Кафедра дискретной математики МФТИ

Курс математической статистики

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Задание №6

№1.

Имеются серверы, которые периодически выходят из строя. Обозначим ξ_i время между i-м и (i+1)-м моментами выхода сервера из строя. Предполагается, что величины ξ_i независимы в совокупности и имеют экспоненциальное распределение с параметром λ.

Обозначим N_t — количество серверов, которые вышли из строя к моменту времени t (в начальный момент времени $N_0 = 0$). В курсе случайных процессов будет доказано, что для любых $s < t$ величина $N_t - N_s \sim Pois(\lambda(t-s))$ и независима с N_s. При этом N_t как функция от t будет называться пуассоновским процессом интенсивности λ.

Необходимо узнать, сколько серверов нужно докупить к моменту времени t взамен вышедших из строя. В момент времени s предсказанием количества серверов, вышедших из строя к моменту времени t, будем считать величину $E(N_t|N_s)$. Напишите программу, которая с момента запуска через каждые t_0 секунд (можно брать $t_0/100$, чтобы не спать перед компьютером) будет выводить уточненное значение предсказания, т.е. $E(N_t|N_{kt_0})$ для $k \in \mathbb{N}$. В текстовых полях jupyter-ноутбука напишите явно вывод формулы для $E(N_t|N_s)$.

В файле 6.csv содержатся сообщения о выходе из строя серверов. По этим данным напишите программу, которая каждые t_0 секунд выдает значение предсказания. Значения параметров t_0, t и λ также находятся в приложенном файле.

In [1]:

```
import numpy as np
```

In [2]:

```
# Открываем и читаем файл.
f = open('6.csv.xls', 'r')
text = f.read()
lines = text.split('\n')
```

In [11]:

```
print(lines[:3])
```

```
['lambda = 66', 't_0 = 200', 't = 70000']
```

In [12]:

```
# Данные из файла:
lamb = 1 / 66
t_0 = 200
t = 70000
sample = np.hstack(([0], np.array([float(line) for line in lines[3:len(lines) -
1]])))
```

Посчитаем в момент s предсказание количества компьютеров, которые выйдут из строя к моменту t:

$$E(N_t|N_s) = E(N_t - N_s + N_s|N_s) = E(N_t - N_s|N_s) + E(N_s|N_s) = E(N_t - N_s) + N_s = \lambda(t-s) + N_s$$

In [28]:

```
# Выведем предсказания для всех s от 0 до t = 70000 с шагом t_0 = 200
# N_s - количество компьютеров, вышедших из строя к моменту времени s.
N_s = 0
for k in np.arange(0, (70000 / 200) + 1):
    s = k * t_0

    # Подсчет N_s.
    flag = False
    while N_s < len(sample) and s > sample[N_s]:
        N_s += 1
        flag = True
    if flag is True:
        N_s -= 1

    # Вывод.
    print('time: %7s' % s, "  broken: %4s" % N_s, "  estimation: %.2f" %  (N_s +
 lamb * (t - s)))
```

```
time:     0.0   broken:    0   estimation: 1060.61
time:   200.0   broken:    3   estimation: 1060.58
time:   400.0   broken:    4   estimation: 1058.55
time:   600.0   broken:    8   estimation: 1059.52
time:   800.0   broken:   11   estimation: 1059.48
time:  1000.0   broken:   12   estimation: 1057.45
time:  1200.0   broken:   19   estimation: 1061.42
time:  1400.0   broken:   29   estimation: 1068.39
time:  1600.0   broken:   32   estimation: 1068.36
time:  1800.0   broken:   33   estimation: 1066.33
time:  2000.0   broken:   38   estimation: 1068.30
time:  2200.0   broken:   40   estimation: 1067.27
time:  2400.0   broken:   44   estimation: 1068.24
time:  2600.0   broken:   47   estimation: 1068.21
time:  2800.0   broken:   52   estimation: 1070.18
time:  3000.0   broken:   55   estimation: 1070.15
time:  3200.0   broken:   60   estimation: 1072.12
time:  3400.0   broken:   62   estimation: 1071.09
time:  3600.0   broken:   66   estimation: 1072.06
time:  3800.0   broken:   71   estimation: 1074.03
time:  4000.0   broken:   76   estimation: 1076.00
time:  4200.0   broken:   79   estimation: 1075.97
time:  4400.0   broken:   80   estimation: 1073.94
time:  4600.0   broken:   82   estimation: 1072.91
time:  4800.0   broken:   83   estimation: 1070.88
time:  5000.0   broken:   84   estimation: 1068.85
time:  5200.0   broken:   87   estimation: 1068.82
time:  5400.0   broken:   91   estimation: 1069.79
time:  5600.0   broken:   93   estimation: 1068.76
time:  5800.0   broken:   97   estimation: 1069.73
time:  6000.0   broken:  100   estimation: 1069.70
time:  6200.0   broken:  101   estimation: 1067.67
time:  6400.0   broken:  105   estimation: 1068.64
time:  6600.0   broken:  106   estimation: 1066.61
time:  6800.0   broken:  111   estimation: 1068.58
time:  7000.0   broken:  113   estimation: 1067.55
time:  7200.0   broken:  113   estimation: 1064.52
time:  7400.0   broken:  117   estimation: 1065.48
time:  7600.0   broken:  119   estimation: 1064.45
time:  7800.0   broken:  119   estimation: 1061.42
time:  8000.0   broken:  122   estimation: 1061.39
time:  8200.0   broken:  123   estimation: 1059.36
time:  8400.0   broken:  128   estimation: 1061.33
time:  8600.0   broken:  131   estimation: 1061.30
time:  8800.0   broken:  133   estimation: 1060.27
time:  9000.0   broken:  136   estimation: 1060.24
time:  9200.0   broken:  139   estimation: 1060.21
time:  9400.0   broken:  145   estimation: 1063.18
time:  9600.0   broken:  149   estimation: 1064.15
time:  9800.0   broken:  150   estimation: 1062.12
time: 10000.0   broken:  150   estimation: 1059.09
time: 10200.0   broken:  153   estimation: 1059.06
time: 10400.0   broken:  155   estimation: 1058.03
time: 10600.0   broken:  160   estimation: 1060.00
time: 10800.0   broken:  162   estimation: 1058.97
time: 11000.0   broken:  166   estimation: 1059.94
time: 11200.0   broken:  167   estimation: 1057.91
time: 11400.0   broken:  168   estimation: 1055.88
time: 11600.0   broken:  171   estimation: 1055.85
time: 11800.0   broken:  174   estimation: 1055.82
time: 12000.0   broken:  179   estimation: 1057.79
```

```
time: 12200.0   broken:  180   estimation: 1055.76
time: 12400.0   broken:  183   estimation: 1055.73
time: 12600.0   broken:  183   estimation: 1052.70
time: 12800.0   broken:  185   estimation: 1051.67
time: 13000.0   broken:  189   estimation: 1052.64
time: 13200.0   broken:  194   estimation: 1054.61
time: 13400.0   broken:  196   estimation: 1053.58
time: 13600.0   broken:  201   estimation: 1055.55
time: 13800.0   broken:  206   estimation: 1057.52
time: 14000.0   broken:  207   estimation: 1055.48
time: 14200.0   broken:  208   estimation: 1053.45
time: 14400.0   broken:  215   estimation: 1057.42
time: 14600.0   broken:  218   estimation: 1057.39
time: 14800.0   broken:  221   estimation: 1057.36
time: 15000.0   broken:  223   estimation: 1056.33
time: 15200.0   broken:  226   estimation: 1056.30
time: 15400.0   broken:  227   estimation: 1054.27
time: 15600.0   broken:  232   estimation: 1056.24
time: 15800.0   broken:  235   estimation: 1056.21
time: 16000.0   broken:  238   estimation: 1056.18
time: 16200.0   broken:  240   estimation: 1055.15
time: 16400.0   broken:  244   estimation: 1056.12
time: 16600.0   broken:  248   estimation: 1057.09
time: 16800.0   broken:  253   estimation: 1059.06
time: 17000.0   broken:  256   estimation: 1059.03
time: 17200.0   broken:  257   estimation: 1057.00
time: 17400.0   broken:  262   estimation: 1058.97
time: 17600.0   broken:  269   estimation: 1062.94
time: 17800.0   broken:  271   estimation: 1061.91
time: 18000.0   broken:  274   estimation: 1061.88
time: 18200.0   broken:  277   estimation: 1061.85
time: 18400.0   broken:  278   estimation: 1059.82
time: 18600.0   broken:  281   estimation: 1059.79
time: 18800.0   broken:  287   estimation: 1062.76
time: 19000.0   broken:  288   estimation: 1060.73
time: 19200.0   broken:  289   estimation: 1058.70
time: 19400.0   broken:  294   estimation: 1060.67
time: 19600.0   broken:  297   estimation: 1060.64
time: 19800.0   broken:  299   estimation: 1059.61
time: 20000.0   broken:  303   estimation: 1060.58
time: 20200.0   broken:  305   estimation: 1059.55
time: 20400.0   broken:  308   estimation: 1059.52
time: 20600.0   broken:  311   estimation: 1059.48
time: 20800.0   broken:  313   estimation: 1058.45
time: 21000.0   broken:  319   estimation: 1061.42
time: 21200.0   broken:  325   estimation: 1064.39
time: 21400.0   broken:  327   estimation: 1063.36
time: 21600.0   broken:  332   estimation: 1065.33
time: 21800.0   broken:  335   estimation: 1065.30
time: 22000.0   broken:  338   estimation: 1065.27
time: 22200.0   broken:  338   estimation: 1062.24
time: 22400.0   broken:  341   estimation: 1062.21
time: 22600.0   broken:  347   estimation: 1065.18
time: 22800.0   broken:  350   estimation: 1065.15
time: 23000.0   broken:  351   estimation: 1063.12
time: 23200.0   broken:  354   estimation: 1063.09
time: 23400.0   broken:  354   estimation: 1060.06
time: 23600.0   broken:  355   estimation: 1058.03
time: 23800.0   broken:  358   estimation: 1058.00
time: 24000.0   broken:  366   estimation: 1062.97
time: 24200.0   broken:  372   estimation: 1065.94
```

time: 24400.0 broken: 377 estimation: 1067.91
time: 24600.0 broken: 377 estimation: 1064.88
time: 24800.0 broken: 380 estimation: 1064.85
time: 25000.0 broken: 384 estimation: 1065.82
time: 25200.0 broken: 386 estimation: 1064.79
time: 25400.0 broken: 388 estimation: 1063.76
time: 25600.0 broken: 391 estimation: 1063.73
time: 25800.0 broken: 394 estimation: 1063.70
time: 26000.0 broken: 395 estimation: 1061.67
time: 26200.0 broken: 396 estimation: 1059.64
time: 26400.0 broken: 403 estimation: 1063.61
time: 26600.0 broken: 405 estimation: 1062.58
time: 26800.0 broken: 408 estimation: 1062.55
time: 27000.0 broken: 415 estimation: 1066.52
time: 27200.0 broken: 418 estimation: 1066.48
time: 27400.0 broken: 421 estimation: 1066.45
time: 27600.0 broken: 424 estimation: 1066.42
time: 27800.0 broken: 427 estimation: 1066.39
time: 28000.0 broken: 429 estimation: 1065.36
time: 28200.0 broken: 433 estimation: 1066.33
time: 28400.0 broken: 436 estimation: 1066.30
time: 28600.0 broken: 437 estimation: 1064.27
time: 28800.0 broken: 440 estimation: 1064.24
time: 29000.0 broken: 443 estimation: 1064.21
time: 29200.0 broken: 445 estimation: 1063.18
time: 29400.0 broken: 448 estimation: 1063.15
time: 29600.0 broken: 448 estimation: 1060.12
time: 29800.0 broken: 449 estimation: 1058.09
time: 30000.0 broken: 453 estimation: 1059.06
time: 30200.0 broken: 453 estimation: 1056.03
time: 30400.0 broken: 455 estimation: 1055.00
time: 30600.0 broken: 458 estimation: 1054.97
time: 30800.0 broken: 460 estimation: 1053.94
time: 31000.0 broken: 460 estimation: 1050.91
time: 31200.0 broken: 462 estimation: 1049.88
time: 31400.0 broken: 465 estimation: 1049.85
time: 31600.0 broken: 467 estimation: 1048.82
time: 31800.0 broken: 470 estimation: 1048.79
time: 32000.0 broken: 472 estimation: 1047.76
time: 32200.0 broken: 473 estimation: 1045.73
time: 32400.0 broken: 473 estimation: 1042.70
time: 32600.0 broken: 476 estimation: 1042.67
time: 32800.0 broken: 476 estimation: 1039.64
time: 33000.0 broken: 479 estimation: 1039.61
time: 33200.0 broken: 481 estimation: 1038.58
time: 33400.0 broken: 482 estimation: 1036.55
time: 33600.0 broken: 484 estimation: 1035.52
time: 33800.0 broken: 485 estimation: 1033.48
time: 34000.0 broken: 487 estimation: 1032.45
time: 34200.0 broken: 490 estimation: 1032.42
time: 34400.0 broken: 492 estimation: 1031.39
time: 34600.0 broken: 492 estimation: 1028.36
time: 34800.0 broken: 496 estimation: 1029.33
time: 35000.0 broken: 497 estimation: 1027.30
time: 35200.0 broken: 497 estimation: 1024.27
time: 35400.0 broken: 500 estimation: 1024.24
time: 35600.0 broken: 504 estimation: 1025.21
time: 35800.0 broken: 509 estimation: 1027.18
time: 36000.0 broken: 512 estimation: 1027.15
time: 36200.0 broken: 515 estimation: 1027.12
time: 36400.0 broken: 517 estimation: 1026.09

time: 36600.0 broken: 519 estimation: 1025.06
time: 36800.0 broken: 525 estimation: 1028.03
time: 37000.0 broken: 528 estimation: 1028.00
time: 37200.0 broken: 532 estimation: 1028.97
time: 37400.0 broken: 535 estimation: 1028.94
time: 37600.0 broken: 538 estimation: 1028.91
time: 37800.0 broken: 542 estimation: 1029.88
time: 38000.0 broken: 543 estimation: 1027.85
time: 38200.0 broken: 548 estimation: 1029.82
time: 38400.0 broken: 553 estimation: 1031.79
time: 38600.0 broken: 556 estimation: 1031.76
time: 38800.0 broken: 558 estimation: 1030.73
time: 39000.0 broken: 560 estimation: 1029.70
time: 39200.0 broken: 562 estimation: 1028.67
time: 39400.0 broken: 564 estimation: 1027.64
time: 39600.0 broken: 565 estimation: 1025.61
time: 39800.0 broken: 567 estimation: 1024.58
time: 40000.0 broken: 569 estimation: 1023.55
time: 40200.0 broken: 573 estimation: 1024.52
time: 40400.0 broken: 577 estimation: 1025.48
time: 40600.0 broken: 581 estimation: 1026.45
time: 40800.0 broken: 585 estimation: 1027.42
time: 41000.0 broken: 588 estimation: 1027.39
time: 41200.0 broken: 594 estimation: 1030.36
time: 41400.0 broken: 600 estimation: 1033.33
time: 41600.0 broken: 605 estimation: 1035.30
time: 41800.0 broken: 606 estimation: 1033.27
time: 42000.0 broken: 609 estimation: 1033.24
time: 42200.0 broken: 613 estimation: 1034.21
time: 42400.0 broken: 615 estimation: 1033.18
time: 42600.0 broken: 617 estimation: 1032.15
time: 42800.0 broken: 620 estimation: 1032.12
time: 43000.0 broken: 625 estimation: 1034.09
time: 43200.0 broken: 625 estimation: 1031.06
time: 43400.0 broken: 627 estimation: 1030.03
time: 43600.0 broken: 627 estimation: 1027.00
time: 43800.0 broken: 631 estimation: 1027.97
time: 44000.0 broken: 633 estimation: 1026.94
time: 44200.0 broken: 634 estimation: 1024.91
time: 44400.0 broken: 636 estimation: 1023.88
time: 44600.0 broken: 639 estimation: 1023.85
time: 44800.0 broken: 643 estimation: 1024.82
time: 45000.0 broken: 643 estimation: 1021.79
time: 45200.0 broken: 647 estimation: 1022.76
time: 45400.0 broken: 650 estimation: 1022.73
time: 45600.0 broken: 651 estimation: 1020.70
time: 45800.0 broken: 654 estimation: 1020.67
time: 46000.0 broken: 657 estimation: 1020.64
time: 46200.0 broken: 659 estimation: 1019.61
time: 46400.0 broken: 663 estimation: 1020.58
time: 46600.0 broken: 667 estimation: 1021.55
time: 46800.0 broken: 670 estimation: 1021.52
time: 47000.0 broken: 673 estimation: 1021.48
time: 47200.0 broken: 676 estimation: 1021.45
time: 47400.0 broken: 680 estimation: 1022.42
time: 47600.0 broken: 684 estimation: 1023.39
time: 47800.0 broken: 686 estimation: 1022.36
time: 48000.0 broken: 687 estimation: 1020.33
time: 48200.0 broken: 689 estimation: 1019.30
time: 48400.0 broken: 692 estimation: 1019.27
time: 48600.0 broken: 695 estimation: 1019.24

time: 48800.0 broken: 700 estimation: 1021.21
time: 49000.0 broken: 704 estimation: 1022.18
time: 49200.0 broken: 708 estimation: 1023.15
time: 49400.0 broken: 710 estimation: 1022.12
time: 49600.0 broken: 713 estimation: 1022.09
time: 49800.0 broken: 715 estimation: 1021.06
time: 50000.0 broken: 719 estimation: 1022.03
time: 50200.0 broken: 721 estimation: 1021.00
time: 50400.0 broken: 724 estimation: 1020.97
time: 50600.0 broken: 728 estimation: 1021.94
time: 50800.0 broken: 732 estimation: 1022.91
time: 51000.0 broken: 736 estimation: 1023.88
time: 51200.0 broken: 738 estimation: 1022.85
time: 51400.0 broken: 740 estimation: 1021.82
time: 51600.0 broken: 741 estimation: 1019.79
time: 51800.0 broken: 743 estimation: 1018.76
time: 52000.0 broken: 746 estimation: 1018.73
time: 52200.0 broken: 753 estimation: 1022.70
time: 52400.0 broken: 756 estimation: 1022.67
time: 52600.0 broken: 757 estimation: 1020.64
time: 52800.0 broken: 760 estimation: 1020.61
time: 53000.0 broken: 765 estimation: 1022.58
time: 53200.0 broken: 766 estimation: 1020.55
time: 53400.0 broken: 772 estimation: 1023.52
time: 53600.0 broken: 772 estimation: 1020.48
time: 53800.0 broken: 774 estimation: 1019.45
time: 54000.0 broken: 776 estimation: 1018.42
time: 54200.0 broken: 778 estimation: 1017.39
time: 54400.0 broken: 779 estimation: 1015.36
time: 54600.0 broken: 782 estimation: 1015.33
time: 54800.0 broken: 783 estimation: 1013.30
time: 55000.0 broken: 784 estimation: 1011.27
time: 55200.0 broken: 788 estimation: 1012.24
time: 55400.0 broken: 790 estimation: 1011.21
time: 55600.0 broken: 794 estimation: 1012.18
time: 55800.0 broken: 798 estimation: 1013.15
time: 56000.0 broken: 800 estimation: 1012.12
time: 56200.0 broken: 801 estimation: 1010.09
time: 56400.0 broken: 803 estimation: 1009.06
time: 56600.0 broken: 807 estimation: 1010.03
time: 56800.0 broken: 814 estimation: 1014.00
time: 57000.0 broken: 821 estimation: 1017.97
time: 57200.0 broken: 824 estimation: 1017.94
time: 57400.0 broken: 824 estimation: 1014.91
time: 57600.0 broken: 828 estimation: 1015.88
time: 57800.0 broken: 832 estimation: 1016.85
time: 58000.0 broken: 837 estimation: 1018.82
time: 58200.0 broken: 838 estimation: 1016.79
time: 58400.0 broken: 841 estimation: 1016.76
time: 58600.0 broken: 843 estimation: 1015.73
time: 58800.0 broken: 843 estimation: 1012.70
time: 59000.0 broken: 844 estimation: 1010.67
time: 59200.0 broken: 847 estimation: 1010.64
time: 59400.0 broken: 848 estimation: 1008.61
time: 59600.0 broken: 851 estimation: 1008.58
time: 59800.0 broken: 853 estimation: 1007.55
time: 60000.0 broken: 857 estimation: 1008.52
time: 60200.0 broken: 857 estimation: 1005.48
time: 60400.0 broken: 859 estimation: 1004.45
time: 60600.0 broken: 866 estimation: 1008.42
time: 60800.0 broken: 868 estimation: 1007.39

```
time: 61000.0   broken:  870   estimation: 1006.36
time: 61200.0   broken:  872   estimation: 1005.33
time: 61400.0   broken:  875   estimation: 1005.30
time: 61600.0   broken:  878   estimation: 1005.27
time: 61800.0   broken:  881   estimation: 1005.24
time: 62000.0   broken:  884   estimation: 1005.21
time: 62200.0   broken:  885   estimation: 1003.18
time: 62400.0   broken:  891   estimation: 1006.15
time: 62600.0   broken:  891   estimation: 1003.12
time: 62800.0   broken:  894   estimation: 1003.09
time: 63000.0   broken:  898   estimation: 1004.06
time: 63200.0   broken:  901   estimation: 1004.03
time: 63400.0   broken:  905   estimation: 1005.00
time: 63600.0   broken:  907   estimation: 1003.97
time: 63800.0   broken:  910   estimation: 1003.94
time: 64000.0   broken:  914   estimation: 1004.91
time: 64200.0   broken:  918   estimation: 1005.88
time: 64400.0   broken:  922   estimation: 1006.85
time: 64600.0   broken:  924   estimation: 1005.82
time: 64800.0   broken:  928   estimation: 1006.79
time: 65000.0   broken:  934   estimation: 1009.76
time: 65200.0   broken:  937   estimation: 1009.73
time: 65400.0   broken:  940   estimation: 1009.70
time: 65600.0   broken:  941   estimation: 1007.67
time: 65800.0   broken:  944   estimation: 1007.64
time: 66000.0   broken:  947   estimation: 1007.61
time: 66200.0   broken:  949   estimation: 1006.58
time: 66400.0   broken:  953   estimation: 1007.55
time: 66600.0   broken:  956   estimation: 1007.52
time: 66800.0   broken:  960   estimation: 1008.48
time: 67000.0   broken:  963   estimation: 1008.45
time: 67200.0   broken:  968   estimation: 1010.42
time: 67400.0   broken:  972   estimation: 1011.39
time: 67600.0   broken:  975   estimation: 1011.36
time: 67800.0   broken:  979   estimation: 1012.33
time: 68000.0   broken:  981   estimation: 1011.30
time: 68200.0   broken:  984   estimation: 1011.27
time: 68400.0   broken:  986   estimation: 1010.24
time: 68600.0   broken:  989   estimation: 1010.21
time: 68800.0   broken:  991   estimation: 1009.18
time: 69000.0   broken:  997   estimation: 1012.15
time: 69200.0   broken: 1000   estimation: 1012.12
time: 69400.0   broken: 1000   estimation: 1009.09
time: 69600.0   broken: 1000   estimation: 1006.06
time: 69800.0   broken: 1000   estimation: 1003.03
time: 70000.0   broken: 1000   estimation: 1000.00
```

Вывод

Мы вывели формулу для величины $E(N_t|N_s)$, которая, как оказалось, очень неплохо предсказывает количество компьютеров, которые нужно докупить к моменту t. Из приведенных выше данных видно, что со временем, при росте числа уже сломавшихся компьютеров, а значит, и при росте информации о процессе, предсказание количества поломок к моменту t постепенно становится точнее и сходится к истинному значению.

In [ ]: