Work

Cash: \$50
Bank: \$100
\$150

→

Cash: \$75
Bank: \$75
\$150

$\xrightarrow{+\$50 \text{ gift}}$

Cash: \$125
Bank: \$75
\$200

Similarly,

KE: 50 J
PE: 100 J
150 J

→

KE: 75 J
PE: 75 J
150 J

+50 J added → by external force

KE: 125 J
PE: 75 J
200 J

↳ 50 J of work was done on the system.

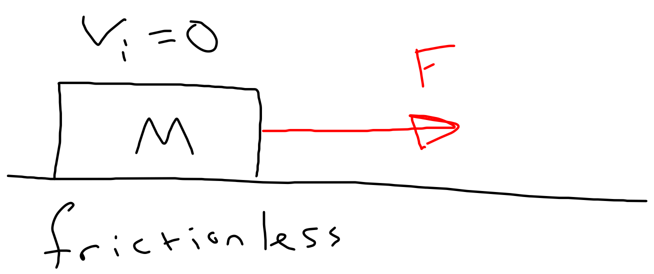
Definition:

$$\boxed{W = \Delta KE}$$

Work = change in kinetic energy

$[W]$ = Joules (same as energy)

Example



constant external force F applied for some time interval t.

- calculate: ΔX, V_f, ΔKE

Solution:

Newton's 2^{nd} Law: $a = \frac{F}{M}$

$$\Rightarrow X_f = \cancel{X_i} + \cancel{V_i}t + \frac{1}{2}at^2 = \boxed{\frac{1}{2}\left(\frac{F}{M}\right)t^2 = \Delta X}$$

$$V_f = \cancel{V_i} + at \quad = \quad \boxed{V_f = \frac{F}{M}t}$$

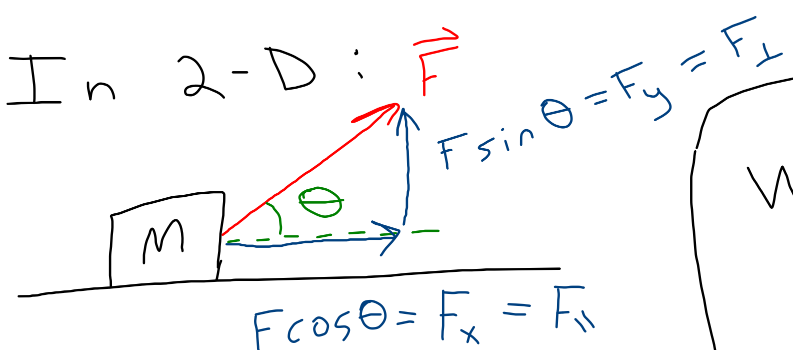
$$\Delta KE = KE_f - \cancel{KE_i} = \frac{1}{2}MV_f^2 = \frac{1}{2}M\left(\frac{F}{M}t\right)^2$$

$$\boxed{\Delta KE = \frac{1}{2}\frac{F^2}{M}t^2} = W$$

$\Rightarrow$ observation: ΔX and W look similar

$$\Rightarrow \boxed{W = F \cdot \Delta X = \Delta KE}$$

In 2-D: $\vec{F}$



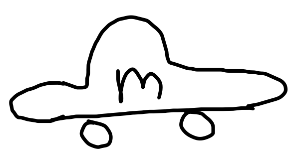
$$\boxed{\begin{aligned} W &= F \cdot \Delta X \cdot \cos\theta \\ &= F_x \cdot \Delta X \\ &= F_\parallel \cdot \Delta X \end{aligned}}$$

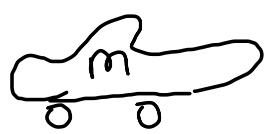
$$= \vec{F} \cdot \Delta\vec{X}$$

dot product between 2 vectors

Power

Consider 2 cars w/ same mass, both starting from rest:

A:  $V_i = 0$ reaches $40\frac{m}{s}$ in 3 s.

B:  $V_i = 0$ reaches $40\frac{m}{s}$ in 8 s.

Which engine did more work?

solution:

A: $W = \Delta KE = \frac{1}{2}m(40)^2$

B: $W = \Delta KE = \frac{1}{2}m(40)^2$

$\boxed{\text{tie}}$

Definition: $\text{Power} = \dfrac{\text{Work}}{\text{time}}$

$$\boxed{P = \frac{W}{\Delta t} = \frac{\Delta KE}{\Delta t}}$$

units: $[P] = \frac{J}{s}$