ASTP 720 Homework Assignment 7

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Task 1

Your first task is to solve for the parameters you choose (it is your signal to model!) via MCMC. Your end goal here is to determine the radius of Kepler 5-b. If you are feeling adventurous, you can try to make one of those fancy corner plots, but if you can show the one-dimensional marginalized distribution for the main variable that determines the true physics of the system (I) and quote your measurement and its uncertainty (MAP? median? some confidence interval range of your choosing?), that will be perfectly fine. Again, please just describe what approach(es) you have taken.

The light curve of the planet is shown in Figure 1.

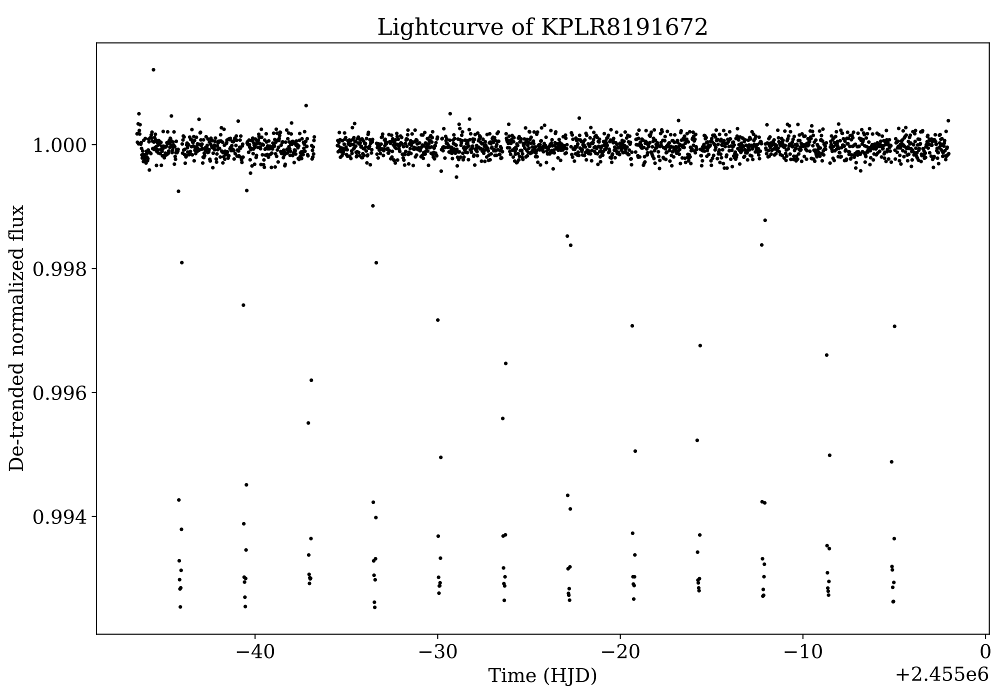

Figure 1: Light curve of planet under examination

The light curve can be folded (sort of like averaging) by calculating new time values using Equation 1

$$t'_i = (t_i = t_0)\%P \tag{1}$$

where t_0 is the first time value, and P is the period, given as 3.5485 days. The folded light curve is shown in Figure 2.

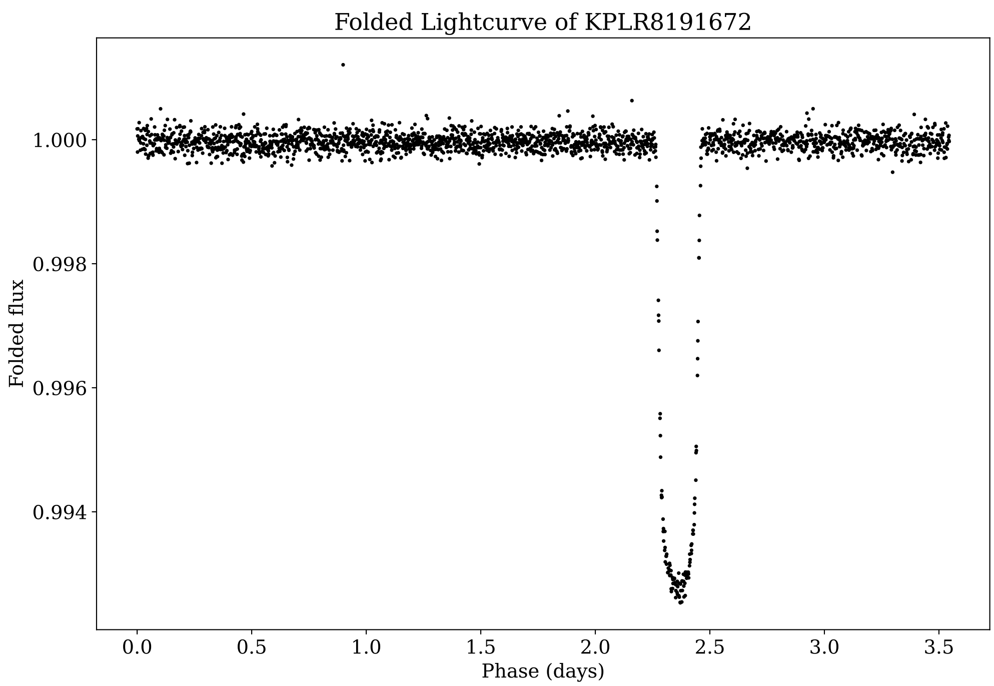

Figure 2: Folded light curve of planet under examination

The equation that governs a model for this folded light curve, can be written as a piece-wise, like a negative boxcar function, shown in Equation 2.

$$\begin{cases} 1 & x < t_{ref} \\ 1 - \Delta I & t_{ref} \leq x \leq t_{ref} + \tau \\ 1 & x > t_{ref} + \tau \end{cases} \tag{2}$$

where t_{ref} is the time at the left edge of the dip, τ is the duration of the dip and ΔI is the change in intensity during the dip phase.

Because I was having a hard enough time getting my code implemented and due to time constraints, I decided to estimate t_{ref} and τ and have my only free parameter I was solving for be ΔI. I wrote a likelihood function,

$$\mathcal{L} = exp\left[ln(\frac{1}{\sqrt{2\pi}}) + \sum -\frac{1}{2}(y_i - f(x_i))^2\right] \tag{3}$$

where y values are the observed flux values, $f(x)$ and are the model values for each data point (Equation 2). Note that typically in Equation 3, there would be a σ, however I considered $\sigma = 1$ so I dropped them.

To implement the Metropolis-Hastings algorithm, I initialized a value for ΔI as 0.005, and then for each iteration of many trials, I drew a value Y from a normal distribution that had a mean equal to the current ΔI value. Then I calculated the Metropolis ratio, and if the ratio was larger than or equal to 1, I accepted Y as the new value for ΔI for the next iteration. If the ratio was less than 1, I drew another value, U from a uniform ratio on the interval [0, 1] and compared it to the ratio. If U was less than the ratio, I accepted Y as the new ΔI. Otherwise, I rejected Y and kept the current value for ΔI. After each step through parameter space, I recorded the final value for ΔI.

The distribution of the values for ΔI is shown in Figure 3. The mean value was 0.00624 and the standard deviation of the value was 0.00291.

To compute the radius of the planet using this result, we need to know the stellar radius of the host star, which is 1.79 $R_\odot$. The radius of the planet can be computed using

$$\frac{\Delta I}{I} = \frac{R_p^2}{R_*^2} \tag{4}$$

yielding a value for the planet's radius of

$$R_p = \left[1.79^2 \ \frac{0.00624 \pm 0.00291}{1}\right]^{1/2} = [0.141 \pm 0.097]R_\odot \tag{5}$$

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This estimate captures the known value of Kepler5b (value looked up on Wikipedia) within tolerance.

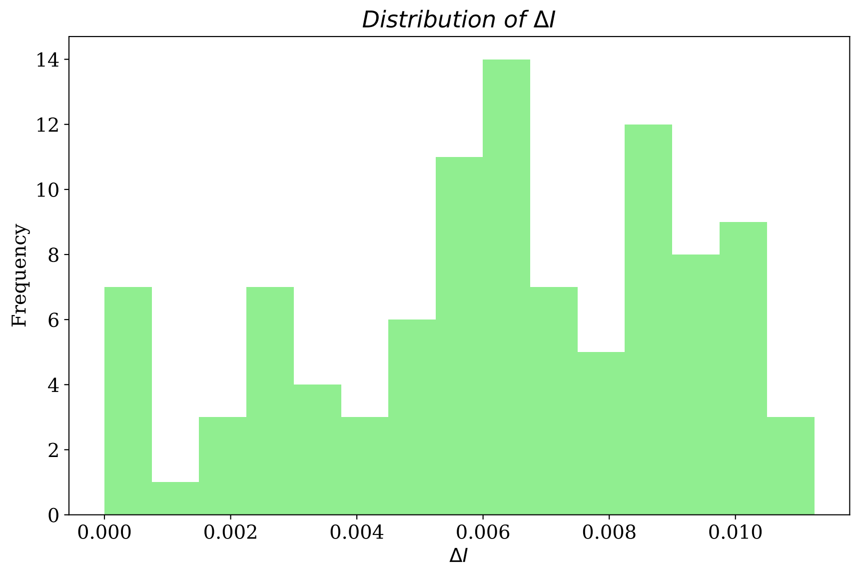

Figure 3: Distribution of ΔI values from implementing the Metropolis-Hasting algorithm.