# ASTP 720 Homework Assignment 3

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## Task 1

*Using the Barnes-Hut algorithm and your favorite symplectic integration method, evolve the system into the future, at least enough for particles to move of order Mpc if not more. You should include force softening of some form. In your write-up, make sure when you describe your method that you describe your parameters. Please include an image that shows the positions of a few of the galaxies at the start to whenever you end your simulation.*

The first step in the Barnes-Hut algorithm, is to recursively subdivide (build quad-tree) the grid into cells until galaxy is in its own cell. In the code, this is done with the `Tree` function. For each cell (both terminal cells – leaves, and parent cells), the coordinates of the lower left and upper right coordinates of the cell are appended to a list called `treeList`. Information about terminal cells, including the coordintes of the boundaries of the cell AND the position of the enclosed singular galaxy are saved in a list called `baseList`. The quad-tree for the initial positions of all the galaxies is shown in Figure 1.

The next step is to compute the forces on each galaxy that is a result of the other gravitational bodies. To avoid having a huge amount of calculations, the Barnes Hut algorithm uses a cutoff criteria,

$$\theta = L/D$$

where L is the length of the side of the cell and d is the distance between the current galaxy and the center of mass of a cell. If theta is sufficiently small (less than unity), all the galaxies in the children cells of the cell being considered can be viewed as a conglomerate body with a combined mass and COM at the position of all of the galaxies in that cell. If theta is greater than 1, forces are considered indivudally. My code handled this part of the algorithm using a function `Forces`, which takes the `treeList` output from the function `Tree`. This function calculates the acceleration components (x and y) for each point, and returns those accelerations.

The final part of the Barnes Hut algorithm I attempted to implement is moving the galaxies. From the computed accelerations and a supplied time step, the new position can be calculated for each galaxy. The final, final part of the code would be to loop through some time rebuilding the Tree and calculating new positions as time progressed to see the evolution over time of the entire system. Unfortunately, I was unable to get this part of

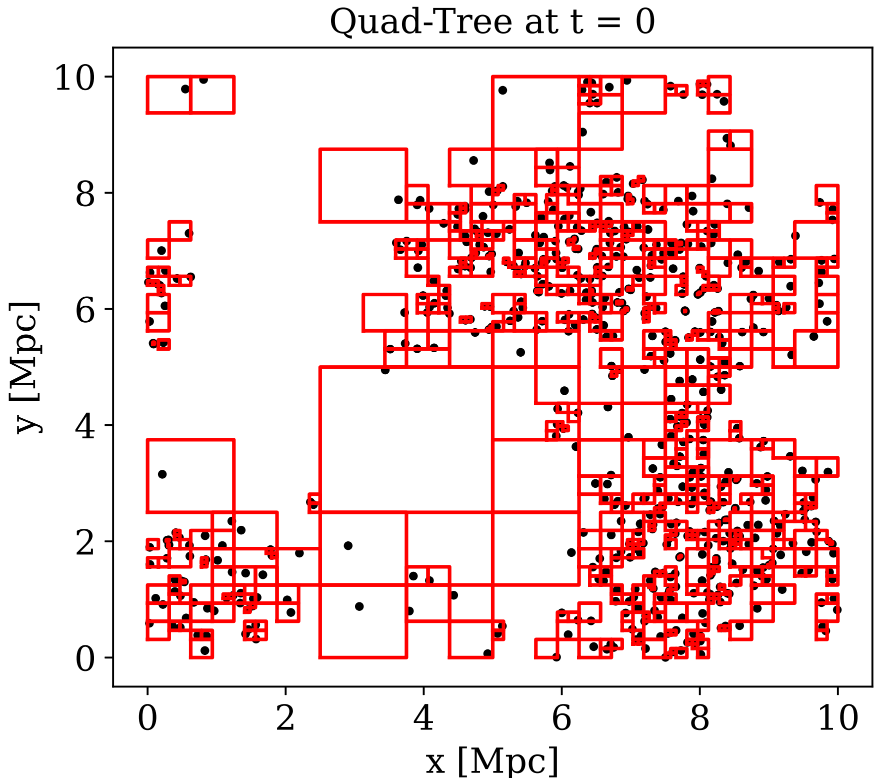


Figure 1: The quad-tree for the grid at time t = 0.

my code working. Had I been able to, I would have used the Verlet method to calculate subsequent positions, implemented as shown in Equation 1.

$$x_{i+1} = 2x_i - x_{i-1} + h^2 a_i \tag{1}$$

where $x$ indices the position, $a$ indices the acceleration and h is a time step. For the first step, I would have started with $x_i$ as the t = 1000 years supplied, and $x_{i-1}$ as t = 0.

So in all, I got most parts of my code working, but was unable to bring them all together due to time constraints.

# Task 2

*You want to understand the crude potential of this cluster to understand how a new galaxy might interact with it. Pretend that all 655 galaxies are distributed into two groups, one centered at (7, 7) Mpc and one centered at (8, 2) Mpc. Take the first clump to have 400 galaxies uniformly distributed in a circle with radius 3 Mpc and the second to have the other 255 distributed in a circle with radius 2 Mpc (i.e.,imagine uniform density "balls"). Calculate the gravitational potential throughout the box and make a 2D plot. Note that at the boundaries of the cube, the potential should be zero.*

I initialize this problem by placing two disks (spheres) on the grid each of mass equal to the number of galaxies in the disk multiplied by the mass of each galaxy.
To evaluate the gravitational potential at each point, we can use Poisson's Law, written as:

$$\nabla^2\phi = 4\pi G\rho \tag{2}$$

.
where G is the gravitational constant and $\rho$ is the density of body. Care must be taken when calculating $\rho$, which is a volume density. So even though we are doing calculations on 2D-grid, we still calculate a volume density versus a surface density in order to preserve accurate units.

In the 2-dimensional square for this problem, Equation 3 can be rewritten as:

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 4\pi G\rho \tag{3}$$

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And by using finite differences, Equation 4 can be written as,

$$\frac{\phi_{i+h,j} + \phi_{i-h,j} - 2\phi_{i,j}}{\delta^2} + \frac{\phi_{i,j+h} + \phi_{i,j-h} - 2\phi_{i,j}}{\delta^2} = 4\pi G\rho \tag{4}$$

where $i$ indexes the rows and $j$ indexes the columns and *delta* is the spatial size step, and $h$ should be equal to 1 for indexing on a grid.

We can rearrange Equation 5 in order to solve the potential at a specific position on the grid, (i, j). That looks like,

$$\phi_{i,j} = \frac{1}{4}(\phi_{i+h,j} + \phi_{i-h,j} + \phi_{i,j+h} + \phi_{i,j-h} - 4\pi G\rho\delta^2) \tag{5}$$

so $\phi$ at each position on the grid can be computed, and you can see that it depends on it's neighbors. Now, it takes many iterations through time to converge on a solution. One method could be checking how similar the current grid was to the last interation's grid. What I did though, was just looped through time steps for a very long time and assumed that was long enough for the solution to converge. After ever time spent, the boundary conditions provided in the problem are reinforced.

Figure 2 shows the potential everywhere. The red circles denote the boundaries of the two galaxy clumps.

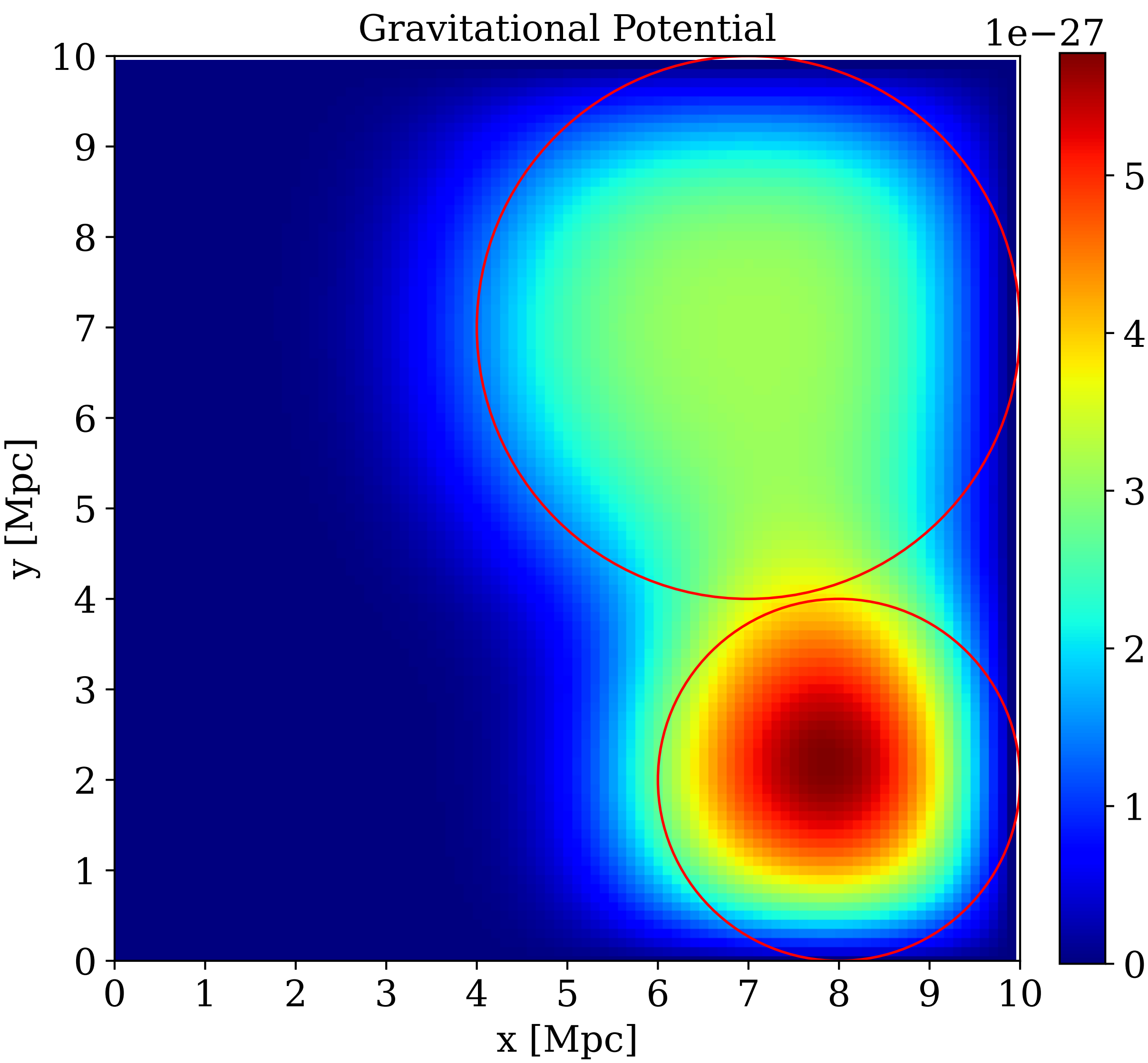


Figure 2: Potential (denoted by colorbar) everywhere (units = $cm^2/s^2$) in grid. The two clumps of galaxies are outlined in red.