Back to all puzzles

Boss Fight

Level 3 Time limit: 10s Solved

There are N warriors, the ith of which has a health of H_i units and can deal D_i units of damage per second. They are confronting a boss who has unlimited health and can deal B units of damage per second. Both the warriors and the boss deal damage continuously — for example, in half a second, the boss deals $B/2$ units of damage.

The warriors feel it would be unfair for many of them to fight the boss at once, so they'll select just two representatives to go into battle. One warrior i will be the front line, and a different warrior j will back them up. During the battle, the boss will attack warrior i until that warrior is defeated (that is, until the boss has dealt H_i units of damage to them), and will then attack warrior j until that warrior is also defeated, at which point the battle will end. Along the way, each of the two warriors will do damage to the boss as long as they are undefeated.

Of course, the warriors will never prevail, but they'd like to determine the maximum amount of damage they could deal to the boss for any choice of warriors i and j before the battle ends.

Note: Your return value must have an absolute or relative error of at most 10^{-6} to be considered correct.

Constraints

$2 \leq N \leq 500{,}000$
$1 \leq H_i \leq 1{,}000{,}000{,}000$
$1 \leq D_i \leq 1{,}000{,}000{,}000$
$1 \leq B \leq 1{,}000{,}000{,}000$

Sample test case #1

```
N = 3
H = [2, 1, 4]
D = [3, 1, 2]
B = 4
```

```
Expected Return Value = 6.500000
```

Sample test case #2

```
N = 4
H = [1, 1, 2, 100]
D = [1, 2, 1, 3]
B = 8
```

```
Expected Return Value = 62.750000
```

Sample test case #3

```
N = 4
H = [1, 1, 2, 3]
D = [1, 2, 1, 100]
B = 8
```

```
Expected Return Value = 62.750000
```

Sample Explanation

In the first case, there are 3 warriors with healths of $[2, 1, 4]$ units, and the ability to deal $[3, 1, 2]$ units of damage respectively. The boss does $B = 4$ units of damage per second.

The optimal solution is to choose warrior 3 as the front line and warrior 1 as backup. Warrior 3 will be defeated after 1 second, dealing 2 units of damage during this time (meanwhile, warrior 1 will have dealt 3 units of damage). Warrior 1 will then step up and last for 0.5 seconds, while dealing another 1.5 units of damage along the way. The total damage dealt will then be $2 + 3 + 1.5 = 6.5$ units.

In each of the second and third cases, it's possible for 62.75 units of damage to be dealt to the boss, though with different configurations of warriors.

⚠ **The code editor for solving puzzles is only available on wider screens.**