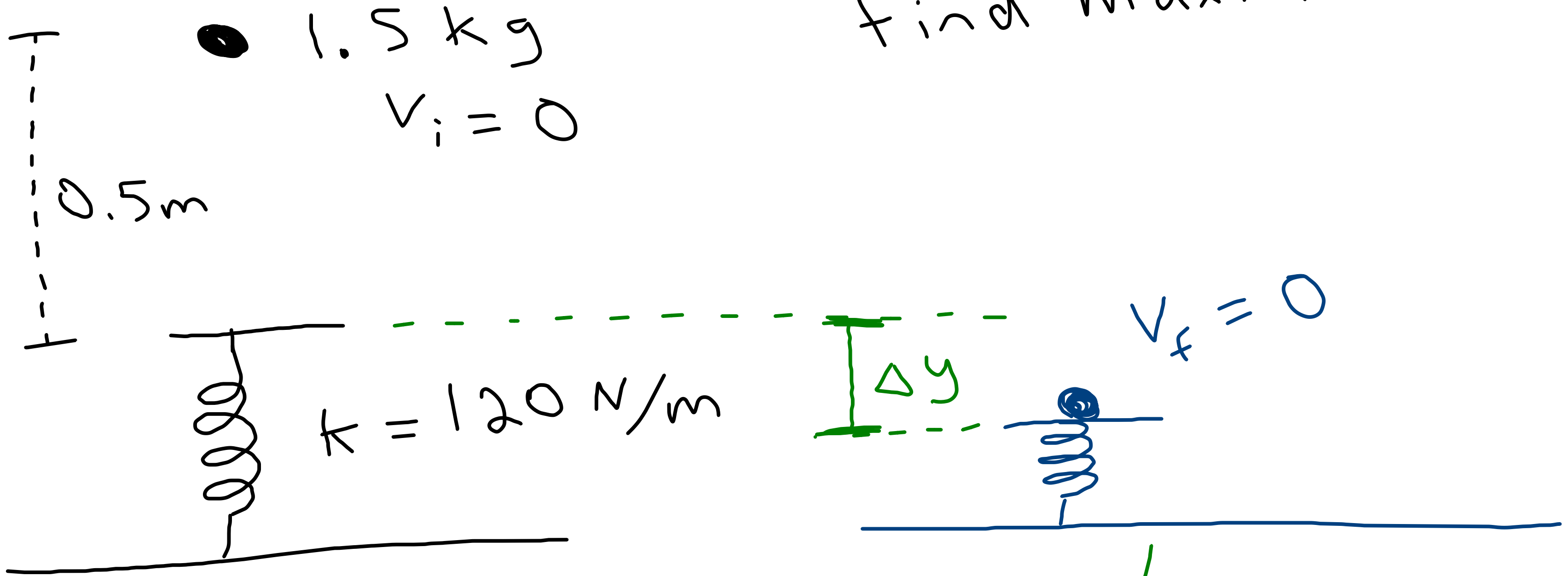
$$T = M\left(\frac{1}{4}g\left(3+\sin\theta\right)\right) - Mg\sin\theta$$

$$= \frac{3}{4}Mg + \frac{1}{4}Mg\sin\theta - Mg\sin\theta$$

$$\boxed{T = \frac{3}{4}Mg\left(1-\sin\theta\right)}$$

Example

find maximum spring compression.



$$E_i = KE_i + PE_{g,i} + PE_{s,i}$$

$$= 0 + (1.5)(9.8)(0.5) + 0$$

$$= 7.35 \text{ J}$$

$$E_f = KE_f + PE_{g,f} + PE_{s,f}$$

$$= 0 + (1.5)(9.8)(-\Delta y) + \frac{1}{2}(120)(\Delta y)^2$$

$$= -14.7\,\Delta y + 60(\Delta y)^2$$

$$E_T = E_i = E_f$$

$$7.35 = -14.7\,\Delta y + 60(\Delta y)^2$$

$$\boxed{\Delta y = 0.49 \text{ m}}$$