Map, Jumble, DLLMap, and SkipMap

Victor Milenkovic

Department of Computer Science
University of Miami

CSC220 Programming II – Spring 2018

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  - Partial implementation is here.

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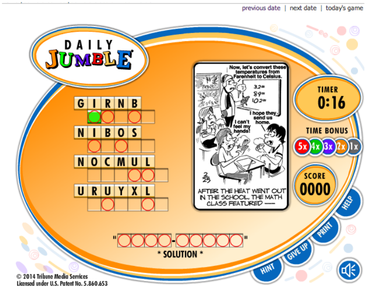

previous date | next date | today's game
DAILY JUMBLE
G I R N B
N I B O S
N O C M U L
U R U Y X L
Now, let's convert these temperatures from Farenheit to Celsius.
32=
89=
102=
I hope they send us home.
I can't feel my hands!
AFTER THE HEAT WENT OUT IN THE SCHOOL, THE MATH CLASS FEATURED ——
* SOLUTION *
TIMER
0:16
TIME BONUS
5x 4x 3x 2x 1x
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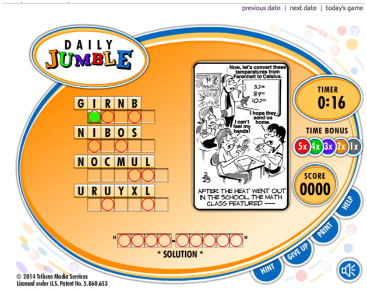
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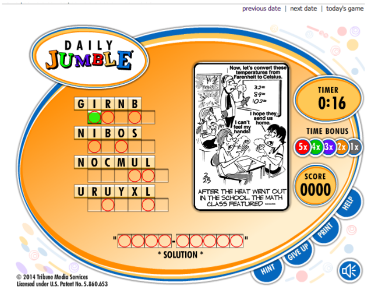
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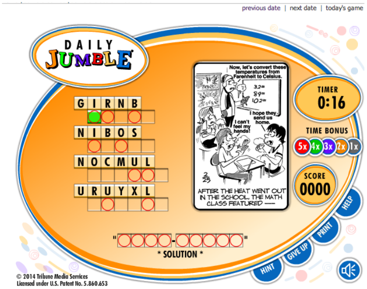
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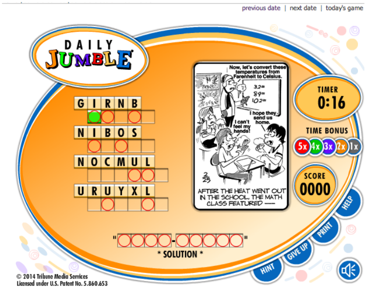
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  - How can a Map help us to do that?

Slow Way

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  - But the number of orderings is 8! = 40,320, bad!.

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  - But we won’t do that this time.

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  - but it takes a while to get there.
- We need a faster way.

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See an example lookup here.

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- $\log_2 n$ steps on average, so $O(\log n)$.

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