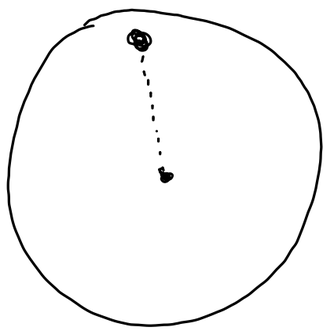
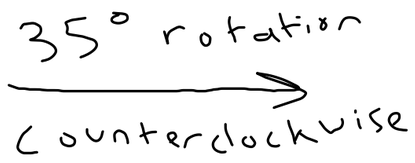
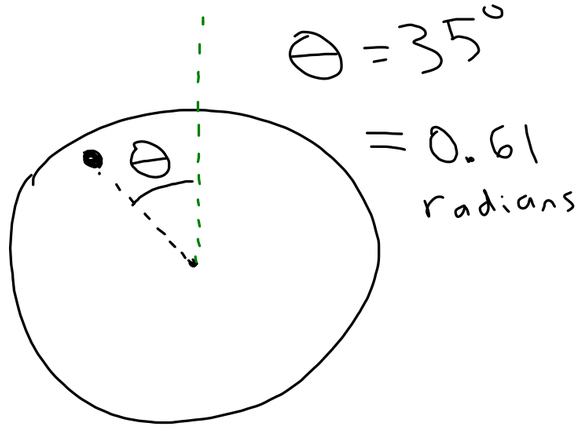
Rotations



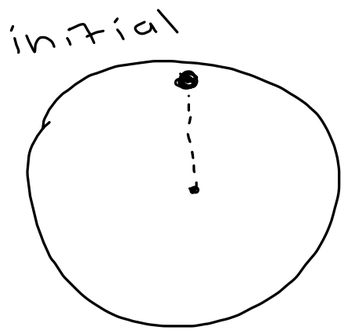
initial

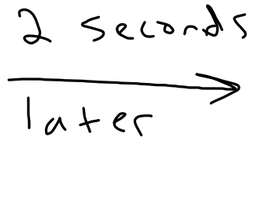


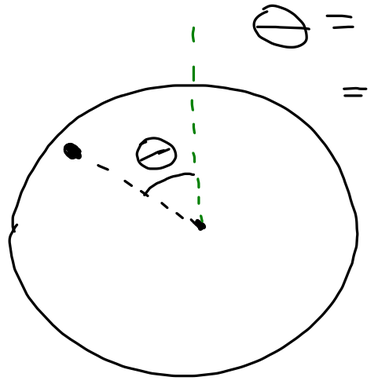


$$\theta\,(\text{in radians}) = \frac{\theta\,(\text{in degrees})\ \pi}{180}$$

angular velocity







$t = 0\,s$

$t = 2\,s$

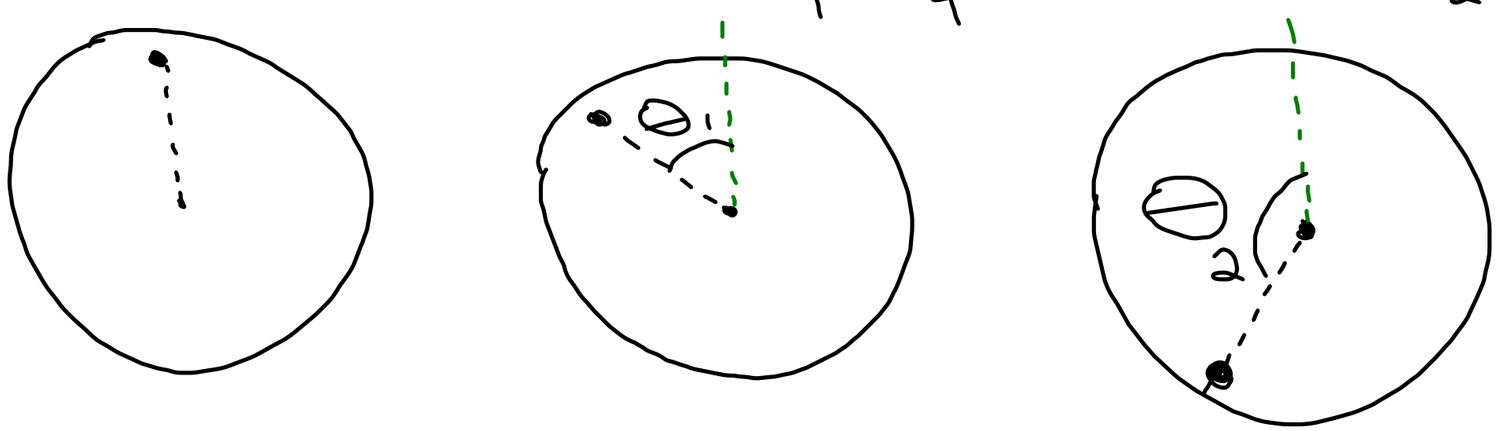
$$\omega_{ave} = \frac{\Delta\theta}{\Delta t} = \frac{\theta_f - \theta_i}{t_f - t_i}$$

above example:

$$\frac{1.05 - 0\ \text{rad}}{2 - 0\ s} = 0.525\ \frac{\text{rad}}{s}$$

angular acceleration:

$\theta_1 = \frac{\pi}{4}\,rad$ $\theta_2 = \frac{3\pi}{4}\,rad$



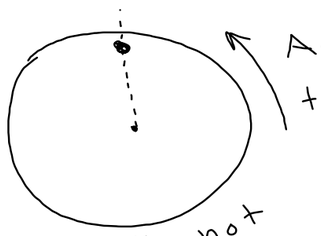
$t = 0\,s$ $t = 1\,s$ $t = 2\,s$

$$\omega_{ave}^{(1)} = \frac{\frac{\pi}{4} - 0}{1 - 0} = \frac{\pi}{4}\,\frac{rad}{s}$$

$$\omega_{ave}^{(2)} = \frac{\frac{3\pi}{4} - \frac{\pi}{4}}{2 - 1} = \frac{\pi}{2}\,\frac{rad}{s}$$

$$\alpha_{ave} = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_i}{t_f - t_i}$$

Example:



$\Rightarrow$ produces $\alpha = 0.15\ rad/s^2$

- Find ω_f after 3s.

$$\omega_f = \omega_i + \alpha t$$

$$\omega_f = 0 + \left(0.15 \frac{rad}{s^2}\right)(3s) =$$

$$\boxed{\omega_f = 0.45 \frac{rad}{s}}$$

- What is θ_f after 3s?

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$$

$$\theta_f = 0 + 0(3) + \frac{1}{2}(0.15)(3)^2$$

$$\boxed{\theta_f = 0.675\ rad} = 38.7^\circ$$

Summary

Analogies:

$x \longrightarrow \theta$

$v \longrightarrow \omega$

$a \longrightarrow \alpha$

$t \longrightarrow t$

$$\omega_f = \omega_i + \alpha t$$

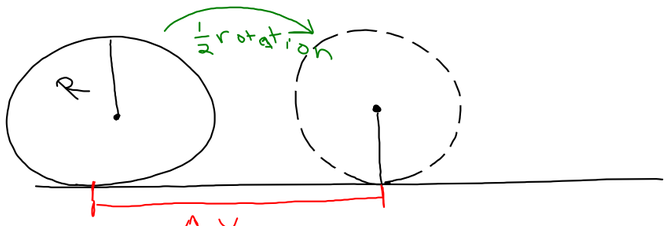
$$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$$

$$\theta_f = \theta_i + \frac{1}{2}(\omega_i + \omega_f) t$$

$$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$$

constant α

Tangential Velocity, acceleration



$\Delta X = \pi R$

$\Delta X = \frac{1}{2}(\text{circumference}) = \frac{1}{2}(2\pi R) = \pi R$

In general:

$\Delta X = (\text{fraction of a full rotation})(\text{circumference})$

$\Delta X = \left(\frac{\Theta \text{ (in radians)}}{2\pi}\right)(2\pi R)$

$\Delta X = \Theta R$ ← in radians

Aside:

$\Delta X = \left(\frac{\Theta(\text{deg.})}{360}\right)(2\pi R)$

$\Delta X = 0.0175\,\Theta R$ ← in degrees

↓ divide by Δt

$\frac{\Delta X}{\Delta t} = \frac{\Theta R}{\Delta t}$

$V = \omega R$ ← in rad/s

↓ divide by Δt again

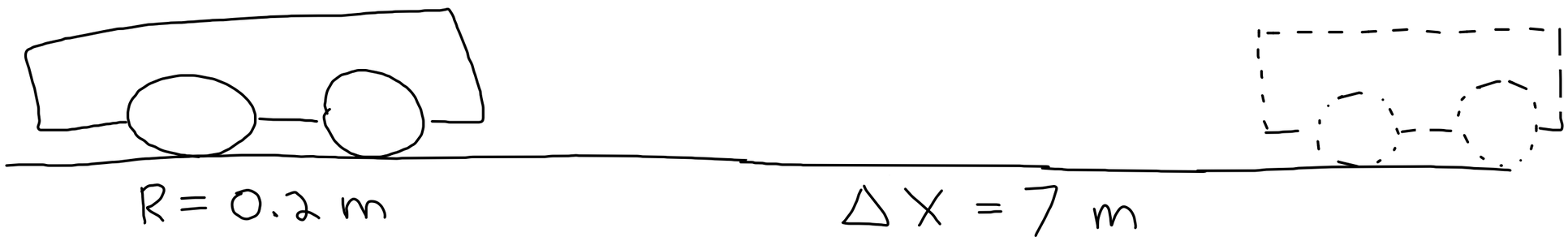
$\frac{V}{\Delta t} = \frac{\omega}{\Delta t} R$

$a = \alpha R$ ← in rad/s²

Example: const. a

$t = 4\ s$

$V_i = 0$



$R = 0.2\ m$

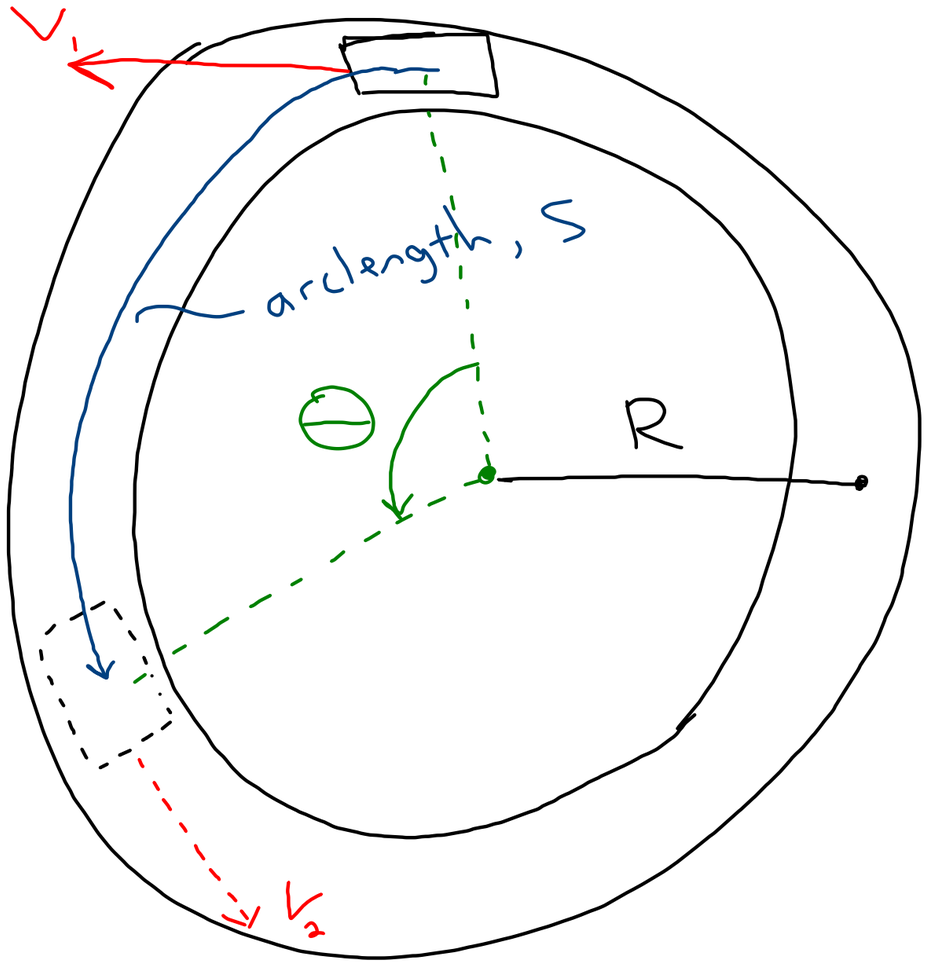
$\Delta X = 7\ m$

- Find α and ω_f for the wheels.
- Find the number of rotations done by the wheels.

Answers: $\alpha = 4.375\ \frac{rad}{s^2}$

$\omega_f = 17.5\ rad/s$

5.57 rotations



Aerial View of race track

$$S = R\theta$$

θ in radians

$$V_T = R\omega$$

$$a_T = R\alpha$$

T = tangential