ASTP 720 Homework Assignment 6

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Task 1

Using linear least squares, estimate the parameters in the boxed equation above, α, β, and γ, and their uncertainties. You can pick your favorite photometric band.

The period-luminosity-metallicity relation of the Cepheids can be expressed as

$$M = \alpha + \beta log(P) + \gamma(Z) \tag{1}$$

where M is the absolute magnitude, P is the period, Z is the metallicity and α, β and γ are the parameters for the model. The data we are given includes the periods (in days), the metallicities, the distances, the apparent magnitudes in different bands, and the color excesses.

In order to calculate the absolute magnitudes, one can use the distance modulus, so that

$$M = m - 5\log(d) + 5 - A \tag{2}$$

where M is the absolute magnitude, m is the apparent magnitude, d is the distance and A is the interstellar extinction. The value of A can be computed from the provided color excesses, as the color excess, $E(B - V)$ can be related to the difference in the interstellar extinction in the B- and V-bands, as shown in the following equation.

$$E(B - V) = A_B - A_V \tag{3}$$

We can also look at extinction curves through the Galaxy, and come up with a dimensionless quantity,

$$R_V = \frac{A_V}{A_B - A_B} = \frac{A_V}{E(B-V)} \tag{4}$$

and by assuming a fixed value of $R_V = 3.1$, the value of A_V can be computed by knowing the the reddening (color excess).

Now, we can perform least squares to get a best-fit line to the data. First, we construct the design matrix, which holds the numerical values associated with each parameter. For example, the first column of the design matrix is 1 because α is not multiplied by anything. Then, we construct a vector $\boldsymbol{y}$ whee $\boldsymbol{y = X\theta}$ where X is the design matrix and θ is the parameter vector which holds α, β and γ. $\boldsymbol{\theta}$ can be solved for as follows:

$$\boldsymbol{\theta = (X^{\dagger}X)^{-1}X^{\dagger}y} \tag{5}$$

The uncertainties on the parameters are the square roots of the variances which are the diagonal elements of the array $\boldsymbol{(X^{\dagger}X)^{-1}}$.

I found the following values for the parameters:

α	12.84018 ± 0.02046
β	-2.12690 ± 0.02810
γ	-0.09353 ± 0.06368

Task 2

Please plot your fit against the data. You can do this in a number of ways, I leave that to you as long as you can demonstrate that the fit is good (this is also a good check for you!).

To test my fit, I plotted M vs. $log(P)$ and then over-plotted my line without the gamma portion, as the extra dependence on metallicity would not allow for a fair comparison. This is shown in Figure 1.

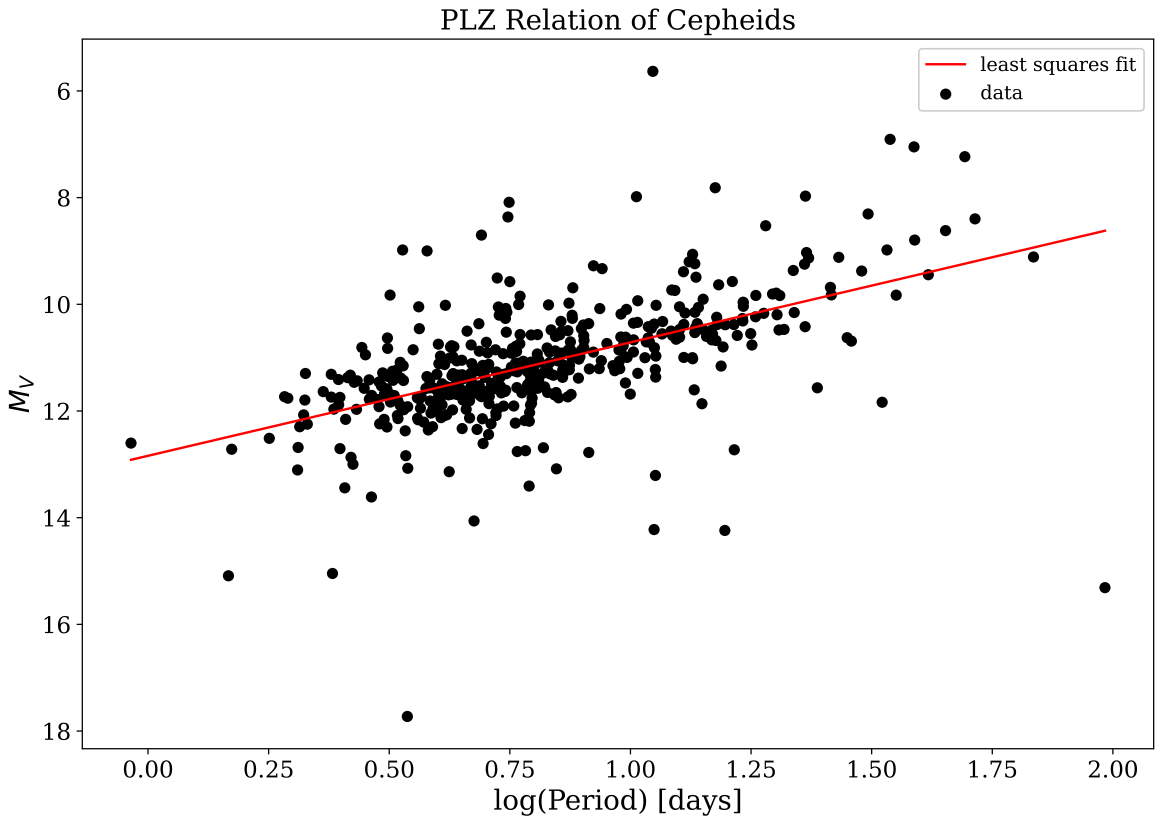

Figure 1: Cepheid Period Luminosity relationship with over-plotted best fit line from least squares. Equation of line is: $M_V = 12.84 - 2.12\ log(P)$.

Task 3

Assume that the errors on the apparent magnitudes are all m = 0.1 mag. Repeat the estimate of the parameters and their uncertainties.

First, the errors on the apparent magnitude must be converted to absolute magnitudes, using the same method in Task 1. Now, the parameter vector can be found as,

$$\boldsymbol{\theta} = (\boldsymbol{X}^{\dagger}\boldsymbol{V}^{-1}\boldsymbol{X})^{-1}\boldsymbol{X}^{\dagger}\boldsymbol{V}^{-1}\boldsymbol{y} \tag{6}$$

where $\boldsymbol{V}$ is the covariance matrix and is equal to

$$\boldsymbol{V} = \boldsymbol{\epsilon}\boldsymbol{\epsilon}^{\dagger} \tag{7}$$

where $\boldsymbol{\epsilon}$ is the error vector. The errors are converted from apparent magnitudes to absolute magnitudes using same procedure as Task 1. The plot of the data with the best fit line is shown in Figure 2, and the parameters are:

α	$14.348126 \pm 1.85637 \times 10^{-18}$
β	$-3.23832 \pm 3.96686 \times 10^{-18}$
γ	$1.33590 \pm 3.20810 \times 10^{-18}$

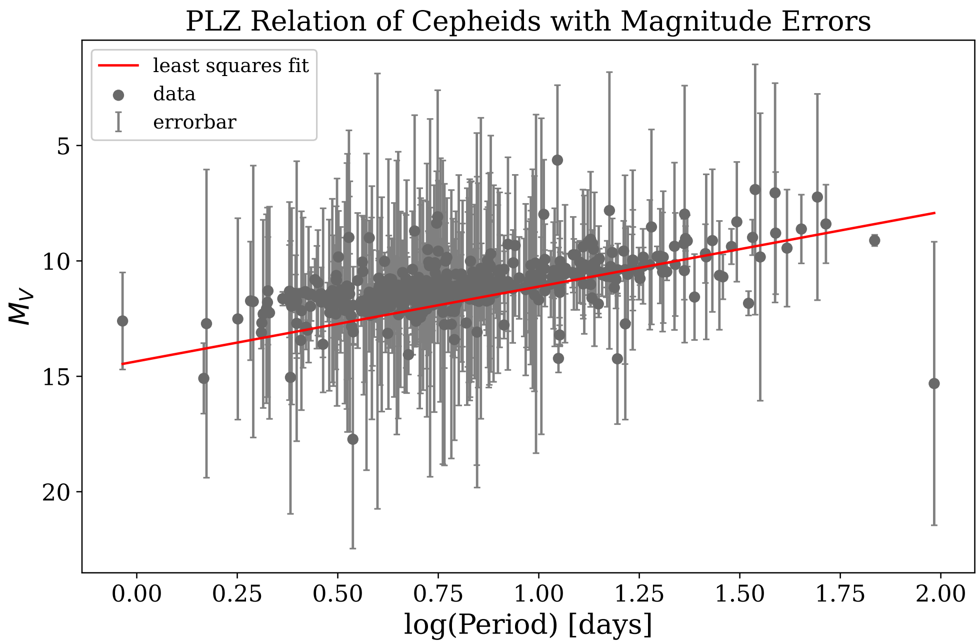

Figure 2: Cepheid Period Luminosity relationship with over-plotted best fit line from least squares. There are uniform errors on the magnitudes. Equation of line is: $M_V = 14.35 - 3.24\ log(P)$.