Fundamentals HW Assignment 5

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November 23, 2020

Task 1

In your write-up, please write down and describe your model.

My model can be expressed as:

$X_t = \phi_1 X_{t-1} + \phi_{12} X_{t-12} + \phi_{132} X_{t-132} + Z_t$

- The first term represents the current value's relation to the previous value.
- The second term represents the current value's relation to the value from one year ago.
- The third term represents the current value's relation to the value from 11 years ago.
- The last term is the noise (shock).

So the parameters I want to fit will be: $\phi_1, \phi_{12}, \phi_{132}$.

Task 2

Using the `emcee` *package and the code snippets provided to you, fit your model to the data. You should show the fit to your model by making a corner plot with the corner package and write down your best-fit parameters. In addition, you should plot both the residuals of the fit and the fit of the time series to the data (remember that data = signal + noise).*

I did this, by defining my model function and completing the code stubs for the likelihood and priors (which I commented in the Jupyter notebook). My best-fit parameters are:

$\phi_1 = 0.50^{+0.23}_{-0.22}$, $\phi_{12} = 0.50^{+0.15}_{-0.26}$, $\phi_{132} = 0.50^{+0.14}_{-0.11}$

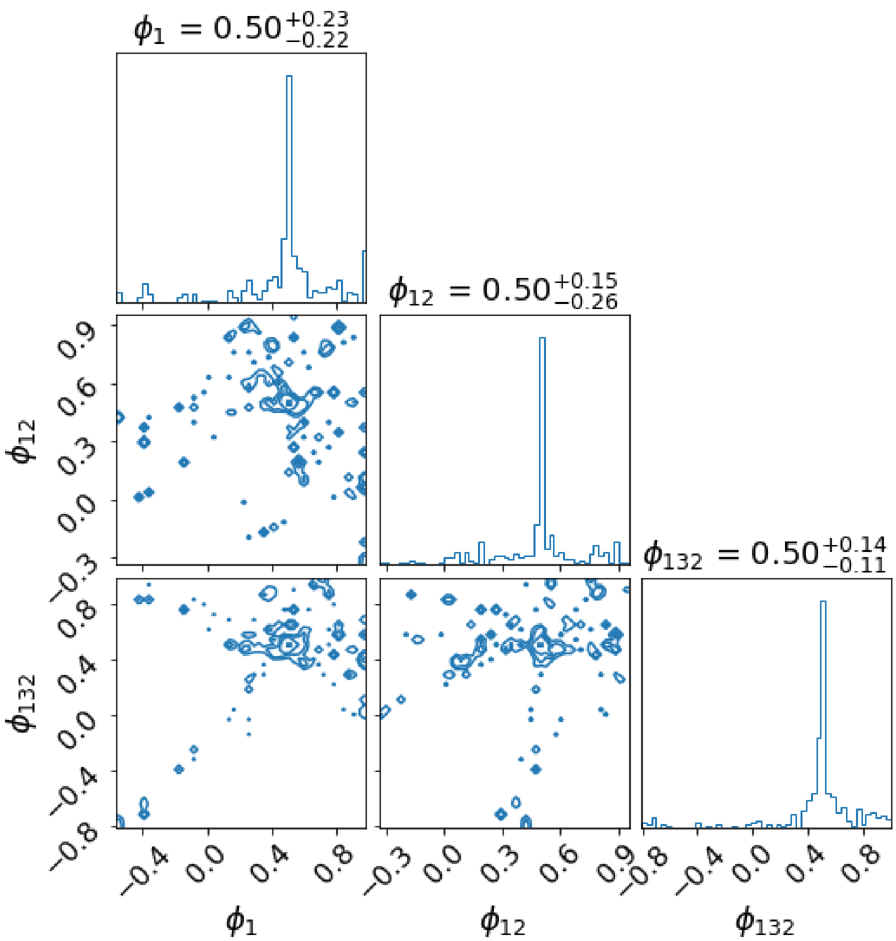

Figure 1: Corner plot of fit to data, showing values for parameters.

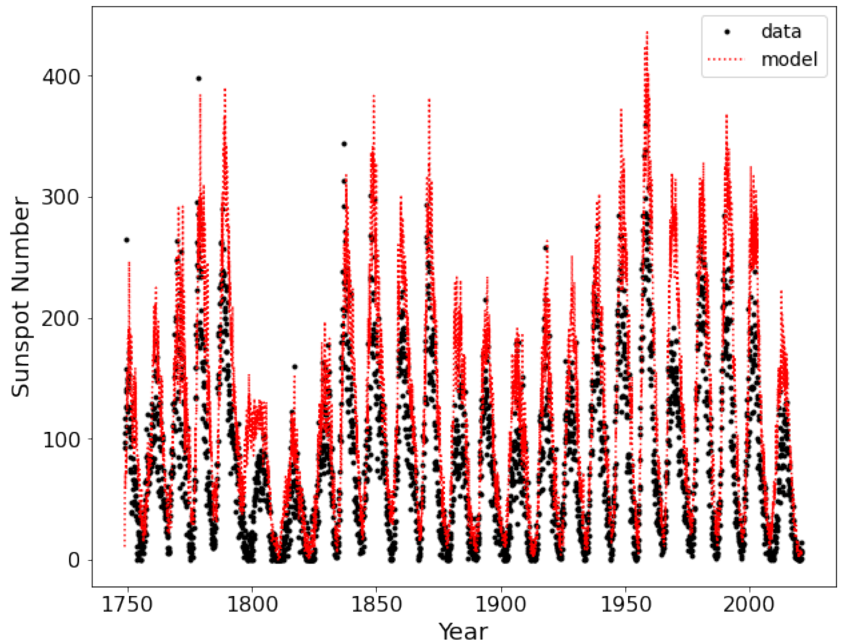

Figure 2: Model overlaid on data.

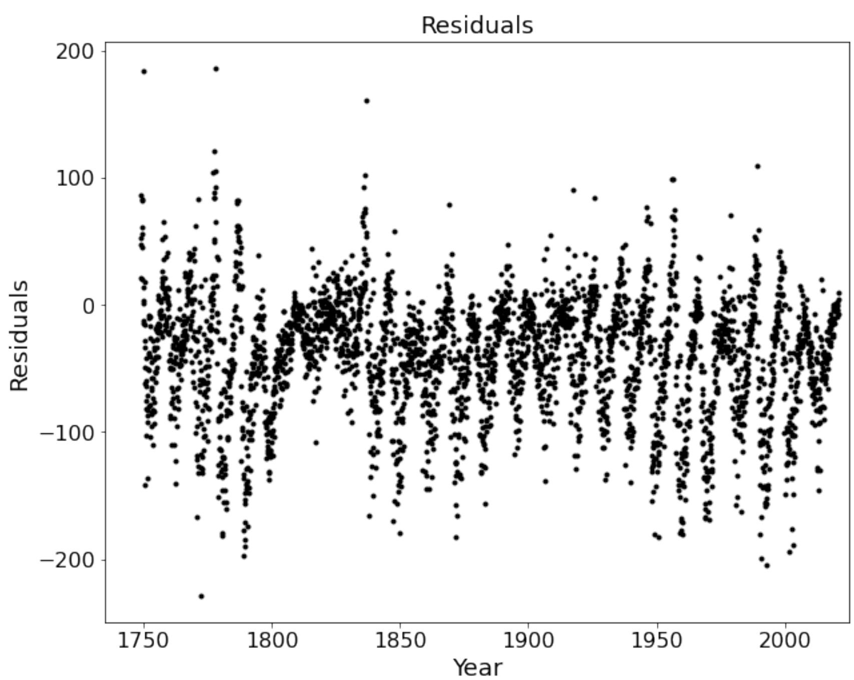

Figure 3: Residuals of model to fit with data.

Task 3

Plot the spectrum of your model. Please describe its behavior(s), for example at low/high/peak frequencies.

At low frequencies, the peaks are much higher, and then stabilize at higher frequencies. I plotted my spectrum by doing:

$$S_x(f) = \sigma_Z^2 \frac{\left|1-\theta_1 e^{-2\pi i f}-\theta_{12} e^{-2\times 12\pi i f}-\theta_{132} e^{-2\times 132\pi i f q}\right|^2}{\left|1-\phi_1 e^{-2\pi i f}-\phi_{12} e^{-2\times 12\pi i f}-\phi_{132} e^{-2\times 132\pi i f}\right|^2}$$

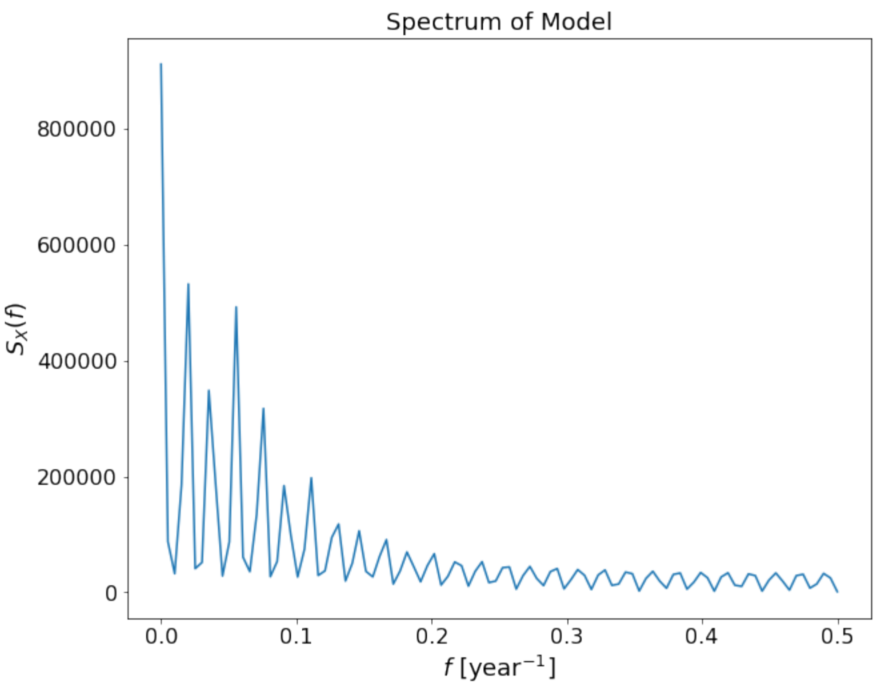

Figure 4: Plot of the spectrum of my model.

Task 4

Using the parameters of your model, as well as σ_Z either derived from your MCMC fit or estimated from the residuals after your fit, predict the sunspot number out to the year 2050.

I constructed an array that was a concatenation of the model values and an array of zeroes for the data points needed to get to the year 2050. Then, I looped through the entries of this array that were unpopulated and filled them in using the same function as my model, now that I had found the parameters this worked.

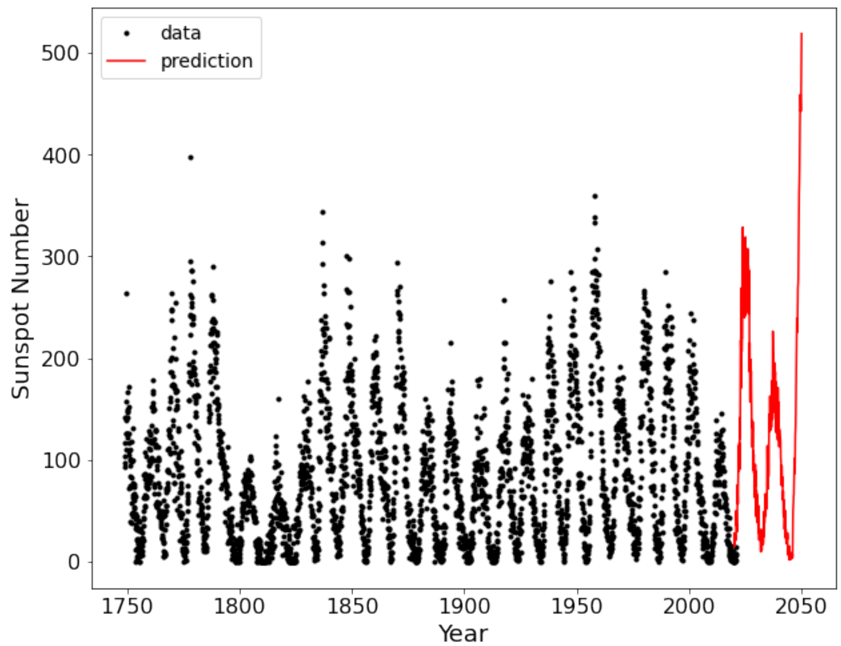

Figure 5: Model and the extrapolated prediction out to the year 2050.