Binary Trees, Binary Search Trees, and Heaps

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  - Corresponds to using all the items in 0 to size-1.

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- We will do a b-tree in prog10.

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