Worksheet 7

February 23, 2023 10:19 AM

1) $y[n] = 0.2(x[n] + x[n-1] + x[n-2] + x[n-3] + x[n-4])$

(a) impulse response, $h[n]$

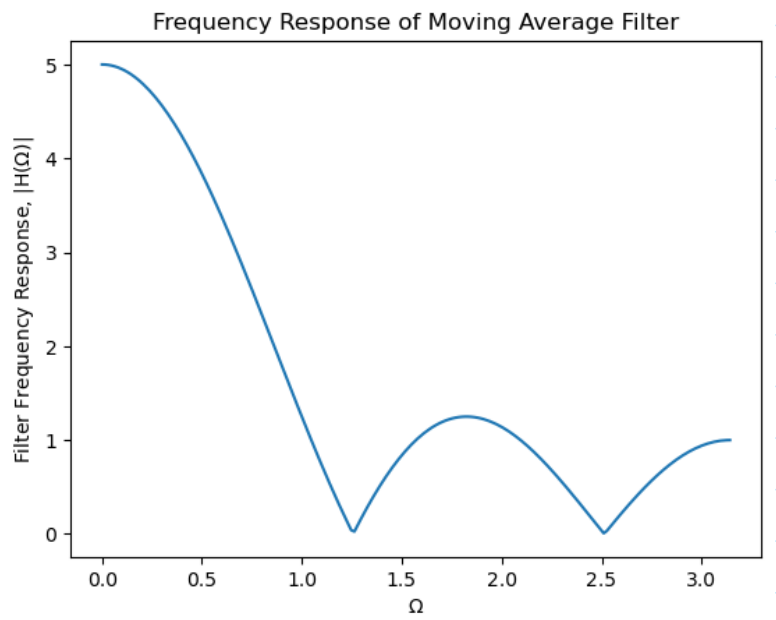
$h[n] = 0.2(\delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3] + \delta[n-4])$

Transfer function, $H(z)$

$H(z) = 0.2(1 + z^{-1} + z^{-2} + z^{-3} + z^{-4})$

(b) Frequency Response, $H(\Omega)$

$H(\Omega) = 0.2(1 + e^{-j\Omega} + e^{-j2\Omega} + e^{-j3\Omega} + e^{-j4\Omega})$



2) $f_{samp} = 10\ kHz$

-3dB freq: $f_{cutoff} = 480\ Hz$

$\Omega = \frac{2\pi f_c}{f_{samp}} = \frac{\pi}{m} \rightarrow m = \frac{f_{samp}}{2f_c} = \frac{10000}{960} = 10.41\overline{6} \rightarrow 10$

$$y[n] = \frac{1}{10}(x[n] + x[n-1] + x[n-2] + x[n-3] + x[n-4] + x[n-5] + x[n-6] + x[n-7] + x[n-8] + x[n-9])$$

Low-Pass Filter Design

3) pass band edge frequency, $f_1 = f_{desired} + \frac{1}{2}$ Transition width

① pass = 2kHz, stop = 3kHz → Transition = stop − pass = 1kHz

$f_1 = 2kHz + 500Hz = 2.5kHz$

② $\Omega_1 = \frac{2\pi f_1}{f_{samp}}$, $f_{samp} = 10kHz$

$= \frac{2\pi 2.5}{10} = \frac{\pi}{2}$

$h_1[n] = \frac{\sin(n\Omega_1)}{n\pi} = \frac{\sin\left(\frac{\pi n}{2}\right)}{n\pi}$

③ stop band attenuation = 40dB

→ Hanning window, attenuation = 44 dB

Window Type	Window Function $\lvert n\rvert \le \frac{N-1}{2}$	Number of Terms In Window, N
Hanning	$0.5 + 0.5\cos\left(\frac{2\pi n}{N-1}\right)$	$3.32\frac{f_s}{\text{T.W.}}$

$N = 3.32\ \frac{10\,kHz}{1kHz} = 33.2 \rightarrow 33$

$w[n] = 0.5 + 0.5\cos\left(\frac{2\pi n}{32}\right)$

$h[n] = h_1[n]w[n]$, $0 < n < N-1$

$h[n] = h_1\left[n - \frac{N-1}{2}\right] w\left[n - \frac{N-1}{2}\right]$

$h[n] = \frac{\sin\left(\frac{\pi}{2}[n-16]\right)}{\pi[n-16]}\left[0.5 + 0.5\cos\left(\frac{\pi[n-16]}{16}\right)\right]$

$$h[n] = \frac{\sin\left(\frac{\pi}{2}[n-16]\right)}{\pi[n-16]}\left[0.5 + 0.5\cos\left(\frac{\pi[n-16]}{16}\right)\right]$$

Band-Pass / High-Pass Filter Design

4) centre frequency, $f_o = 4\text{kHz}$

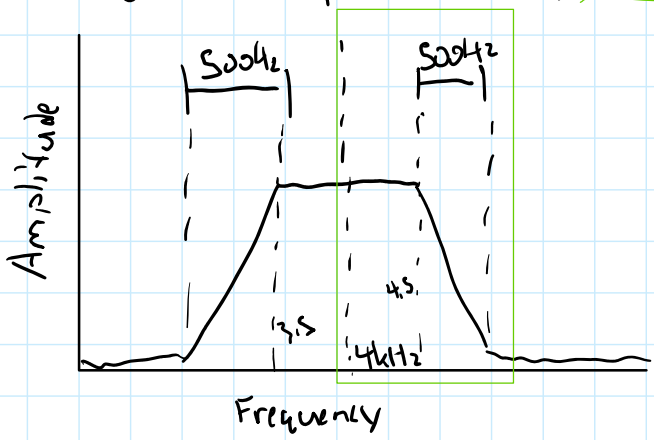
Transition width = 500 Hz

stop-band attenuation = 50dB

$f_{samp} = 22\text{kHz}$

$h_{band} = h_{low}\cos(n\omega_o)$

① design low-pass first, relative to centre frequency of band pass



$f_1 = 500 + \frac{500}{2} = 750\text{Hz}$

$\omega_1 = 2\pi\frac{f_1}{f_s} = 2\pi\frac{750}{22k} = \frac{3\pi}{44}$

$h_1[n] = \frac{\sin\left(\frac{3\pi}{44}n\right)}{\pi n}$

Hamming window

Window Type	Window Function $\lvert n\rvert \le \frac{N-1}{2}$	Number of Terms In Window, N	Filter Stop Band Attenuation (dB)
Hamming	$0.54 + 0.46\cos\left(\frac{2\pi n}{N-1}\right)$	$3.44\frac{f_s}{\text{T.W.}}$	55

$$N = 3.44\,\frac{22kHz}{500Hz} = 151.36 \rightarrow 151 \text{ terms}$$

$$w[n] = 0.54 + 0.46\cos\left(\frac{2\pi n}{150}\right)$$

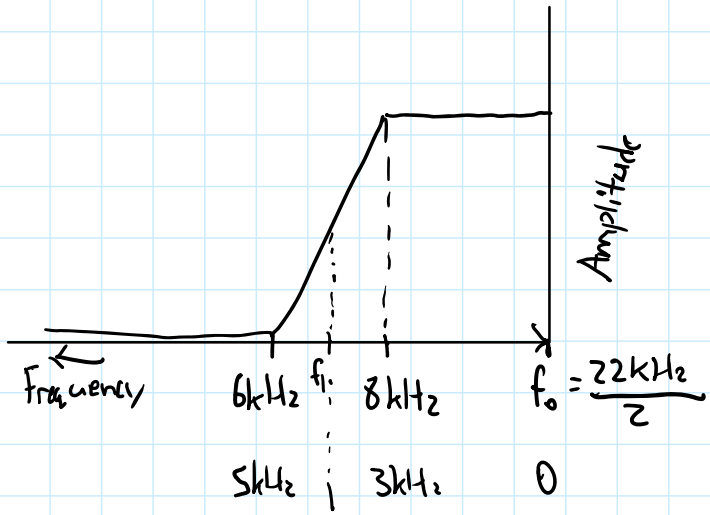
Band-pass $\Omega_0 = 2\pi\,\frac{f_0}{f_s} = 2\pi\,\frac{4}{22} = \frac{4\pi}{11}$

$$h[n] = \frac{\sin\left(\frac{3\pi}{44}n\right)}{n\pi}\left[0.54 + 0.46\cos\left(\frac{2\pi n}{150}\right)\right]\cos\left(\frac{4\pi}{11}n\right)$$

shift n

$$h[n] = \frac{\sin\left(\frac{3\pi}{44}[n-75]\right)}{\pi[n-75]}\left[0.54 + 0.46\cos\left(\frac{2\pi[n-75]}{150}\right)\right]\cos\left(\frac{4\pi}{11}[n-75]\right)$$

5) $h_{high} = (-1)^n h_{low}$



$f_1 = 4kHz$?

$$\Omega_1 = 2\pi\,\frac{f_1}{f_s} = 2\pi\,\frac{4}{22} = \frac{4\pi}{11}$$

$$h_1[n] = \frac{\sin\left(\frac{4\pi}{11}n\right)}{\pi n}$$

stop band attenuation > 40 dB

→ Hanning window

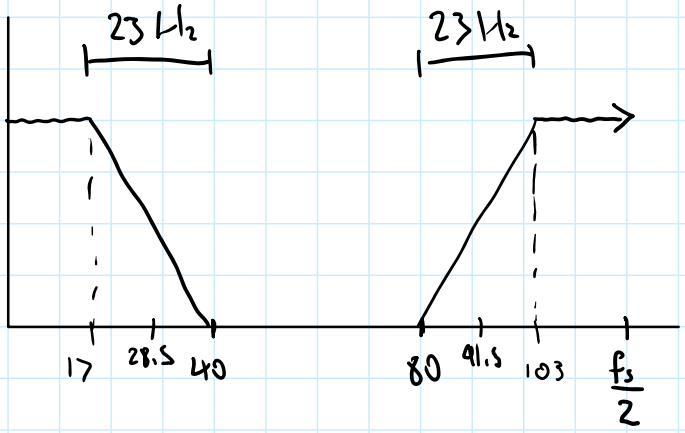
$$N = 3.32 \frac{f_s}{\pi w} = 3.32 \frac{22000}{(8-6)1000} = 36.52 \rightarrow 37$$

$$w[n] = 0.5 + 0.5\cos\left(\frac{2\pi n}{36}\right)$$

$$h_{low}[n] = \frac{\sin\left(\frac{4\pi}{11} n\right)}{n\pi}\left[0.5 + 0.5\cos\left(\frac{2\pi n}{36}\right)\right]$$

$$h_{high}[n] = (-1)^{n-18}\frac{\sin\left(\frac{4\pi}{11}[n-18]\right)}{\pi[n-18]}\left[0.5 + 0.5\cos\left(\frac{2\pi[n-18]}{36}\right)\right]$$

6)



$f_{samp} = 1\,kHz$

Low pass

$f_1 = 28.5\,Hz$

$\Omega_1 = 2\pi\frac{28.5}{1000} = \frac{57\pi}{1000}$

$$h_1[n] = \frac{\sin\left(\frac{57\pi}{1000} n\right)}{n\pi}$$

Hamming window

$$N = 3.44\frac{f_s}{\pi w} = 3.44\frac{1000}{23} = 149.565 \rightarrow 150 \rightarrow 151$$

$L = \frac{1}{2} \rightarrow 75$ shift

$$w[n] = 0.54 + 0.46\cos\left(\frac{2\pi n}{150}\right)$$

$$h_{low}[n] = \frac{\sin\left(\frac{57\pi}{1000}[n-75]\right)}{[n-75]\pi} w[n-75]$$

High-pass

$f_1 = \text{pass edge} + \frac{TW}{2} = \left(\frac{f_s}{2} - 103\right) + \frac{23}{2} = 908.5$

$\omega_1 = 2\pi \frac{908.5}{1000} = 1.817\pi$

$h_1[n] = \frac{\sin(1.817\pi n)}{n\pi}$ use same window

$h_{high}[n] = \frac{\sin(1.817\pi[n-75])}{[n-75]\pi} w[n-75]$

$h_{band}[n] = h_{low} + h_{high}$

$= \frac{\sin\left(\frac{57\pi}{1000}[n-75]\right) + \sin(1.817\pi[n-75])}{[n-75]\pi}\left[0.54 + 0.46\cos\left(\frac{2\pi[n-75]}{150}\right)\right]$