Кафедра дискретной математики МФТИ

Курс математической статистики

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Задание №2

№2. (К теоретической задаче 5)

Сгенерируйте выборку $X_1, \dots, X_N$ из экспоненциального распределения с параметром $\theta = 1$ для $N = 10^4$. Для всех $n \leq N$ посчитайте оценку $(k!/\overline{X^k})^{1/k}$ параметра θ. Проведите исследование, аналогичное предыдущей задаче, и выясните, при каком k оценка ведет себя лучше (рассмотрите не менее 10 различных значений k).

In [1]:

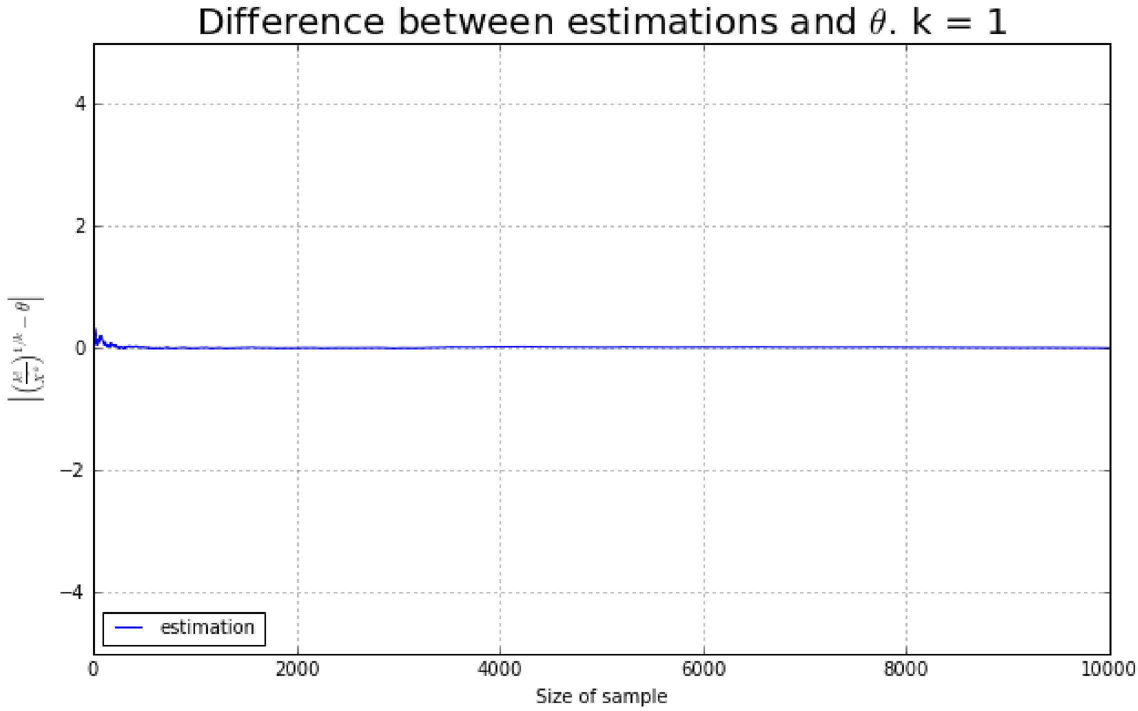
```
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import expon
from math import factorial

%matplotlib inline
```

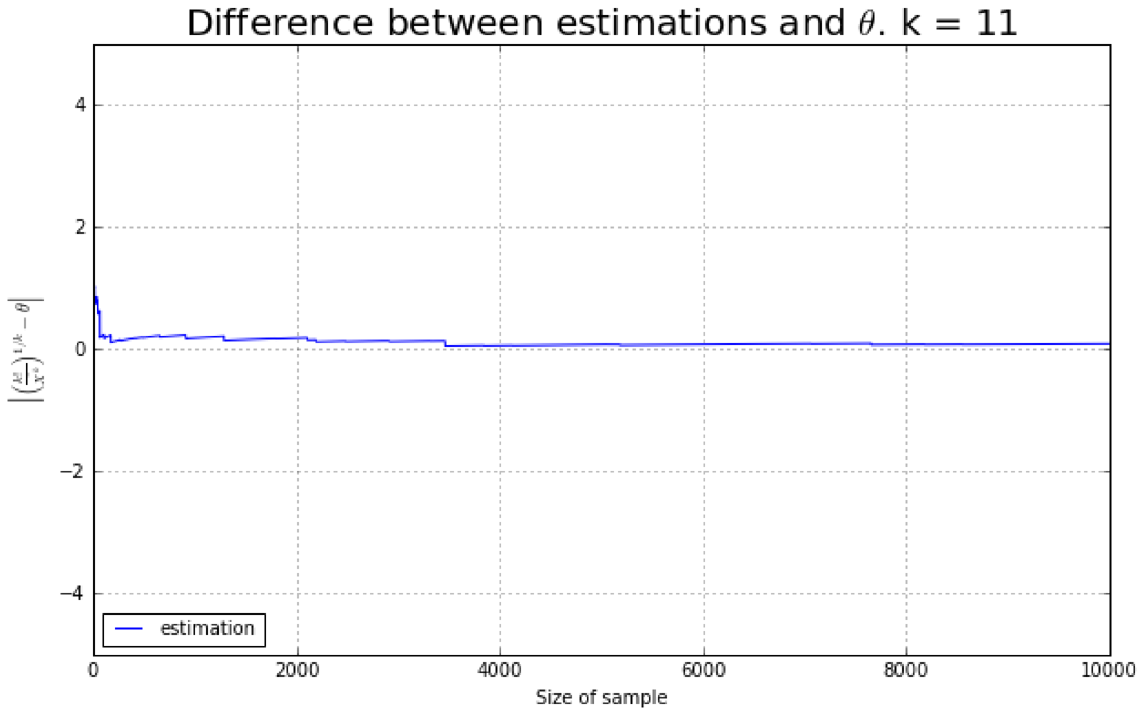
In [2]:

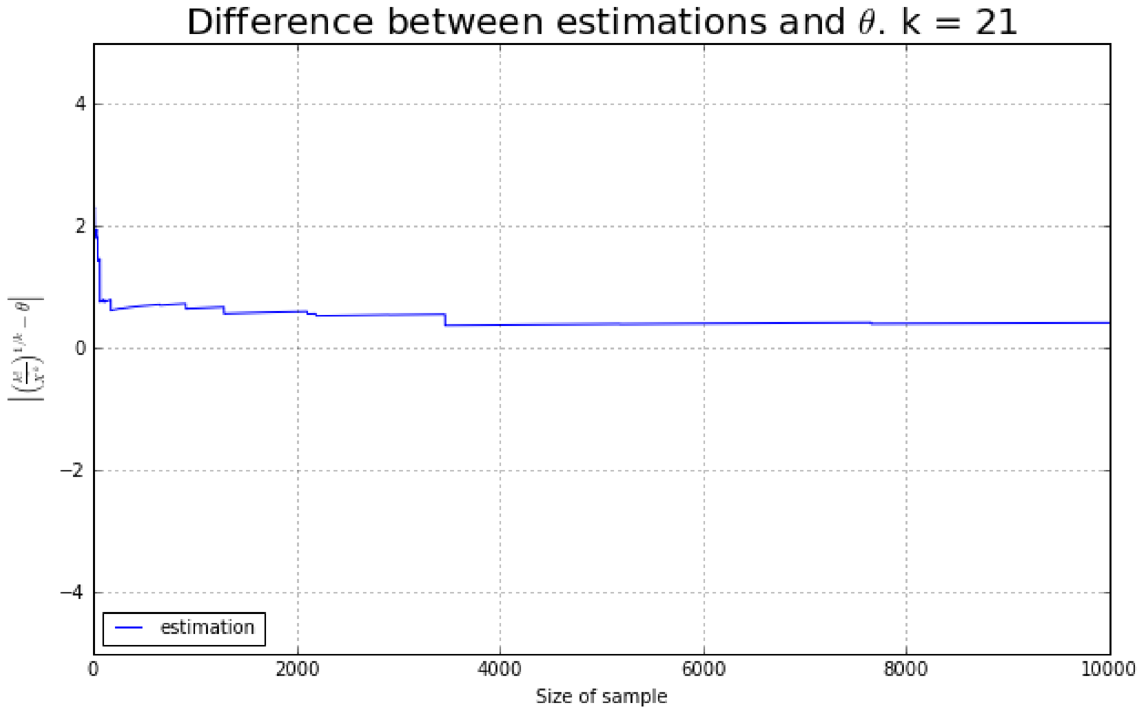
```
N = 10000

# Сгенерируем выборку с параметром theta = 1.
theta = 1
sample = expon.rvs(size=N, scale=theta)
```

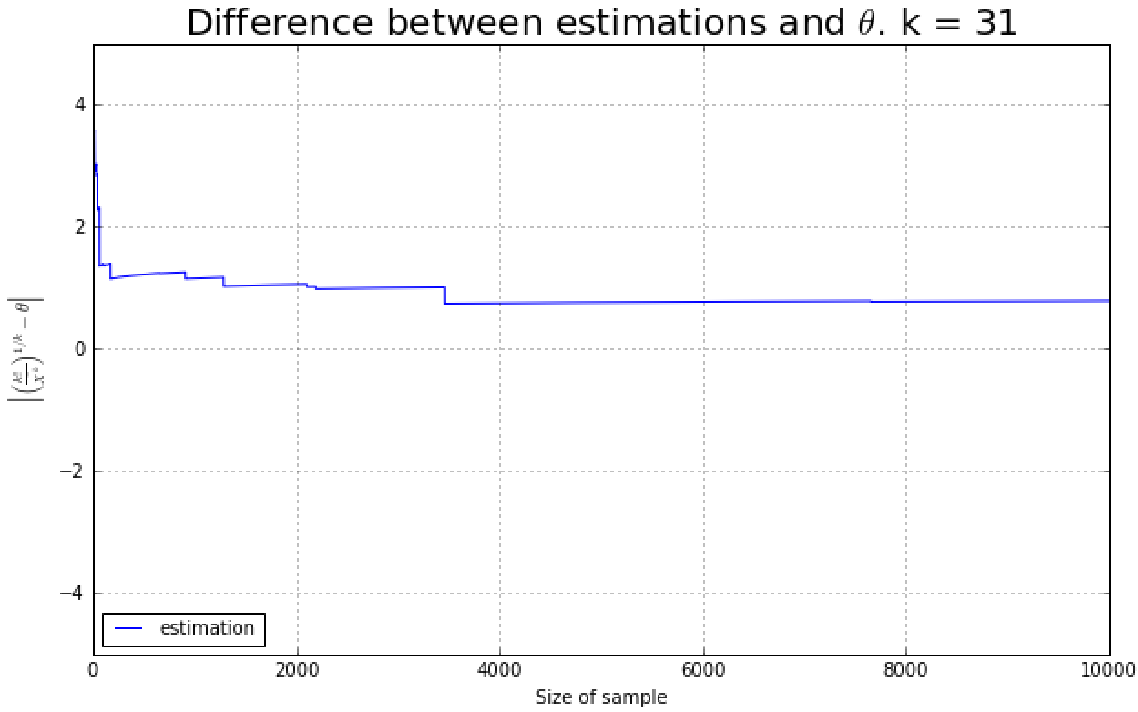
In [3]:

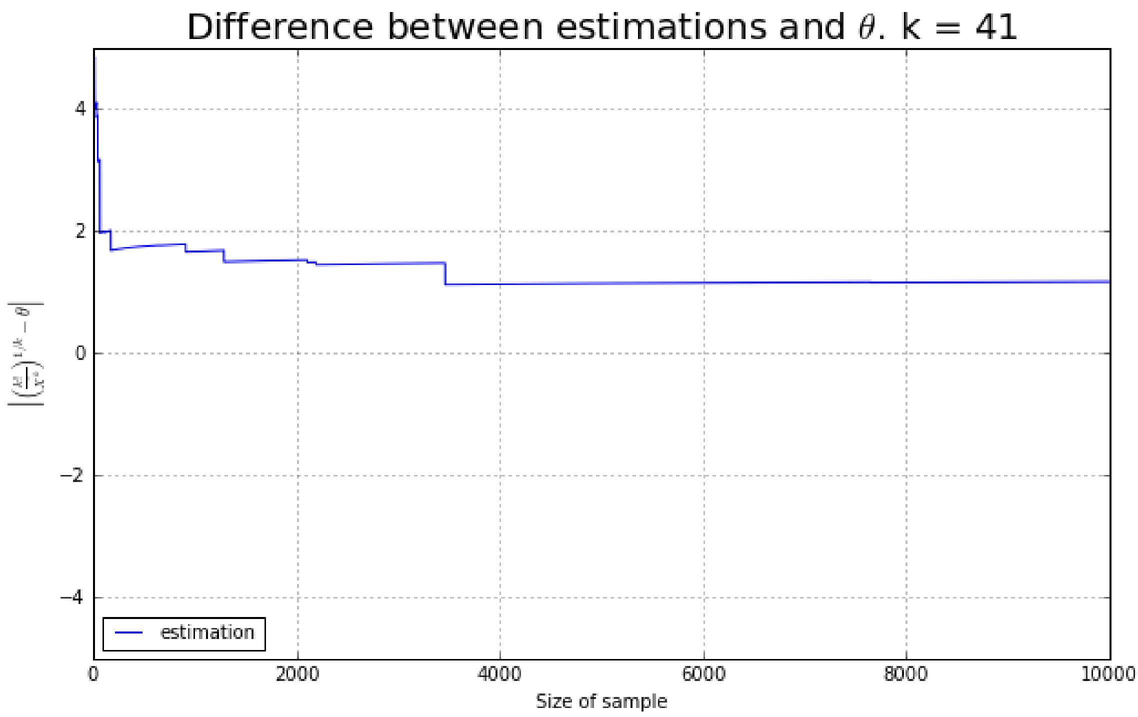
```
# Для различных k посмотрим на разность оценки и истинного значения theta.
for k in range(1, 50, 10):
    # Считаем оценку параметра theta для всех n.
    k_moment = sample ** k
    estimation = (factorial(k) / (k_moment.cumsum() / [n for n in range(1, N + 1, 1)]) ) ** (1/k)

    # Строим график функции модуля разности оценки и истинного значения theta.
    plt.figure(figsize=(10, 6))
    plt.plot(np.linspace(1, N, N), abs(estimation - theta), label="estimation")

    plt.title(r'Difference between estimations and $\theta$. k = ' + str(k), fontsize=20)
    plt.xlabel(r'Size of sample', fontsize='10')
    plt.ylabel(r'$\left|\left(\frac{k!}{\bar{X^k}}\right)^{1/k} - \theta\right|$', fontsize='10')
    plt.legend(fontsize=10, loc=3)
    plt.ylim(-5, 5)
    plt.grid()
    plt.show()
```


Difference between estimations and θ. k = 1
$\left|\left(\frac{k!}{\bar{X}^k}\right)^{1/k} - \theta\right|$
4
2
0
−2
−4
estimation
0
2000
4000
6000
8000
10000
Size of sample


Difference between estimations and θ. k = 11
$\left|\left(\frac{k!}{\bar{X}^k}\right)^{1/k} - \theta\right|$
4
2
0
−2
−4
estimation
0
2000
4000
6000
8000
10000
Size of sample


Difference between estimations and θ. k = 21
$\left|\left(\frac{\mu}{X^k}\right)^{1/k} - \theta\right|$
Size of sample
estimation
4
2
0
−2
−4
0
2000
4000
6000
8000
10000


Difference between estimations and θ. k = 31
$\left|\left(\frac{\mu}{X^k}\right)^{1/k} - \theta\right|$
Size of sample
estimation
4
2
0
−2
−4
0
2000
4000
6000
8000
10000

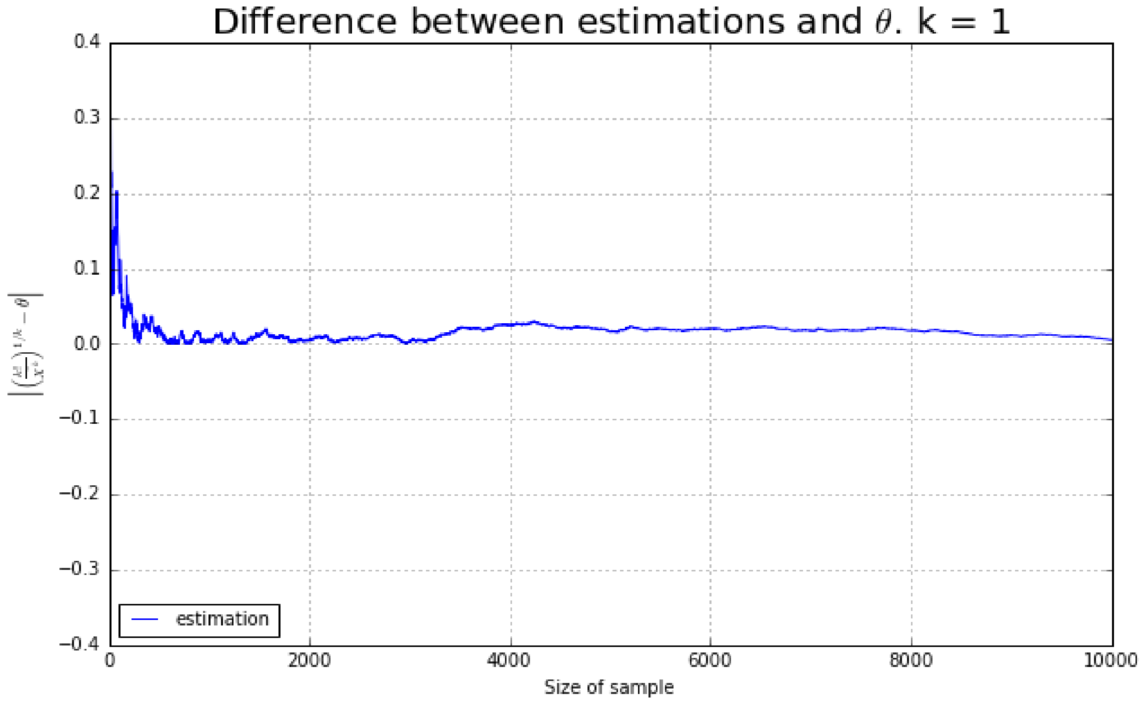


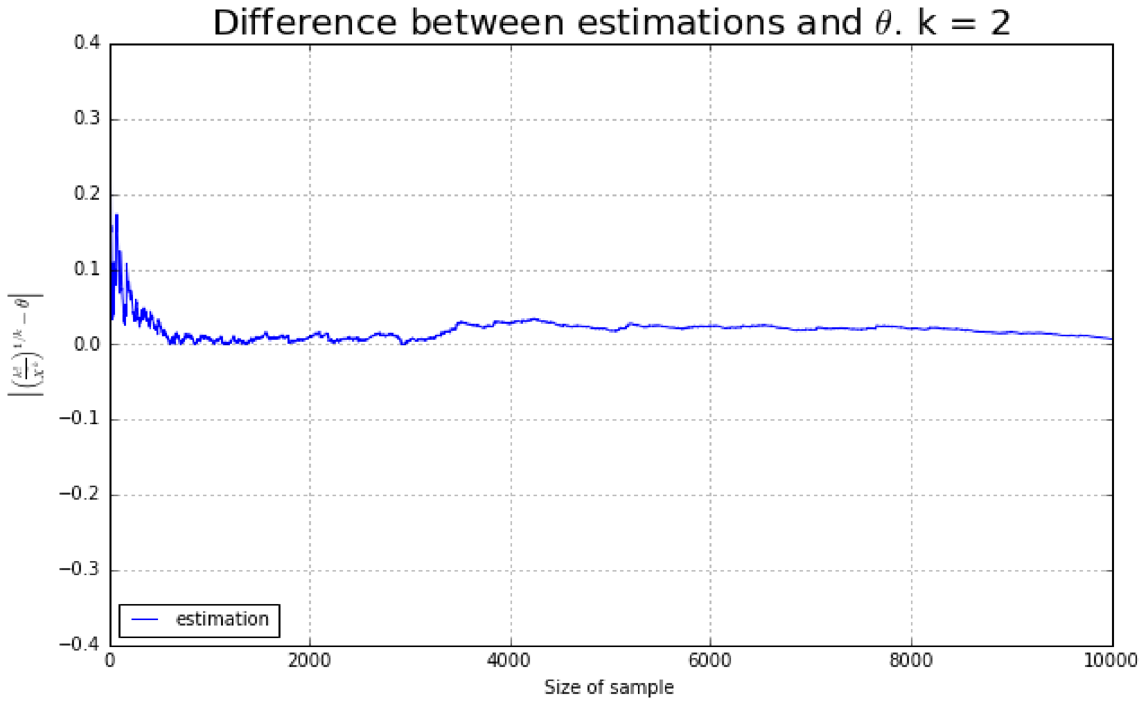
Из графиков видно, что более точная оценка достигается при $k \in \{1, \ldots, 15\}$.

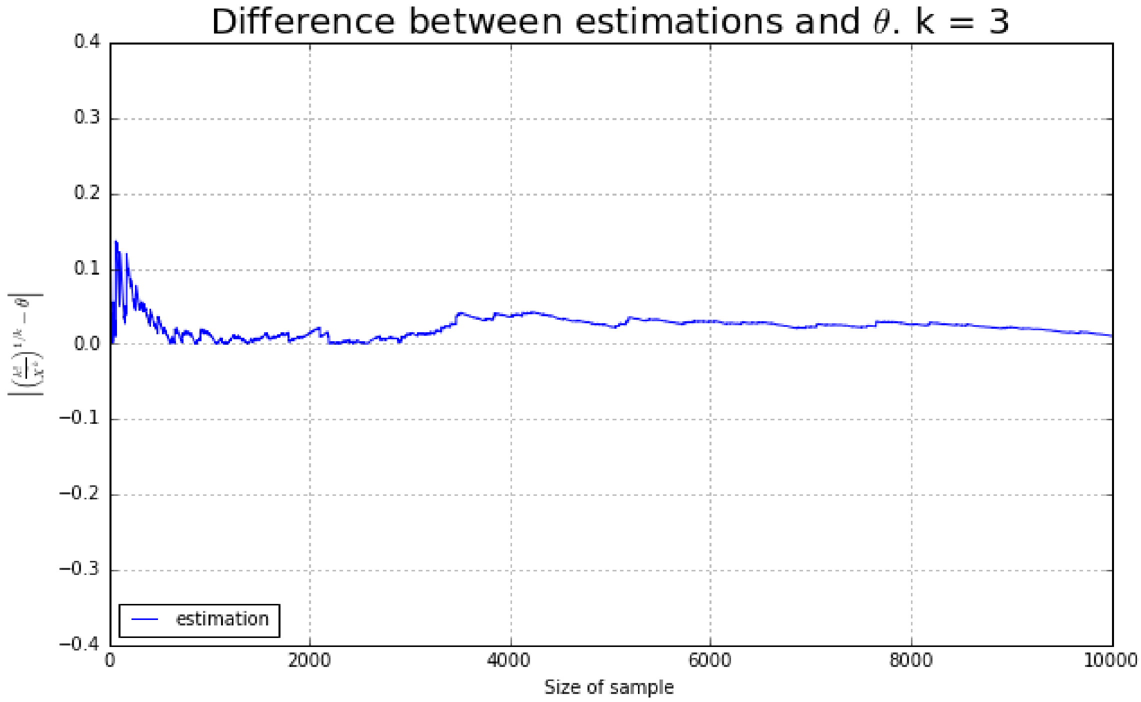
Построим графики для каждого из этих значений k в новом масштабе.

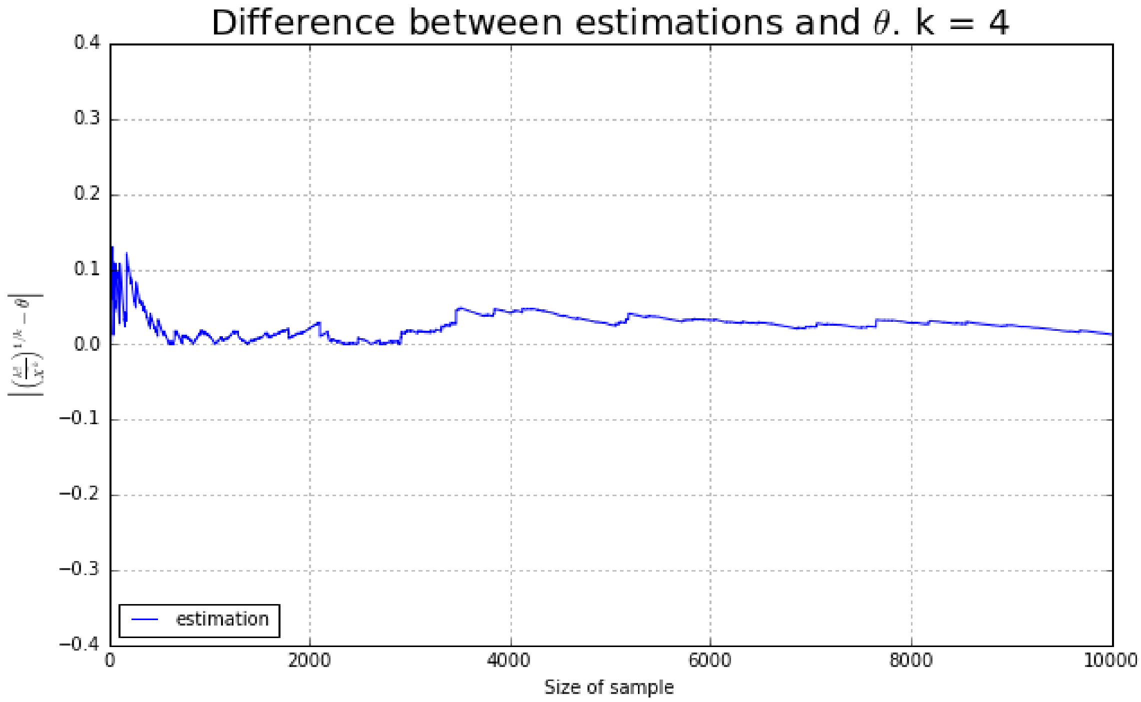
In [4]:

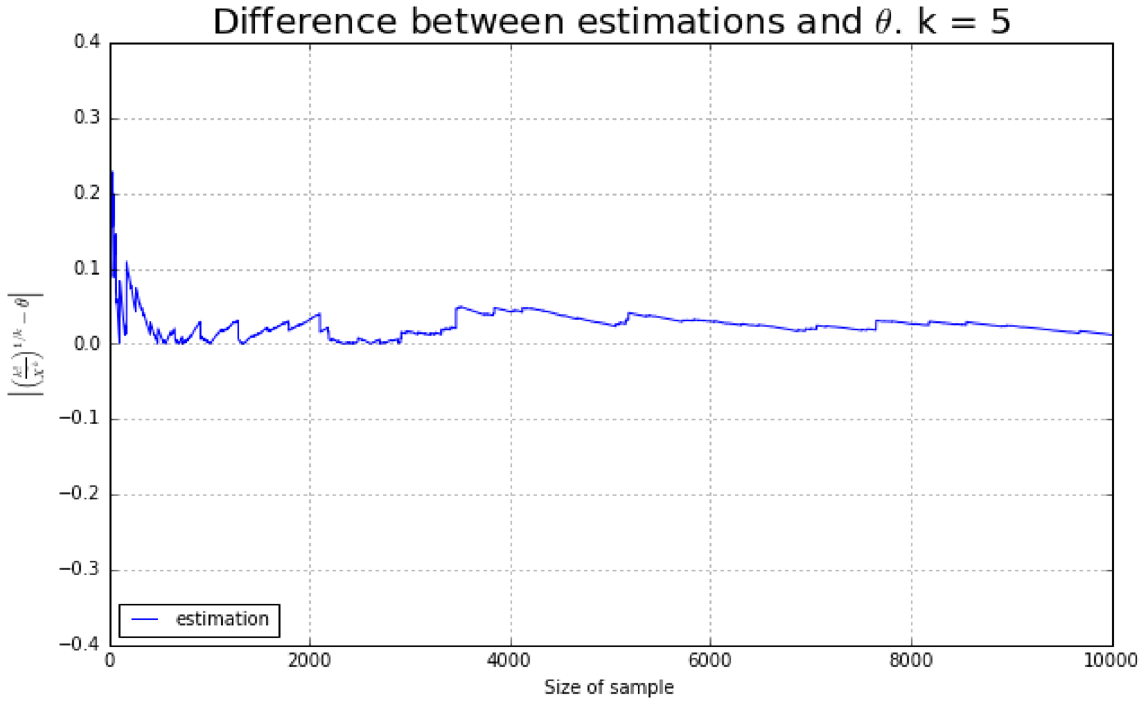
```
for k in range(1, 16, 1):
    # Считаем оценку параметра theta для всех n.
    k_moment = sample ** k
    estimation = (factorial(k) / (k_moment.cumsum() / [n for n in range(1, N +
1, 1)]) ) ** (1/k)

    # Строим график функции модуля разности оценки и истинного значения theta.
    plt.figure(figsize=(10, 6))
    plt.plot(np.linspace(1, N, N), abs(estimation - theta), label="estimation")

    plt.title(r'Difference between estimations and $\theta$. k = ' + str(k), fon
tsize=20)
    plt.xlabel(r'Size of sample', fontsize='10')
    plt.ylabel(r'$\left|\left(\frac{k!}{\bar{X^k}}\right)^{1/k} - \theta\right|
$', fontsize='10')
    plt.legend(fontsize=10, loc=3)
    plt.ylim(-0.4, 0.4)
    plt.grid()
    plt.show()
```


Difference between estimations and θ. k = 1
$\left|\left(\frac{k!}{\bar{X^k}}\right)^{1/k} - \theta\right|$
Size of sample
estimation
0.4
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
−0.4
0
2000
4000
6000
8000
10000

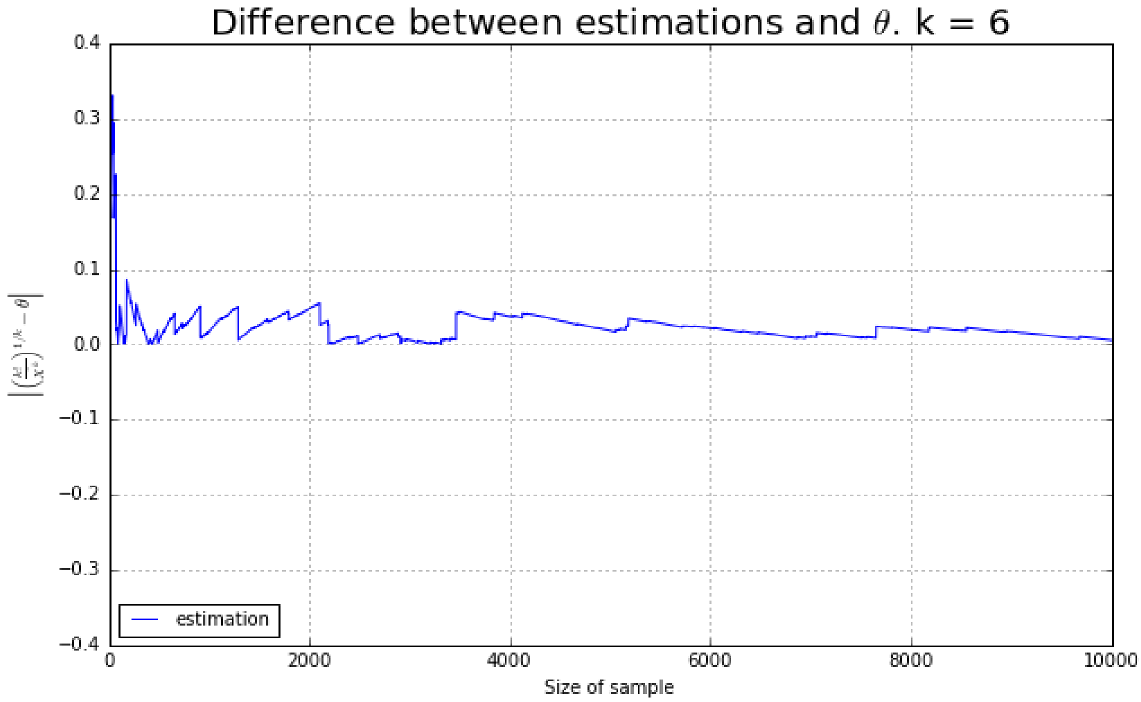

Difference between estimations and θ. k = 2
$\left|\left(\frac{k!}{\bar{X^k}}\right)^{1/k} - \theta\right|$
Size of sample
estimation
0.4
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
−0.4
0
2000
4000
6000
8000
10000


Difference between estimations and θ. k = 3
$\left|\left(\frac{k!}{\bar{X^k}}\right)^{1/k} - \theta\right|$
Size of sample
estimation

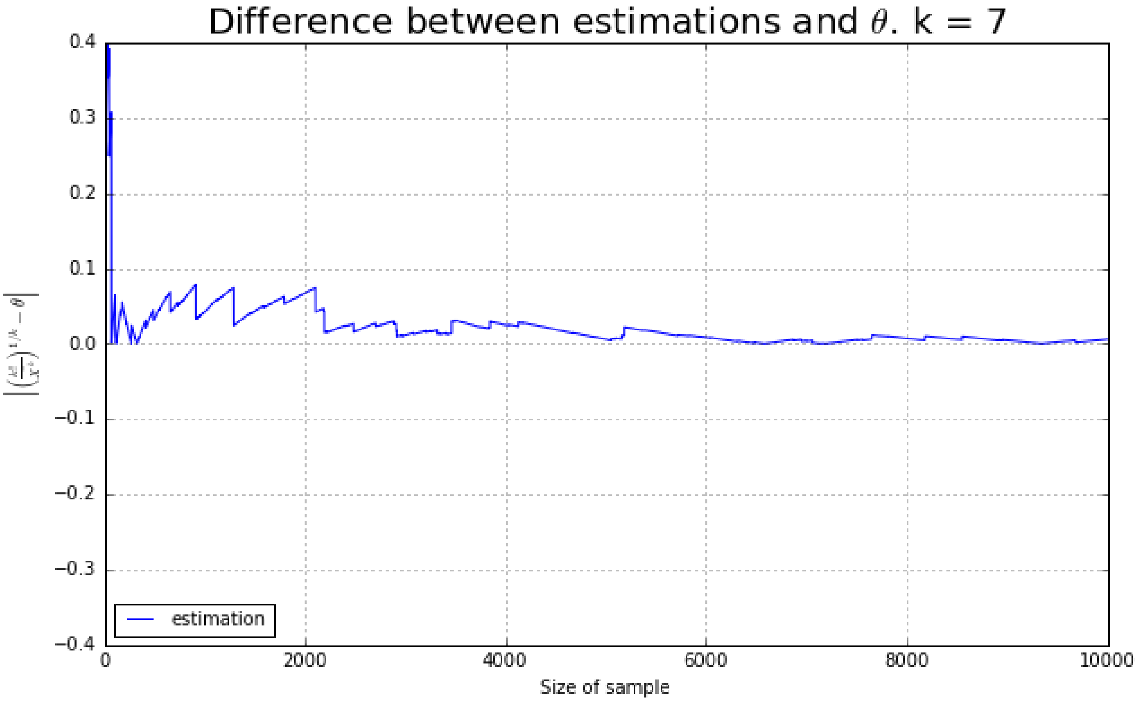

Difference between estimations and θ. k = 4
$\left|\left(\frac{k!}{\bar{X^k}}\right)^{1/k} - \theta\right|$
Size of sample
estimation

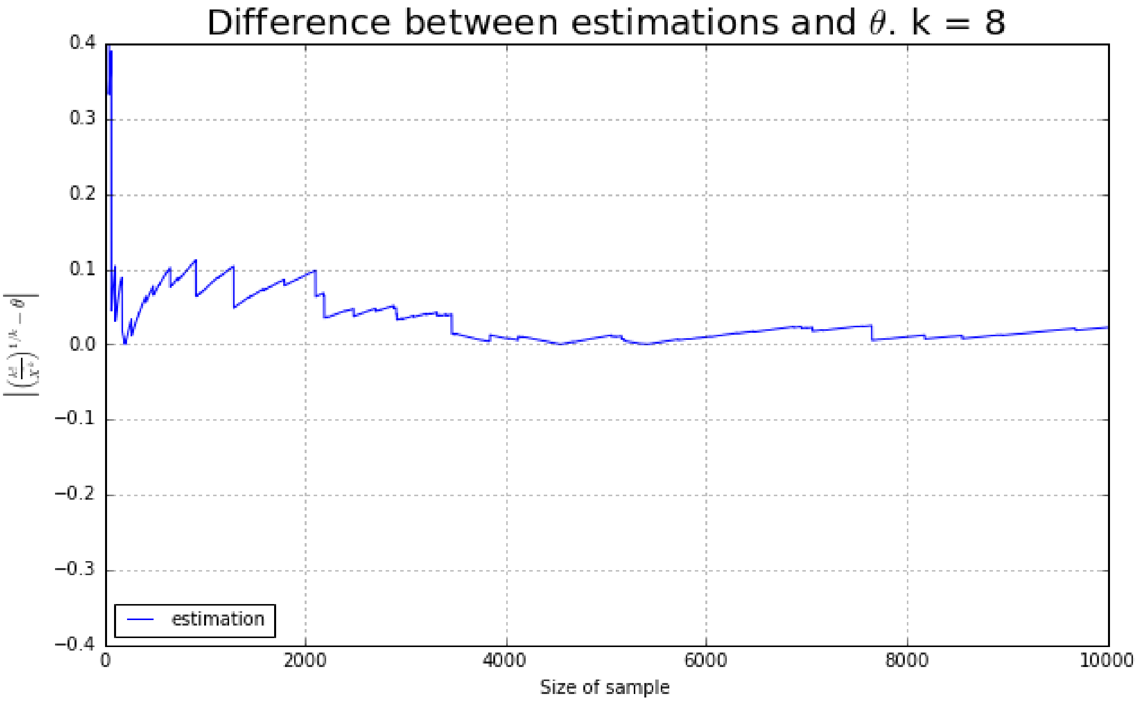


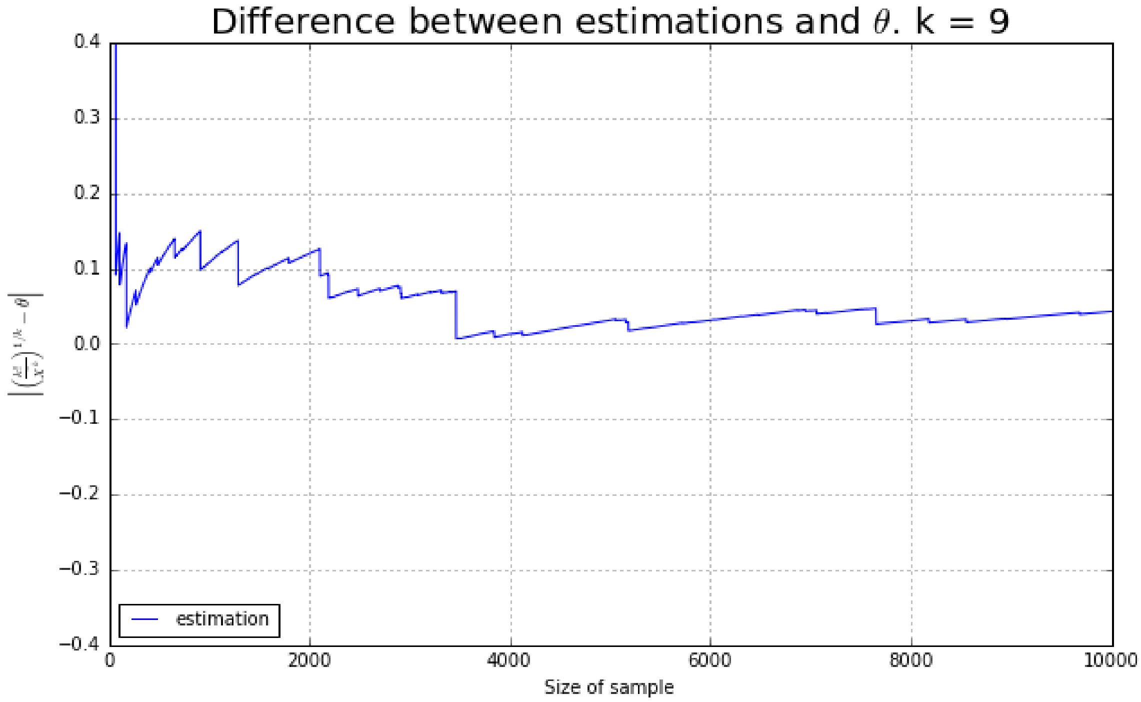
Difference between estimations and θ. k = 5
$\left|\left(\frac{k!}{\bar{X}^k}\right)^{1/k} - \theta\right|$
Size of sample
estimation
0.4
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
−0.4
0
2000
4000
6000
8000
10000

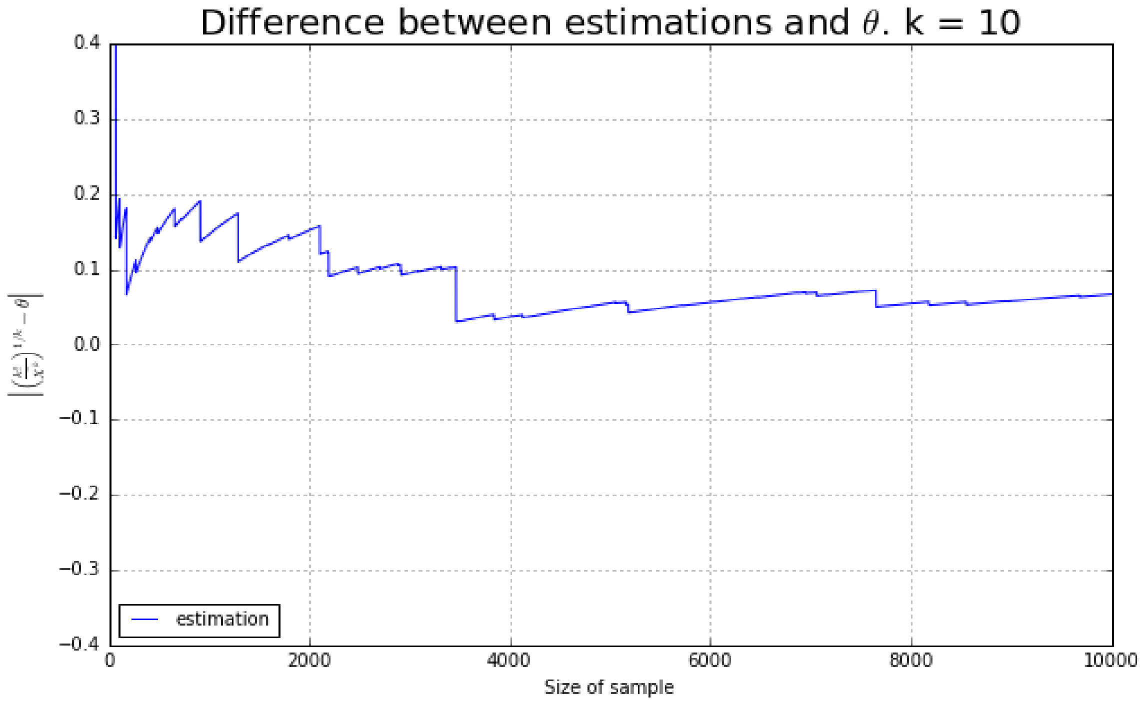


Difference between estimations and θ. k = 6
$\left|\left(\frac{k!}{\bar{X}^k}\right)^{1/k} - \theta\right|$
Size of sample
estimation
0.4
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
−0.4
0
2000
4000
6000
8000
10000

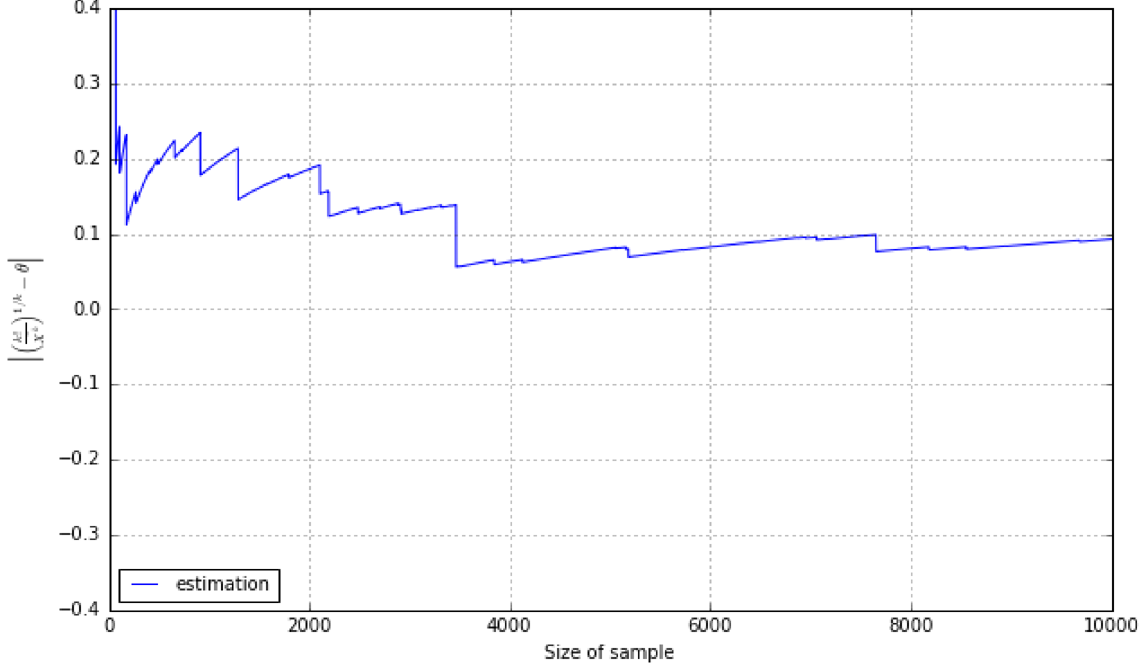

Difference between estimations and θ. k = 7
0.4
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
−0.4
$\left|\left(\frac{k!}{\overline{X^k}}\right)^{1/k} - \theta\right|$
estimation
0
2000
4000
6000
8000
10000
Size of sample


Difference between estimations and θ. k = 8
0.4
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
−0.4
$\left|\left(\frac{k!}{\overline{X^k}}\right)^{1/k} - \theta\right|$
estimation
0
2000
4000
6000
8000
10000
Size of sample

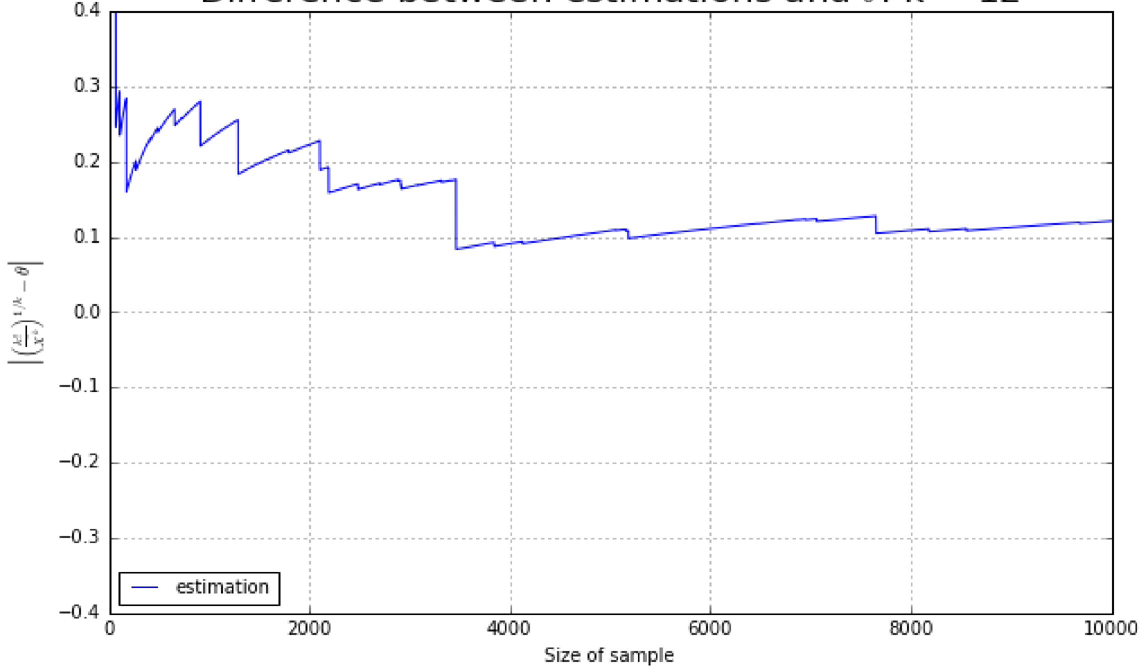

Difference between estimations and θ. k = 9
0.4
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
−0.4
$\left|\left(\frac{k!}{\bar{X}^k}\right)^{1/k}-\theta\right|$
estimation
0
2000
4000
6000
8000
10000
Size of sample

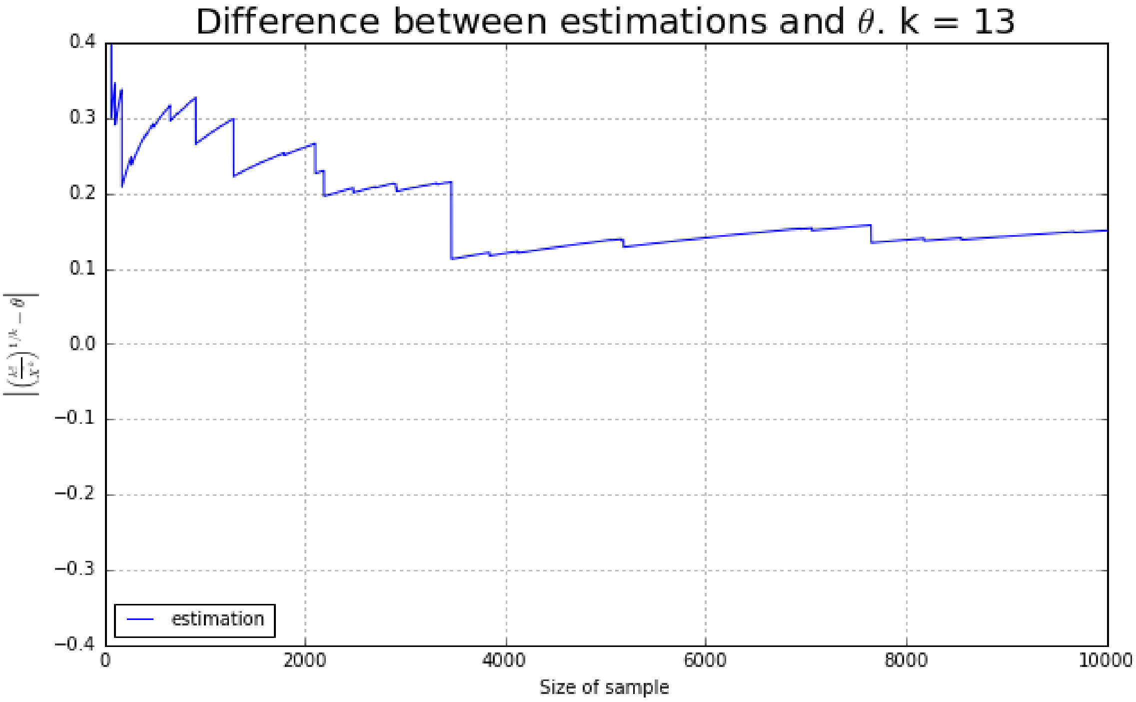

Difference between estimations and θ. k = 10
0.4
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
−0.4
$\left|\left(\frac{k!}{\bar{X}^k}\right)^{1/k}-\theta\right|$
estimation
0
2000
4000
6000
8000
10000
Size of sample

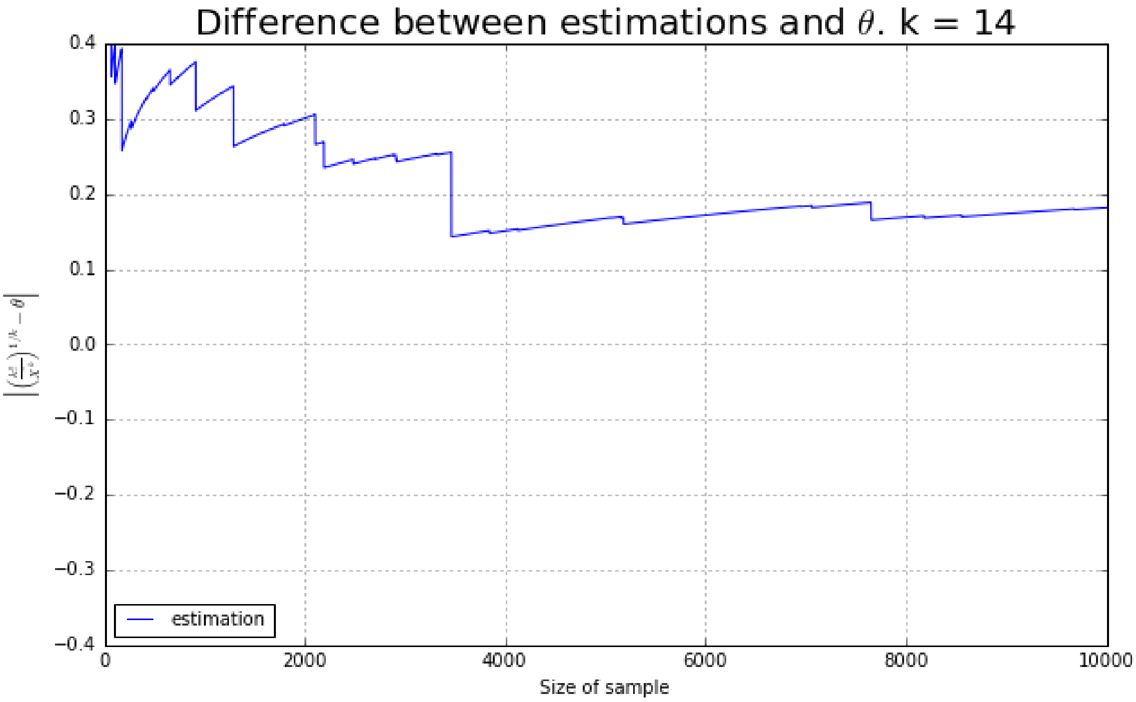
Difference between estimations and θ. k = 11

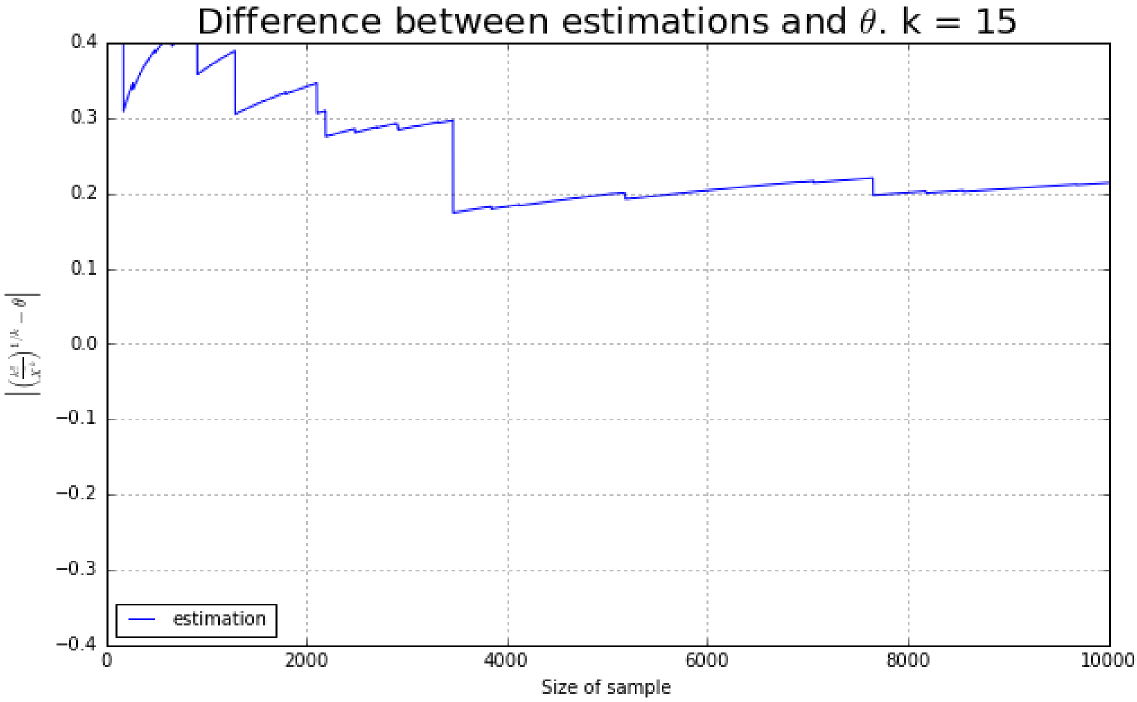


Difference between estimations and θ. k = 12




Difference between estimations and θ. k = 13
0.4
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
−0.4
$\left|\left(\frac{k!}{X^k}\right)^{1/k} - \theta\right|$
estimation
0
2000
4000
6000
8000
10000
Size of sample


Difference between estimations and θ. k = 14
0.4
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
−0.4
$\left|\left(\frac{k!}{X^k}\right)^{1/k} - \theta\right|$
estimation
0
2000
4000
6000
8000
10000
Size of sample



Вывод:

Мы провели эксперименты по исследованию оценки $(k!/\bar{X^k})^{1/k}$ для выборки $X_1 \dots X_N \sim Exp(\theta),\ \theta = 1.$

По полученным графикам можно сделать вывод, что при достаточно больших n данная оценка получается наиболее точной при $k \in \{1, \dots, 7\}$.