Worksheet 8

February 27, 2023 12:18 PM

1) (a) $z = \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$

$$s \leftrightarrow 2f_s\frac{z-1}{z+1} = 2f_s\frac{\frac{1}{\sqrt{2}}-1+j\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}+1+j\frac{1}{\sqrt{2}}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = 2f_s\frac{1-\sqrt{2}+j}{1+\sqrt{2}+j}$$

↳ not "equals"
s in rad/s, f_s in Hz

⇒ multiply denominator by conjugate

$$= 2f_s\frac{(1-\sqrt{2}+j)(1+\sqrt{2}-j)}{(1+\sqrt{2}+j)(1+\sqrt{2}-j)} = 2f_s\frac{1+\sqrt{2}-j-\sqrt{2}-2+j\sqrt{2}+j+j\sqrt{2}+1}{(1+\sqrt{2})^2+1}$$

$$= 2f_s\frac{2j\sqrt{2}}{1+2\sqrt{2}+2+1} = 8f_s\frac{2j\sqrt{2}}{4+8\sqrt{2}} = f_s\frac{2j\sqrt{2}}{2+\sqrt{2}}$$

$$= 2f_s j\frac{\sqrt{2}}{2+\sqrt{2}}$$

(b) $z = 0 \rightarrow s = 2f_s\frac{z-1}{z+1} = 2f_s\frac{-1}{1} = -2f_s$

(c) $z = 2+j \rightarrow s = 2f_s\frac{2+j-1}{2+j+1} = 2f_s\frac{1+j}{3+j}\frac{(3-j)}{(3-j)}$

$$= 8f_s\frac{3+2j+1}{9+10s} = f_s\frac{4+2j}{5}$$

2) $H(s) = \frac{\omega_{p1}}{s+\omega_{p1}}$ $\quad f_{-3dB} = 2000\ \text{rad/s} = \omega_{p1}$, $f_s = 1500\ \text{Hz}$

$$H(z) = \frac{\omega_{p1}}{2f_s\frac{z-1}{z+1}+\omega_{p1}} = \frac{2000\ \text{rad/s}}{2\cdot 1500\text{Hz}\frac{z-1}{z+1}+2000\ \text{rad/s}}$$

$\frac{\text{rad}}{\text{s}} \rightarrow \text{Hz}$, $\frac{1}{2\pi}$

$$H(z) = \frac{\omega_{p1}}{2f_s\frac{z-1}{z+1} + \omega_{p1}} = \frac{}{2 \cdot 1500\text{Hz}\frac{z-1}{z+1} + 2000\text{rad/s}}$$

$$\frac{2000}{2\pi} = \frac{1000}{\pi} = 318.31\ \text{Hz}$$

$$H(z) = \frac{2000(z+1)}{3000(z-1) + 2000(z+1)} = \frac{2000z + 2000}{5000z - 1000}$$

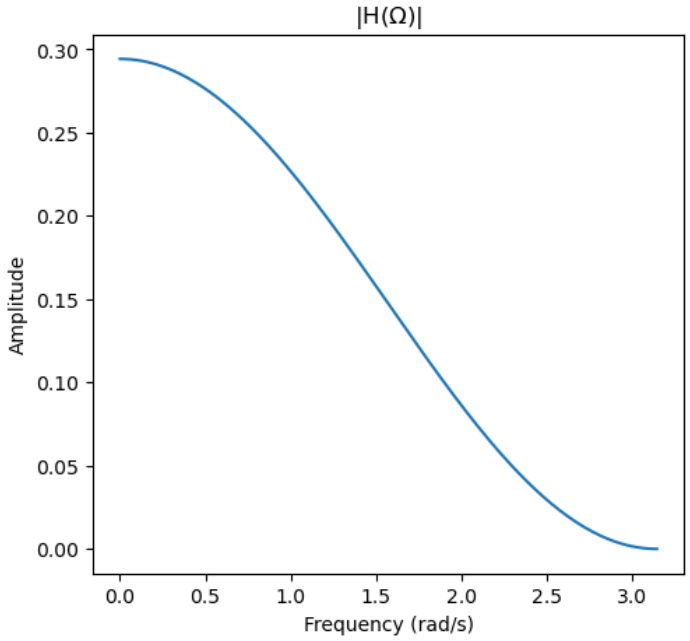
$$= \frac{2z+2}{5z+1}$$

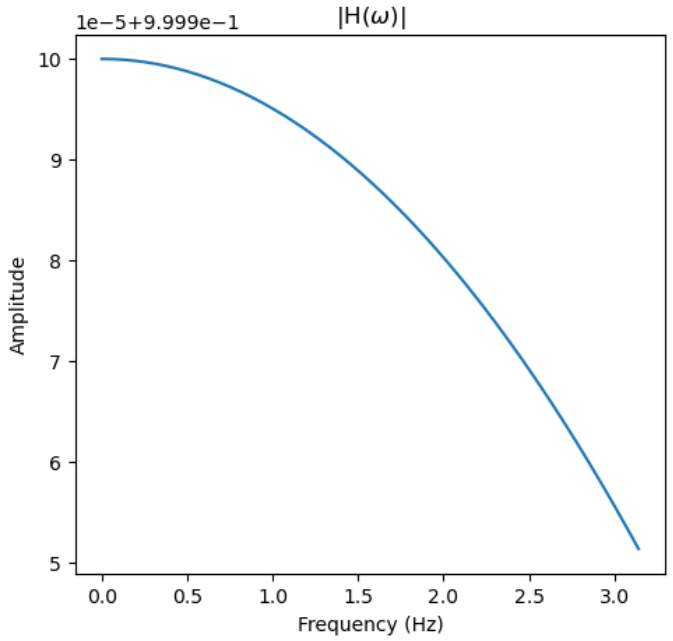
$$H(\Omega) = \frac{2e^{j\Omega} + 2}{5e^{j\Omega} + 1}$$

Plotting $|H(\Omega)|$ and $|H(\omega)|$ against freq in Hz.

$$|H(\omega)| = \frac{1}{\sqrt{\left(\frac{\omega}{\omega_{p1}}\right)^2 + 1}} \quad \text{(for specified pass band)}$$

$$= \frac{1}{\sqrt{\left(\frac{\omega}{318.31}\right)^2 + 1}} \quad \text{(for units of Hz)}$$





3) (a) $\omega_{p1} = 2000 \text{ rad/s} = 318.31 \text{ Hz}$

$f_s = 1500 \text{ Hz}$